Hadronic Tensor and Neutrino-Nucleon Scattering

Jian Liang, Terrence Draper, Keh-Fei Liu, Alexander Rothkopf and Yi-Bo Yang

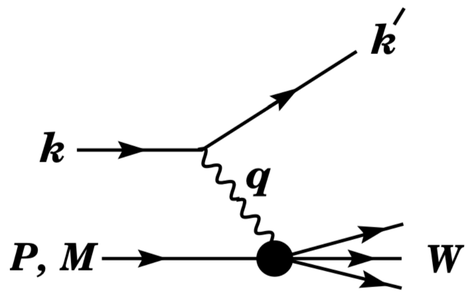
(χQCD collaboration)

06/17/2019 Lattice2019@Wuhan, China

arXiv:1906.05312

- **Introduction**
- Formalism of calculating the hadronic tensor on the lattice
- Numerical results

Hadronic tensor



for lepton-nucleon scatterings, in leading order perturbation: $\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4}\sum_j \eta_j L_j^{\mu\nu} W_{\mu\nu}^j$

the hadronic tensor $W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s \left| \left[J_\mu^\dagger(z)\, J_\nu(0)\right] \right| p,s \right\rangle$, which is the imaginary part of the forward amplitude: $W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}\left[T_{\mu\nu}\right]$

further decomposed into structure functions $W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2(x,Q^2)$ (for unpolarized case with vector currents)

The hadronic tensor and structure functions encode the nonperturbative nature of the nucleon.

Observables to study the nucleon structures and lepton-nucleon scatterings:

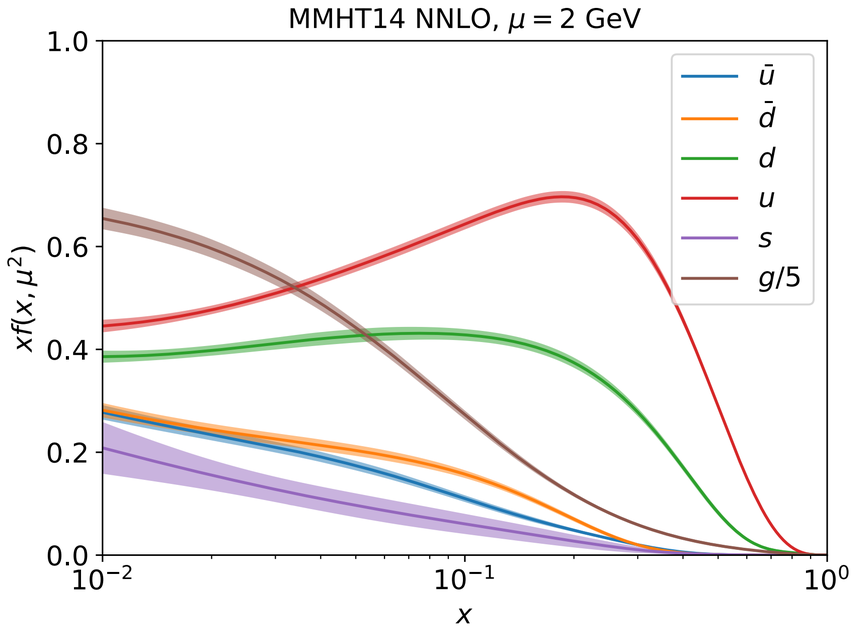
- For high energy scatterings (DIS), PDFs are extracted through factorization: $F_i = \sum_a C_i^a \otimes f_a$
- For lower energy scatterings, e.g. neutrino-nucleon scatterings, inclusive hadronic tensor enters the cross sections.
- For elastic scatterings, they are combination of form factors: $F_2^{\mathrm{el}} = \delta(q^2 + 2m_N\nu)\frac{2m_N}{1 - q^2/4m_N{}^2}\left(G_E^2(q^2) - \frac{q^2}{4M_N^2}G_M^2(q^2)\right)$

Hadronic tensor and parton physics

$$F_i = \sum_a C_i^a \otimes f_a$$

Lattice efforts of studying *x*-dependent parton physics:

- Quasi-PDFs and LaMET — *X. Ji, PRL110, 262002 (2013)*; *arXiv:1803.04393*; *H.W. Lin et. al., PRL121, 242003 (2018)*; *C. Alexandrouetal et. al., PRL121, 112001 (2018)*
- Puedo-PDFs — *A. V. Radyushkin, PRD96, 034025 (2017)*; *K. Orginos et. al., PRD96, 094503 (2017)*
- Lattice cross sections — *Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)*; *R. S. Sufian et. al., PRD99, 074507 (2019)*
- Compton amplitude — *A. J. Chambers et. al., PRL118, 242001(2017)*
- …



L. A. Harland-Lang et. al., EPJ C75, 204 (2015)

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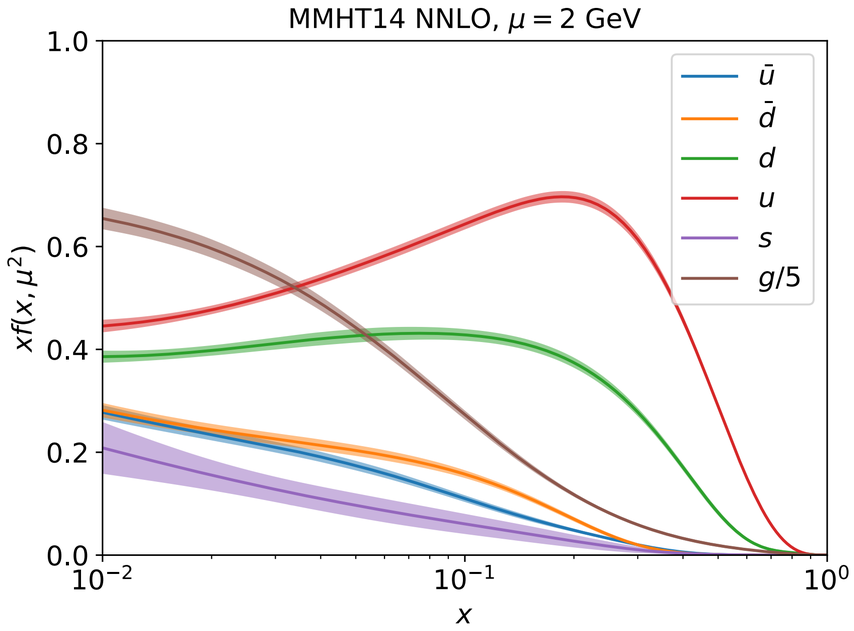


L. A. Harland-Lang et. al., EPJ C75, 204 (2015)

Advantages of calculating the hadronic tensor

- Hadronic tensor is **scale independent**! No need to do renormalization!
- Structure functions are **frame independent**! No need of having large nucleon momentum!
- Explicit study of the **higher-twist** effects
- Hadronic tensor provides a direct way to reveal the **Gottfried sum rule violation**.

