

Nucleon isovector charges from physical mass domain-wall QCD

Shigemi Ohta^{*†‡} for LHP+RBC+UKQCD

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Domain-wall fermions (DWF) lattice Quantum Chromodynamics (QCD):

- preserves both **chiral and flavor symmetries**,
- started by RIKEN-BNL-Columbia Collaboration 21 years ago, using purpose-built parallel supercomputers.

Joint RBC+UKQCD Collaborations have been generating **2+1-flavor dynamical DWF** ensembles:

- for more than a decade, and at **physical mass** for several years,
- with a range of momentum cuts off, 1-3 GeV, and volumes $m_\pi L \sim 4$.

We have been calculating **pion, kaon, $(g - 2)_\mu$, and nucleon electroweak matrix elements**.

Updates to my isovector nucleon charges report in Lattice 2018.

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RBC/UKQCD $N_f = 2 + 1$ -flavor dynamical DWF ensembles¹ with $a^{-1} = 1.730(4)$ and $2.359(7)$ GeV :

- $L \sim 5.5$ fm, with pion mass of 139.2(4) and 139.2(5) MeV respectively,
- $m_\pi L \sim 4$, small volume corrections.

Chiral and continuum limit with good flavor and chiral symmetries (and so renormalizations wherever needed):

- meson decay constants: $f_\pi = 130.2(9)$ MeV, $f_K = 155.5(8)$ MeV, $f_K/f_\pi = 1.195(5)$;
- quark mass: $m_s^{\overline{\text{MS}}(3\text{GeV})} = 81.6(1.2)$ MeV, $m_{ud}^{\overline{\text{MS}}(3\text{GeV})} = 3.00(5)$ MeV, $m_s/m_{ud} = 27.34(21)$;
- chiral condensate $\Sigma^{1/3}(\overline{\text{MS}}, 3\text{GeV}) = 0.285(2)_{\text{stat.}}(1)_{\text{pert.}}$ GeV;
- kaon mixing parameter: $B_K^{\overline{\text{RGI}}} = 0.750(15)$, $B_K^{\overline{\text{MS}}(3\text{GeV})} = 0.530(11)$,
- $K_{l3}^2 f_+(0) = 0.9685(34)_{\text{stat.}}(14)_{\text{FV}}$, $|V_{us}| = 0.2233(5)_{\text{exp.}}(9)_{\text{lat}}$;
- $SU(2)$ low-energy constants³ $B^{\overline{\text{MS}}}$, f , $\Sigma^{1/3, \overline{\text{MS}}}$, $f_\pi/f = 1.064(2)(5)$, $l_{1,2,3,4,7}$;
- $SU(3)$ -breaking ratios for D- and B-mesons⁴, $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$;
- BSM kaon mixing⁵ are also being calculated, testing the SM, or constraining the BSM.

Contribute to **determining SM parameters** from meson calculations and **constraining the BSM**⁶.

¹T. Blum et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 074505, arXiv:1411.7017 [hep-lat].

²D. Murphy et al., RBC and UKQCD Collaborations, PoS LATTICE2014 (2015) 369

³P.A. Boyle et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 054502, arXiv:1511.01950 [hep-lat].

⁴P.A. Boyle et al., RBC and UKQCD Collaborations, arXiv:1812.08791 [hep-lat].

⁵P.A. Boyle et al., arXiv:1812.04981 [hep-lat].

⁶S. Aoki et al., Eur.Phys.J. C77 (2017) 112 DOI: 10.1140/epjc/s10052-016-4509-7 e-Print: arXiv:1607.00299 [hep-lat].

In contrast, systematics in the baryon sector is not well understood yet:

- Proton mean squared charge radius,
- Nucleon axial charge, g_A ,
- Nucleon electroweak form factors, $F_V(q^2)$, $F_T(q^2)$, $F_A(q^2)$, $F_P(q^2)$,
- Nucleon structure functions and parton distribution functions,
- Proton spin puzzle,

despite potentials for new physics:

- dark matter via g_T and g_S ,
- neutron electric dipole moment,
- proton decay,
- $n\bar{n}$ mixing, ...

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N} F_T(q^2) \right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_5 \gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2) \right] u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2} F_2, G_M = F_1 + F_2.$$

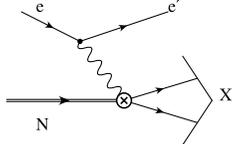
Related to

- mean-squared charge radii, $F_V = F_V(0) - \frac{1}{6}\langle r_E^2 \rangle Q^2 + \dots$
- anomalous magnetic moment, $F_2(0)$,
- $g_A = F_A(0) = 1.2732(23)g_V$ ($g_V = F_V(0) = G_{\text{Fermi}} \cos \theta_{\text{Cabibbo}}$).

$\langle r_E^2 \rangle$ and g_A , in particular, are yet to be understood:

- $\sqrt{\langle r_E^2 \rangle} = 0.875(6)$ fm from electron scattering, $0.8409(4)$ from muonic atom;
- $g_A/g_V = 1.264(2)$ pre 2002 (“cold neutron,”) $1.2755(11)$ post, (“ultra cold neutron.”)

The latter, with Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, determines much of nuclear physics, such as primordial and neutron-star nucleosyntheses.

Deep inelastic scatterings  : $\left| \frac{\mathcal{A}}{4\pi} \right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}, W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

unpolarized: $W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu \right) \left(P^\nu - \frac{\nu}{q^2} q^\nu \right) \frac{F_2(x, Q^2)}{\nu},$

polarized: $W^{[\mu\nu]}(x, Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\sigma}{\nu^2} g_2(x, Q^2) \right),$

with $\nu = q \cdot P, S^2 = -M^2, x = Q^2/2\nu.$

Traditionally, moments of the structure functions, $F_i(x, Q^2)$, are accessible on the lattice:

$$2 \int_0^1 dx x^{n-1} F_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$

$$2 \int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu)) d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2),$$

- c_1, c_2, e_1 , and e_2 are the **Wilson coefficients** (perturbative),
- $\langle x^n \rangle_q(\mu), \langle x^n \rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is transversity, $\langle 1 \rangle_{\delta q}(\mu) = \langle P, S | \bar{q} i \gamma_5 \sigma_{\mu\nu} q | P, S \rangle,$
- and scalar density g_S .

Now we have better lattice access to **PDFs**^{7 8 9}.

⁷T. Ishikawa, Y. Q. Ma, J. W. Qiu and S. Yoshida, Phys. Rev. D **96**, 094019 (2017).

⁸X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. Lett. **120**, 112001 (2018).

⁹T. Izubuchi, X. Ji, L. Jin, I. W. Stewart and Y. Zhao, arXiv:1801.03917 [hep-ph].

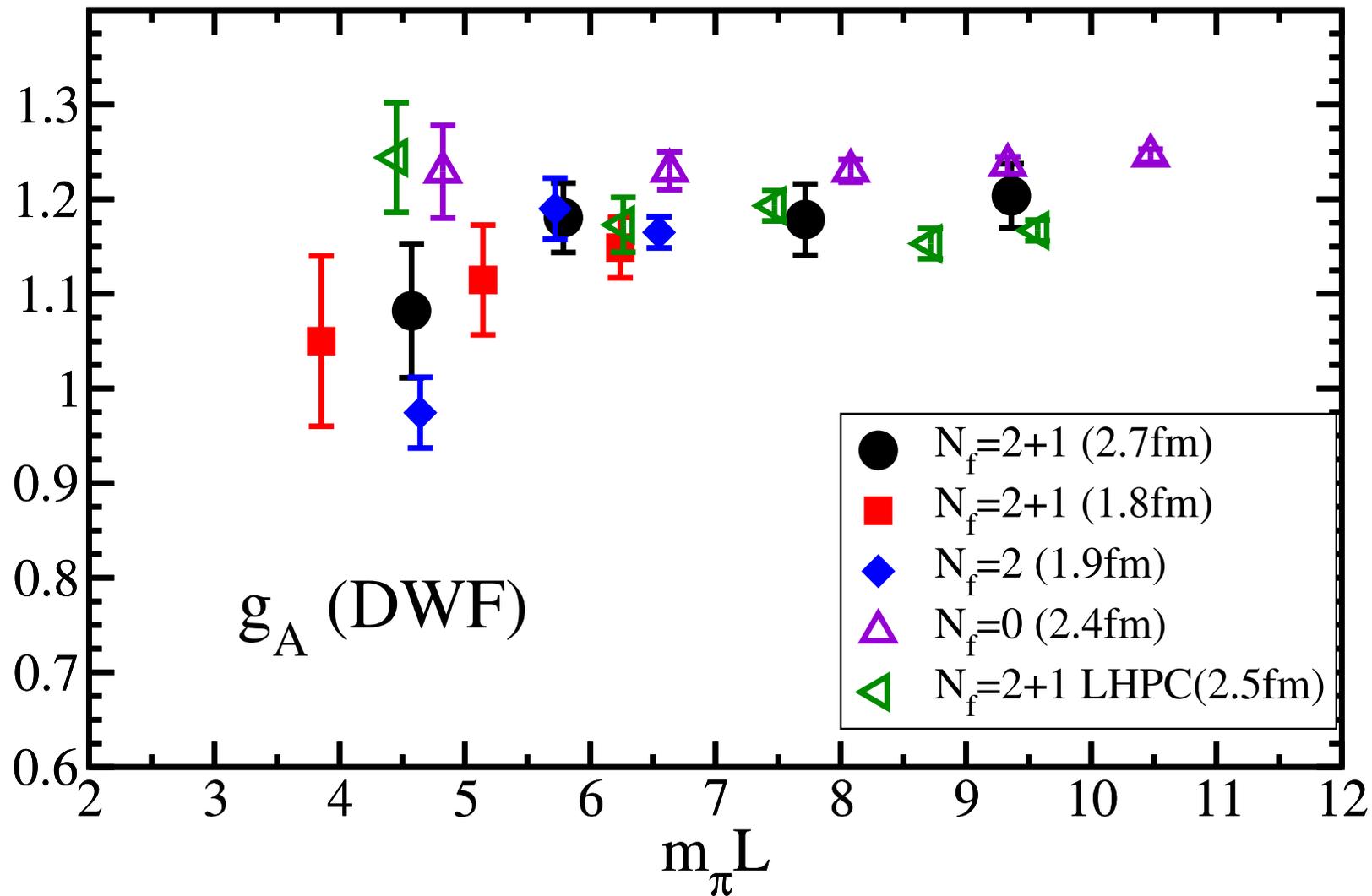
On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$, ratio of two- and three-point correlators such as $\frac{C_{3\text{pt}}^{\Gamma,O}(t_{\text{sink}}, t)}{C_{2\text{pt}}(t_{\text{sink}})}$ with

$$C_{2\text{pt}}(t_{\text{sink}}) = \sum_{\alpha,\beta} \left(\frac{1 + \gamma_t}{2} \right)_{\alpha\beta} \langle N_\beta(t_{\text{sink}}) \bar{N}_\alpha(0) \rangle,$$

$$C_{3\text{pt}}^{\Gamma,O}(t_{\text{sink}}, t) = \sum_{\alpha,\beta} \Gamma_{\alpha\beta} \langle N_\beta(t_{\text{sink}}) O(t) \bar{N}_\alpha(0) \rangle,$$

give a plateau in t for a lattice bare value $\langle O \rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1 + \gamma_t)/2$ or $(1 + \gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

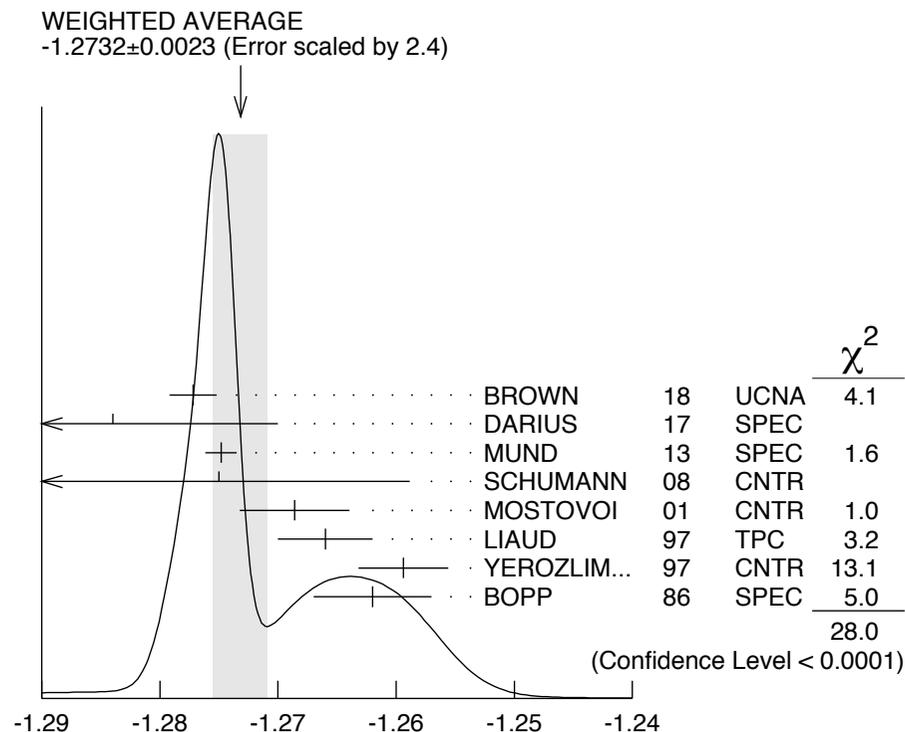
A bit of back ground: In 2007 Takeshi Yamazaki reported **unexpectedly large deficit** in lattice calculation ¹⁰:



¹⁰T. Yamazaki *et al.* [RBC+UKQCD Collaboration], Phys. Rev. Lett. **100**, 171602 (2008).

Why?

Difficult history¹¹



The lifetime has been almost monotonically increasing since the first measurement > 21 minutes^{12 13}: the more recent peak from the ultra-cold neutrons, $1.2764(6)$ ¹⁴ and $1.2772(20)$ ¹⁵, appears more reliable¹⁶.

Lattice calculations appear to follow a parallel path.

¹¹M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.

¹²A.H. Snell and L.C. Miller in APS Washington Meeting, Spring 1948.

¹³Dirk Dubbers, arXiv:1807.07026 [hep-ph].

¹⁴B. Märkisch et al., arXiv:1812.04666 [nucl-ex].

¹⁵M. A.-P. Brown et al. (UCNA Collaboration) Phys. Rev. C 97, 035505.

¹⁶A. Czarnecki, W.J. Marciano, and A. Sirlin, Phys.Rev.Lett. 120 (2018) 202002.

Why?

Difficult history:

Non-relativistic quark model: $5/3$. Very bad, but some “large- N_c ” conform?

And with absurd “relativistic” correction: $5/4$, really?

