Hyperon couplings from $N_f = 2 + 1$ lattice QCD

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For RQCD collaboration
The nucleon axial charge $g_A^N$

- Axial charge $g_A^N/g_V^N = 1.2732(23)$ [PDG: Phys. Rev. D 98, 030001] is experimentally well known from neutron $\beta$-decay
- Computed via the matrix element $\langle p | \bar{u} \gamma_5 \gamma_\mu u - \bar{d} \gamma_5 \gamma_\mu d | p \rangle$
- Extensively studied on the lattice and serves as a benchmark quantity for lattice QCD calculations
Motivation

Hyperon axial charges $g_A^B$ for octet baryons ($B = \Sigma, \Xi$) are less well known.

- Can be easily computed from lattice QCD in the same way as for the nucleon
- Possibility to give some predictions from ab initio lattice QCD calculations for experiments
- Interesting to study $SU(3)$ flavor symmetry breaking
- $SU(3)$ flavor symmetry relations used in phenomenology
Motivation

Other charges which are interesting to compute also for hyperons:

- **Vector:** \((g_V, \langle x \rangle_{u-d})\)
- **Axial vector:** \(g_A, \langle x \rangle_{\Delta u-\Delta d}\)
- **Tensor:** \(g_T, \langle x \rangle_{\delta u-\delta d}\)
- **Scalar:** \(g_S\)

Not very well known from experiment or lattice QCD studies at present.
**CLS Gauge Ensembles**

- $N_f = 2 + 1$ flavors of non-perturbatively $O(a)$ improved dynamical Wilson fermions and tree-level Symanzik improved gauge action
- (Mostly) Open boundary conditions in time to avoid freezing of the topological charge [Lüscher: 1105.4749]

\[
\text{tr } M = \text{const} \quad \Rightarrow \quad m_s = m_s^{\text{phys}} \quad \Rightarrow \quad m_l = m_s
\]

[Bruno: JHEP02, 043 (2015)]


[CLS: https://wiki-zeuthen.desy.de/CLS/]
The two trajectories $\text{tr} \, M$ and $m_s = m_s^{\text{phys}}$ extrapolate to the physical point.

The symmetric line $m_s = m_l$ to the chiral limit.
Lattice Setup

- Pion masses $m_\pi$ from $\sim 410$ MeV down to $\sim 200$ MeV
- Three different lattice spacings $a = [0.0854, 0.0644, 0.05]$ fm
- Volumes $m_\pi L$ between 4.1 and 6.4
- So far 8 ensembles on three trajectories

Volumes:
- H: $32^3 \times 96$
- X: $48^3 \times 64$
- N: $48^3 \times 128$
- D: $64^3 \times 128$
Matrix Elements

Isovector charges are defined by matrix elements of local operators at zero momentum transfer

\[ g^B_J = \langle B| O(\Gamma_J)|B\rangle, \quad J \in \{V, A, T, S\} \]
\[ m_B\langle x\rangle_J = \langle B| O(\Gamma_J)|B\rangle, \quad J \in \{u - d, \Delta u - \Delta d, \delta u - \delta d\}\]

\[ \langle x\rangle_J : \Gamma_J \text{ with derivatives} \]

with current insertion

\[ O(\Gamma) = \bar{u}\Gamma_J u - \bar{d}\Gamma_J d \]

and baryon interpolators (with flavor structure)

\[ B = N(uud), \Sigma(uus), \Xi(ssu) \]

For the isovector combination only the connected three-point functions are needed since disconnected contributions cancel.
Stochastic Estimators

All-to-all propagator computed by stochastic (timeslice-to-all) propagator (wiggly line)

\[ G_U(x', y)_{\delta'}^{\delta} = \frac{1}{N_{sto}} \sum_{i=1}^{N_{sto}} \gamma_5^{\delta} s_i^* U(y)_d \eta_i(x')^{\sigma} \gamma_5^{\sigma} \delta'. \]
Factorization into spectator and insertion part:

\[ C(p', q, x_4', y_4, x_4)_{UDUU}^{\alpha' \alpha \beta' \beta \delta' \delta \gamma' \gamma} = \]

\[ \frac{1}{N_{\text{sto}}} \sum_{i=1}^{N_{\text{sto}}} \sum_{c=1}^{3} \left( S_{UD}(p', x_4', r_4)_{ic}^{\alpha' \alpha \beta' \beta \delta'} \times I_{UU}(q, y_4, r_4)_{ic}^{\delta \gamma' \gamma} \right) \]
Stochastic Estimators

