Hadronic vacuum polarization in finite volume using NNLO ChPT

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Lattice 19

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Outline

Quick review of g-2
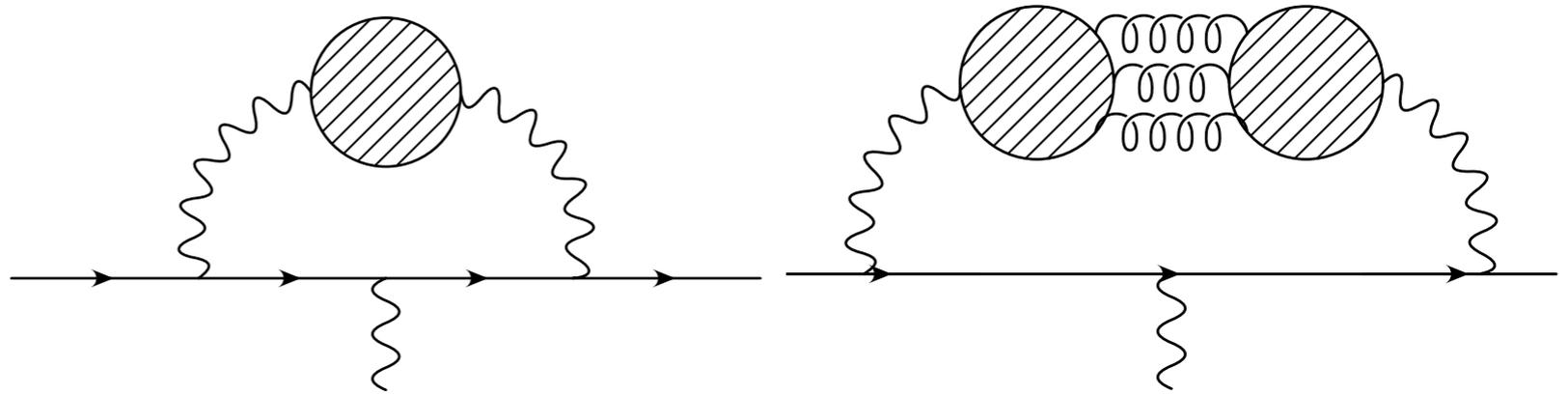Lattice Simulation details
Finite volume ChPT
Results

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(quick) g-2 background

A quick summary

T. Blum, PRL 91, 052001 (2003)

$$a_\mu^{\text{HVP}} = 4\alpha^2 \int_0^\infty dq^2 f(q^2)\hat{\Pi}(q^2)$$

$$f(q^2) = \frac{m_\mu^2 q^2 Z^3(1 - q^2 Z)}{1 + m_\mu^2 q^2 Z^2}$$

$$Z = -\frac{q^2 - \sqrt{q^4 + 4m_\mu^2 q^2}}{2m_\mu^2 q^2}$$

$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2}t^2 \right) C(t)$$

$$C(t) = \frac{1}{3} \sum_{\vec{x},i} \langle j^i(\vec{x}, t)j^i(0) \rangle$$

$$w(t) = 4\alpha^2 \int_0^\infty d\omega^2 f(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right]$$

D. Bernecker and H. B. Meyer
EPJ A47, 148 (2011)

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$$a_\mu^{\text{HVP}}(T) = \sum_{t=-T/2}^{T/2} w(t)C(t) = 2\sum_{t=0}^{T/2} w(t)C(t)$$

(quick) g-2 background

A quick summary

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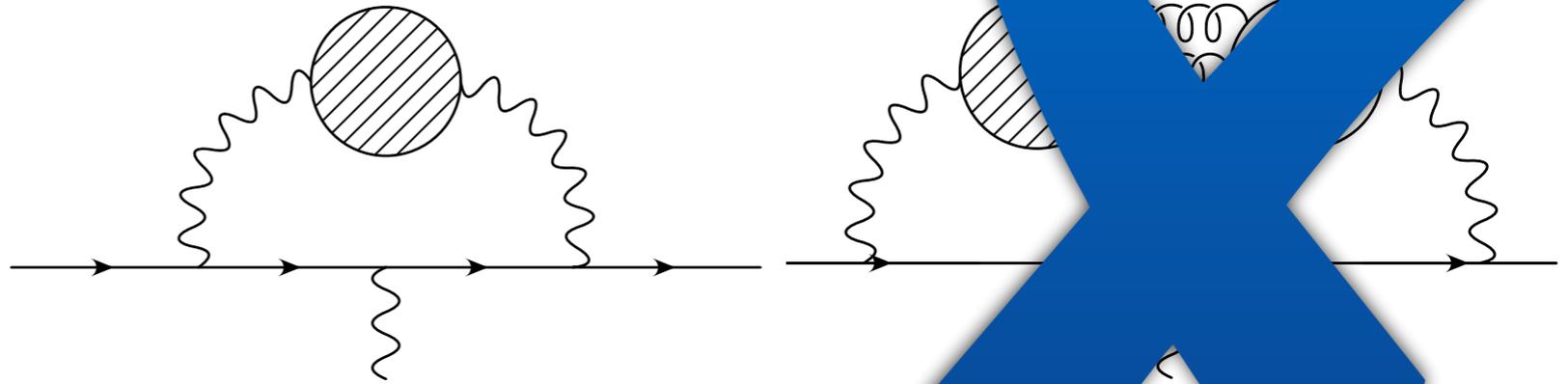$$Z = -\frac{q^2 - \sqrt{q^4 + 4m_\mu^2 q^2}}{2m_\mu^2 q^2}$$