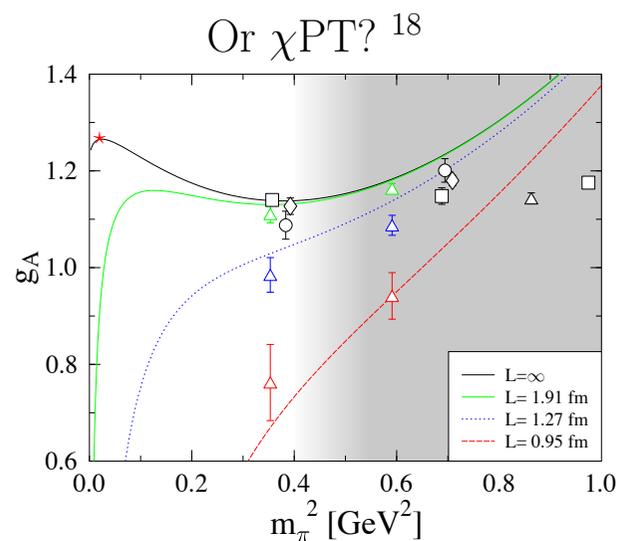
Without pion,

MIT bag model: 1.09, as good(!) as lattice but when experiment was 1.22.¹⁷

With only pion,

Skyrmion: 0.61(!) with a peculiar geometry but when experiment was 1.23.

Accurate reproduction of the ‘pion cloud’ geometry seems essential.



¹⁷Assuming a growth rate of 0.001 per year.

¹⁸A. A. Khan *et al.*, PoS LAT **2005**, 349 (2006).

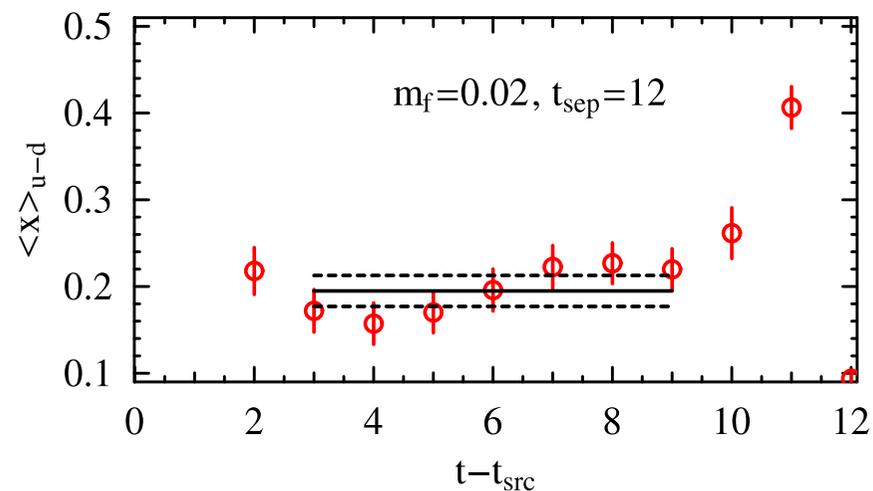
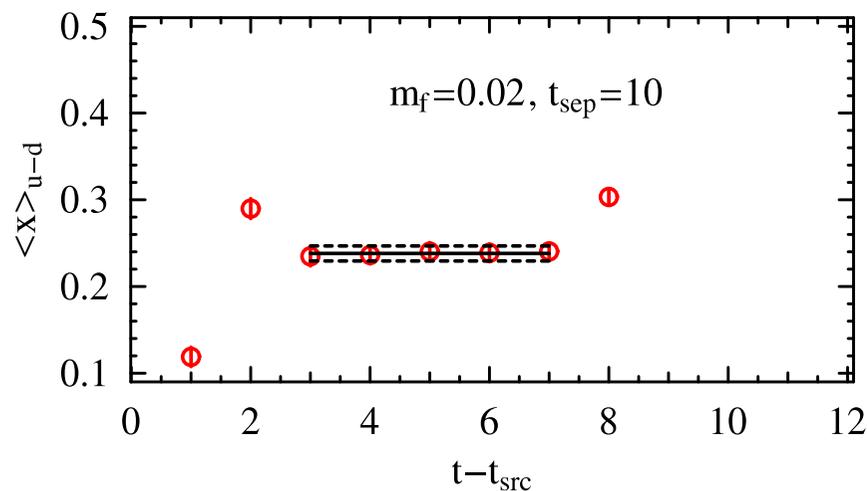
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

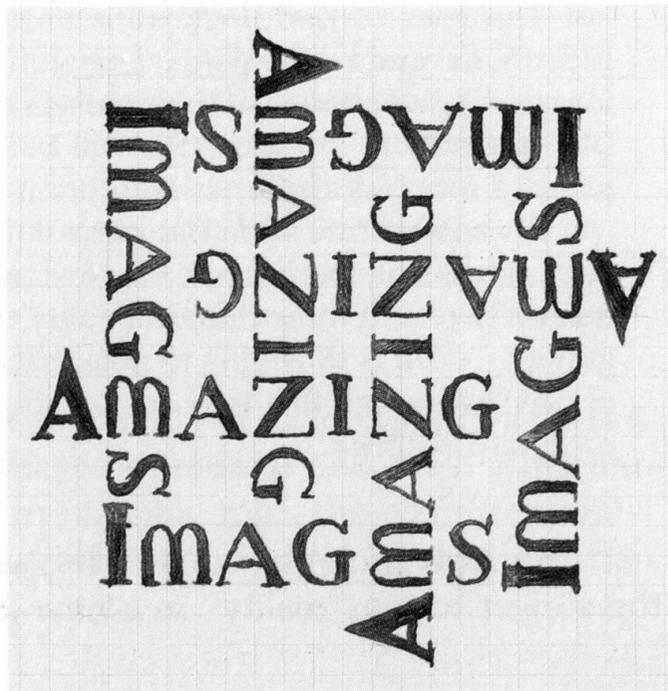


- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

The “AMA” trick¹⁹ helped a lot. It provides $\times 10\text{--}20$ acceleration by allowing

- cruder,
- but cheaper,

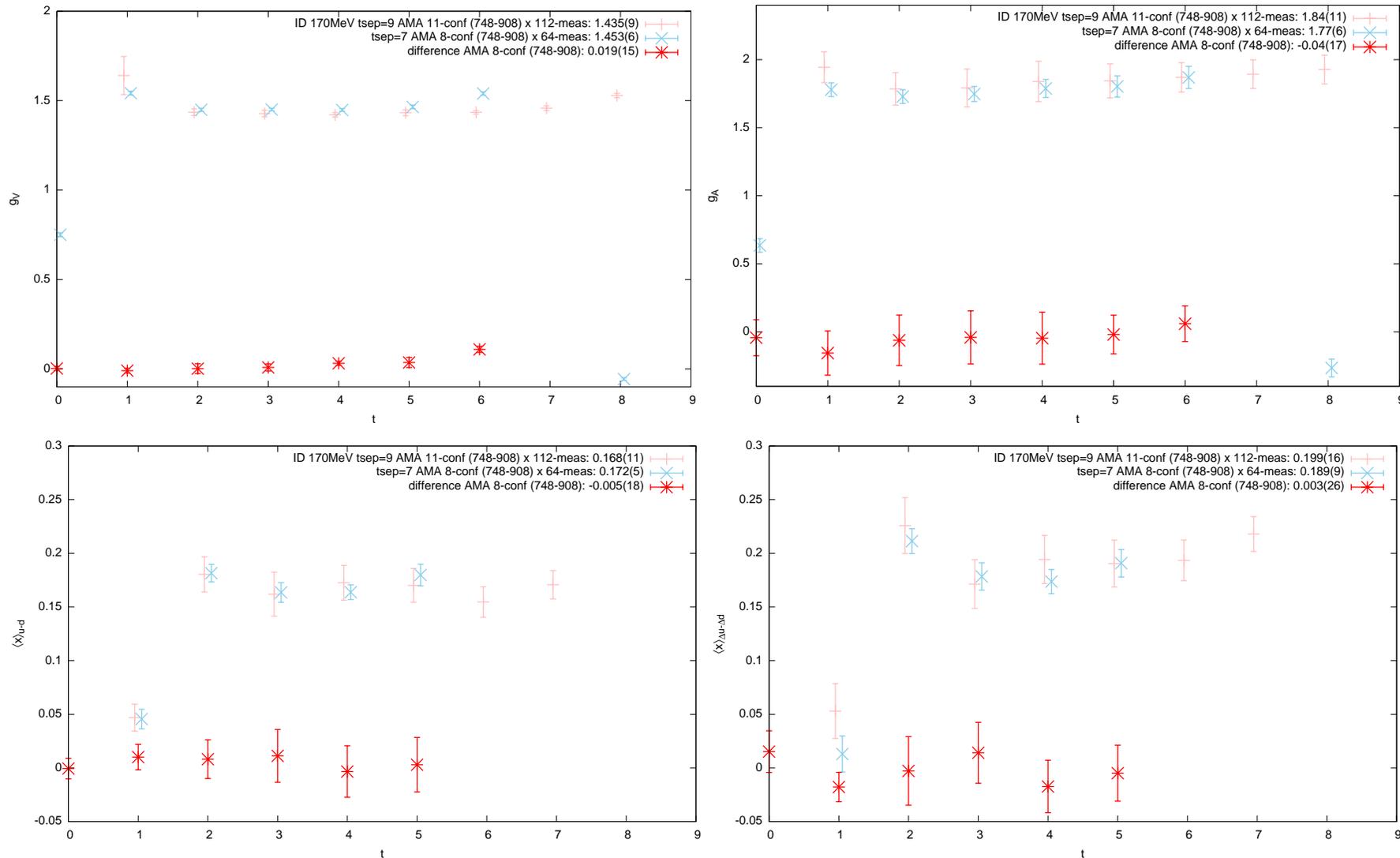
independent statistical sampling at much higher frequency, by taking advantage of point-group symmetries of the lattice to organize many such cruder but independent and equivalent measurements:



$$\langle O \rangle_{\text{AMA}} = \frac{1}{N_{\text{sloppy}}} \sum_s^{N_{\text{sloppy}}} \langle O \rangle_{\text{sloppy}}^s + \frac{1}{N_{\text{accurate}}} \sum_a^{N_{\text{accurate}}} (\langle O \rangle_{\text{accurate}}^a - \langle O \rangle_{\text{sloppy}}^a)$$

¹⁹E. Shintani, R. Arthur, T. Blum, T. Izubuchi, C. Jung and C. Lehner, Phys. Rev. D **91**, 114511 (2015).

With the AMA we established no excited-state contamination is present in any of our 170-MeV calculations:



When compared with the same configurations, the difference is always consistent with 0.

$A_1 \langle 1|O|0 \rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0 \rangle$.

In agreement with many other groups' experiences in controlling this systematics.

More recently from RBC: results from a heavier and coarser "I24" ensembles

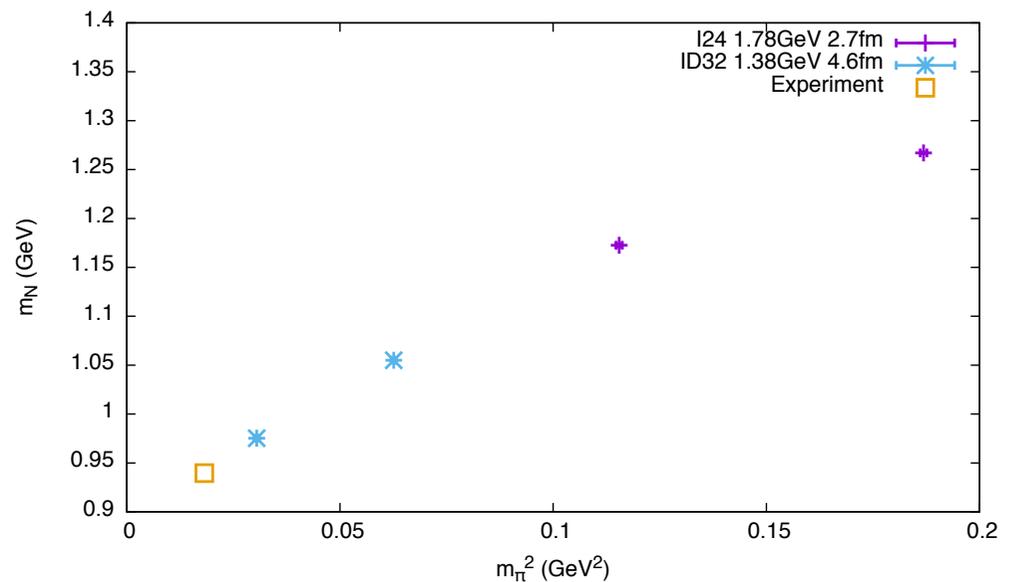
- with Iwasaki \times dislocation-suppressing-determinant-ratio (DSDR) gauge action at $\beta = 1.75$, $a^{-1} = 1.378(7)$ GeV, and pion mass of about 249 and 172 MeV.

We also improved AMA statistics for "I24" ensembles

- with Iwasaki gauge action at $\beta = 2.13$, corresponding the inverse lattice spacing of $a^{-1} = 1.7848(5)$ GeV, and pion mass values of about 432 and 340 MeV.