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD62, 074501 (2000)

J. Liang et. al., arXiv:1901.07526

related proposals:

U. Aglietti et. al., PLB432, 411 (1998)

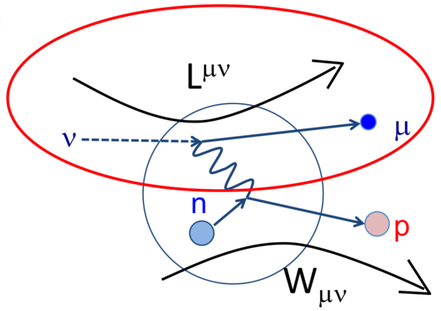
W. Detmold and C. J. D. Lin, PRD73, 014501 (2006)

M. T. Hansen et. al., PRD96, 094513 (2017)

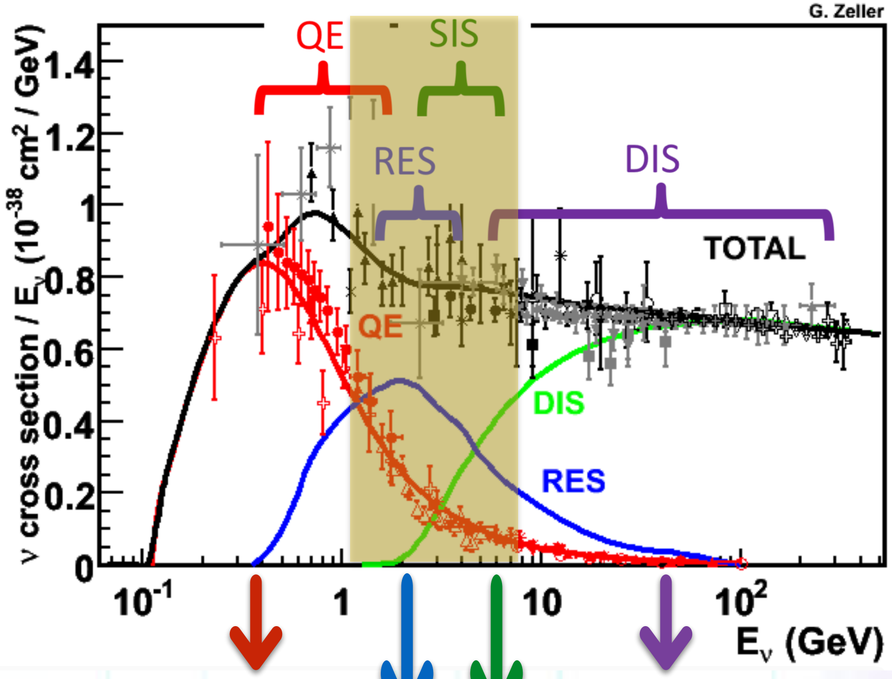
Hadronic tensor and neutrino-nucleus scattering

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi y\alpha^2}{Q^4}\sum_j \eta_j L_j^{\mu\nu} W_{\mu\nu}^j$$

- To study the properties of neutrinos (mass hierarchy, oscillation…), new long-baseline neutrino experiments are in preparation: T2K, NOvA, PINGU, ORCA, Hyper-Kamiokande, DUNE...
- Besides nuclear effects and modeling, input of fundamental **neutrino-nucleon scattering** is needed.
- Challenge: at different neutrino energies, different contributions dominate the cross section.



Teppei Katori's talk at NuSTEC nuS&DIS workshop (2018)



elastic form factors

inclusive hadronic tensor!

inclusive hadronic tensor!

Parton distribution functions

J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

Hadronic tensor and neutrino-nucleus scattering

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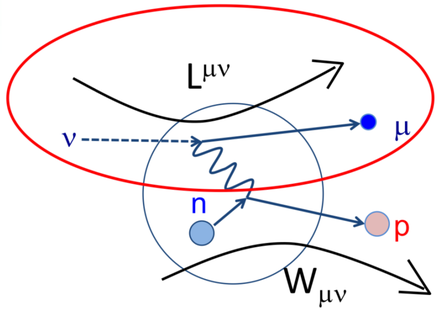

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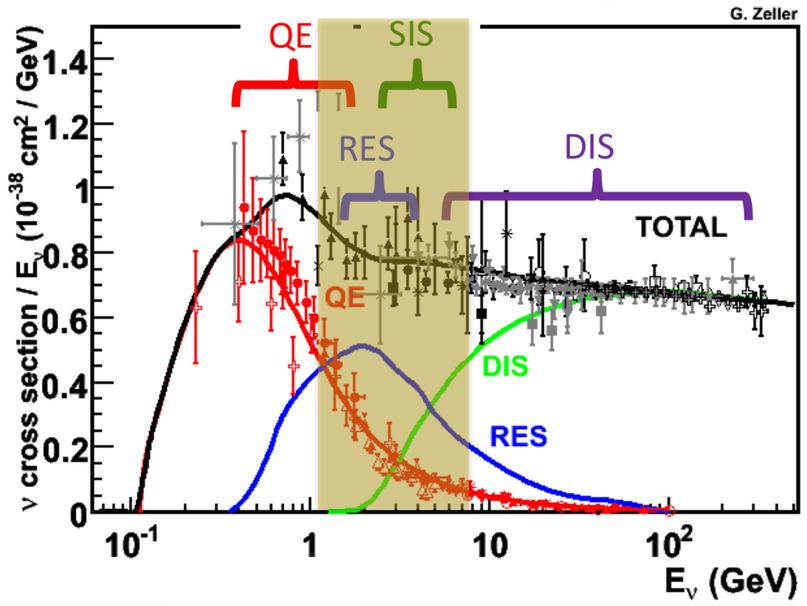

J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

Lattice QCD and Neutrino-Nucleus Scattering

Andreas S. Kronfeld,[1,*] David G. Richards,[2,†] William Detmold,[3] Rajan Gupta,[4] Huey-Wen Lin,[5] Keh-Fei Liu,[6] Aaron S. Meyer,[7] Raza Sufian,[2] and Sergey Syritsin[8]

(USQCD Collaboration)

- **This is the only way that lattice QCD can help in all these energy regions.**

- Introduction
- **Formalism of calculating the hadronic tensor on the lattice**
- Numerical results

Calculating hadronic tensor on the lattice

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s \left| \left[J_\mu^\dagger(z)\, J_\nu(0)\right] \right| p,s \right\rangle$$

Lattice QCD: **Euclidean** field theory using the path-integral formalism: **time dependent matrix** elements are problematic.