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Simulation Details

Noise reduction techniques developed by RBC/UKQCD including all-mode and full volume low-mode averaging

Blum et al, PRD88, 094503; PRL 121, 022003, *etc.*

m_π (MeV)	a (fm)	size	L (fm)	$m_\pi L$	LM	AMA srcs	measurements (approx-exact-LMA)
133	0.12121(64)	$48^3 \times 64$	5.82	3.91	3000	$4^3 \times 4$	26-26-26
130	0.08787(46)	$64^3 \times 96$	5.62	3.66	3000	$4^3 \times 4$	36-36-40
134	0.05684(30)	$96^3 \times 192$	5.46	3.73	2000	$3^3 \times 8$	21-21-22

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2+1+1-flavor HISQ ensembles generated by the MILC Collaboration

Calculate $C(t)$ in FV ChPT to correct result for g-2 at each lattice spacing, then take continuum limit.

The full two-loop ChPT calculation can be done in finite volume for $C(t)$, and specifically we do the following:

– Correct FV results using NLO SChPT, including taste-breaking effects,
– Extrapolate to the continuum
– Apply NNLO continuum ChPT FV correction, without taste-breaking effects (irrelevant here)

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Finite volume ChPT

We calculate *C(t)* in ChPT and in what follows evaluate:

$$\Delta a_\mu^{\mathrm{HVP}} = \lim_{L\to\infty} a_\mu^{\mathrm{HVP}}(L) - a_\mu^{\mathrm{HVP}}(L)$$

The pion contribution to the electromagnetic current (including only terms we need) is:

$$j_\mu(x) = i\left(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-\right)\left(1 - \frac{1}{3F^2}\left((\pi^0)^2 + 2\pi^+\pi^-\right)\right)$$
$$- \frac{2i\ell_6}{F^2}\,\partial_\nu\left(\partial_\mu\pi^+\partial_\nu\pi^- - \partial_\nu\pi^+\partial_\mu\pi^-\right)$$

Finite volume ChPT

NLO part

$$C(t) = \frac{10}{9}\frac{1}{3}\left(\frac{1}{L^d}\sum_{\vec{p}}\frac{\vec{p}^2}{E_p^2}\,e^{-2E_p t}\left[1 - \frac{2}{F^2}D(m_\pi^2) - \frac{8(\vec{p}^2 + m_\pi^2)}{F^2}\,\ell_6\right]\right.$$

$$\left. + \frac{1}{2dF^2}\frac{1}{L^{2d}}\sum_{\vec{p},\vec{k}}\frac{\vec{p}^2\vec{k}^2}{E_p^2 E_k^2}\frac{E_k e^{-2E_p t} - E_p e^{-2E_k t}}{\vec{k}^2 - \vec{p}^2}\right),$$

$$E_p = \sqrt{m_\pi^2 + \vec{p}^2}\,,$$

$$D(m_\pi^2) = \frac{1}{L^d}\sum_{\vec{k}}\frac{1}{2E_k}\,,$$

$$\ell_6 = \ell_6^r(\mu) - \frac{1}{3}\frac{1}{16\pi^2}\left(\frac{1}{\epsilon} - \log\mu - \frac{1}{2}(\log(4\pi) - \gamma + 1)\right)$$

Finite volume ChPT - NLO

At NLO we have

$$\Delta a_\mu^{\mathrm{HVP,\,NLO}} = \frac{10}{9} \frac{\alpha^2}{6\pi^2} \sum_{n^2=1}^{\infty} \frac{Z_{00}(0, n^2)}{nL} \int_0^\infty dp\, \frac{p^3}{E_p^2}\, \sin\left(npL\right) F(p^2)$$

$$F(p^2) = -\frac{8E_p^2 - m_\mu^2}{2E_p^3 m_\mu^2} + \frac{8(2E_p^2 - m_\mu^2)}{E_p m_\mu^4} \log\left(\frac{2E_p}{m_\mu}\right) + \frac{\left(8E_p^4 - 8E_p^2 m_\mu^2 + m_\mu^4\right)}{E_p^2 m_\mu^4 \sqrt{E_p^2 - m_\mu^2}} \log\left(\frac{-2E_p\sqrt{E_p^2 - m_\mu^2} + 2E_p^2 - m_\mu^2}{m_\mu^2}\right)$$

$$Z_{00}(0, n^2) = -\sum_{\vec{m},\,\vec{m}^2=\vec{n}^2} 1$$

Finite volume ChPT - NLO

At NLO we have:

$$\Delta a_\mu^{\mathrm{HVP,\,NLO}} = \begin{cases} 20.59 \times 10^{-10} \,, & L/a = 96 \\ 21.60 \times 10^{-10} \,, & L/a = 64 \\ 18.08 \times 10^{-10} \,, & L/a = 48 \end{cases}$$

Averaging over pion tastes:

$$\Delta a_\mu^{\mathrm{HVP}} = \begin{cases} 15.6 \times 10^{-10} \,, & L/a = 96 \\ 6.9 \times 10^{-10} \,, & L/a = 64 \\ 2.1 \times 10^{-10} \,, & L/a = 48 \end{cases}$$

The effect of taste breaking in infinite volume can also be determined:

$$\Delta_{\mathrm{taste}} a_\mu^{\mathrm{HVP}} = \begin{cases} 9.5 \times 10^{-10} \,, & a = 0.06 \text{ fm} \\ 34.2 \times 10^{-10} \,, & a = 0.12 \text{ fm} \\ 51.6 \times 10^{-10} \,, & a = 0.09 \text{ fm} \end{cases}$$