From these we estimate the nucleon mass:

a^{-1} [GeV]	$m_q a$	$m_N a$	m_N [GeV]
1.378(7)	0.001	0.7077(08)	0.9752(11)
	0.0042	0.76557(16)	1.0550(20)
1.7848(5)	0.005	0.6570(9)	1.1726(16)
	0.01	0.7099(5)	1.2670(09)



Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t} |0\rangle + A_1 e^{-E_1 t} |1\rangle + \dots,$$

resulting in dependence on source-sink separation, $t_{\text{sep}} = t_{\text{sink}} - t_{\text{source}}$,

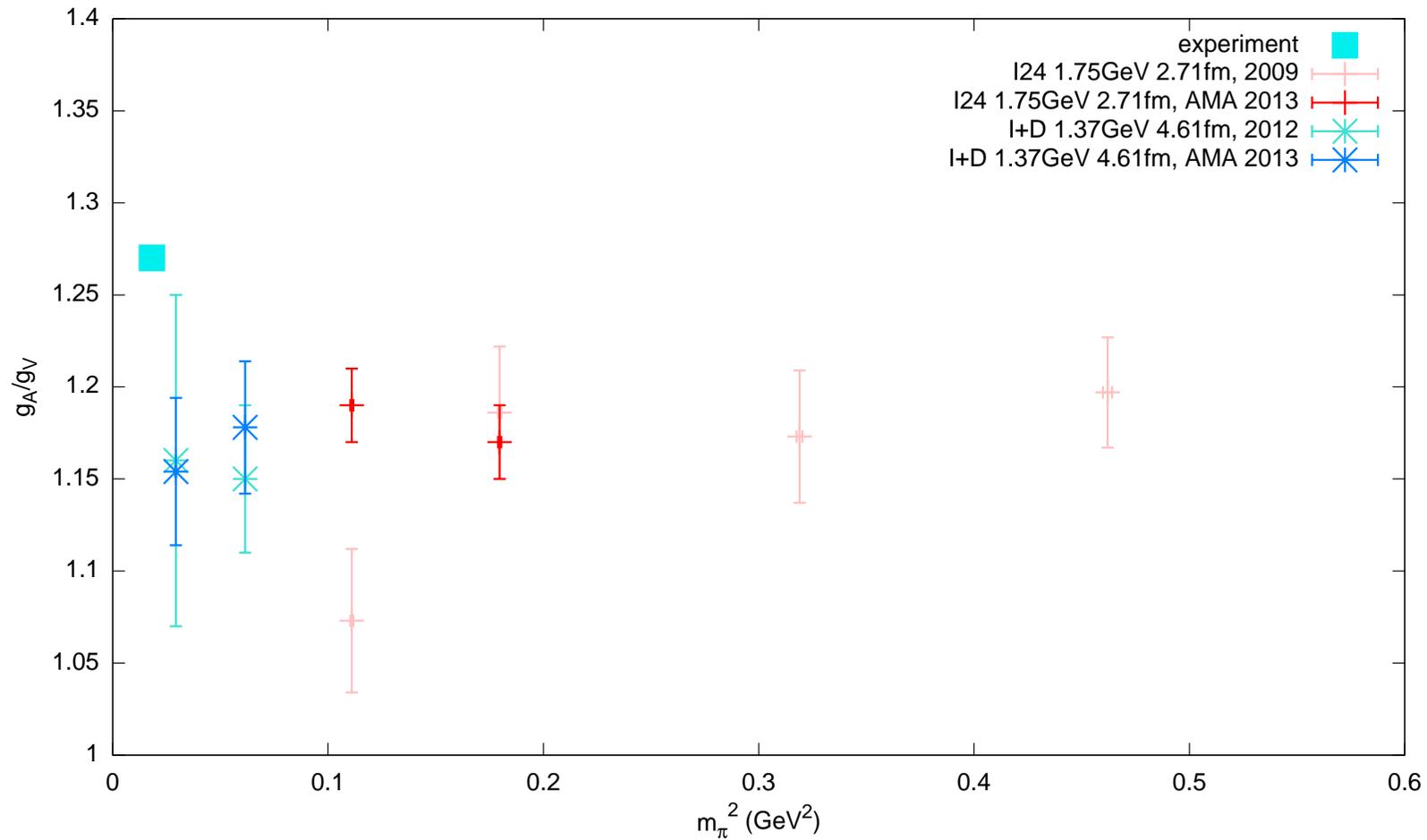
$$\langle 0 | O | 0 \rangle + A_1 e^{-(E_1 - E_0) t_{\text{sep}}} \langle 1 | O | 0 \rangle + \dots$$

Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1 | Q | 0 \rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

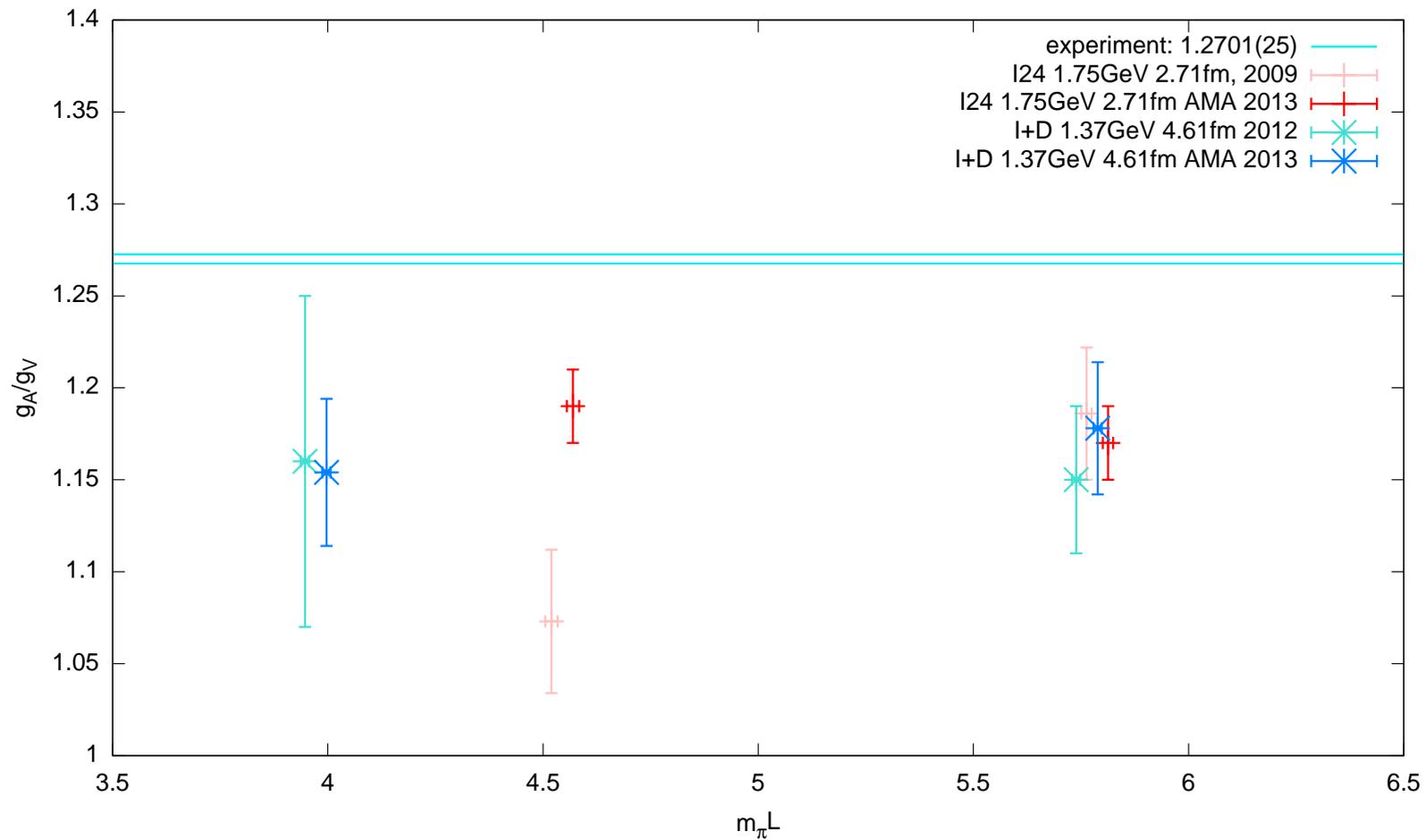
We can optimize the source so that A_1 is small, and we take sufficiently large t_{sep} : Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

With AMA and other statistical improvements, g_A/g_V vs m_π^2 then looked like the following:



Moves away from the experiment as m_π approaches the experimental value.

About 10-% deficit in g_A/g_V seems solid except perhaps for $O(a^2)$ error:



Excited-state contamination now is unlikely the cause.

Appears like monotonically decreasing with $m_\pi L$.

In agreement with the great majority of other groups.

Why?

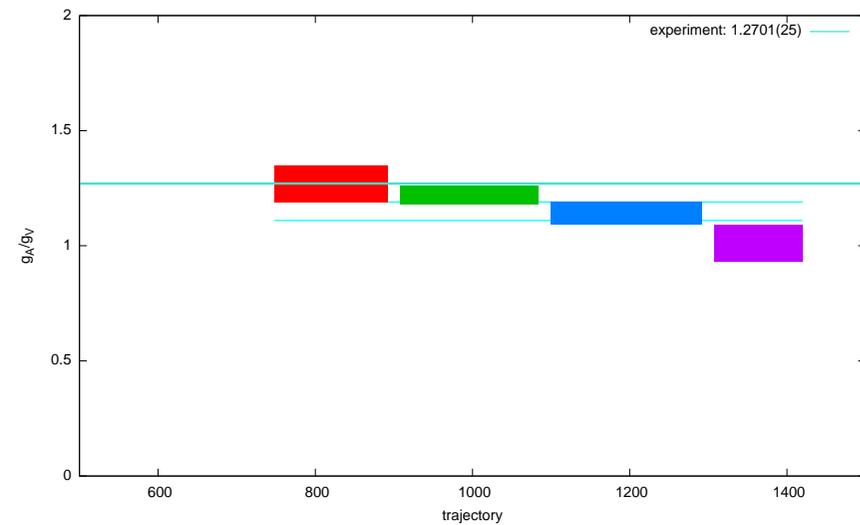
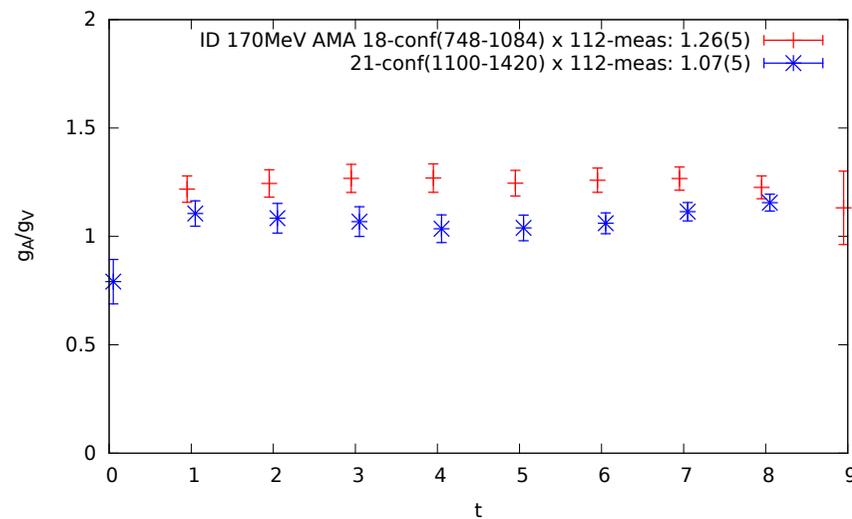
There appear long-range autocorrelations in axial charge but not in others:

Blocked jackknife analysis				
	bin size			
	1	2	3	4
g_V	1.447(8)	1.447(6)	-	-
g_A	1.66(6)	1.66(7)	1.71(8)	1.65(4)
g_A/g_V	1.15(4)	1.15(5)	1.15(6)	1.14(3)
$\langle x \rangle_{u-d}$	0.146(7)	0.146(8)	0.146(8)	-
$\langle x \rangle_{\Delta u-\Delta d}$	0.165(9)	0.165(11)	0.165(10)	-
$\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$	0.86(5)	0.86(4)	-	-
$\langle 1 \rangle_{\delta u-\delta d}$	1.42(4)	1.42(6)	1.42(6)	1.41(3)

except in perhaps transversity.

But the difference may be hard to notice by standard blocked jackknife analysis.

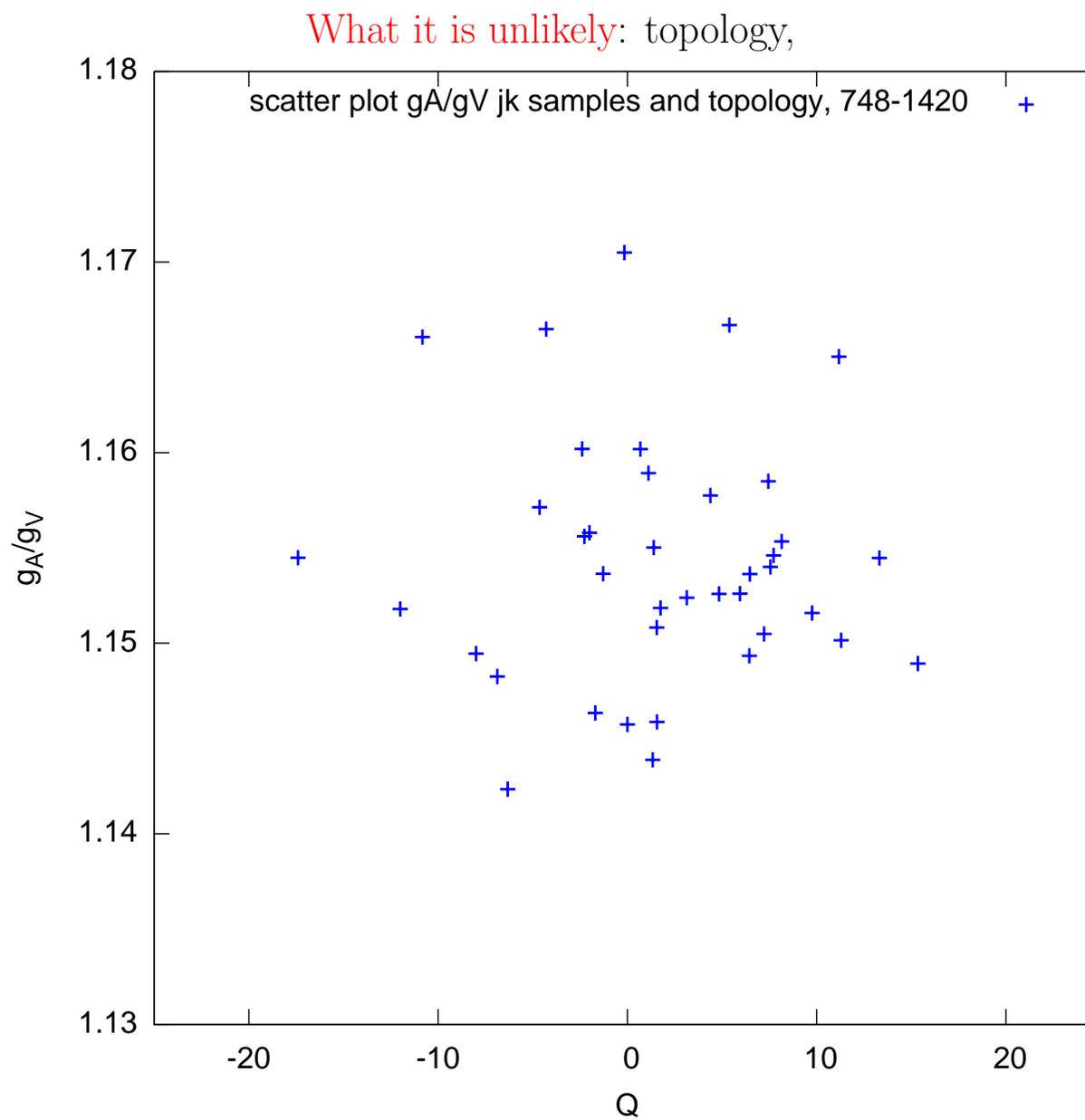
Long-range auto-correlation seen in g_A/g_V :



Non-AMA analyses are much noisier but not inconsistent with these:

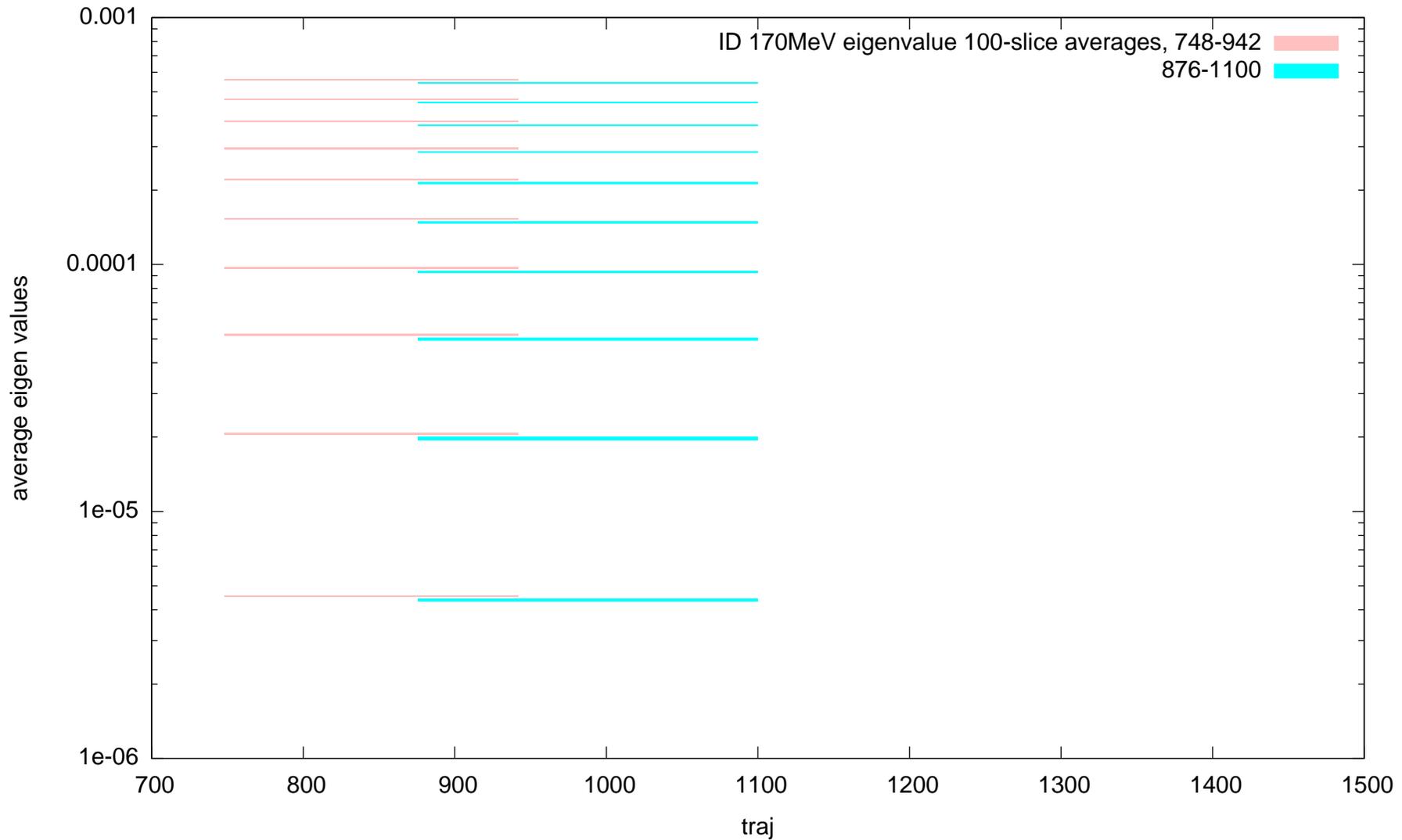
Indicative of inefficient sampling, but only in g_A and g_A/g_V .

Why?



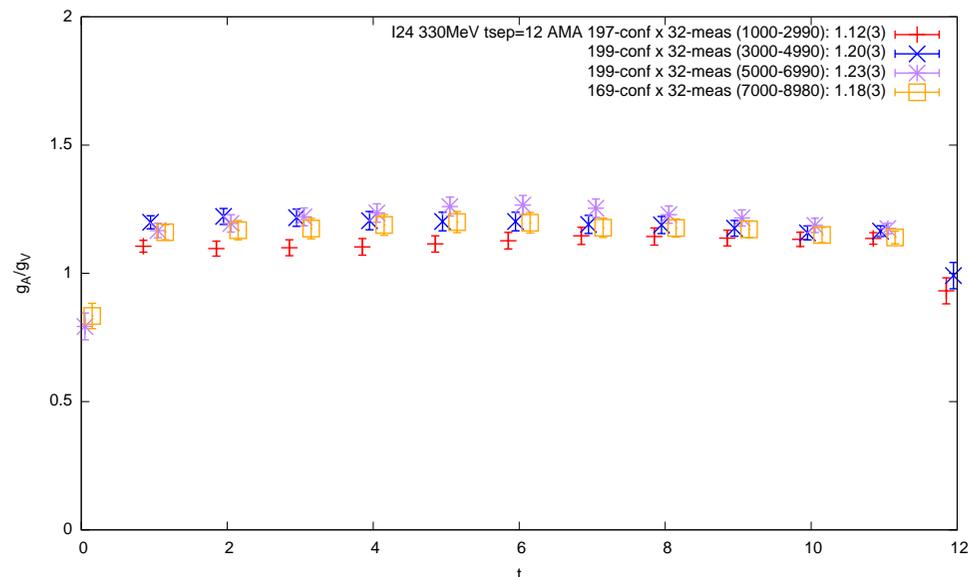
Scatter plot of g_A/g_V jackknife samples against gauge topological charge: No correlation is seen.

What it is unlikely: low-eigenmode deflation,



Deflation eigenmode statistics for the two quarters where data are available: there is no difference in the lowest 100 modes averaged. Some insignificant difference emerges in higher modes.

Why?



A similar long-range auto-correlation is also seen in g_A/g_V in $m_\pi = 340$ MeV ensemble:

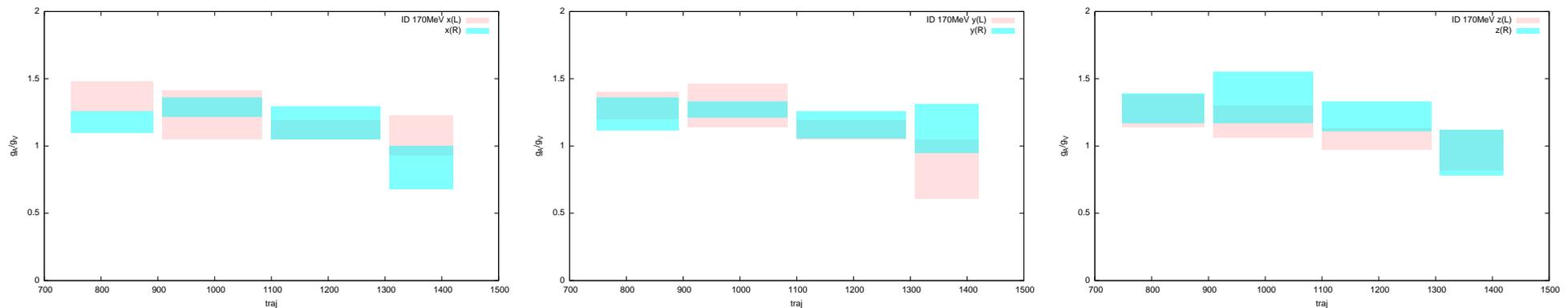
However no such sign of under-sampling is seen in the other two ensembles.

Under-sampling is seen at the smaller finite-size scaling parameters of $m_\pi L \sim 4.00(6)$ and $4.569(15)$,
but not in larger values of $m_\pi L \sim 5.79(6)$ or $5.813(12)$.

This of course does not prove the problem is caused by the finite lattice spatial volume, but suggests so.

How?

Divide the AMA samples into two halves along each spatial axes such as $0 \leq x < L/2$ and $L/2 \leq x < L$:



Spin polarization is along the z -axis.

The calculation appears to fluctuate spatially.
Larger spatial volume would stabilize the calculation better.

2017: there were deficit in nucleon g_A/g_V calculated in lattice QCD with small volumes.

Yet a validation of lattice QCD: As of Lattice 2017, with similar quark mass and lattice cuts off,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: $\sim 1.2^{20}$,
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
 - PACS, $1.16(8)^{21}$,
 - QCDSF $\sim 1.1^{22}$,
- and even a Wilson valence on HISQ, PNDME²³, ~ 1.2 ,
- except the then latest DWF valence²⁴ on HISQ staggered ensembles after an extrapolation.

g_A from different actions “blindedly” agree with deficit once $O(a)$ systematics is removed,

²⁰J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].

²¹A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].

²²J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].

²³T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].

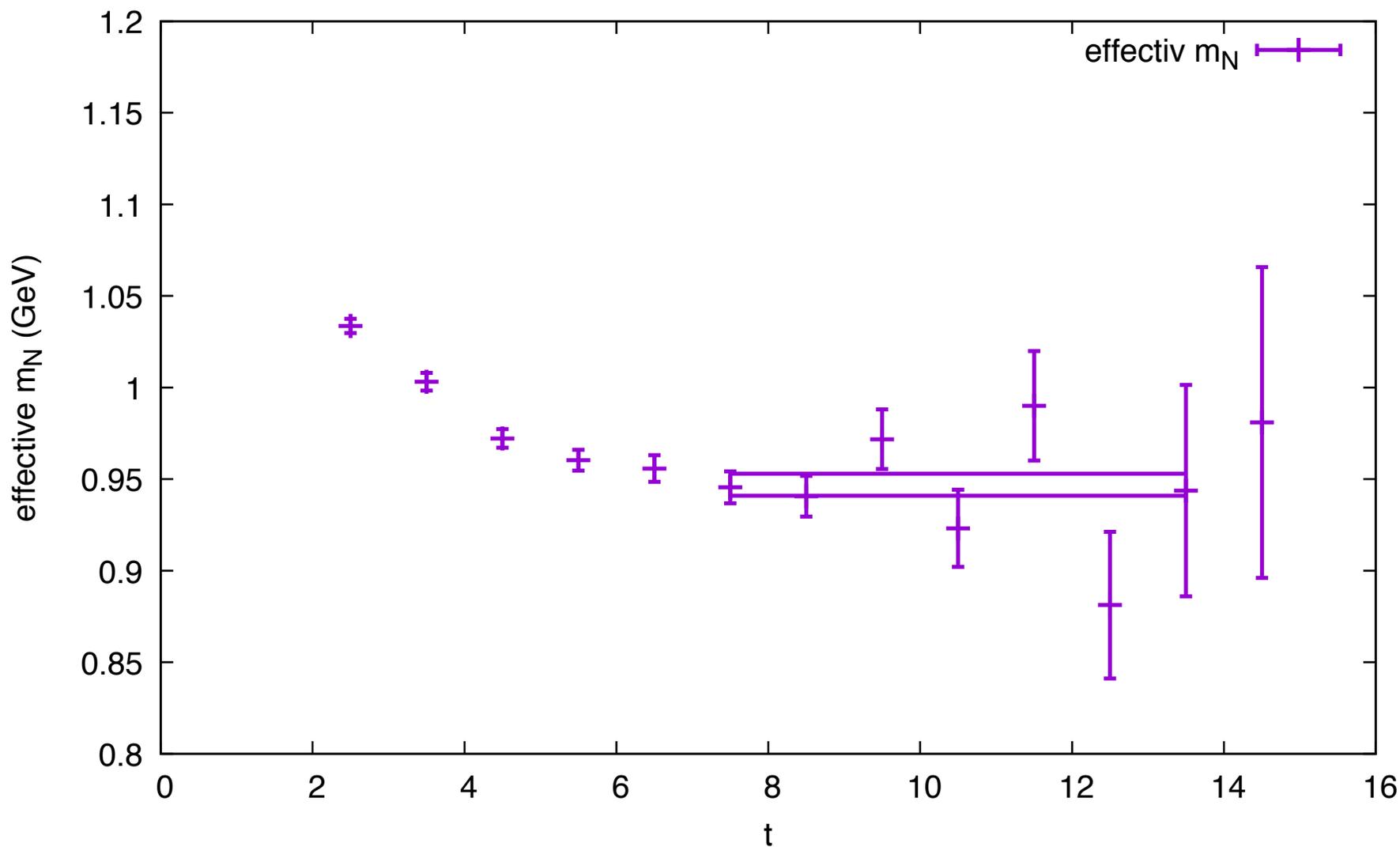
²⁴E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

2018: I reported results for isovector quark bilinears: vector charge g_V , $O = \bar{q}\gamma_t q$, axial charge g_A , $O = \bar{q}\gamma_5\gamma_z q$, transversity, g_T , $O = \bar{q}\gamma_5\gamma_z\gamma_t q$, and scalar “charge,” g_S , $O = \bar{q}q$, from RBC+UKQCD “48I” ensemble:

- with Iwasaki gauge action at $\beta = 2.13$, $a^{-1} = 1.730(4)$ GeV, and pion mass of about 139.2(4) MeV,
- 130 configurations at trajectory (620-980)/20 and (990-2160)/10
 - except 1050, 1070, 1150, 1170, 1250, 1270, and 1470,
- each deflated with 2000 low-lying eigenvalues,
- each with $4^4 = 256$ AMA sloppy calculations unbiased by 4 precision ones.

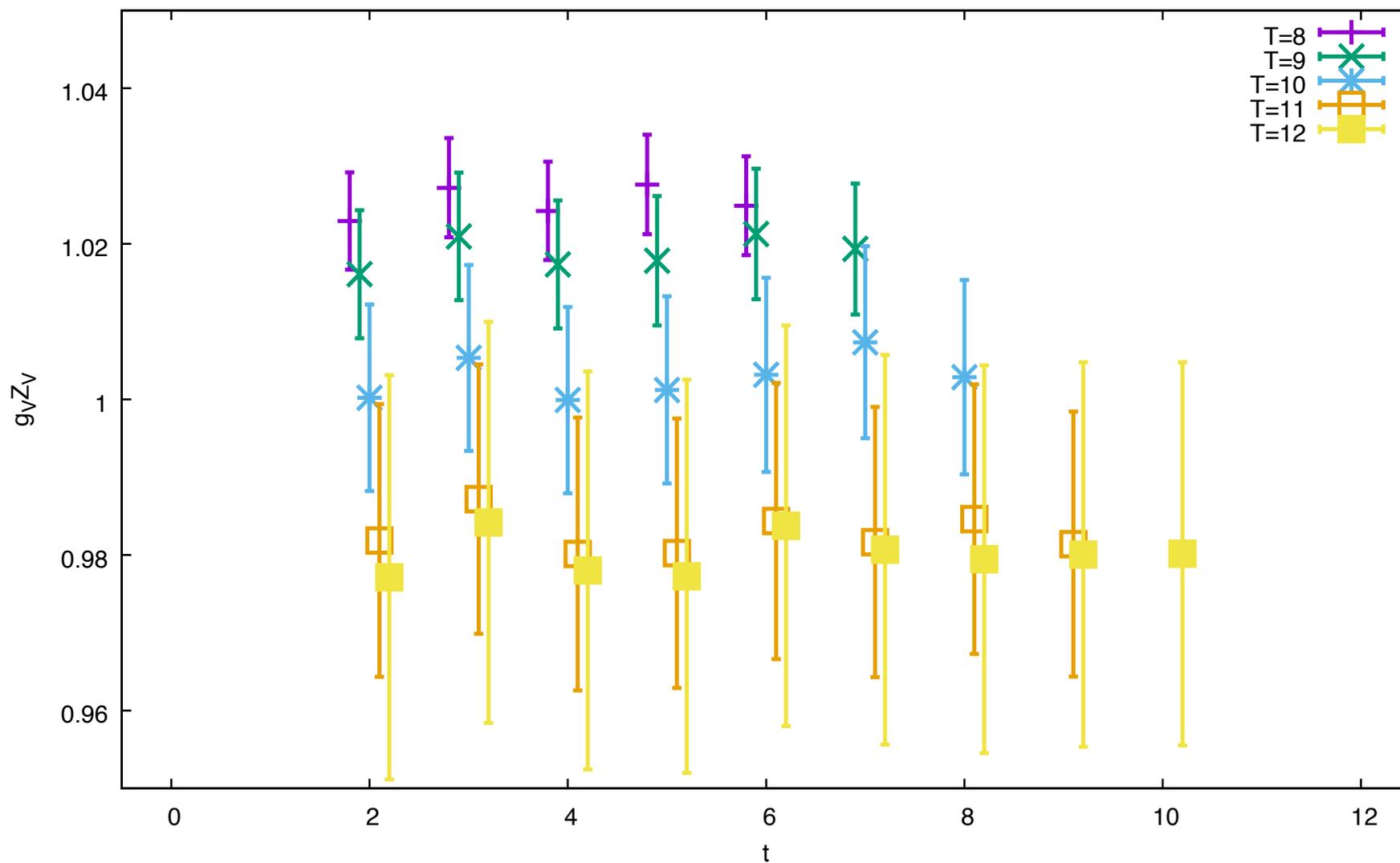
with similar Gaussian smearing as in earlier RBC studies.