Minkowski case:

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\left\langle p,s \left| \left[J_\mu^\dagger(z)J_\nu(0)\right] \right| s,s \right\rangle = \frac{1}{2}\sum_n \int \prod_i^n \left[\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right] \langle p,s|J_\mu^\dagger(0)|n\rangle\langle n|J_\nu(0)|p,s\rangle (2\pi)^3 \delta^4(q-p_n+p)$$

Euclidean case:

$$W'_{\mu\nu} = \frac{1}{4\pi}\sum_n \int dt e^{\left(\nu-(E_n-E_p)\right)t}\int d^3\vec{z} e^{i\vec{q}\cdot\vec{z}}\langle p,s|J_\mu^\dagger(\vec{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle = \frac{1}{4\pi}\sum_n \boxed{\frac{e^{\left(\nu-(E_n-E_p)\right)T}-1}{\nu-(E_n-E_p)}}\int d^3\vec{z} e^{i\vec{q}\cdot\vec{z}}\langle p,s|J_\mu^\dagger(\vec{z})|n\rangle\langle n|J_\nu(0)|p,s\rangle$$

A simple change from **Fourier transform** to **Laplace transform** in the time direction **leads to divergences when** $\nu-(E_n-E_p)>0$.

Calculating hadronic tensor on the lattice

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A simple change from **Fourier transform** to **Laplace transform** in the time direction **leads to divergences when** $\nu-(E_n-E_p)>0$.

Define Euclidean hadronic tensor:

$$\tilde{W}_{\mu\nu}(\vec{p},\vec{q},\tau) = \frac{E_p}{m_N}\frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \to \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s|J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)|p,s\rangle = \sum_n A_n e^{-\nu_n\tau},\ \tau\equiv t_2-t_1,\ \nu_n = E_n-E_p$$

The energy transfer is determined by the energy of the intermediate states.

$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f}\sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\left\langle \chi_N(\vec{x}_f,t_f)J_\mu^\dagger(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\bar{\chi}_N(\vec{0},t_0)\right\rangle$$

$$C_2 = \sum_{x_f} e^{-i\vec{p}\cdot\vec{x}_f}\left\langle \chi_N(\vec{x}_f,t_f)\bar{\chi}_N(\vec{0},t_0)\right\rangle$$

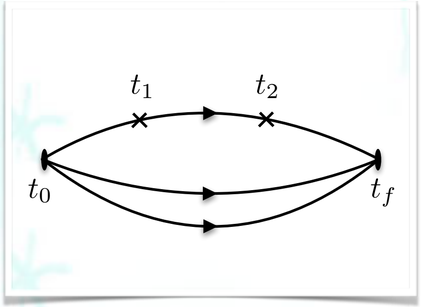
K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD62, 074501 (2000)

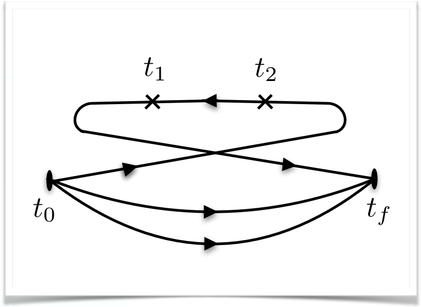
J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

Contractions

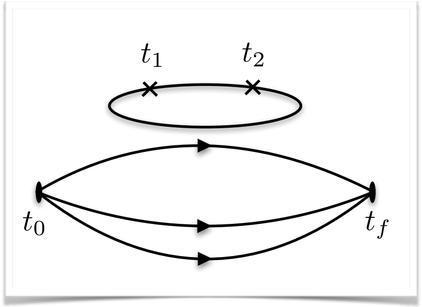
$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \left\langle \chi_N(\vec{x}_f, t_f) J_\mu^\dagger(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{\chi}_N(\vec{0}, t_0) \right\rangle$$



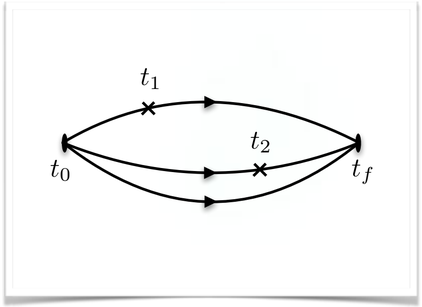
valence and connected-sea (CS) parton

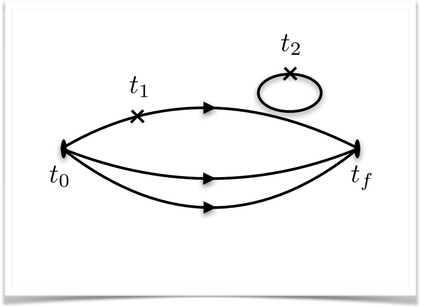


CS anti-parton **(Gottfried sum rule violation)**

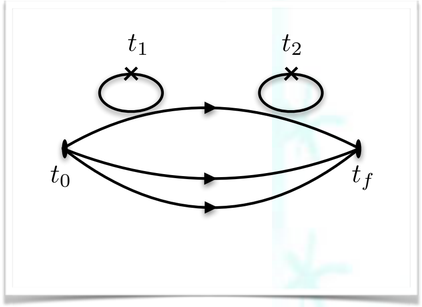


disconnected-sea parton and anti-parton





leading-twist plus higher-twist contributions



pure higher-twist contribution

The CS anti-partons are supposed to be responsible for the **Gottfried sum rule violation**.

The latter three are **suppressed** when the momentum and energy transfers are large.