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Finite volume ChPT - NNLO

The full NNLO FV corrections to the correlator can be (using the Poisson summation formula):
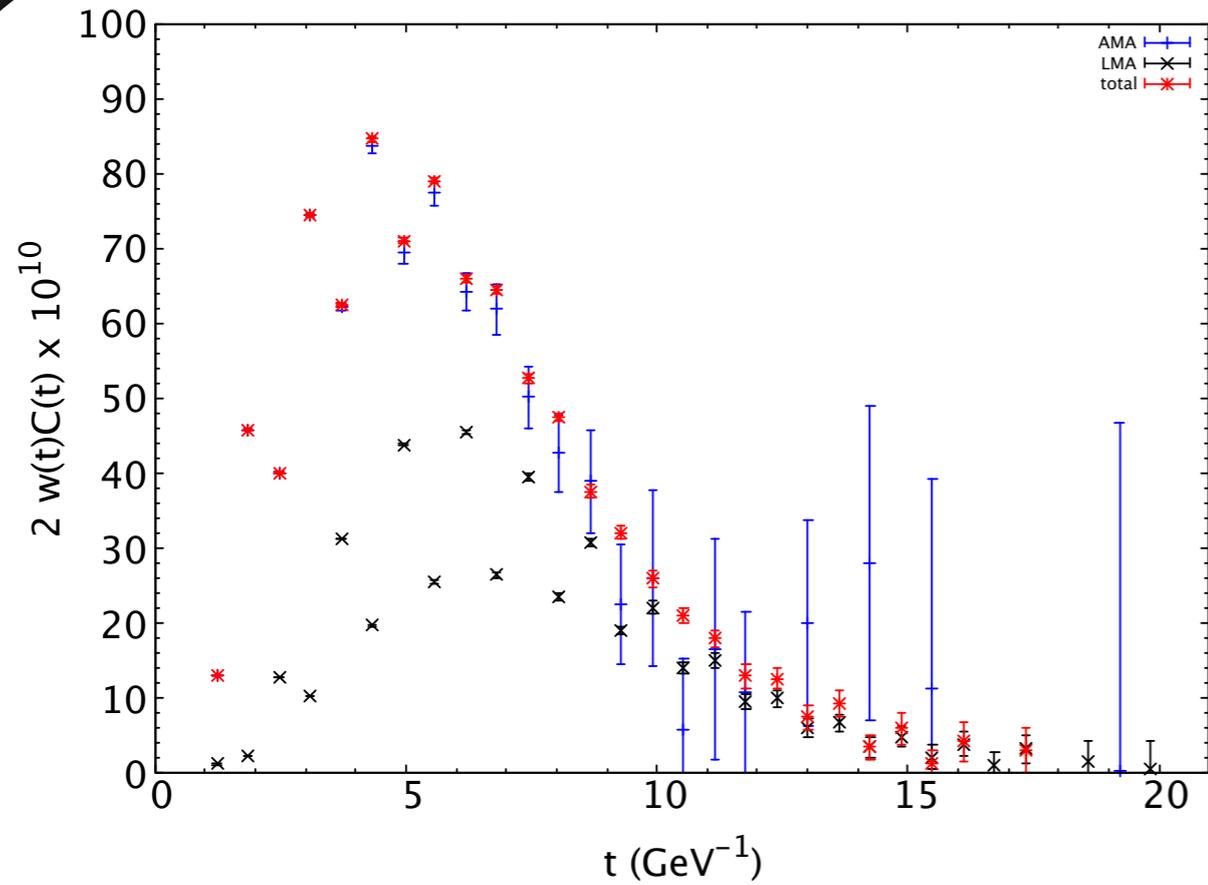
$$
\Delta C(t) = -\frac{10}{9} \left(\frac{1}{3} \int \frac{d^3 p}{(2\pi)^3} \sum_{\vec{n} \neq 0} e^{i\vec{n} \cdot \vec{p} L} \frac{\vec{p}^2}{E_p^2} e^{-2E_p t} \left[1 - \frac{m_\pi^2}{36\pi^2 F^2} + \frac{5\vec{p}^2}{36\pi^2 F^2} + \frac{\vec{p}^2 + m_\pi^2}{12\pi^2 F^2} \overline{\ell}_6 \right.\right.
$$

$$
\left.\left. - \frac{\vec{p}^2}{6\pi^2 F^2} \sqrt{\frac{\vec{p}^2}{\vec{p}^2 + m_\pi^2}} \log \left(\sqrt{\frac{\vec{p}^2}{m_\pi^2}} + \sqrt{\frac{\vec{p}^2}{m_\pi^2} + 1} \right) \right] \right.
$$

$$
\left. - \frac{1}{3F^2} \int \frac{d^d p}{(2\pi)^d} \sum_{\vec{n} \neq 0} e^{i\vec{n} \cdot \vec{p} L} \frac{\vec{p}^2}{E_p^2} e^{-2E_p t} \left[\int \frac{d^d k}{(2\pi)^d} \sum_{\vec{m} \neq 0} \frac{e^{i\vec{m} \cdot \vec{k} L}}{E_k} \right] \right.
$$

$$
\left. - \frac{1}{3F^2} \int \frac{d^d p}{(2\pi)^d} \frac{\vec{p}^2}{E_p^2} e^{-2E_p t} \left[\int \frac{d^d k}{(2\pi)^d} \sum_{\vec{n} \neq 0} \frac{e^{i\vec{n} \cdot \vec{k} L}}{E_k} \left(1 - \frac{1}{d} \frac{\vec{k}^2}{\vec{k}^2 - \vec{p}^2} \right) \right] \right.
$$