Nucleon mass: our mass estimate is $m_N = 947(6)$ MeV from 7–13.



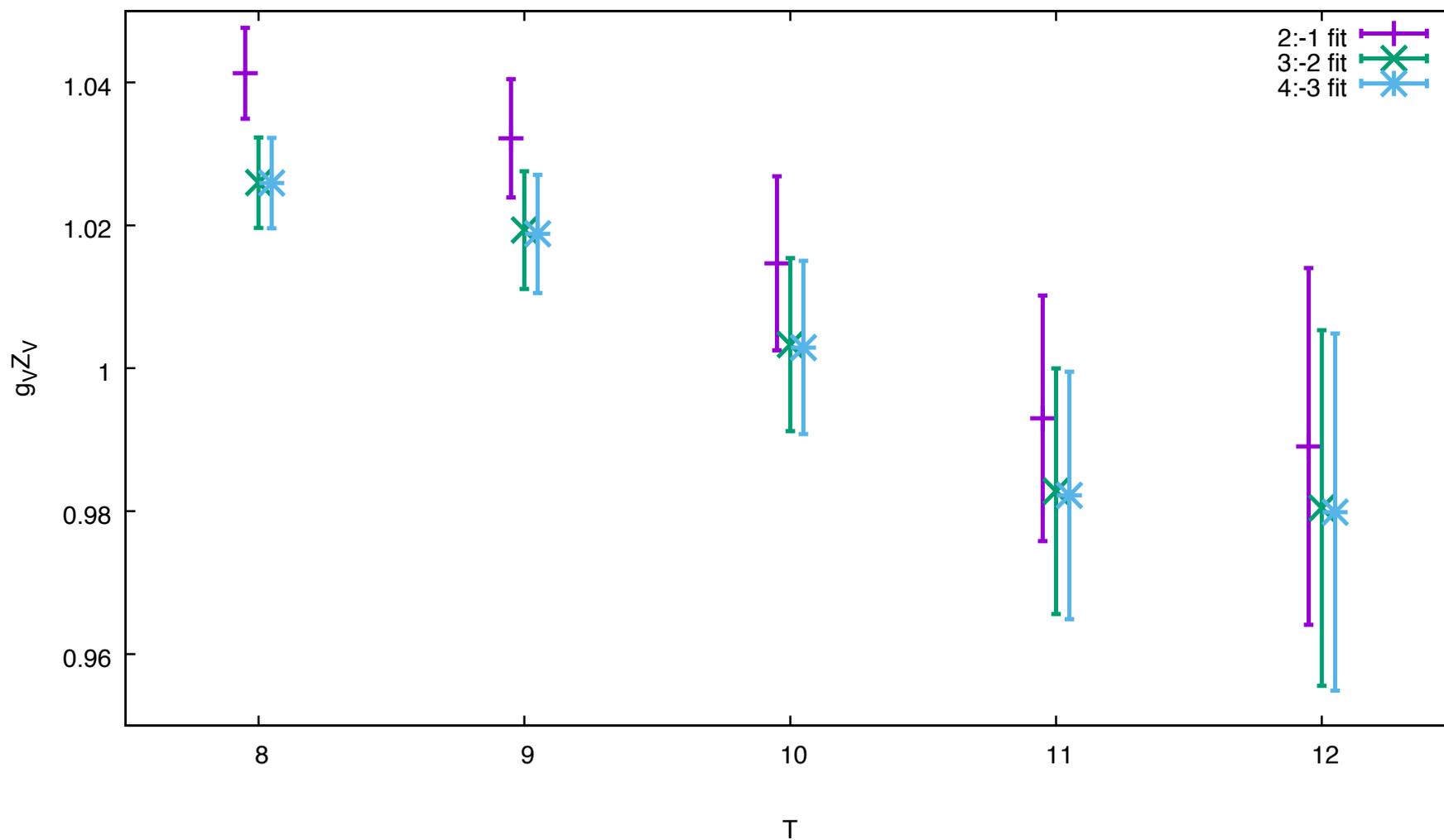
We set source-sink separations of $T = 8, 9, 10, 11$, and 12 lattice units, or (0.9-1.4) fm.

Isvector vector charge, g_V , renormalized with meson-sector $Z_V^{\text{meson}} = 0.71076(25)$:



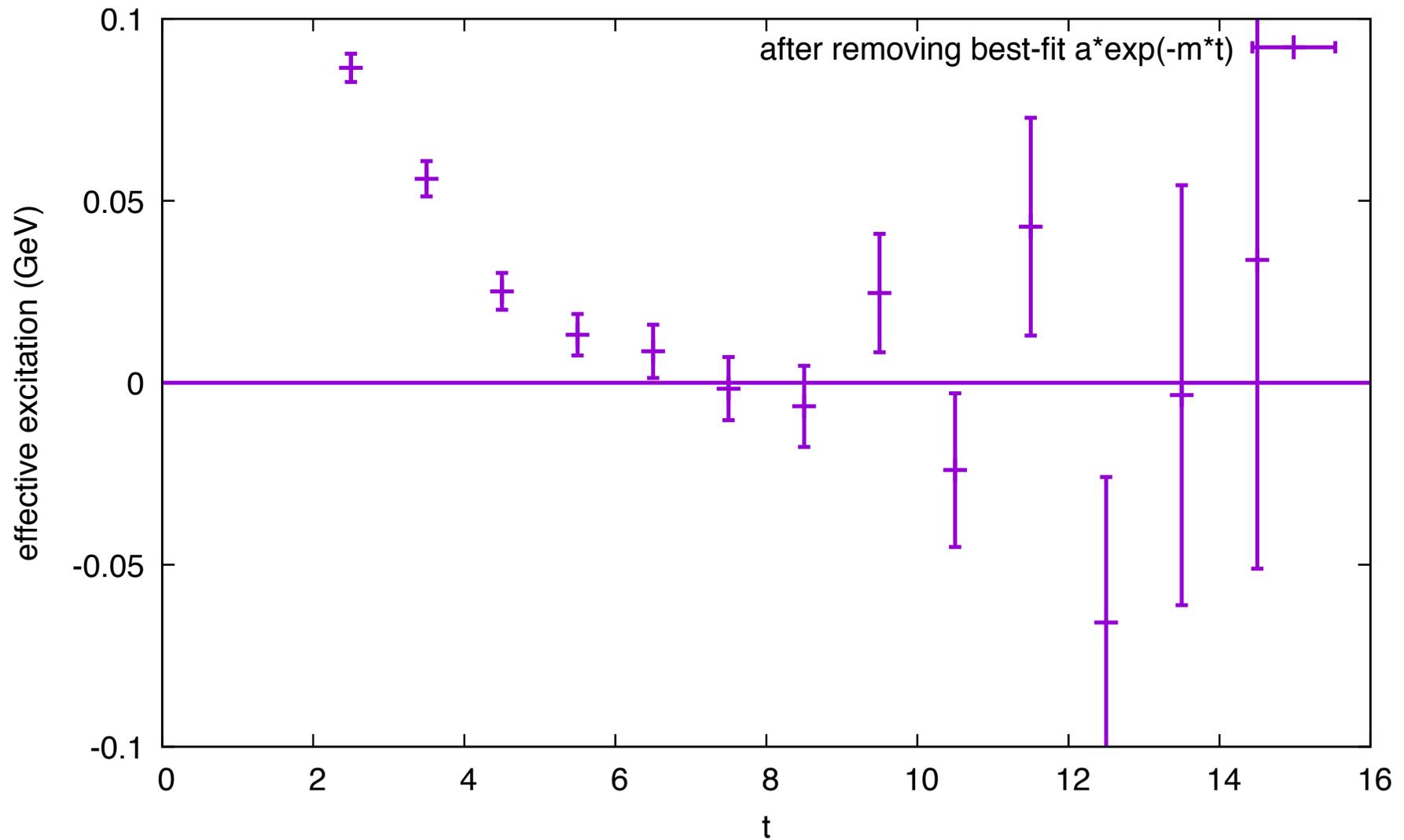
Sub-percent statistical accuracy exposes $O(a^2)$ systematics, at a couple of percent, as expected.

Isovector vector charge, g_V , renormalized with meson-sector $Z_V^{\text{meson}} = 0.71076(25)$:



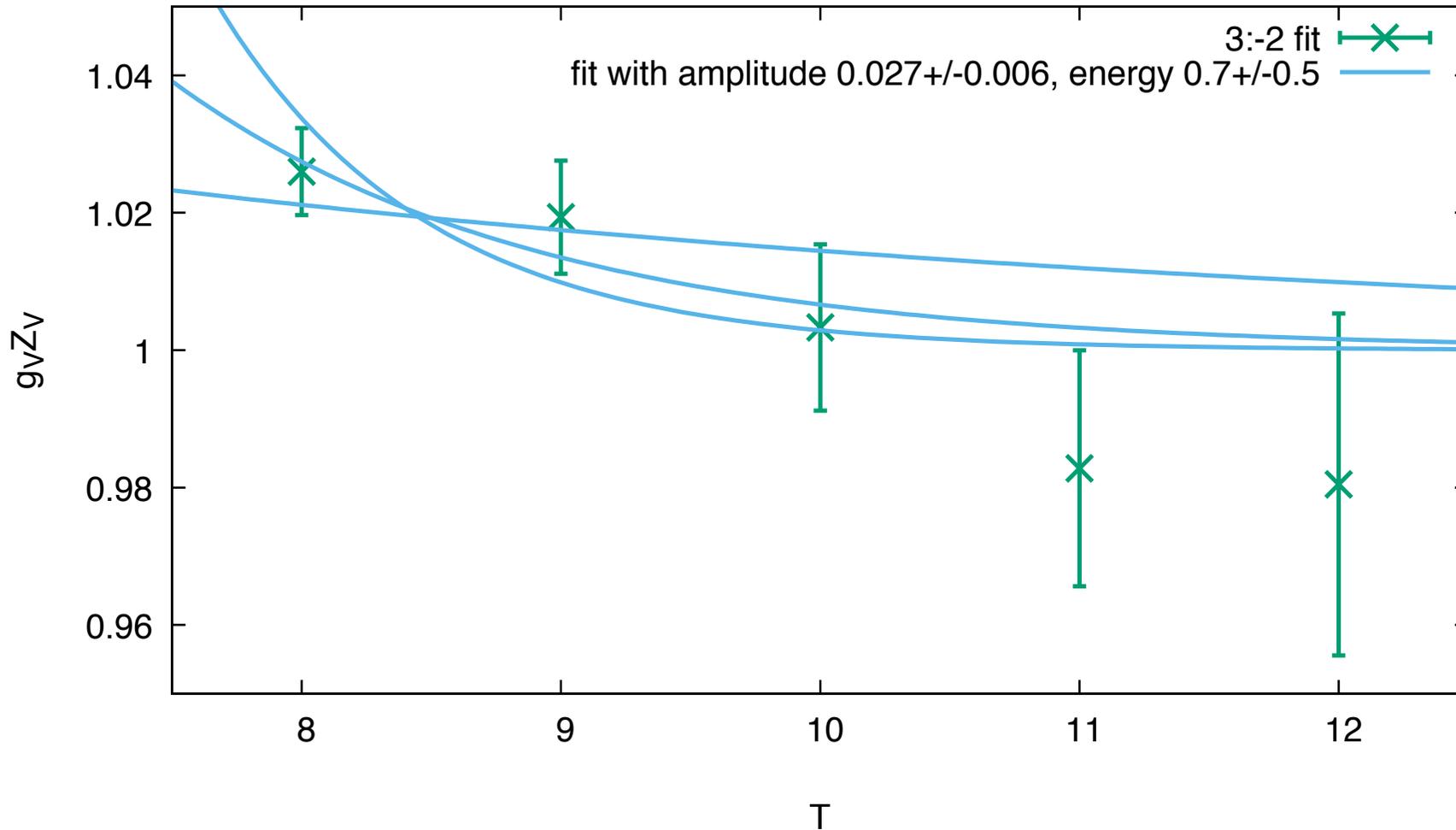
We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.