K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD62, 074501 (2000)

Back to Minkowski space

Formally, **inverse Laplace transform**: $W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu) = \frac{1}{i}\int_{c-i\infty}^{c+i\infty} d\tau e^{\nu\tau}\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau)$

Practically, need to solve the **inverse problem** of the Laplace transform $\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \int d\nu W_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\nu)e^{-\nu\tau}$

several (O(10)) discrete data points

continuous function w.r.t. ν

lack of information, an ill-posed problem

solving the inverse problem

- **Backus-Gilbert (BG)** — bad resolution but very smooth
- **Maximum Entropy (ME)** — better resolution than BG
- **Bayesian Reconstruction (BR)** — best resolution, may have oscillations in the flat regions
- **Smoothed BR** — balanced resolution and smoothness
- **...**

G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)

E Rietsch et. al., JOURNAL OF GEOPHYSICS, 42:489 (1977)

M. Asakawa et. al., Prog. Part. Nucl. Phys. 46, 459 (2001)

Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

C. S. Fischer el. al., PRD98:014009 (2018)

Inverse problems are ubiquitous

- Extracting spectral functions from lattice data: $c_2(t) = \int d\omega e^{-\omega t} \rho(\omega)$

  Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

- Global fittings of PDFs: $F_i = \sum_a C_i^a \otimes f_a$

- Lattice calculation of Quasi-PDFs: $\tilde{q}(x, P_3) = \frac{2P_3}{4\pi} \sum_{z=-z_{\max}}^{z_{\max}} e^{-ixP_3 z} h_\Gamma(P_3, z)$

  J. Karpie et. al., JHEP04, 057 (2019)
  C. Alexandrouar et. al., Xiv:1902.00587

$$q_{\gamma^0}(x, \bar{\mu}) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} C_{\gamma^0}^{\overline{\text{MS}}} \left(\frac{x}{y}, \frac{\bar{\mu}}{p_3}, \frac{\bar{\mu}}{\mu_F} \right) \tilde{q}_{\gamma^0}(y, p_3, \bar{\mu})$$

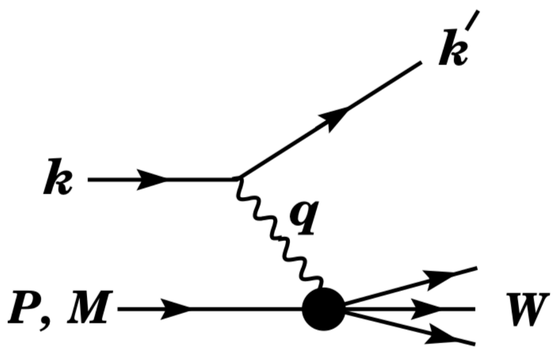
- Lattice calculation of Pseudo-PDFs: $\mathfrak{M}_R(\nu, \mu^2) \equiv \int_0^1 dx \, \cos(\nu x) \, q_v(x, \mu^2)$

  K. Orginos et al., PRD96, 094503 (2017)

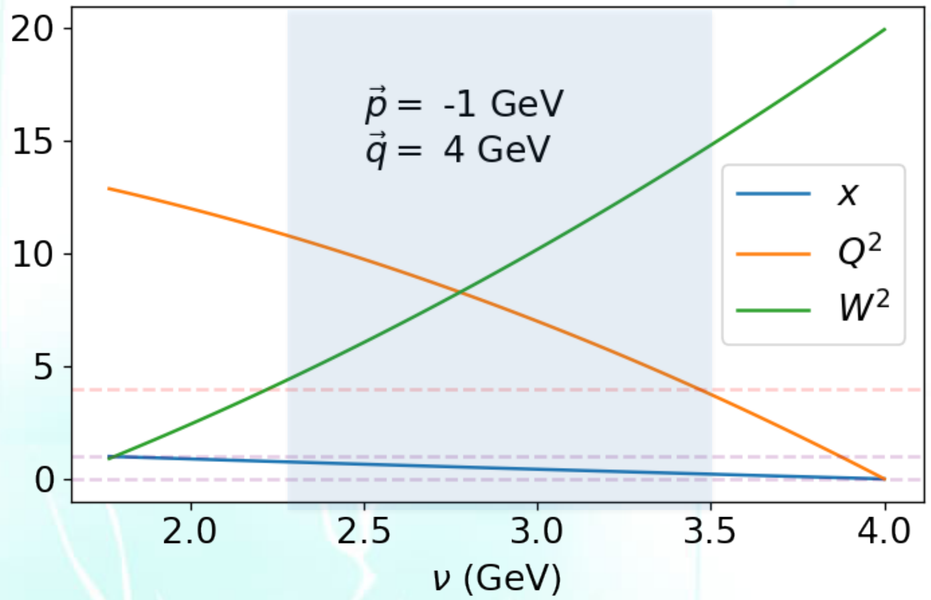
- Lattice cross sections: $\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{\text{QCD}}^2)$

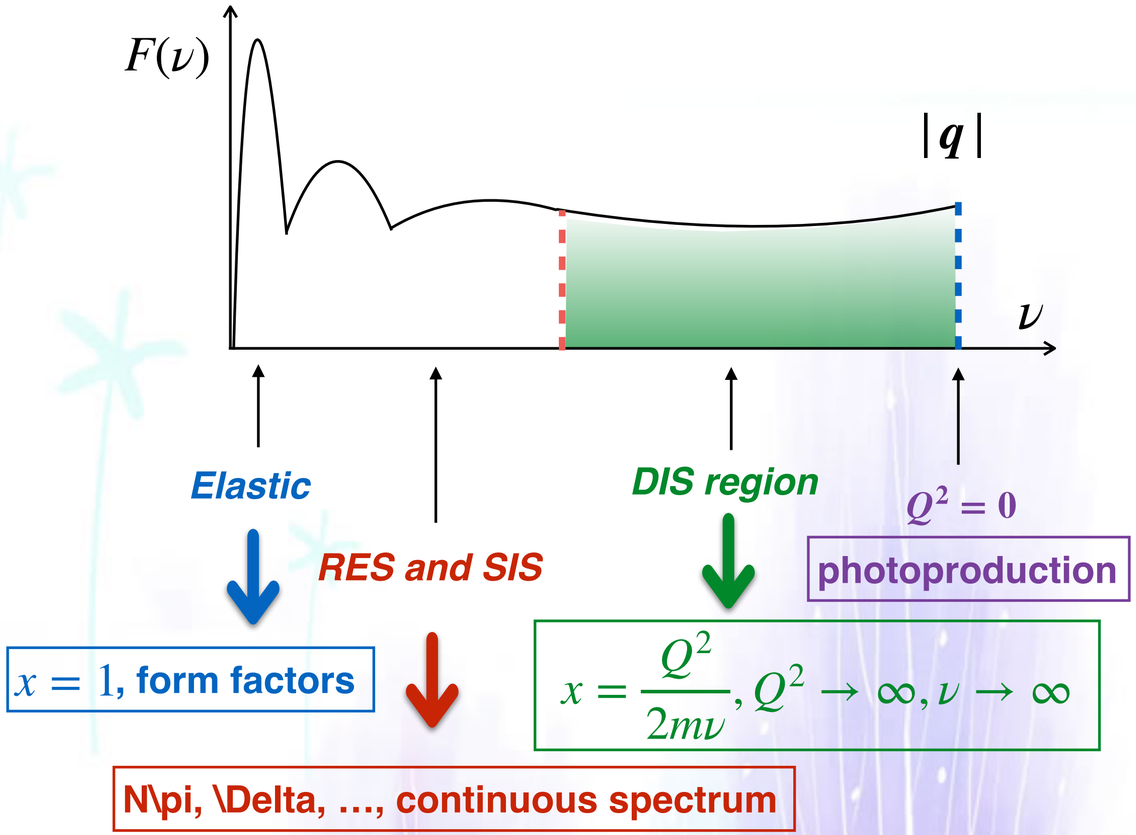
  Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)