$$
\left. + \frac{1}{6dF^2} \int \frac{d^d p}{(2\pi)^d} \int \frac{d^d k}{(2\pi)^d} \sum_{\vec{n} \neq 0, \vec{m} \neq 0} e^{i\vec{n} \cdot \vec{p} L + i\vec{m} \cdot \vec{k} L} \frac{\vec{p}^2 \vec{k}^2}{E_p^2 E_k^2} \frac{E_k e^{-2E_p t} - E_p e^{-2E_k t}}{\vec{k}^2 - \vec{p}^2} \right) ,
$$

and we get:

$$
\Delta a_\mu^{\mathrm{HVP, NNLO}} = \begin{cases} 9.13 \times 10^{-10} \ , & L/a = 96 \\ 9.01 \times 10^{-10} \ , & L/a = 64 \\ 7.40 \times 10^{-10} \ , & L/a = 48 \end{cases}
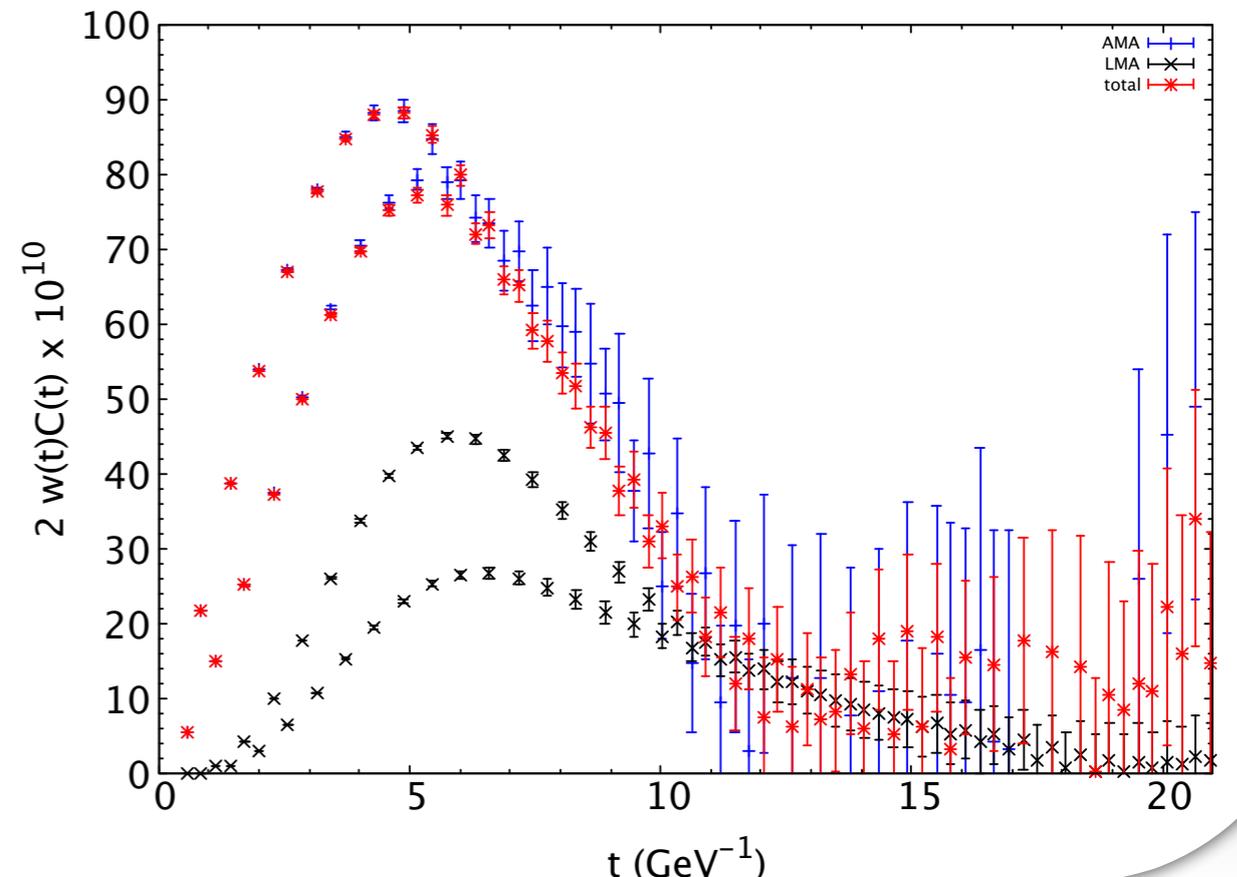$$