We may be losing the signal at as early as $T = 10$ or 1.1 fm: “effective” excitation,



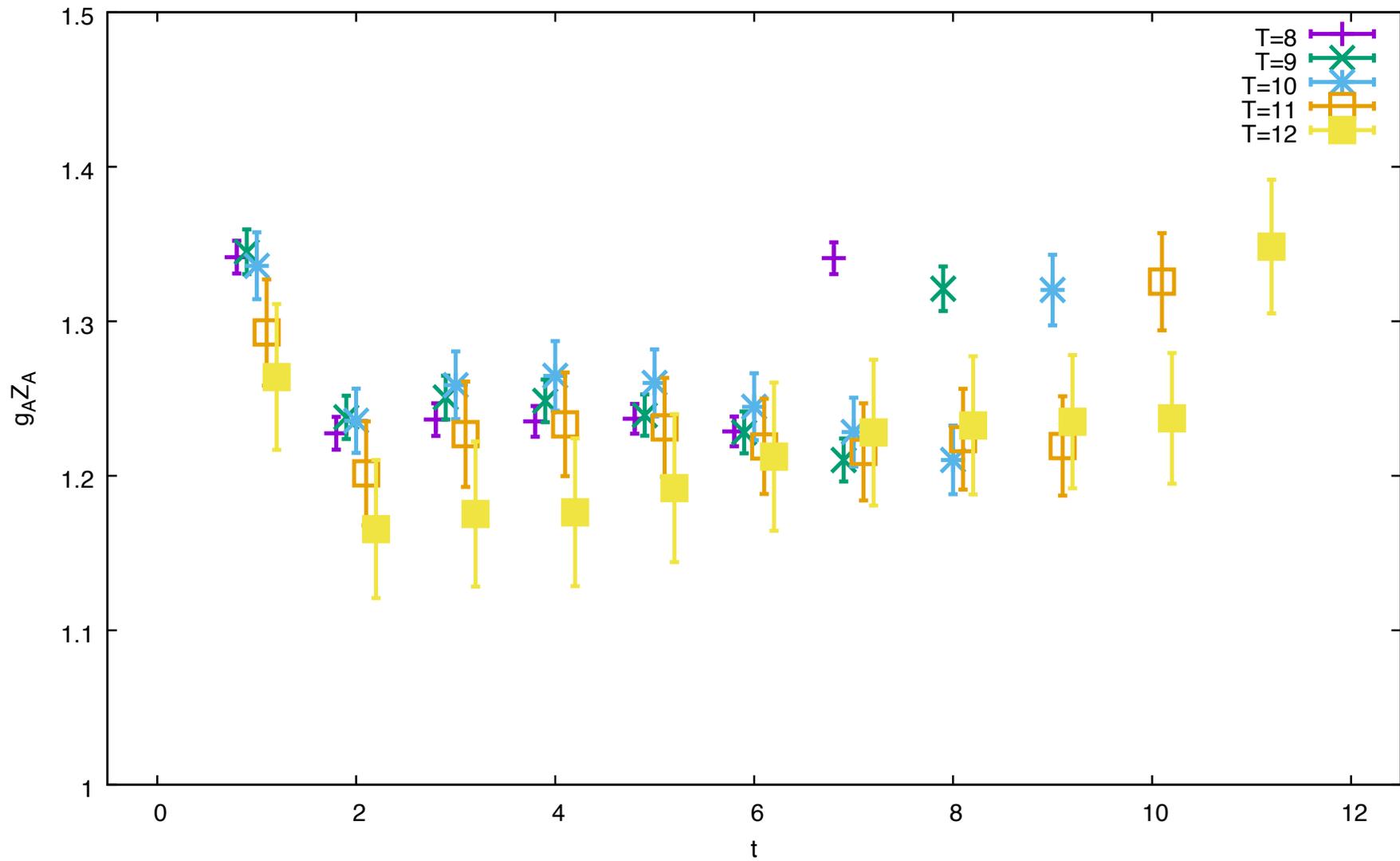
after removing the best-fit $(6.3(2) \times 10^{-9} \exp(-0.547(3)t)$.

Isovector vector charge, g_V , at $T = 8$ and 9, deviates from unity: possibly $O(a^2)$ mixing with excited states,

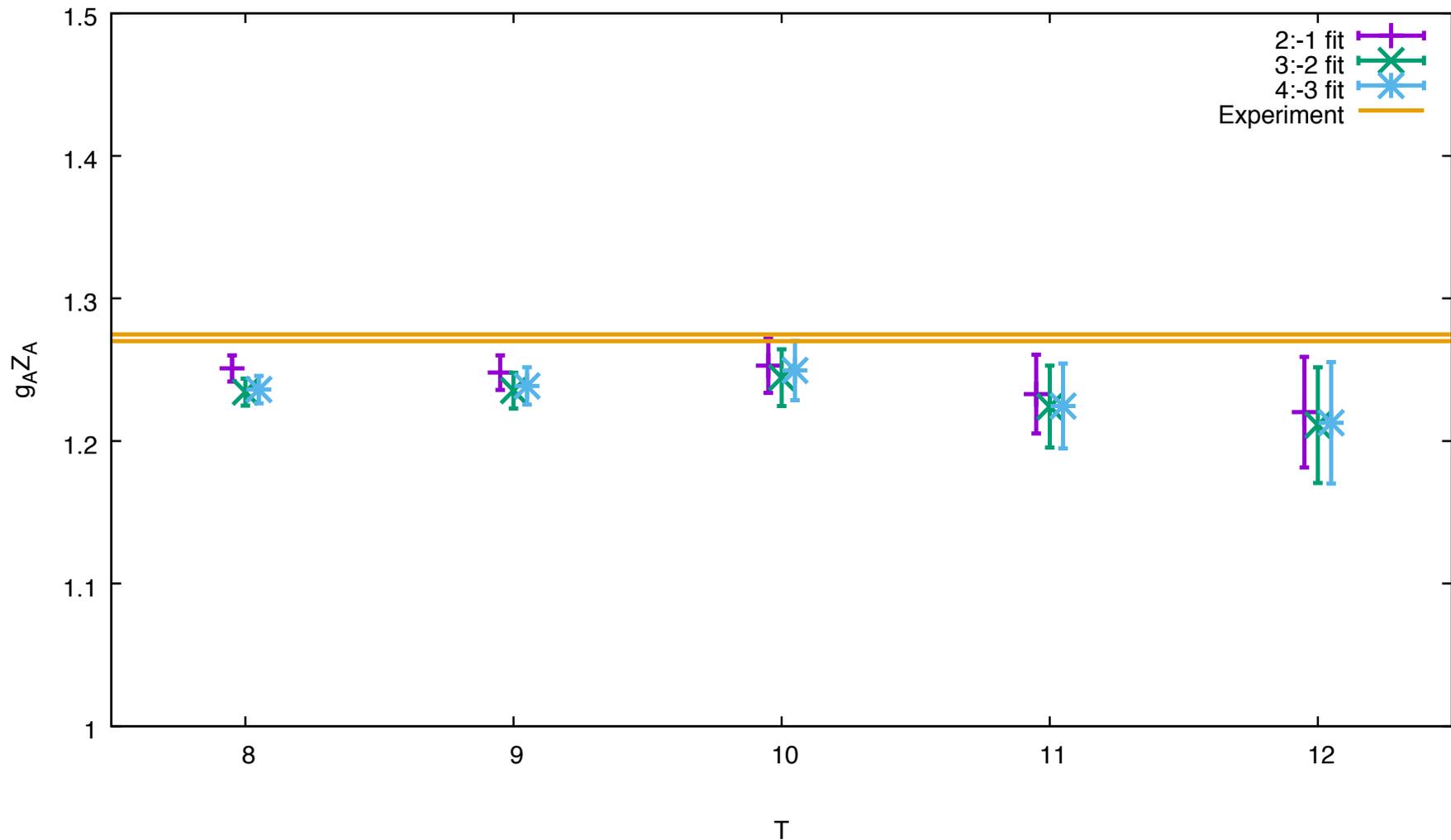


single-excitation fit is not so precise: we need shorter $T = 7$ and 6 calculations for further investigation.

Isovector axialvector charge, g_A , renormalized with meson-sector $Z_A^{\text{meson}} = 0.71191(5)$:

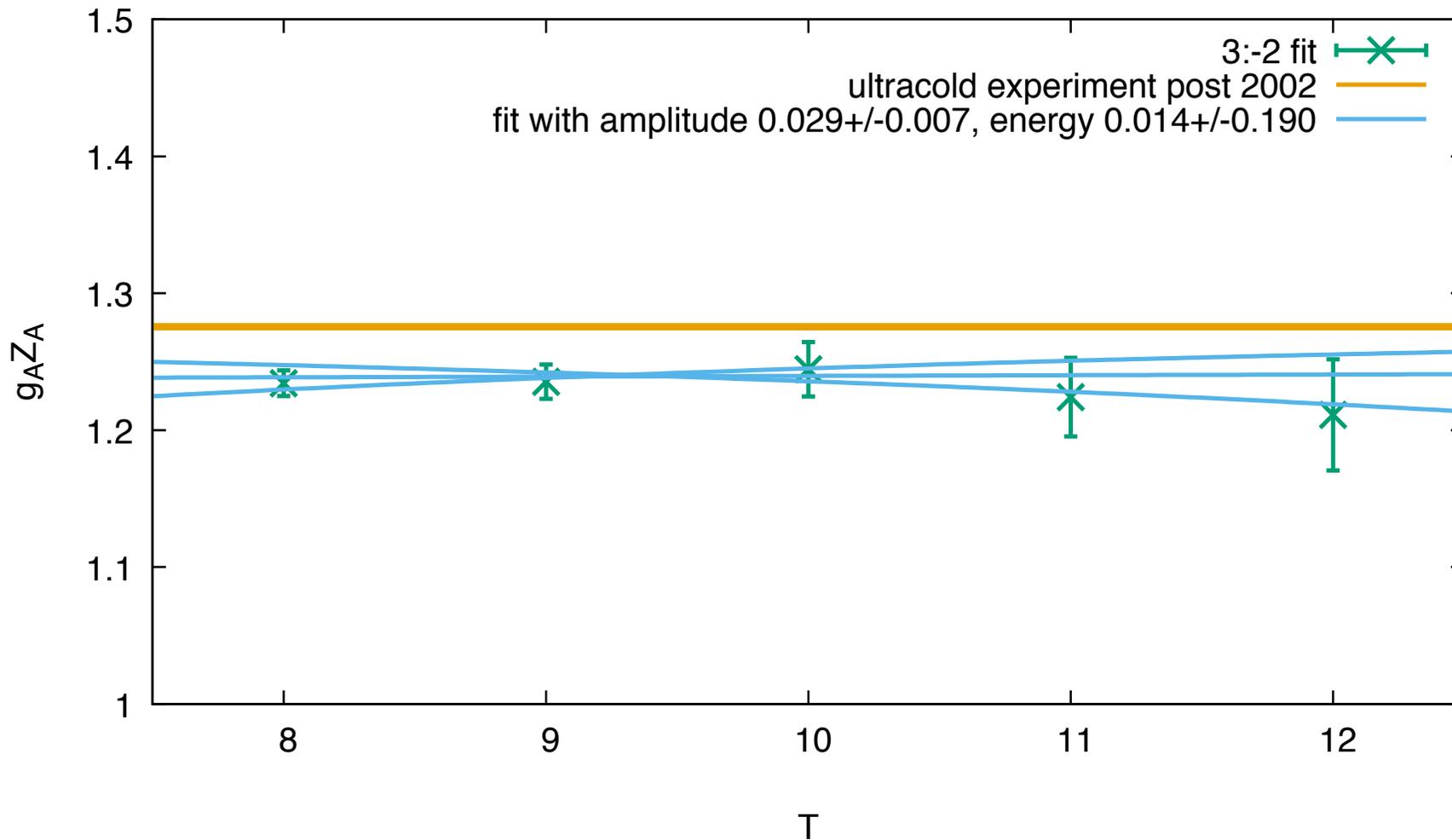


Isvector axialvector charge, g_A , renormalized with meson-sector $Z_A^{\text{meson}} = 0.71191(5)$:



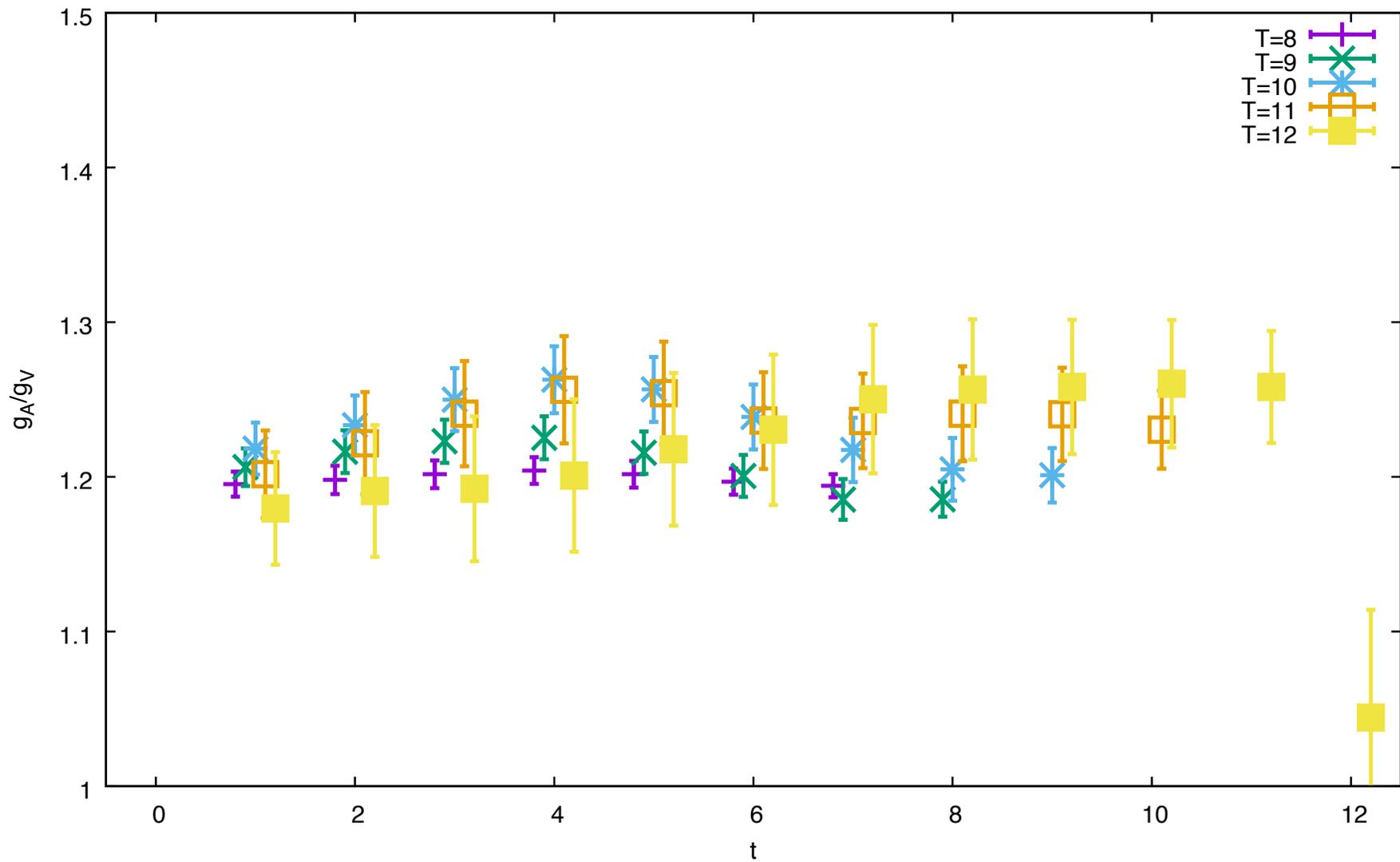
Undershoots the experiment by a few times the statistical error without dependence on source-sink separation, T . Percent-level statistical accuracy, but not quite in agreement with g_A/g_V in the following either.

Isovector axialvector charge, g_A , renormalized with Z_A^{meson} , undershoots the experiment by a few percent.