Kinematics



$$Q^2 = -q^2 = \nu^2 - |\vec{q}|^2 \qquad x = \frac{Q^2}{2(E\nu - \vec{p}\cdot\vec{q})}$$





- Introduction
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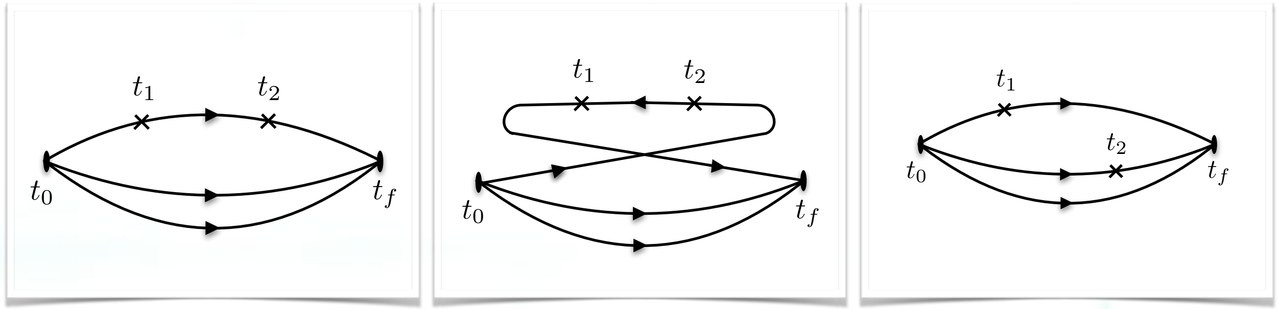
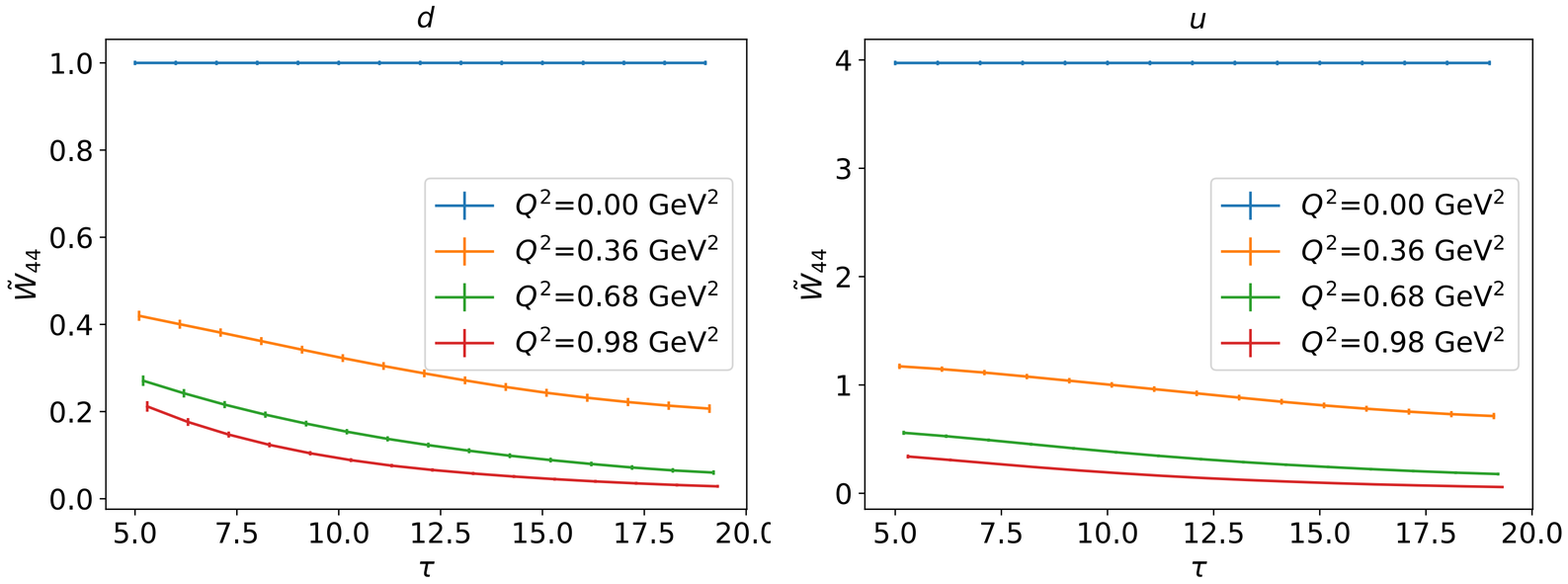
Case 1: Euclidean hadronic tensor and elastic FFs

T. Blum et al., PRD93:074505 (2016)

$$\tilde{W}_{44}(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s\,|\,J_4(\vec{x}_2,t_2)J_4(\vec{x}_1,t_1)\,|\,p,s\rangle = \sum_n A_n e^{-\nu_n\tau}$$



$$A_0 = \langle p,s\,|\,J_4(\vec{q})\,|\,n=0\rangle\langle n=0\,|\,J_4(-\vec{q})\,|\,p,s\rangle = G_E^2(Q^2)$$



- 32IF lattice (a ~0.063 fm, pion mass ~370 MeV), clover on domain wall, proton at rest, the first 4 momentum transfers, vector currents, g4.
- **Constant** for zero momentum transfer, **exponentials** for non-zero momentum transfers.
- **Nucleon elastic form factors squared** can be extracted by fitting for the spectral weight of the lowest-lying state.