Results - Summand
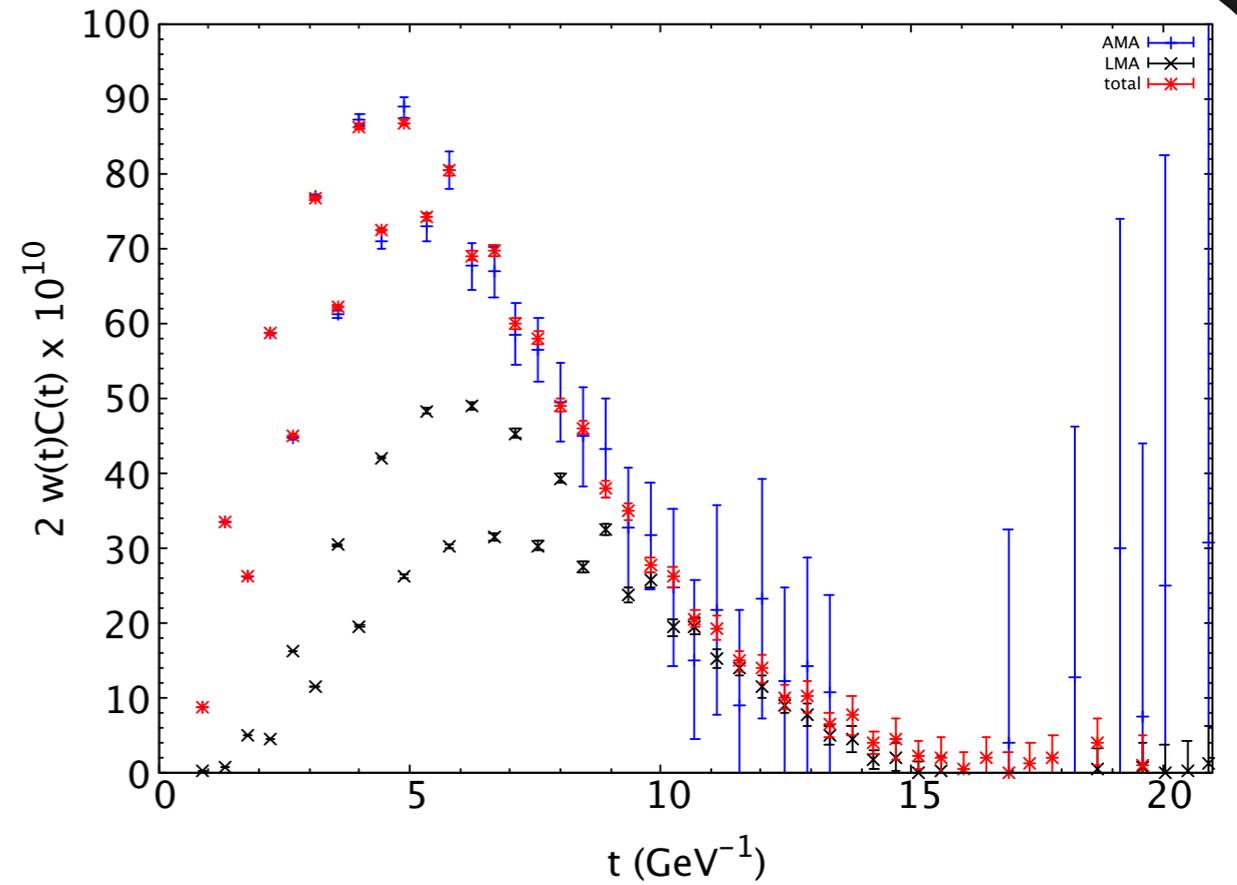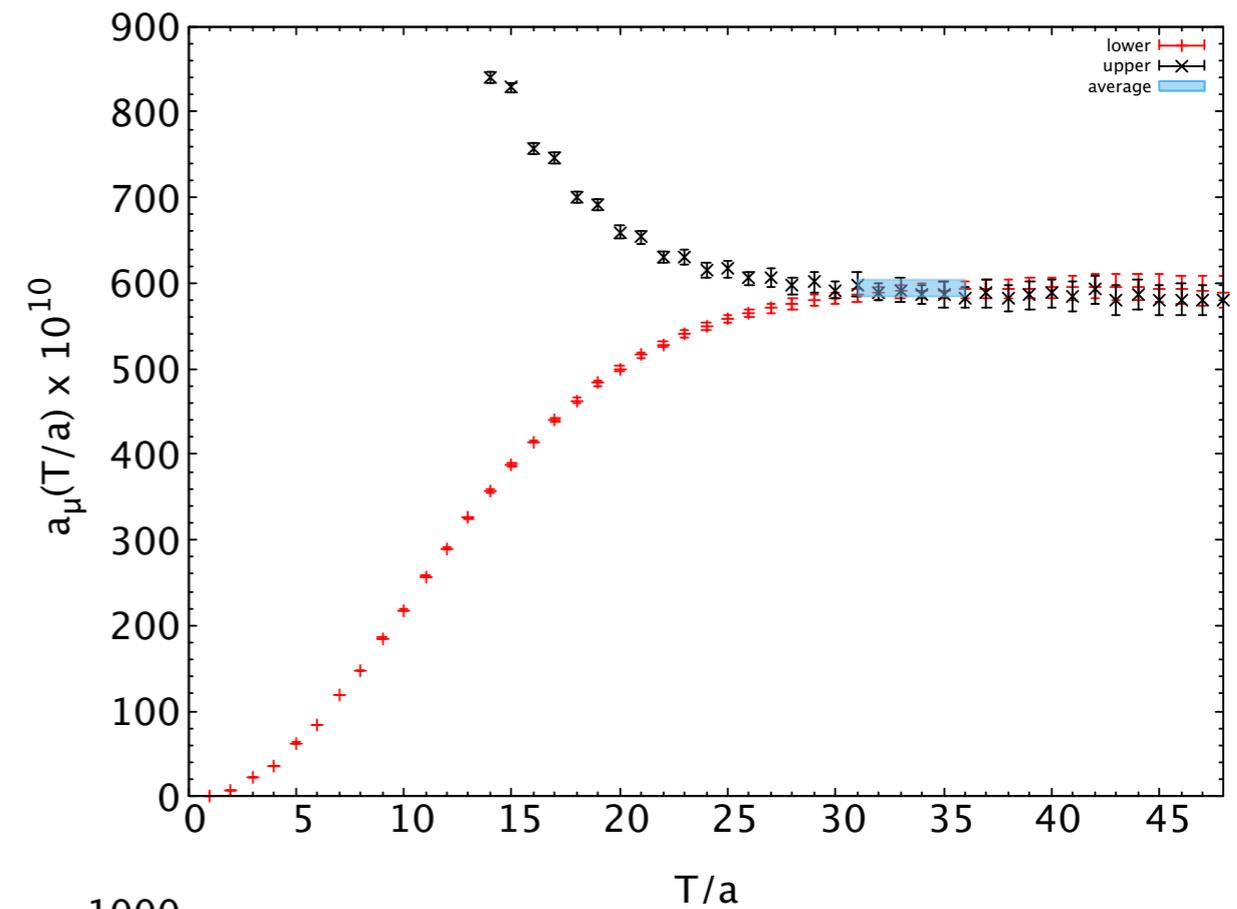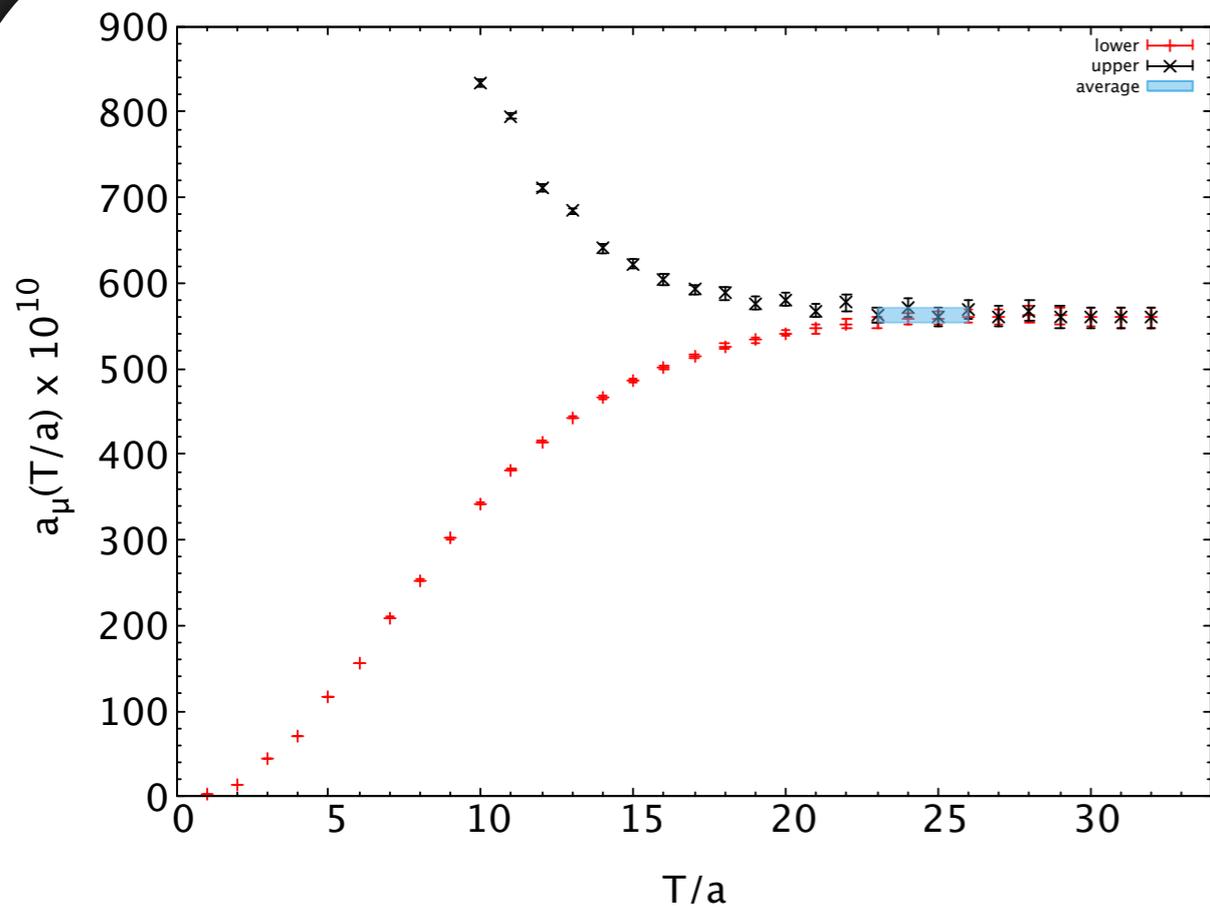

