The hadronic vacuum polarization of the muon from four-flavor lattice QCD

Fermilab Lattice, HPQCD, and MILC Collaborations

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Methodology

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K_E(Q^2)\, \widehat{\Pi}(Q^2)$$

$$\widehat{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$$

Taylor coefficients

$$\widehat{\Pi}(\omega^2) = \frac{4\pi^2}{\omega^2}\int_0^\infty dt\, G(t)\left[\omega^2 t^2 - 4\sin^2\left(\frac{\omega t}{2}\right)\right]$$

$$G_{2n} \equiv a^4 \sum_t \sum_{\mathbf{x}} \sum_f t^{2n} Z_V^2 q_f^2 \langle j_i^f(\mathbf{x},t) j_i^f(0)\rangle$$

$$G(t) = \frac{1}{3}\int d\mathbf{x} \sum_f q_f^2\, Z_V^2 \langle j_i^f(\mathbf{x},t) j_i^f(0,0)\rangle$$

$$\widehat{\Pi}(Q^2) = \sum_{j=1}^{\infty} Q^{2j}\Pi_j$$

- Calculate $G_{2n} \rightarrow \Pi_{n-1}$
- Use Padé to extend $\widehat{\Pi}(Q^2)$ to small Q^2
- Integrate to get $a_\mu^{\mathrm{HVP,LO}}$

Lattice parameters

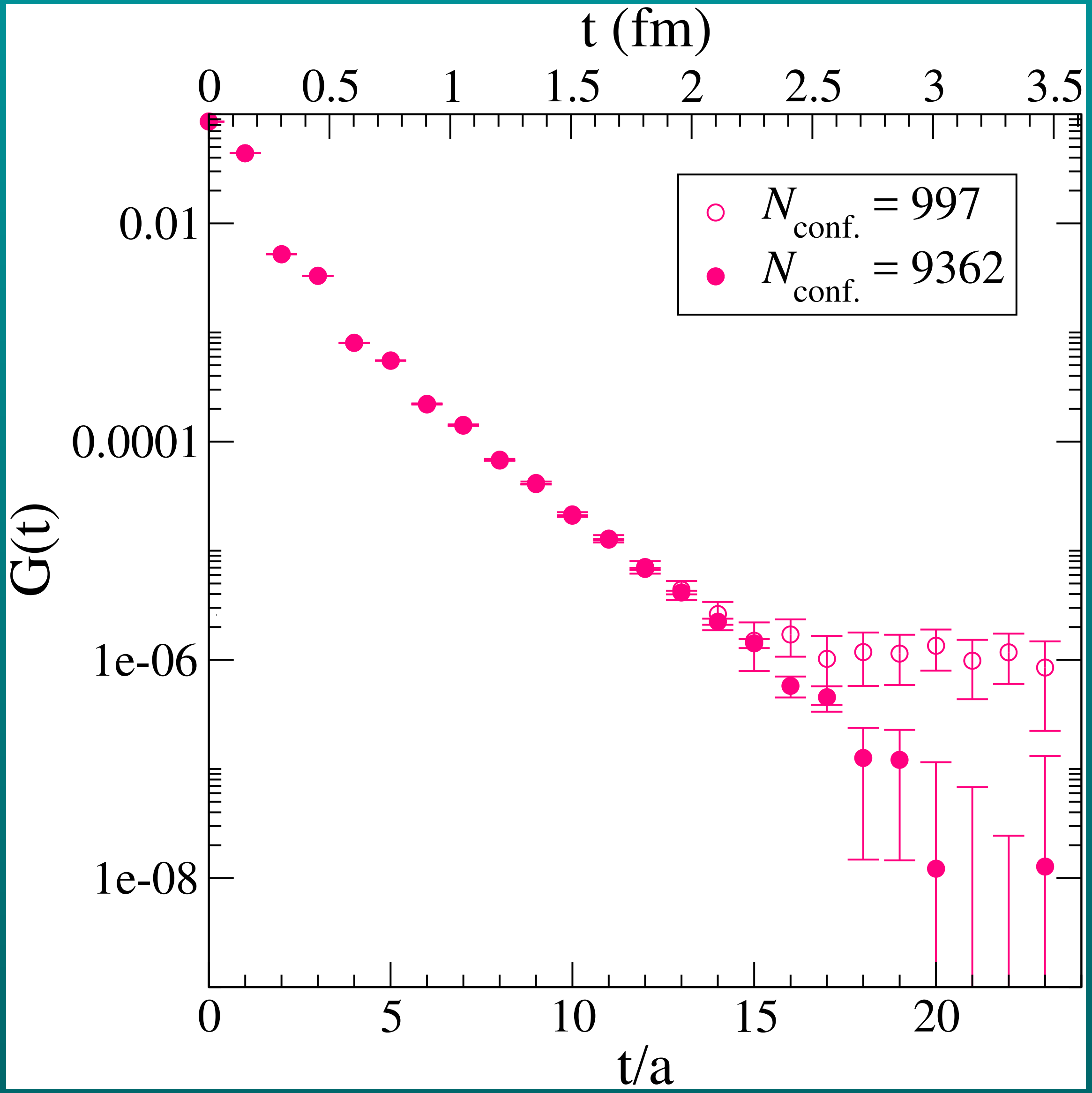
- Ensembles
- 2+1+1 sea

$\approx a$(fm)	M_π(MeV)	size	$N_{\rm config}$
0.15	133	32^3x48	997
0.15	135	32^3x48	9362
0.12	133	48^3x64	998
0.09	128	64^3x96	1557
0.06	134	96^3x192	1230

- Valence: here only $m_u = m_d = m_\ell$ so only $a_\mu^{\ell\ell}$
- Lattice spacing a/w_0 is known to 5 per mille.