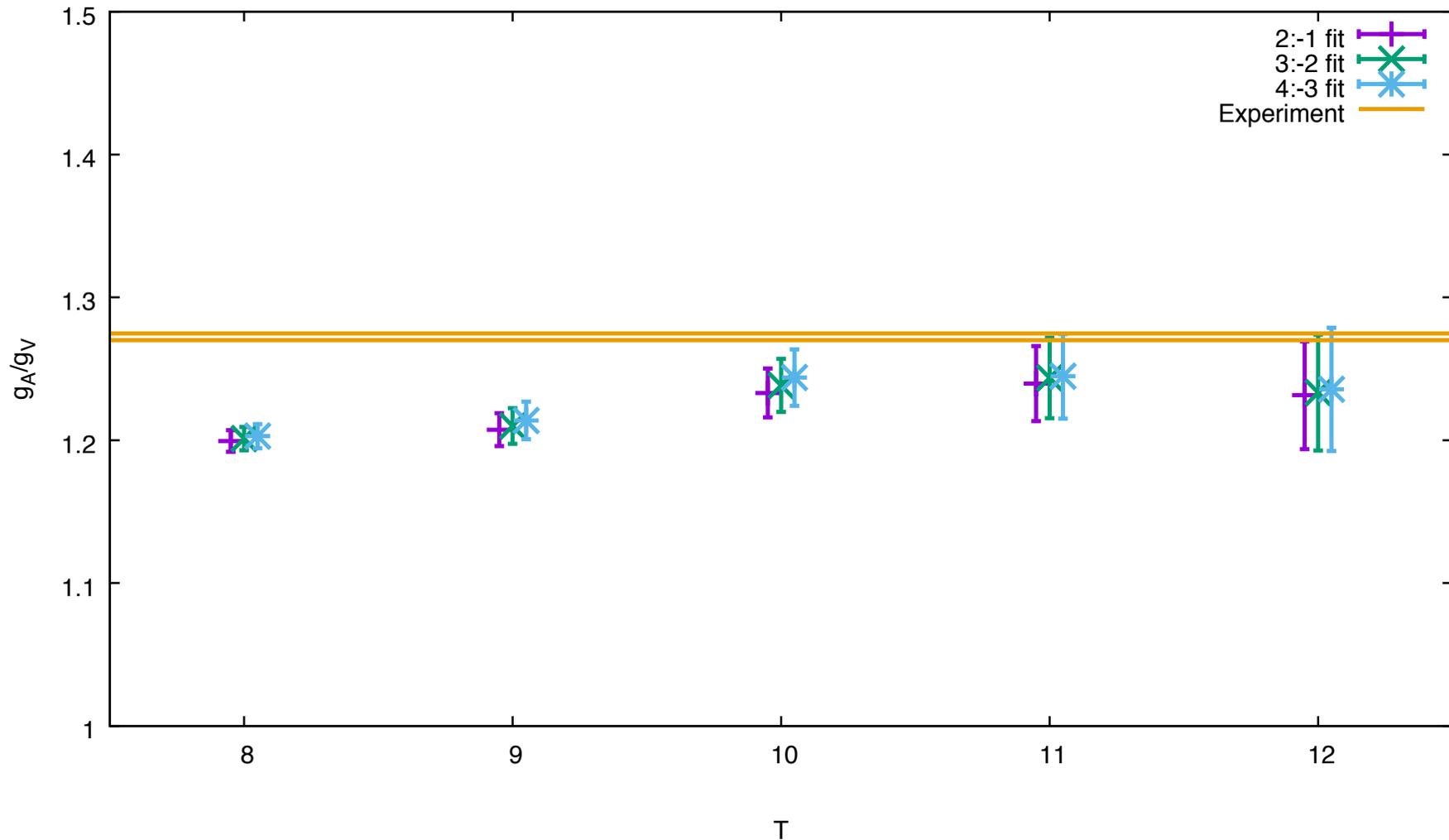
Excitation consistent with 0: this deficit appears independent of excited state contamination.

Isovector axialvector to vector charge ratio, g_A/g_V :



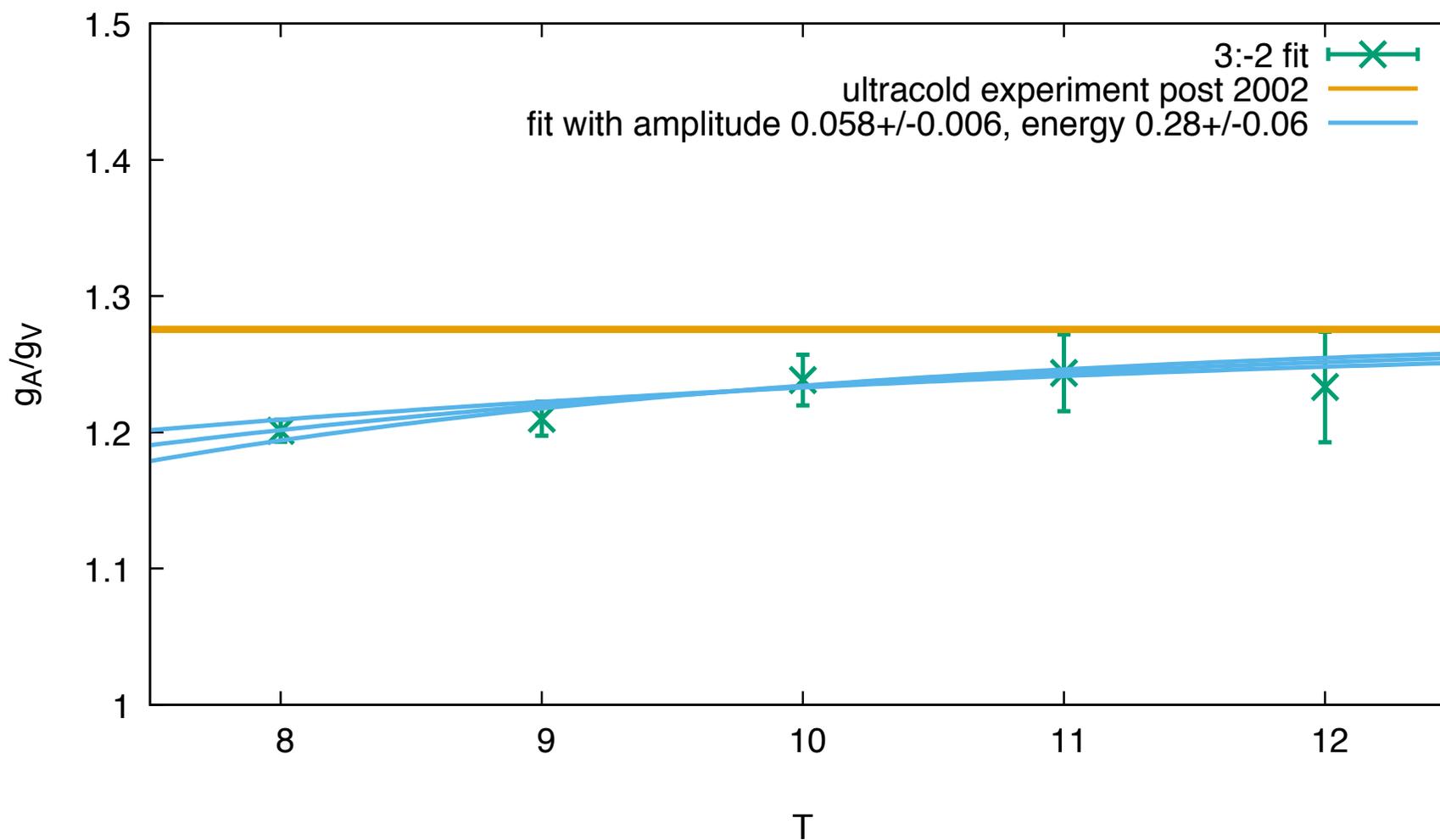
Percent-level statistical accuracy, again.

Isovector axialvector to vector charge ratio, g_A/g_V :



Undershoots the experiment by several times the statistical error, so rather different from $g_A Z_A$ in the above. We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-10 slope appears steeper than 8-9.

Isovector axialvector to vector charge ratio, g_A/g_V , undershoots the experiment by several percent.



On top of the g_V T -dependence, a better-precision.

Validation of lattice QCD:

As of Lattice 2017, with similar quark mass and lattice cuts off, and small volumes,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: $\sim 1.2^{25}$,
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
 - PACS, $1.16(8)^{26}$,
 - QCDSF $\sim 1.1^{27}$,
- and even a Wilson valence on HISQ, PNDME²⁸, ~ 1.2 ,
- except the then latest DWF valence²⁹ on HISQ staggered ensembles after an extrapolation.

g_A from different actions “blindedly” agree with deficit once $O(a)$ systematics is removed,

This stayed true, as of Lattice 2018, for raw data with similar quark mass, cuts off, and small volumes,

- only the values are now more refined with better statistical errors,
- and clustering around ~ 1.2 , but **up to $O(a^2)$ systematics**,
 - including DWF-valence/HISQ raw data, which agree with Wilson-valence/HISQ raw,
 - but except the newest, low-statistics Wilson-fermion “PACS10.”

²⁵J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].

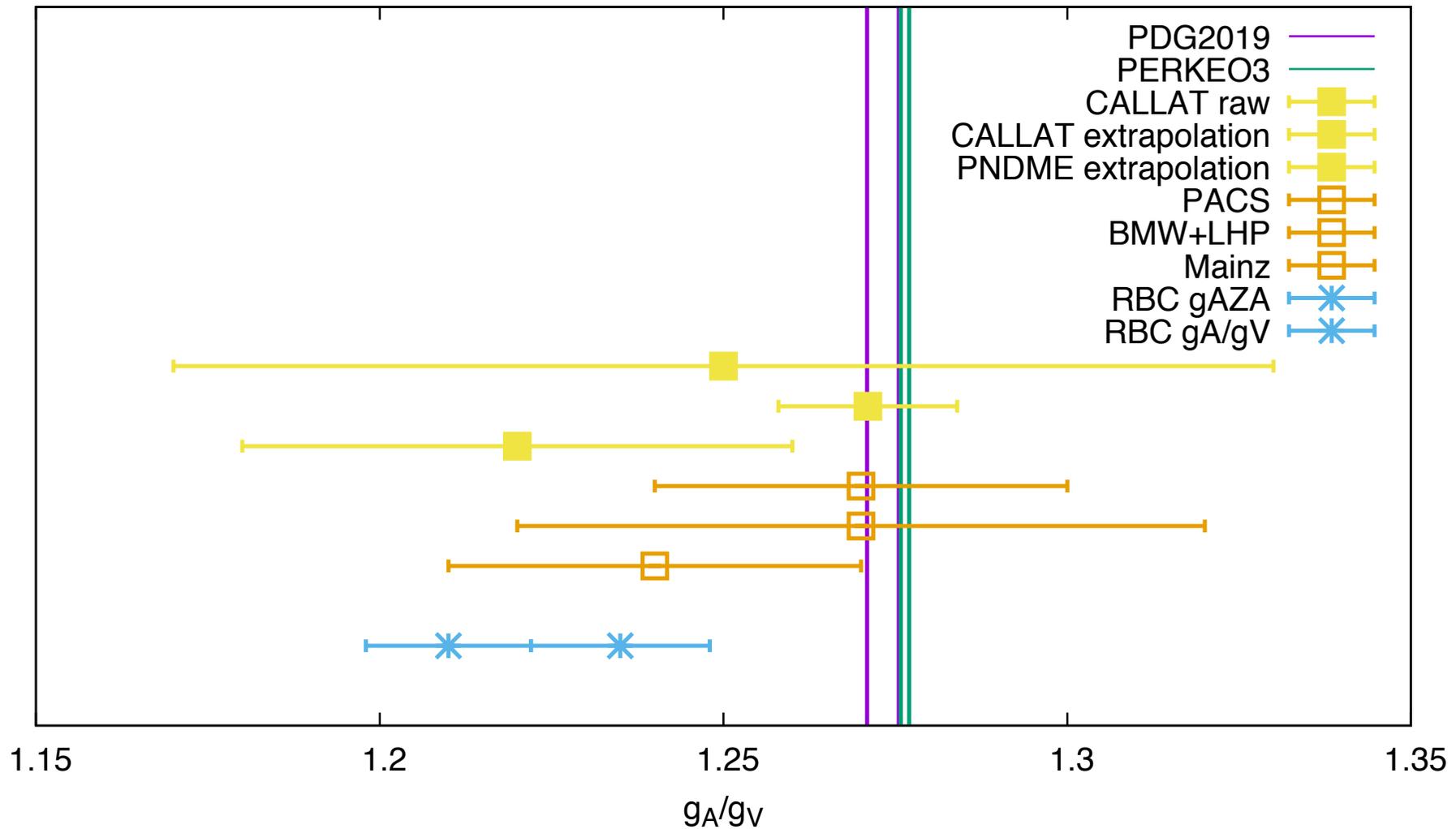
²⁶A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].

²⁷J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].

²⁸T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].

²⁹E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

As of the first half of June, 2019, before Lattice 2019,

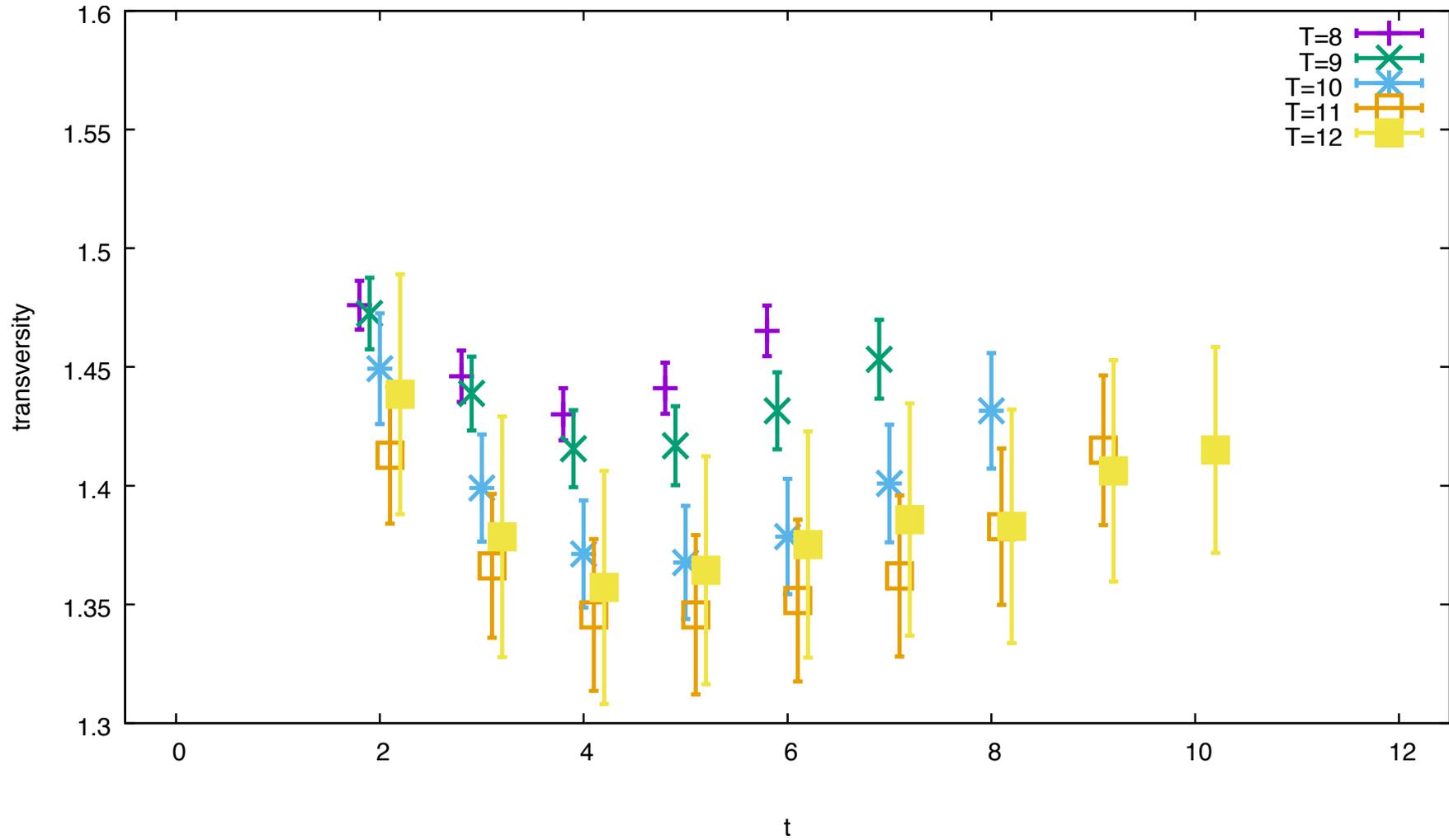


Volumes are mostly small, some physical mass, some extrapolations, some errors are large, ...