- Flexibility: Correlator with all spin, flavor and momentum indices open
- Contractions for various insertion gamma structures and hadron interpolators at analysis stage
  ⇒ compute the wanted matrix elements
- No additional inversions for different final momentum (main advantage in terms of computer time)
- Disadvantage: additional stochastic noise
- But forward-backward averaging and averaging over equivalent polarizations and momenta (if with momentum transfer) almost for free

Previous work:

Matrix Elements

\[ R_B(\tau, t, \Gamma_J) = \frac{C_{3pt,B}(\tau, t, q = 0, \Gamma_J)}{C_{2pt,B}(t, p = 0)} \]

\[ \xrightarrow{t \to \infty} \langle B | O(\Gamma_J) | B \rangle^{lattice} \]

Discretisation effects of \( O(a^2)/O(a) \) of matrix elements without \((g_J)/with derivatives \((\langle x \rangle_J)\).

Renormalization with improvement

\[ Z_J(g^2, a\mu) \left( 1 + am_q b_J(g^2) + 3a\tilde{m}b_J(g^2) \right) \]

(present estimates of \( \tilde{b}_J \) compatible with zero) \([1609.09477]\).
Excited State Analysis

In some channels excited state contributions are clearly visible

\[ N203 - g^N_t \]
Set of four source-sink separations $\sim [0.7, 0.8, 1.0, 1.2]$ fm for all ensembles.
Excited State Analysis

Simultaneous fit to the two-point function and ratios for different source-sink separations, varying the fit range.

Fit ansatz:

\[ C_{2pt,B}(t) = A_0 e^{-m_B t} \left( 1 + A_1 e^{-\Delta m_B t} \right) \]

\[ R_B(\tau, t) = B_0 + B_1 e^{-\Delta m_B t/2} \cosh(\Delta m_B t) + B_2 e^{-\Delta m_B t} \]

with mass difference \( \Delta m_B = m'_B - m_B \)
Hyperon $g_A^B$

Renormalization and improvement factors cancel from ratios $g_A^B / g_A^N$

Splitting of symmetric line (red) and tr $M$ line (blue)

Leftmost point corresponds to ensemble on $m_s = m_{s}^{\text{phys}}$
line close to tr $M$ ($m_K = 500$ MeV instead of $m_K = 480$ MeV)
Further results e.g. [Erkol: 0911.2447] (heavier pion masses) and [Savanur: 1901.00018] not included.
**SU(3) Flavor Symmetry**

Decomposition of charges in couplings $F$ and $D$:

$$g_A^N = F + D \quad g_A^\Sigma = 2F \quad g_A^\Xi = F - D$$

Linear extrapolation in $m_{\pi}^2$ to chiral limit

$$g_A$$

Similar to $F = 0.438(7)_{\text{stat}}(6)_{\text{sys}}$, $D = 0.708(10)_{\text{stat}}(6)_{\text{sys}}$

[Savanur: 1901.00018]
**SU(3) Flavor Symmetry breaking**

For $SU(3)$ flavor symmetry one finds

\[
g_A^N = F + D \quad g_A^{\Sigma} = 2F \quad g_A^{\Xi} = F - D
\]

\[
\Rightarrow \quad \frac{\text{"N-charge"}}{\text{"\Sigma-charge"}} = 1 + \delta_{SU(3)}
\]

Axial charges:

\[
m_K^2 - m_{\pi}^2 \sim m_s - m_l = \delta s
\]

Systematic investigation of flavor symmetry breaking: [QCDSF: 1212.2564]
$SU(3)$ Flavor Symmetry breaking

**PRELIMINARY**

- $g_T$

![Graph showing $g_T$ vs. $m_K^2 - m_{\pi}^2$ [GeV^2]](image)

- $g_S$

![Graph showing $g_S$ vs. $m_K^2 - m_{\pi}^2$ [GeV^2]](image)

- $\langle X \rangle_{u-d}$

![Graph showing $\langle X \rangle_{u-d}$ vs. $m_K^2 - m_{\pi}^2$ [GeV^2]](image)

Ensembles:

- N202
- H102
- N203
- N302
- N200
- D201
Summary and Outlook

Summary

- Computation of (generalized) hyperon charges in the vector, axial vector, scalar and tensor channel
- Chiral extrapolation of couplings $F$ and $D$
- Investigation of $SU(3)$ flavor symmetry breaking

Outlook

- Continuum limit and extrapolation to physical quark masses

Thank you for your attention!