Correlator example

- 0.15 fm analysis
- Higher statistics has obviously improved the signal in the tail



Dealing with noisy data at large t

- Bounding method: 2 pion contribution for $t > t_C$

  [RBC/UKQCD, BMW, Mainz/CLS (improved)]

- Hybrid R: Replace with contribution from

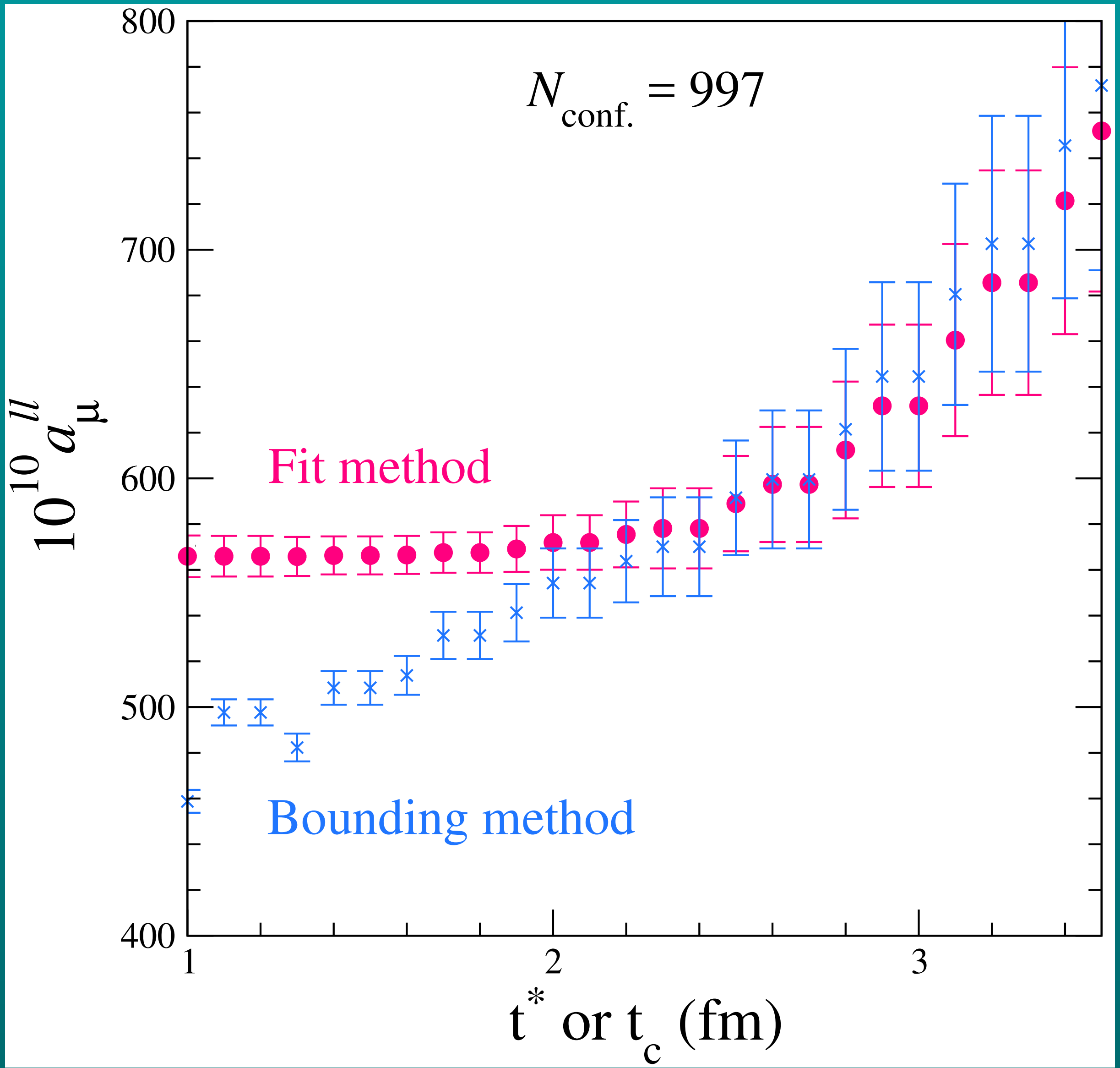
  $$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

  [RBC/UKQCD]

- (Here) Fit method: Fit low t data and replace high t > t* values with an extrapolation