$$a_\mu^{\mathrm{HVP}}(T) = 2 \sum_{t=0}^{T/2} w(t)C(t)$$
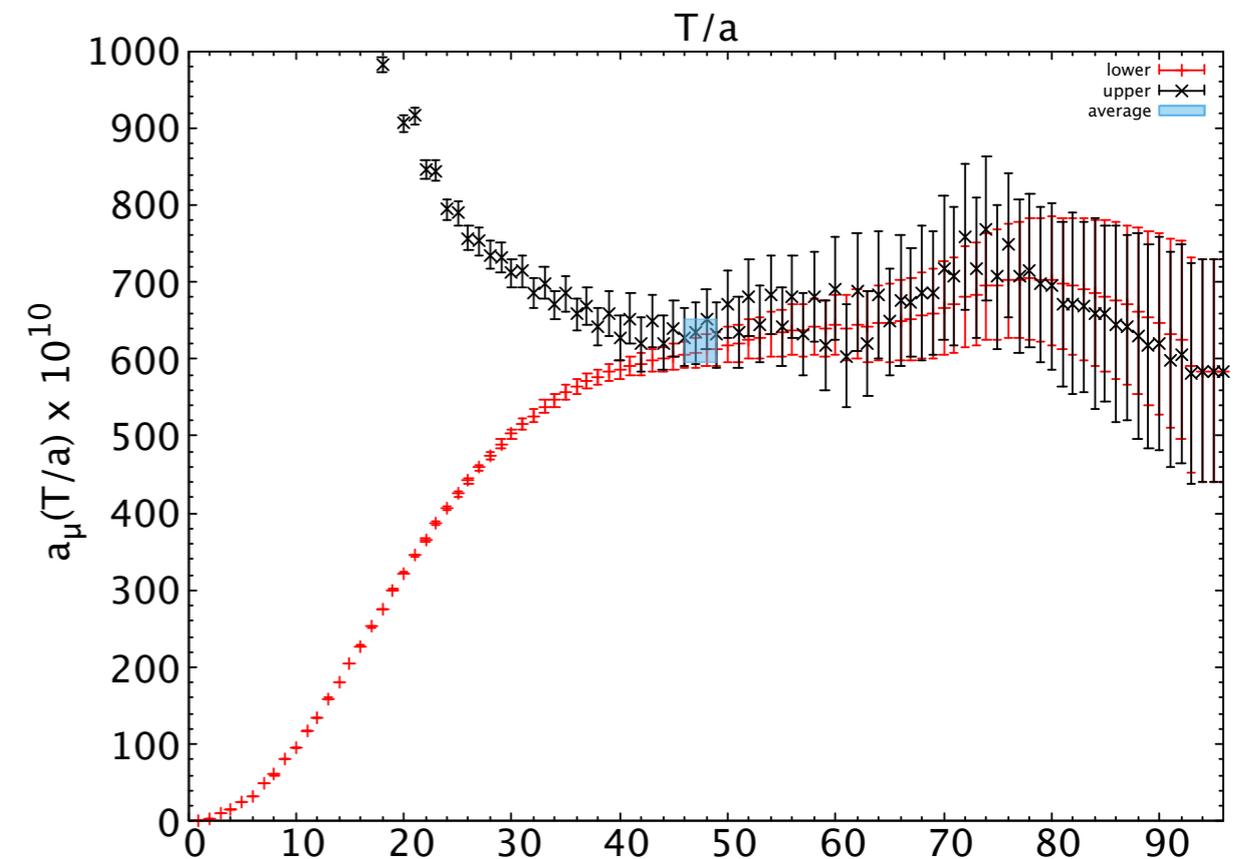
The effects of the staggered parity partners are apparent