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$$\tilde{W}_{44}(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s|J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)|p,s\rangle = \sum_n A_n e^{-\nu_n\tau}$$

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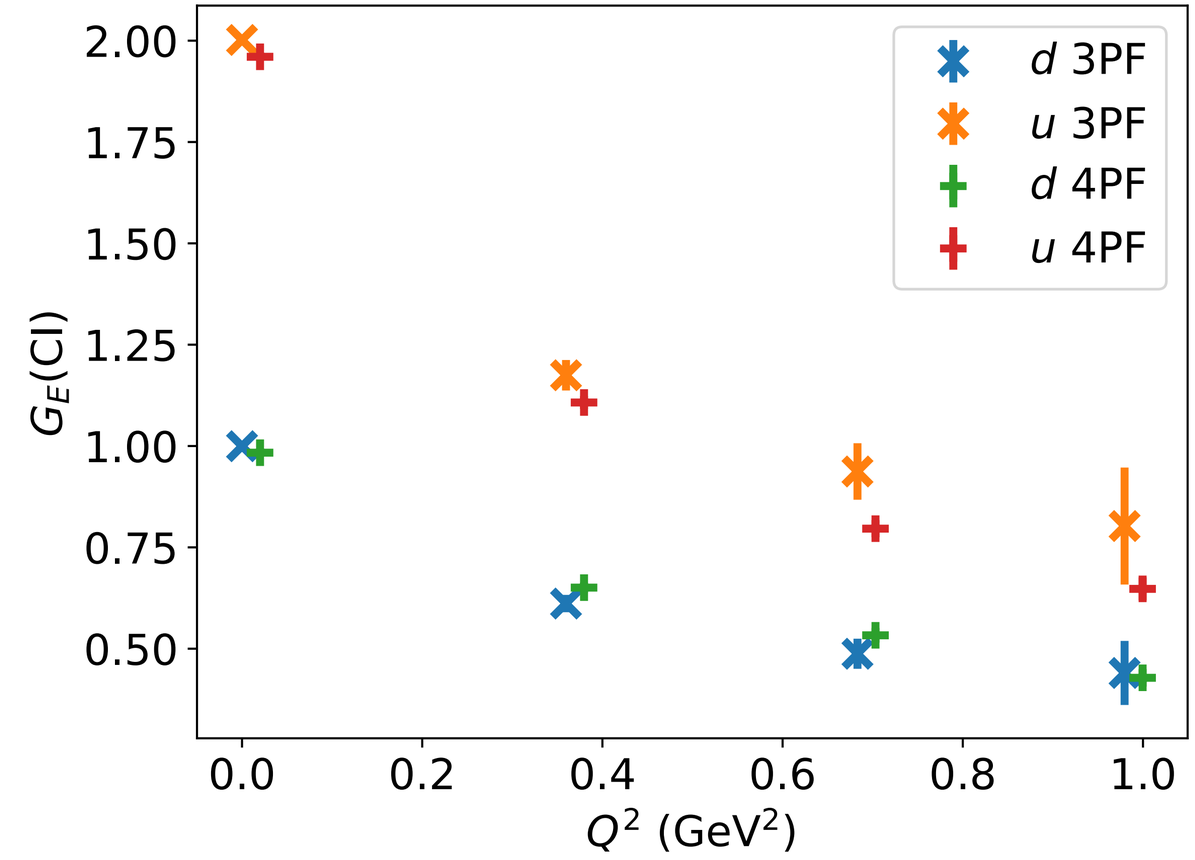

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- **Constant** for zero momentum transfer, **exponentials** for non-zero momentum transfers.
- **Nucleon elastic form factors squared** can be extracted by fitting for the spectral weight of the lowest-lying state.
- FFs extracted from 3-point functions and 4-point functions shows **consistency**.

Case 2: hadronic tensor with large momentum transfer

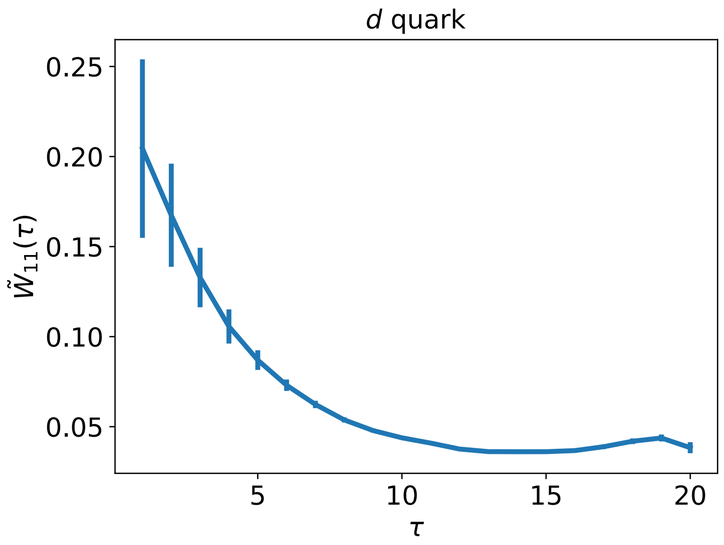
clover anisotropic lattice, $24^3 \times 128$, $a_t \sim 0.035$ fm, $\xi = 3.7$, $m_\pi \sim 380$ MeV, $\frac{2\pi}{L} \sim 0.42$ GeV

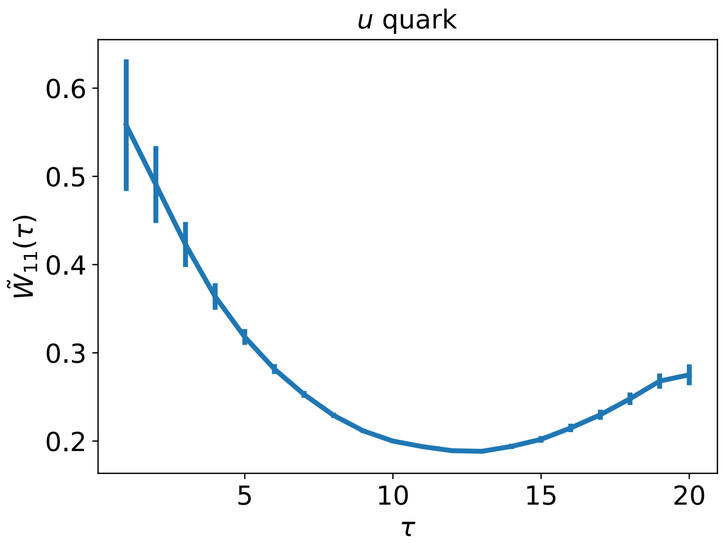
H.-W. Lin et al., PRD 79, 034502 (2009)

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \qquad \boldsymbol{p} + \boldsymbol{q} = -\boldsymbol{p} \qquad E_0 = (m_N^2 + |\boldsymbol{p} + \boldsymbol{q}|^2) = E_p$$

$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\lvert\boldsymbol{q}\rvert$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]