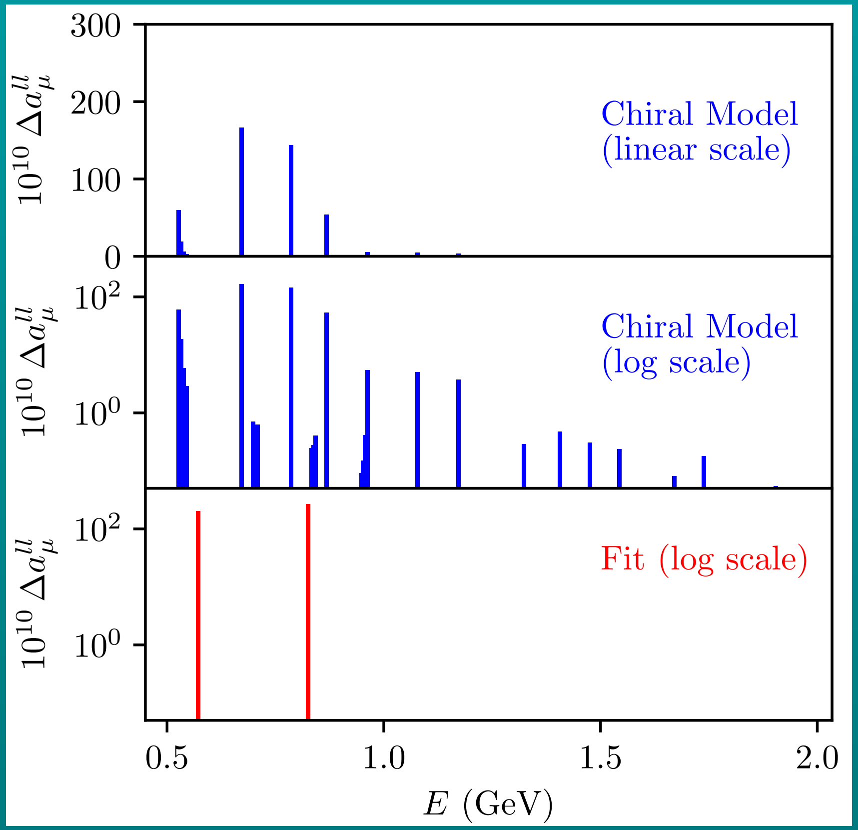
Compare bounding and fit methods

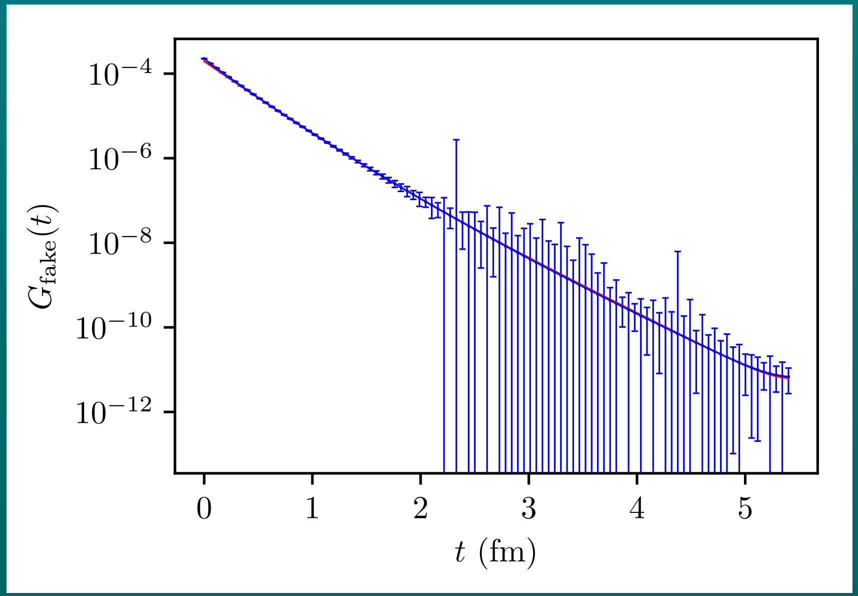
- Agree for t > 2.3 fm
- Fit method stable for 1 < t* < 2 fm



Synthetic data test of fit method

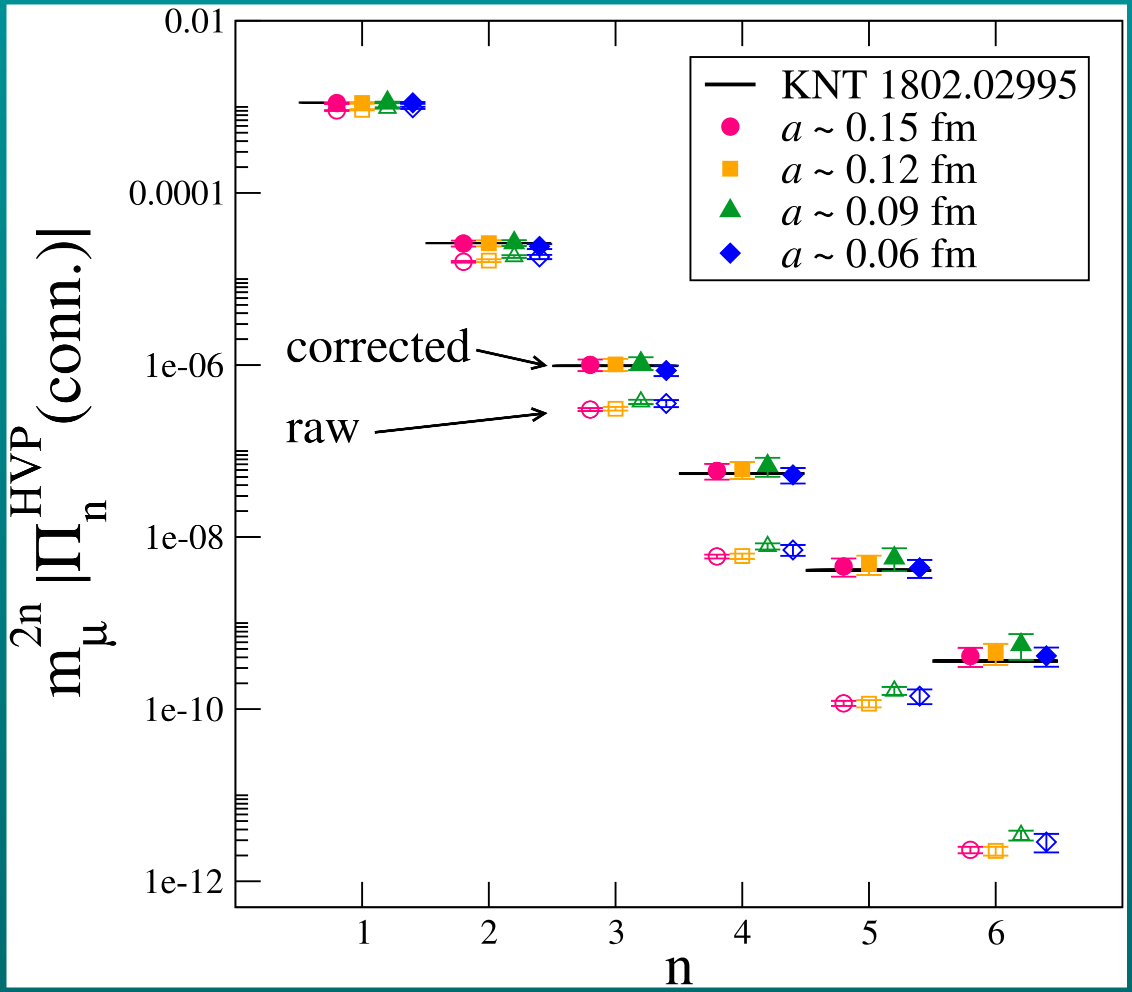
- Generate synthetic data set based on a chiral model with rho plus staggered 2 pion states
- Fit with only two states (red curve) agrees perfectly with model (blue curve)





