Hyperon couplings from $N_f = 2 + 1$ lattice QCD

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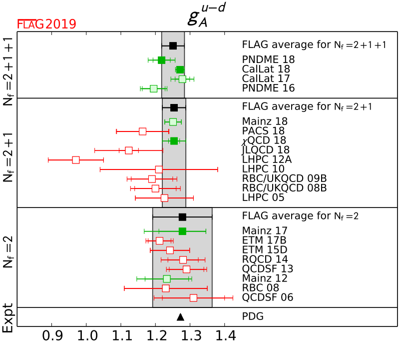
June 20, 2019

For RQCD collaboration

Motivation

The nucleon axial charge g_A^N

- Axial charge $g_A^N/g_V^N = 1.2732(23)$ [PDG: Phys. Rev. D 98, 030001] is experimentally well known from neutron β-decay
- Computed via the matrix element $\langle p|\bar{u}\gamma_5\gamma_\mu u - \bar{d}\gamma_5\gamma_\mu d|p\rangle$
- Extensively studied on the lattice and serves as a benchmark quantity for lattice QCD calculations

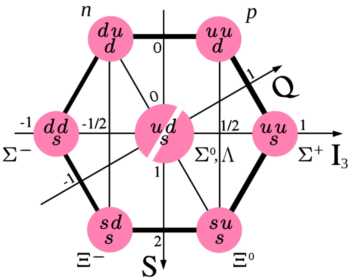


[FLAG 2019]

Motivation

Hyperon axial charges g_A^B for octet baryons ($B = \Sigma, \Xi$) are less well known.

- Can be easily computed from lattice QCD in the same way as for the nucleon
- Possibility to give some predictions from ab initio lattice QCD calculations for experiments
- Interesting to study $SU(3)$ flavor symmetry breaking
- $SU(3)$ flavor symmetry relations used in phenomenology



[Wikipedia.org]

Motivation

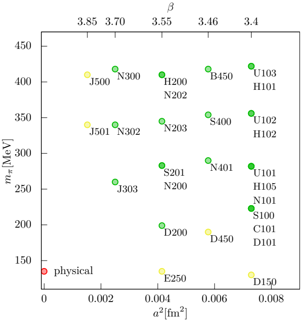
Other charges which are interesting to compute also for hyperons:

- Vector: (g_V,) $\langle x \rangle_{u-d}$
- Axial vector: g_A, $\langle x \rangle_{\Delta u - \Delta d}$
- Tensor: g_T, $\langle x \rangle_{\delta u - \delta d}$
- Scalar: g_S

Not very well known from experiment or lattice QCD studies at present.

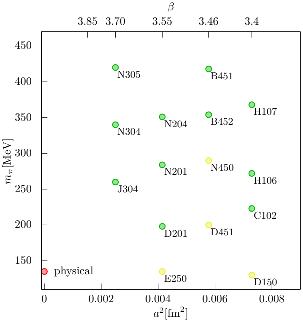
CLS Gauge Ensembles

- $N_f = 2+1$ flavors of non-perturbativly $\mathcal{O}(a)$ improved dynamical Wilson fermions and tree-level Symanzik improved gauge action
- (Mostly) Open boundary conditions in time to avoid freezing of the topological charge [Lüscher: 1105.4749]



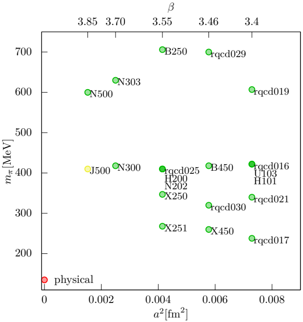
tr M = const

[Bruno: JHEP02, 043 (2015)]



$m_s = m_s^{phys}$

[Bali: Phys. Rev. D 94, 074501]