Results - Bounding Method

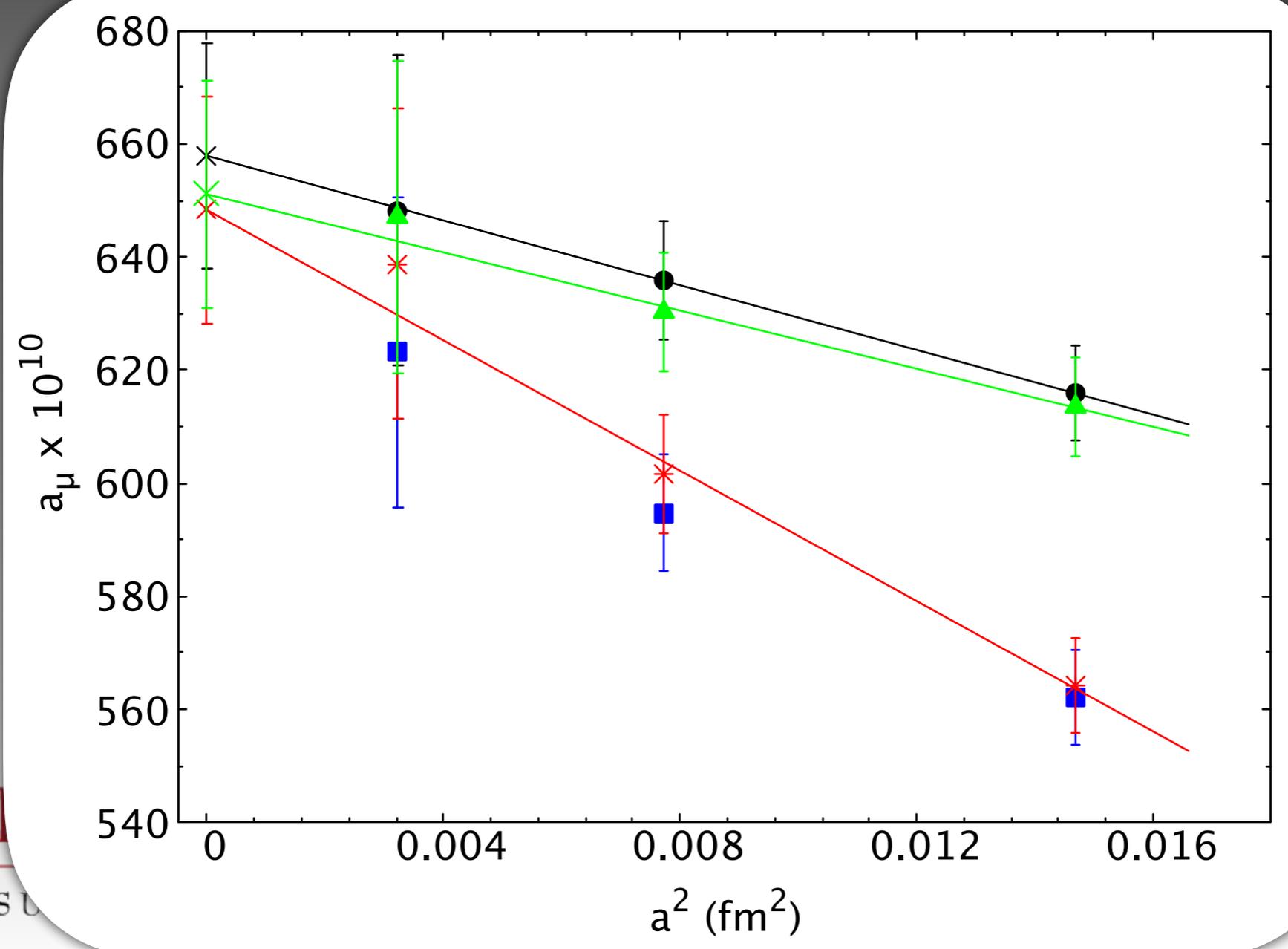$$C(t > T) = \begin{cases} 0 & \text{lower} \\ C(T)e^{-E_0(t-T)} & \text{upper} \end{cases}$$