Corrections

- Use staggered chiral model to correct for
  - Finite volume effects
  - Discretization due to taste-splitting
  - Quark mass tuning
- Subtract lattice $\pi - \pi$ contributions and replace with physical $\pi^0 - \pi^0$ contributions.
- Nice agreement with phenomenological R method (black lines)



n^{th} Taylor coefficient

Light-light extrapolation

$$a_\mu^{ll}(\text{latt.}) = a_\mu^{ll}(\text{conn.})\left(1 + c_s \sum_{f=l,l,s,c} \frac{\delta m_f}{\Lambda} + c_{a^2}\frac{(a\Lambda)^2}{\pi^2}\right)$$

- $\Lambda = 500$ MeV

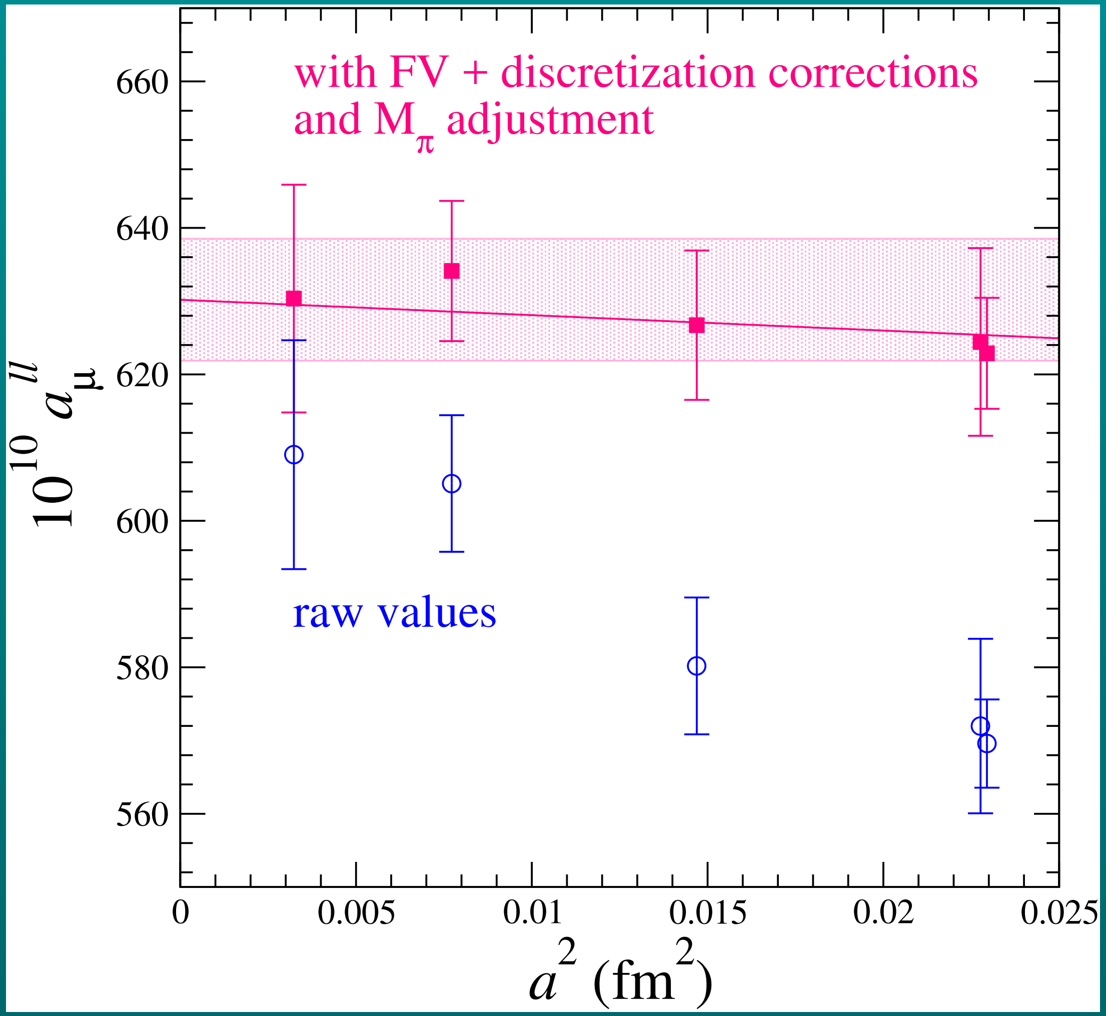
- Priors

  $c_s = 0.0(3)$

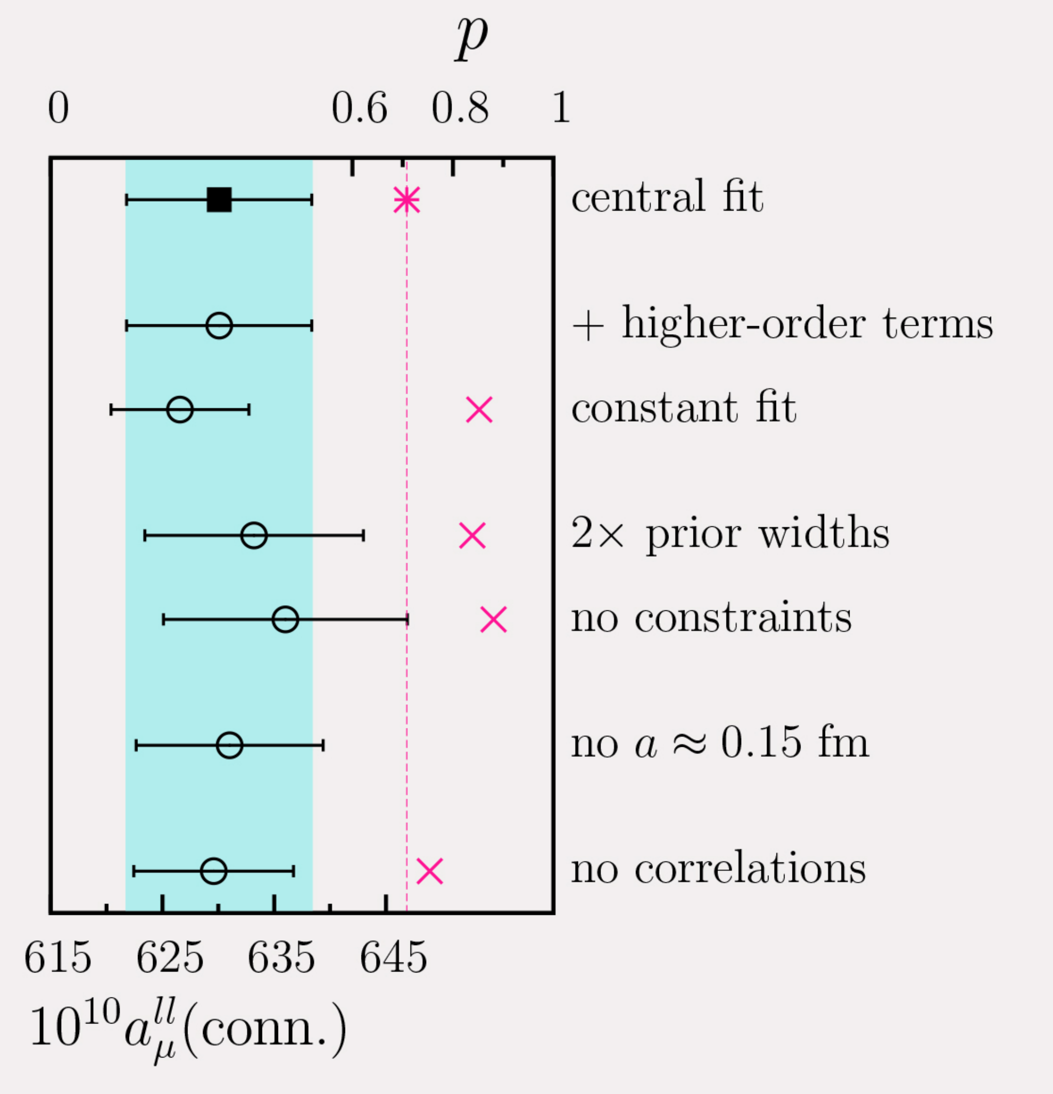
  $c_{a^2} = 0(1)$

- Result

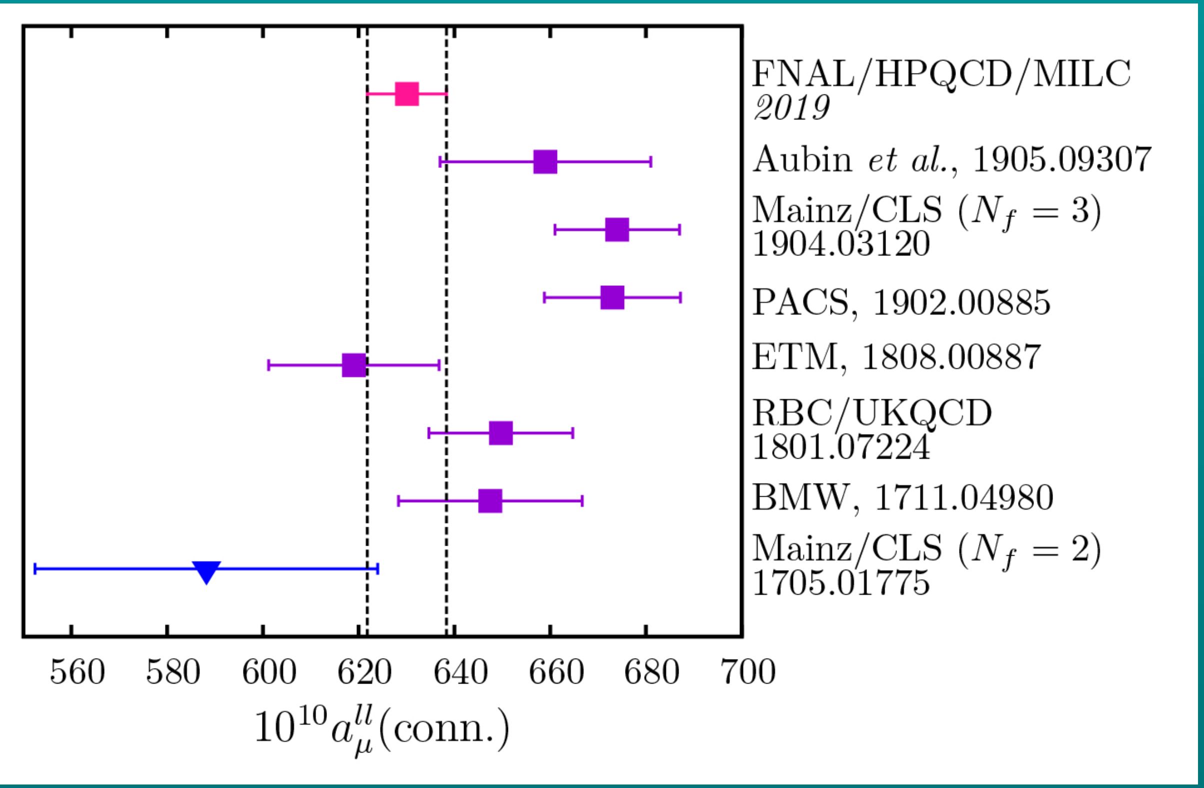
  $a_\mu^{\ell\ell}(\text{conn}) = 630.1(8.3)$