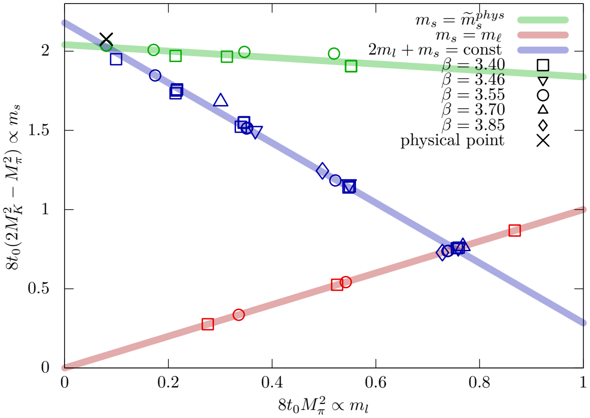


$m_l = m_s$

[CLS: https://wiki-zeuthen.desy.de/CLS/]

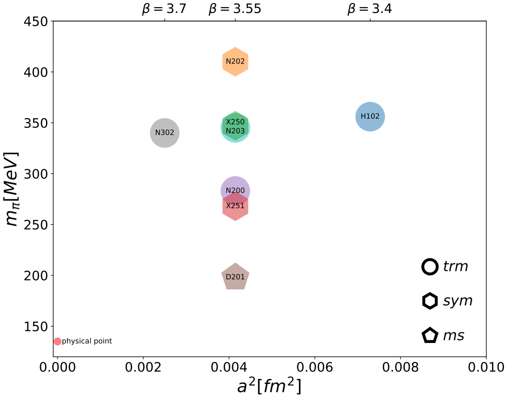
CLS Gauge Ensembles

- The two trajectories tr M and $m_s = m_s^{phys}$ extrapolate to the physical point
- The symmetric line $m_s = m_l$ to the chiral limit



Lattice Setup

- Pion masses m_π from ~ 410 MeV down to ~ 200 MeV
- Three different lattice spacings $a = [0.0854, 0.0644, 0.05]$ fm
- Volumes $m_\pi L$ between 4.1 and 6.4
- So far 8 ensembles on three trajectories



<u>Volumes:</u>

H: $32^3 \times 96$

X: $48^3 \times 64$

N: $48^3 \times 128$

D: $64^3 \times 128$

Matrix Elements

Isovector charges are defined by matrix elements of local operators at zero momentum transfer

$$g_J^B = \langle B|O(\Gamma_J)|B\rangle, \quad J \in \{V, A, T, S\}$$
$$m_B\langle x\rangle_J = \langle B|O(\Gamma_J)|B\rangle, \quad J \in \{u-d, \Delta u - \Delta d, \delta u - \delta d\}$$

$\langle x\rangle_J$: Γ_J with derivatives

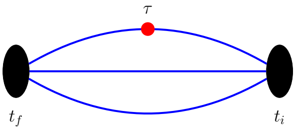
with current insertion

$$O(\Gamma) = \bar{u}\Gamma_J u - \bar{d}\Gamma_J d$$

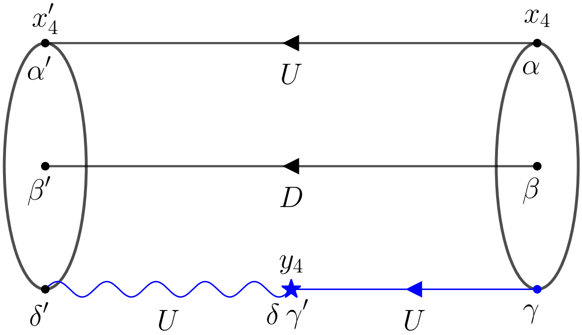
and baryon interpolators (with flavor structure)

$$B = N(uud),\ \Sigma(uus),\ \Xi(ssu)\ .$$

For the isovector combination only the connected three-point functions are needed since disconnected contributions cancel.



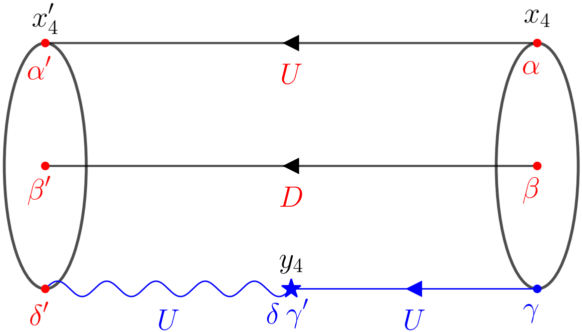
Stochastic Estimators



All-to-all propagator computed by stochastic (timeslice-to-all) propagator (wiggly line)

$$G_U(x',y)^{\delta'\,\delta}_{c'\,d} \;=\; \frac{1}{N_{\rm sto}} \sum_{i=1}^{N_{\rm sto}} \gamma_5^{\delta\,\rho}\; s^{*}_{i,U}(y)^{\rho}_{d}\; \eta_i(x')^{\sigma}_{c'}\; \gamma_5^{\sigma\,\delta'}\;.$$