... 獨酌無相親 舉杯邀明月 ... 醉後各分散 永結無情遊 相期遙雲漢³⁰.

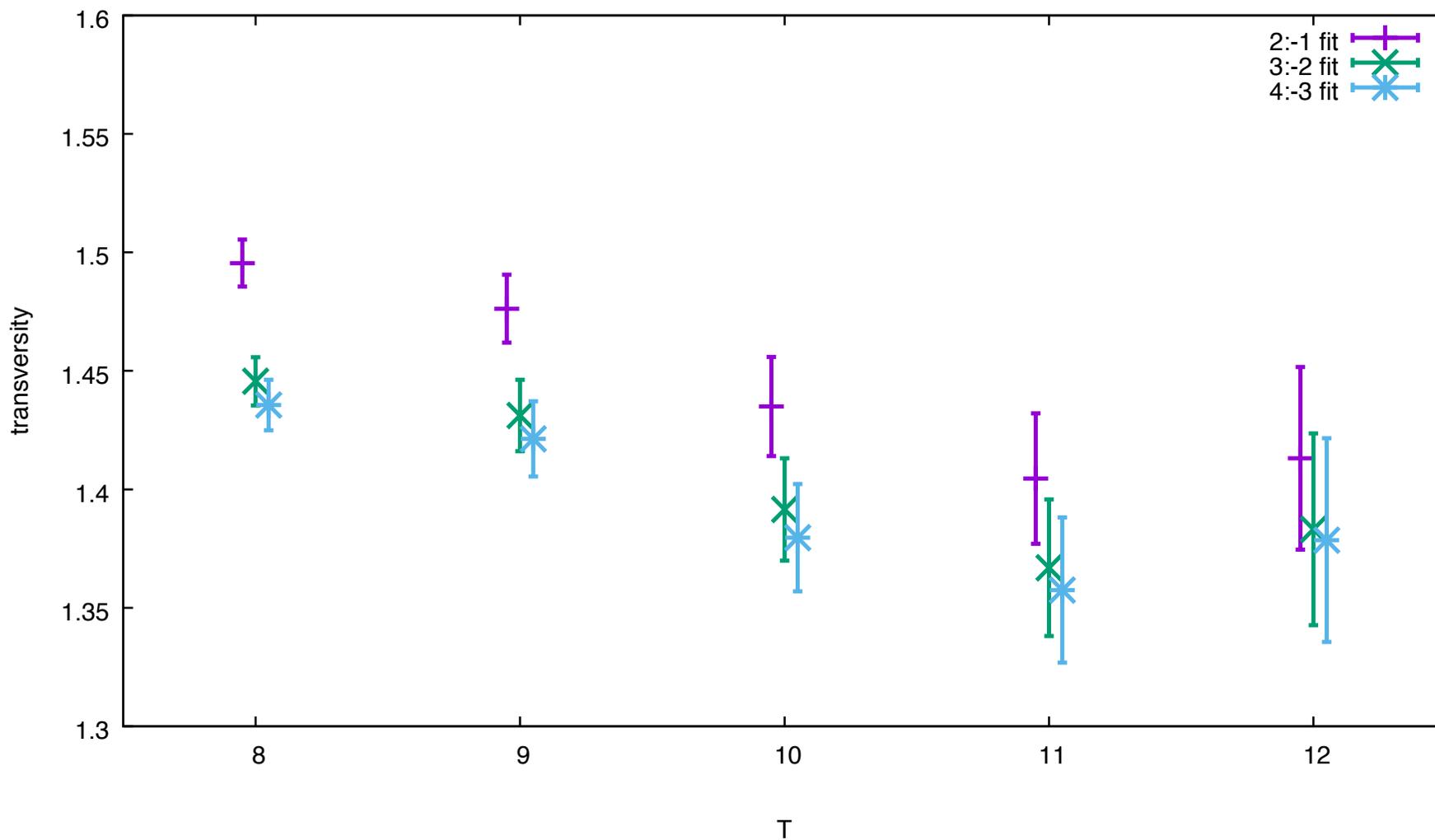
³⁰from 李白's 月下独酌; who also sang 黃鶴樓送孟浩然之廣陵“故人西辭黃鶴樓 烟花三月下揚州 孤帆遠影碧空盡 唯見長江天際流”

Isvector transversity, bare:



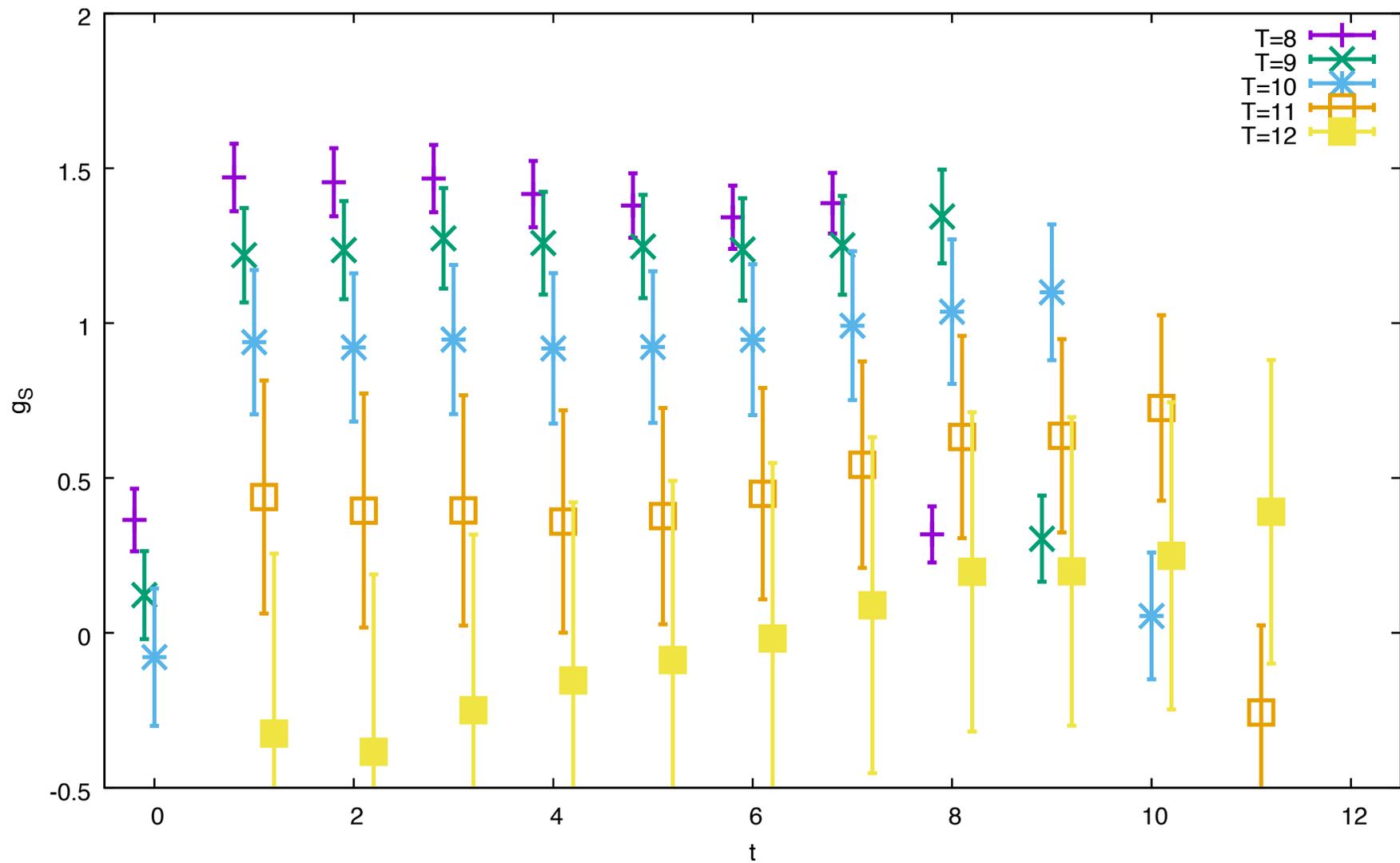
Clear dependence on source-sink separation, T .

Isvector transversity, bare:



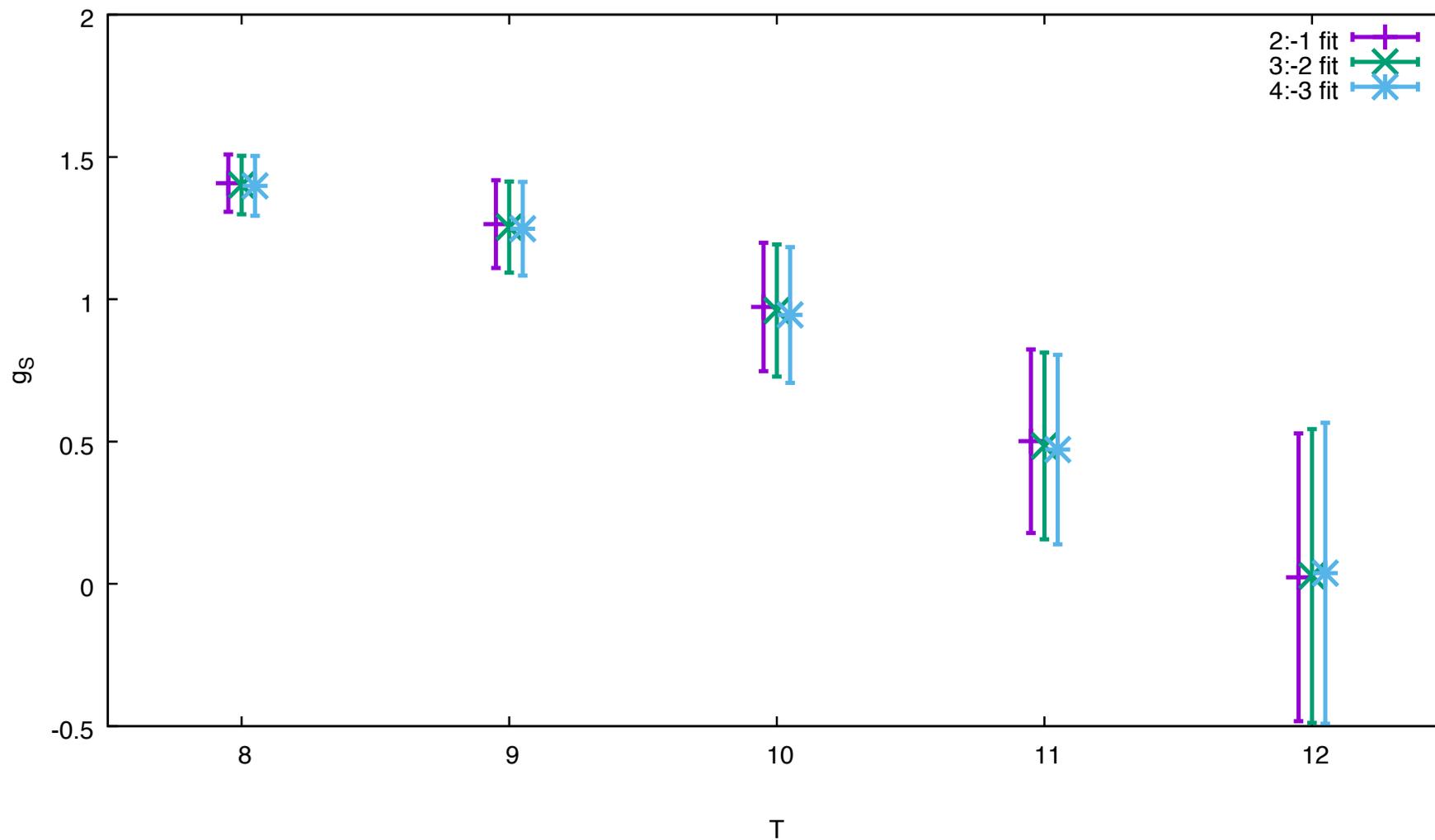
We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.
We are yet to work out the renormalization, Z_T .

Isvector scalar “charge,” g_S , bare:



Clear dependence on source-sink separation, T .

Isovector scalar “charge,” g_S , bare:



We know the renormalization, $Z_S = 1/Z_m$.

We may be losing the signal at as early as $T = 10$ or 1.1 fm: 9-11 slope appears steeper than 8-9.

Nucleon “charges” from RBC+UKQCD 2+1-flavor dynamical DWF ensemble at physical mass, 48I: $a^{-1} = 1.730(4)$ GeV, 130 configurations, 2000 eigenvalues, 256/4 AMA samples each, $T = 8, 9, 10, 11, 12$.

Nucleon mass: 947(6) MeV.

Vector charge: sub-percent statistical accuracy,

- expected $O(a^2)$ systematics can be fit by single-excitation,
- excitation energy consistent with $m_\pi + 2\pi/La$ or $2m_\pi$ though poorly determined,
- $T = 8$ and 9 alone give excitation energy of $\sim 0.3a^{-1}$.

Axial charge: around-a-percent statistical accuracy,

- $g_A Z_A$ and g_A/g_V both undershoot g_A^{exp} but do not agree with each other.
- Systematics is yet to be understood.

Signals in transversity and scalar “charge” with dependence on source-sink separation, T , are seen.

We may be losing the signals as early as $T = 10$, or 1.1 fm:

- steeper slopes at later T ,
- so we are yet to understand $O(a^2)$ or excited-state systematics.

Shorter T such as 7 and 6 would help, as well as another coupling such as in a finer 64I or coarser IDs.