Stability of the continuum extrapolation



Comparison of light-quark-connected contribution

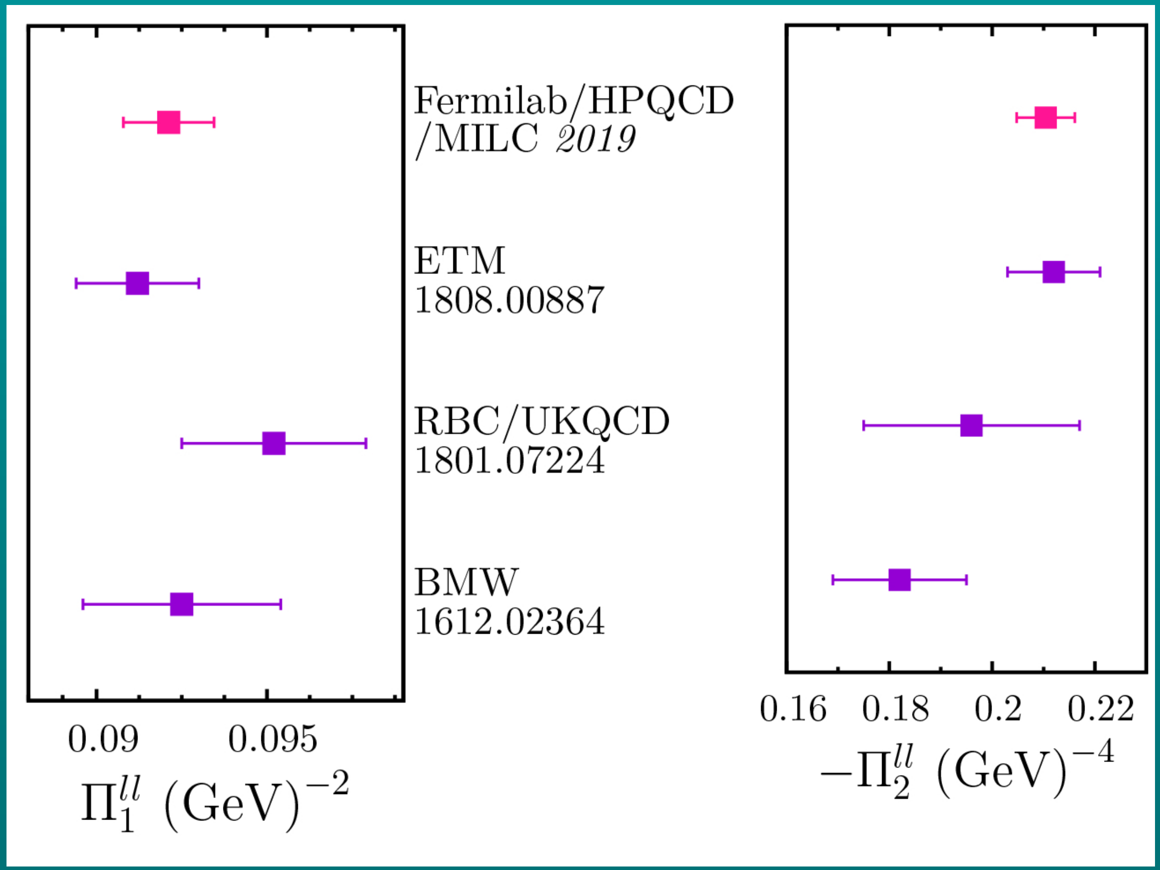


- Small error results from small error in fit-method extrapolation and finite-volume correction

Error budget - light-quark connected

Lattice spacing uncertainty (w_0)	0.8%
Monte Carlo statistics	0.7
Continuum extrapolation	0.7
Finite volume and discretization corrections	0.3
Current renormalization	0.1
Chiral interpolation	0.1
Strange sea quark mass adjustment	0.1
Pion mass uncertainty	0.0
Total	**1.3%**

Comparison of light-light Taylor coefficients



Adjusting to get the total u-d HVP contribution

All values $\times 10^{-10}$

$M(\pi^0) \to M(\pi^+)$	-4.3
$\pi - \pi$ disconnected	-7.9
Total $\pi - \pi$	-12(3)
$\rho - \omega$ disconnected	-5(5)
Strong isospin breaking	10(10)
Electromagnetism	0(5)
Total adjustment	**-7(13)**