Stochastic Estimators



Factorization into spectator and insertion part:

$$C(\boldsymbol{p}', \boldsymbol{q}, x_4', y_4, x_4)^{\alpha'\,\alpha\,\beta'\,\beta\,\delta'\,\delta\,\gamma'\,\gamma}_{U\,D\,U\,U} =$$

$$\frac{1}{N_{\mathrm{sto}}} \sum_{i=1}^{N_{\mathrm{sto}}} \sum_{c=1}^{3} \Big(S_{UD}(\boldsymbol{p}', x_4', r_4)^{\alpha'\,\alpha\,\beta'\,\beta\,\delta'}_{ic} \times I_{UU}(\boldsymbol{q}, y_4, r_4)^{\delta\,\gamma'\,\gamma}_{ic} \Big)$$

Stochastic Estimators

- Flexibility: Correlator with all spin, flavor and momentum indices open
- Contractions for various insertion gamma structures and hadron interpolators at analysis stage
  $\Rightarrow$compute the wanted matrix elements
- No additional inversions for different final momentum (main advantage in terms of computer time)
- Disadvantage: additional stochastic noise
- But forward-backward averaging and averaging over equivalent polarizations and momenta (if with momentum transfer) almost for free

Previous work:

[arXiv: 1008.3293, arXiv: 1311.1718, ETMC: arXiv:1302.2608, χQCD: 1509.04616]

Matrix Elements

$$R_B(\tau, t, \Gamma_J) = \frac{C_{3pt,B}(\tau, t, q=0, \Gamma_J)}{C_{2pt,B}(t, p=0)}$$

$$\xrightarrow[t\to\infty]{} \langle B|O(\Gamma_J)|B\rangle^{lattice}$$

Discretisation effects of $\mathcal{O}(a^2)/\mathcal{O}(a)$ of matrix elements without (g_J)/with derivatives ($\langle x \rangle_J$).

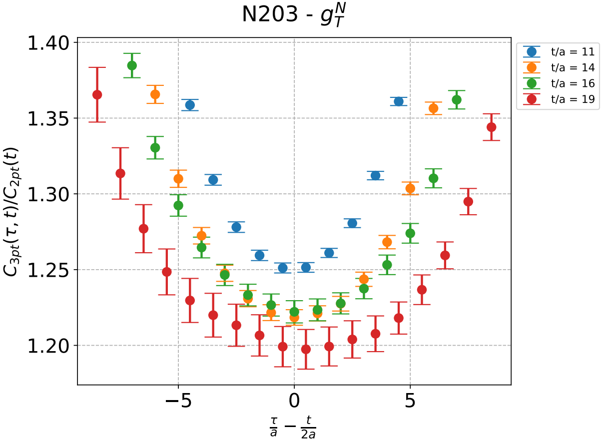
Renormalization with improvement

$$Z_J(g^2, a\mu)\left(1 + am_q b_J(g^2) + 3a\bar{m}\tilde{b}_J(g^2)\right)$$

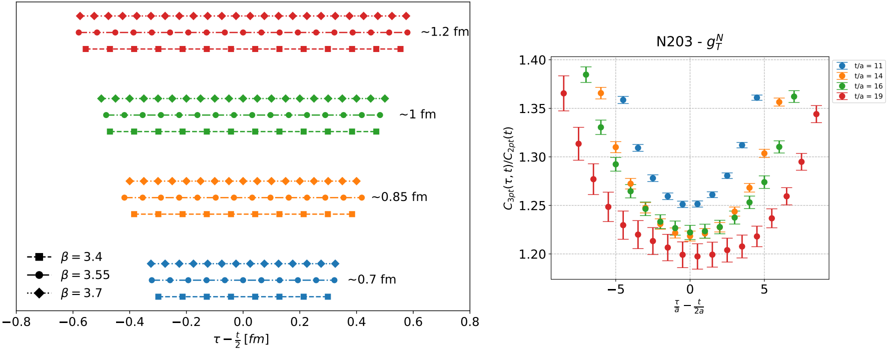
(present estimates of $\tilde{b}_J$ compatible with zero) [1609.09477].