Case 2: hadronic tensor with large momentum transfer

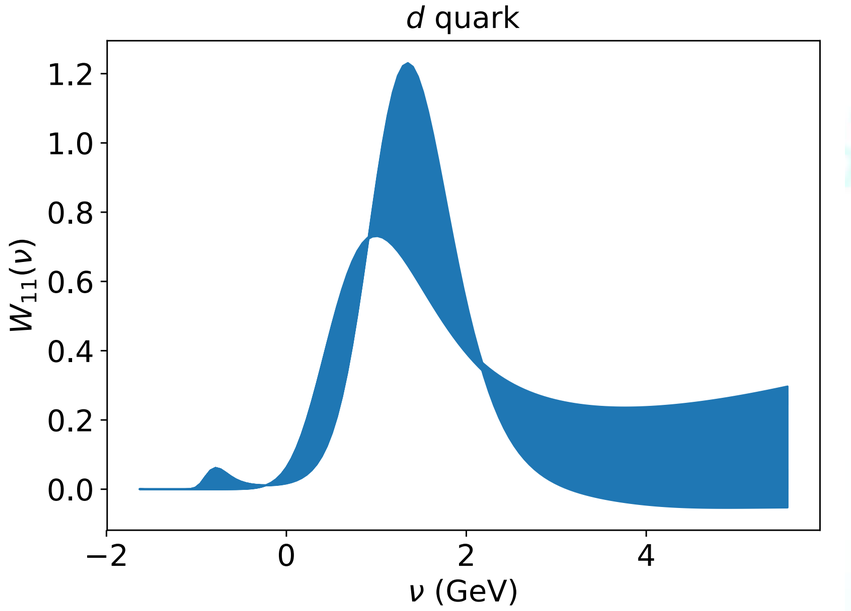
clover anisotropic lattice, $24^3 \times 128$, $a_t \sim 0.035$ fm, $\xi = 3.7$, $m_\pi \sim 380$ MeV, $\frac{2\pi}{L} \sim 0.42$ GeV

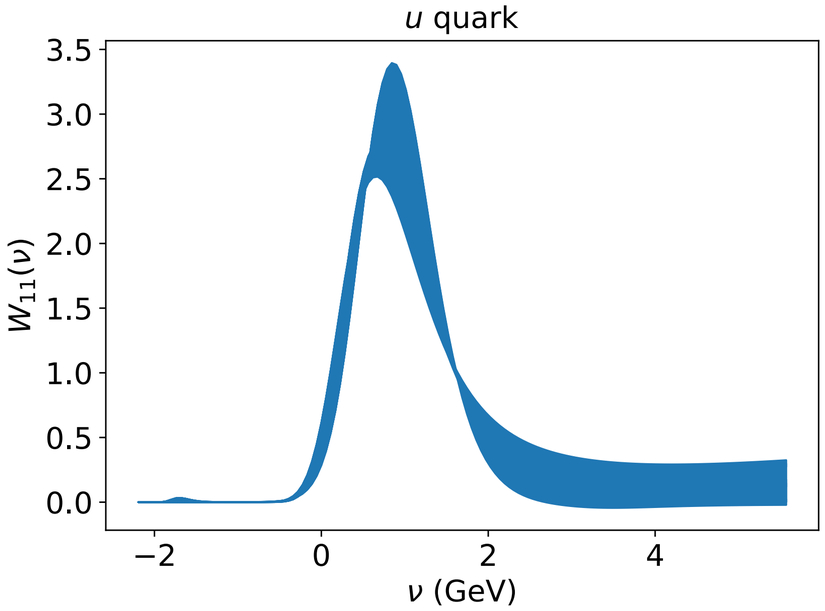
H.-W. Lin et al., PRD 79, 034502 (2009)

$\mu = \nu = 1$ and $p_1 = q_1 = 0$ $\quad W_{11}(\nu) = F_1(x, Q^2)$

$$\tilde{W}_{\mu\nu}(\boldsymbol{p}, \boldsymbol{q}, \tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \qquad \boldsymbol{p} + \boldsymbol{q} = -\boldsymbol{p} \qquad E_0 = (m_N^2 + |\boldsymbol{p} + \boldsymbol{q}|^2) = E_p$$

$\boldsymbol{p}$	$\boldsymbol{q}$	E_p	$E_{n=0}$	$\lvert\boldsymbol{q}\rvert$	ν	Q^2	x
(0,3,3)	(0,-6,-6)	2.15	2.15	3.57	[2.96, 3.28]	[4, 2]	[0.16, 0.07]





- Elastic contribution at \nu=0 is suppressed by the large momentum transfer.

$$G^2(0) \propto \frac{1}{\left(1 + \frac{Q_{\rm el}^2}{\Lambda^2}\right)^4}$$

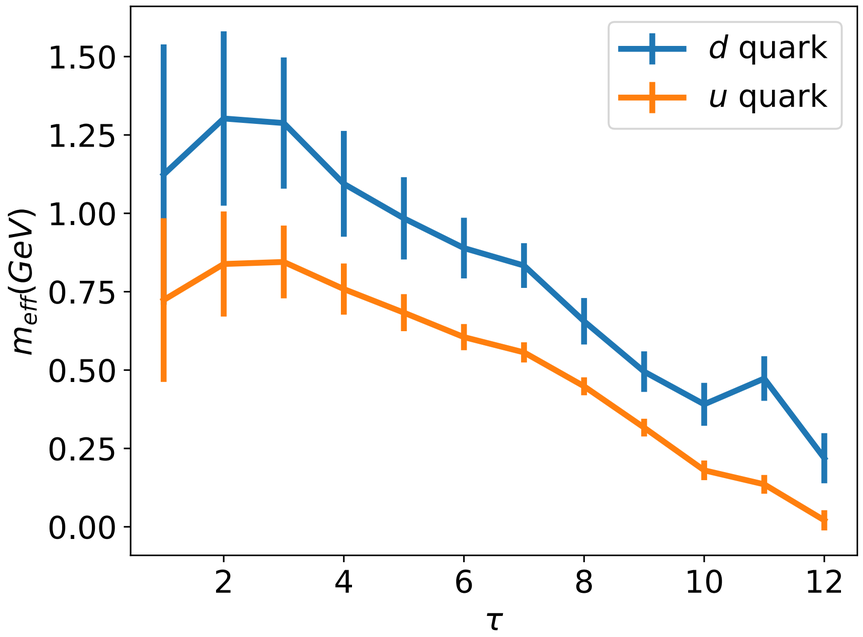
$$Q^2 \sim 13\ \text{GeV}^2,\ G^2(0) \sim 10^{-5}$$

- RES contribution at ~ 1 GeV is large and relatively stable.
- Large error in the SIS and DIS region (> 2 GeV), no enough constraint from the data.

Check of effective energy

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \quad E_p \sim 2.15 \text{ GeV}$$

$$m_{\text{eff}} = \log[\tilde{W}(\tau)/\tilde{W}(\tau+1)]$$



$$\nu \sim E_n - E_p \sim 1 \text{ GeV} \quad E_n \sim 3.2 \text{ GeV}$$

lattice artifacts: **finite volume** (discrete momenta and discrete spectrum)? **finite lattice spacing** (UV cutoff)? and/or **unphysical pion mass** (unphysical multi-particle states)?