Net $a_\mu^{ud} = 623.1(8.3)(13.0)$

Including the s, c, b contributions

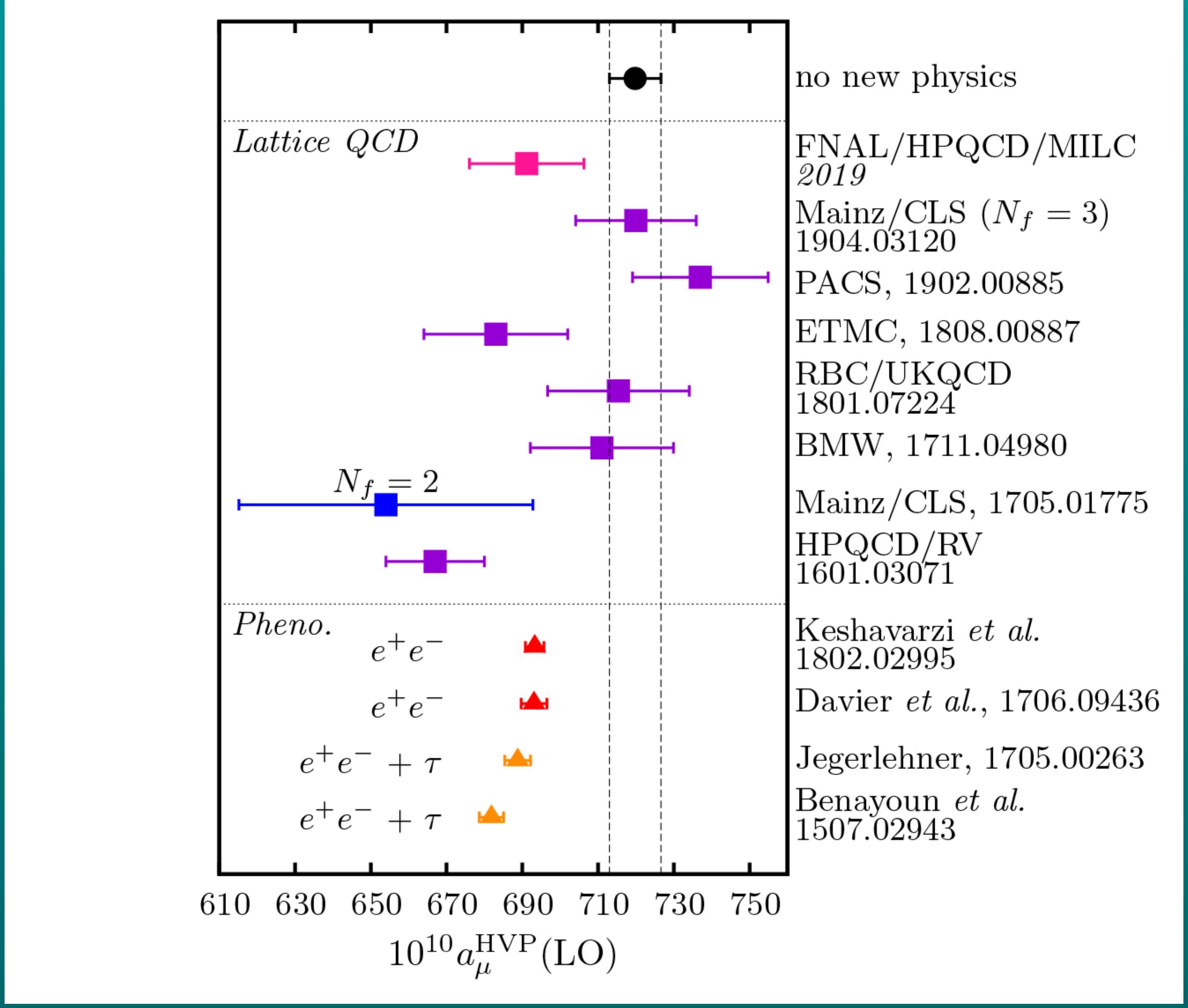
light	623.1(8.3)(13)
strange	53.40(60)
charm	14.40(40)
bottom	0.270(40)