Excited State Analysis

In some channels excited state contributions are clearly visible



Excited State Analysis



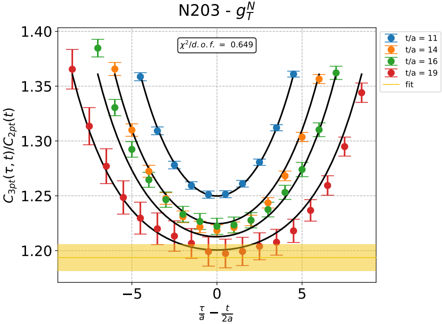
Set of four source-sink separations $\sim [0.7, 0.8, 1.0, 1.2]$ fm for all ensembles.

Excited State Analysis

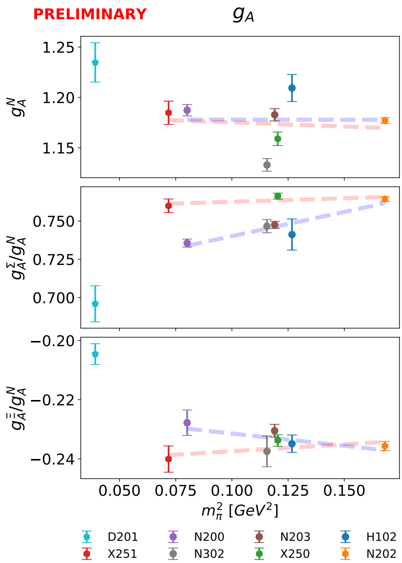
Simultaneous fit to the two-point function and ratios for different source-sink separations, varying the fit range.
Fit ansatz:

$$C_{2pt,B}(t) = A_0 e^{-m_B t}\left(1 + A_1 e^{-\Delta m_B t}\right)$$
$$R_B(\tau, t) = B_0 + B_1 e^{-\Delta m_B t/2}\cosh(\Delta m_B t) + B_2 e^{-\Delta m_B t}$$

with mass difference
$\Delta m_B = m'_B - m_B$

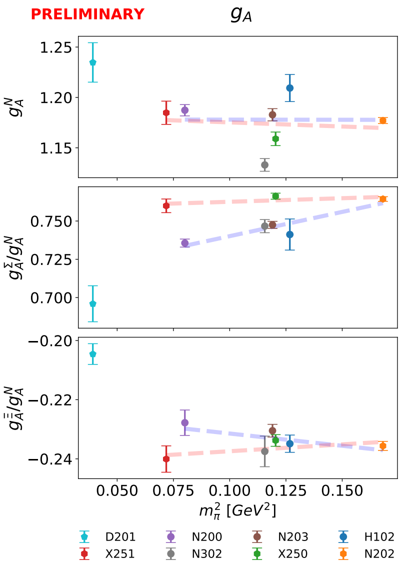


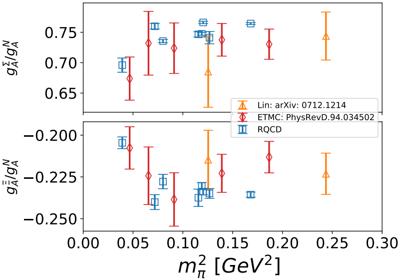
Hyperon g_A^B



- Renormalization and improvement factors cancel from ratios g_A^B / g_A^N
- Splitting of symmetric line (red) and tr M line (blue)
- Leftmost point corresponds to ensemble on $m_s = m_s^{phys}$ line close to tr M ($m_K = 500$ MeV instead of $m_K = 480$ MeV)

Hyperon g_A^B





Further results e.g.
[Erkol: 0911.2447] (heavier pion masses) and
[Savanur: 1901.00018]