Check of effective energy

$$\tilde{W}_{\mu\nu}(\boldsymbol{p},\boldsymbol{q},\tau) = \sum_n A_n e^{-(E_n - E_p)\tau} \quad E_p \sim 2.15 \text{ GeV}$$

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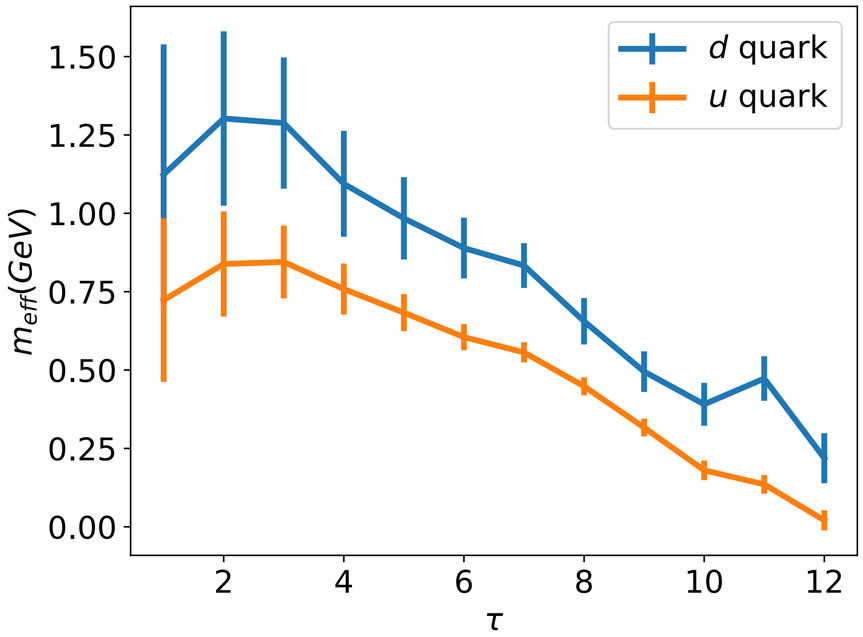
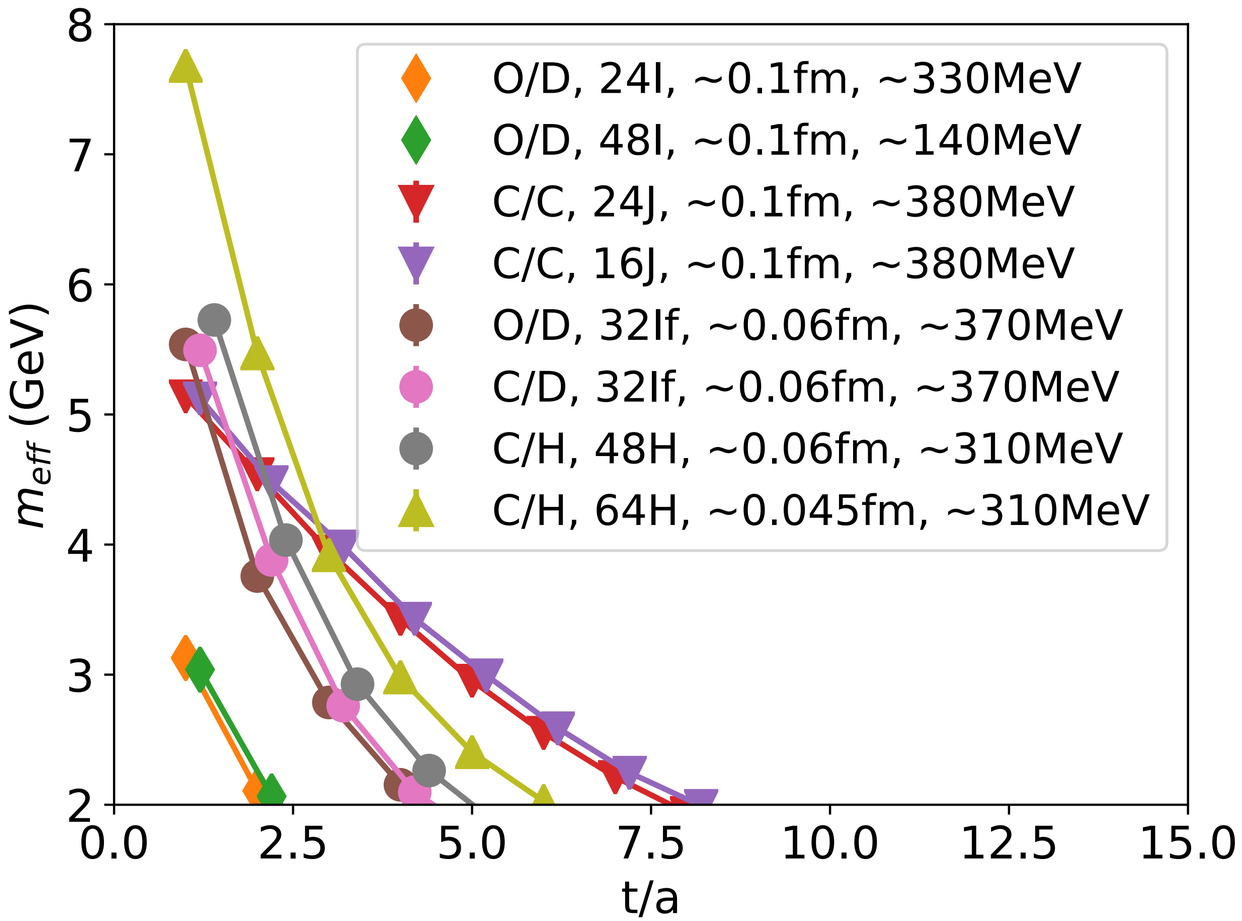


$$\nu \sim E_n - E_p \sim 1 \text{ GeV} \quad E_n \sim 3.2 \text{ GeV}$$



Summary and outlook

- Calculating the hadronic tensor on the lattice is helpful to study **nucleon structures** and **neutrino-nucleus scatterings**.
- This is the only lattice approach so far that gives **inclusive results** in both the RES and SIS regions.
- We now have good results for the **elastic form factors**.
- To study physics with larger energy transfers, **finer lattice spacings** are essential.
- We are working on lattices with smaller lattice spacings (~0.045 fm) to have better results for the neutrino-nucleon scatterings.
- Great approach to studying parton physics, $\nu > 6\ \mathrm{GeV}$.

Thank you!