arXiv:1403.1778
arXiv:1208.2855
arXiv:1408.5768

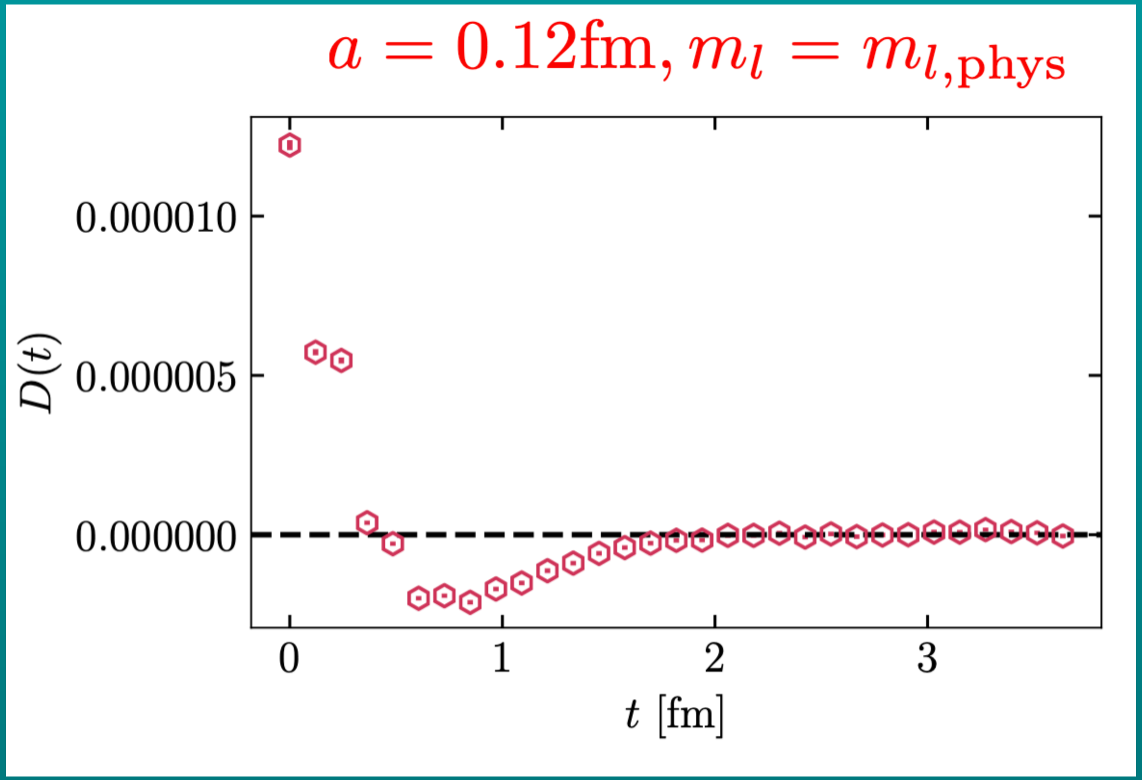
Dan Hatton Mon 4:30 PM

$$\text{Net } a_\mu^{\mathrm{HVP-LO}} = 691(15)$$

Total LO HVP result



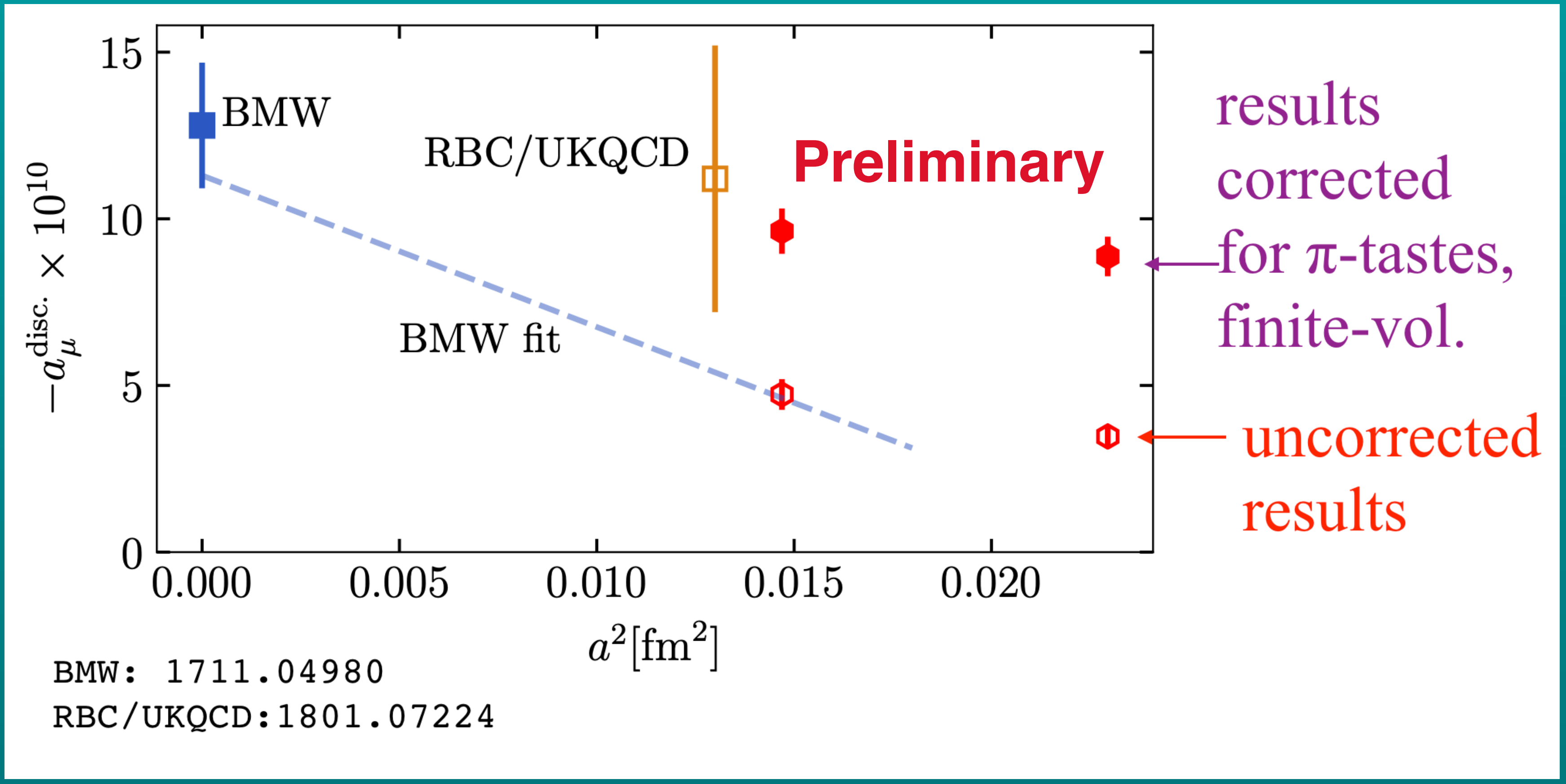
Progress with disconnected HVP



Fit to "ω-like" states minus "ρ-like" states

Replace data with fit for t > 2 fm
80% of result is from data.

Progress with disconnected HVP



See less α dependence than for BMW.
Results at 0.09 fm will allow an accurate continuum result.
We find isospin breaking effects in disconnected correlaters are relatively large.

Outlook

- Further reductions in the LO HVP require
  - More accurate lattice spacing determination
  - Better control of strong isospin breaking
  - Better control of electromagnetic effects
  - Better statistics -> better control of the continuum extrapolation
- Reaching 0.5% uncertainty is feasible over the next couple of years

Backup

Stability of fit method

- Stability against variations in the number of states and t_{min}
- a ~ 0.09 fm