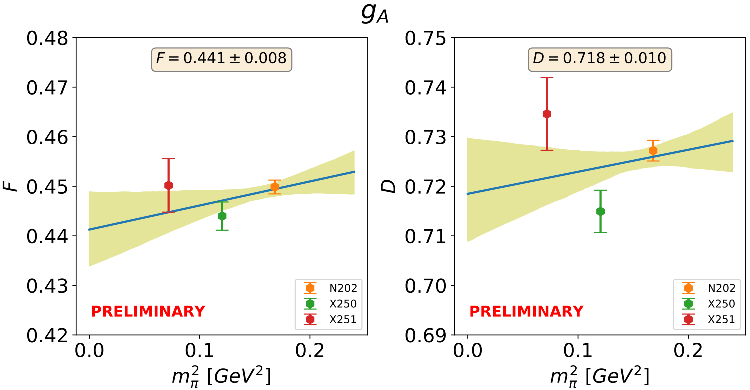
not included.

$SU(3)$ Flavor Symmetry

Decomposition of charges in couplings F and D:

$$g_A^N = F + D \qquad g_A^\Sigma = 2F \qquad g_A^\Xi = F - D$$

Linear extrapolation in m_π^2 to chiral limit



Similar to $F = 0.438(7)_{stat}(6)_{sys}$, $D = 0.708(10)_{stat}(6)_{sys}$

[Savanur: 1901.00018]

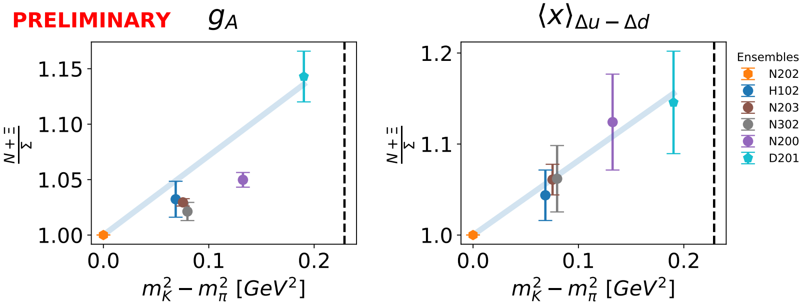
$SU(3)$ Flavor Symmetry breaking

For $SU(3)$ flavor symmetry one finds

$$g_A^N = F + D \qquad g_A^\Sigma = 2F \qquad g_A^\Xi = F - D$$

$$\Rightarrow \frac{\text{"}N\text{-charge"} + \text{"}\Xi\text{-charge"}}{\text{"}\Sigma\text{-charge"}} = 1 + \delta_{SU(3)}$$

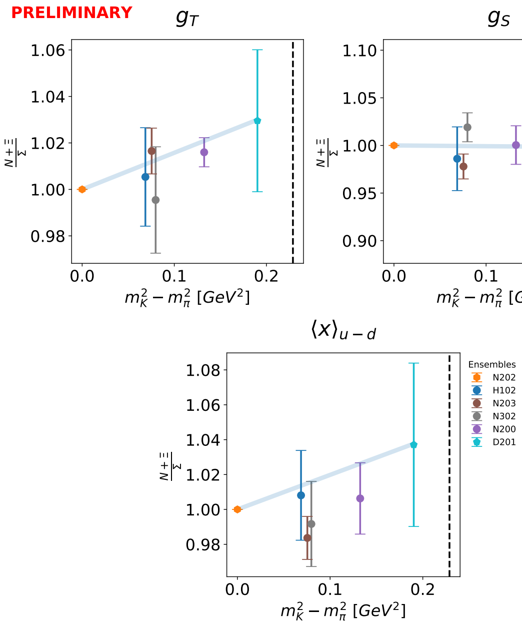
Axial charges:



$m_K^2 - m_\pi^2 \sim m_s - m_l = \delta s$

Systematic investigation of flavor symmetry breaking: [QCDSF: 1212.2564]

$SU(3)$ Flavor Symmetry breaking



Summary and Outlook

Summary

- Computation of (generalized) hyperon charges in the vector, axial vector, scalar and tensor channel
- Chiral extrapolation of couplings F and D
- Investigation of $SU(3)$ flavor symmetry breaking

Outlook

- Continuum limit and extrapolation to physical quark masses

Thank you for your attention!