$$E_0 = 2\sqrt{m_\pi^2 + (2\pi/L)^2}$$

Results

| a (fm)	lattice value	FV corr.	FV + taste corr.	FV+taste+m_π corr.
0.12121(64)	562.1(8.4)	564.2(8.4)	615.8(8.4)	613.6(8.4)
0.08787(46)	594.8(10.4)	601.7(10.4)	635.9(10.4)	630.2(10.4)
0.05684(30)	623.1(27.5)	638.7(27.5)	648.2(27.5)	647.1(27.5)
0		648.3(20.0)	657.9(20.0)	651.1(20.1)

Results

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$$(659 \pm 20 \pm 5 \pm 5 \pm 4) \times 10^{-10} = 659(22) \times 10^{-10}$$
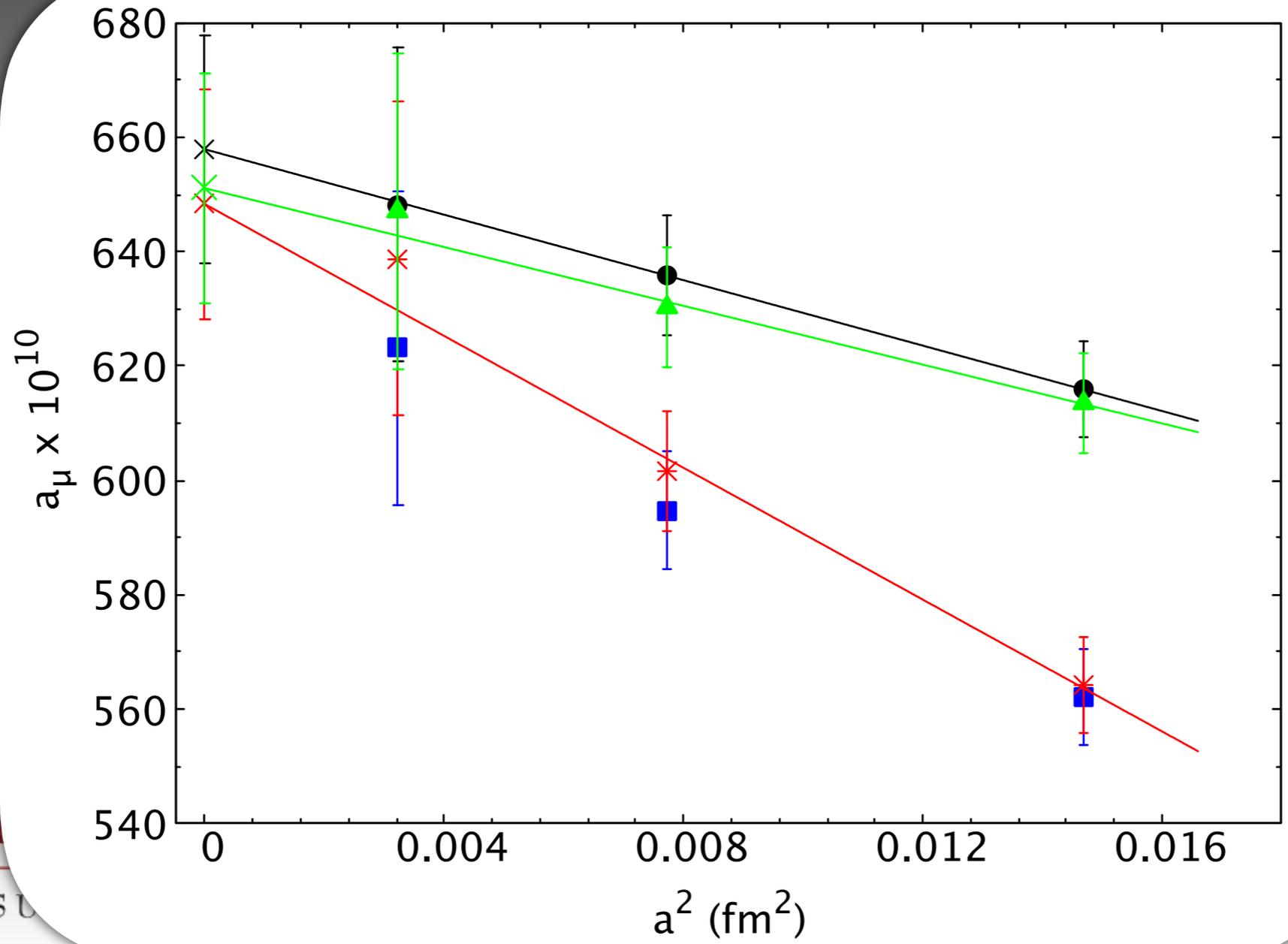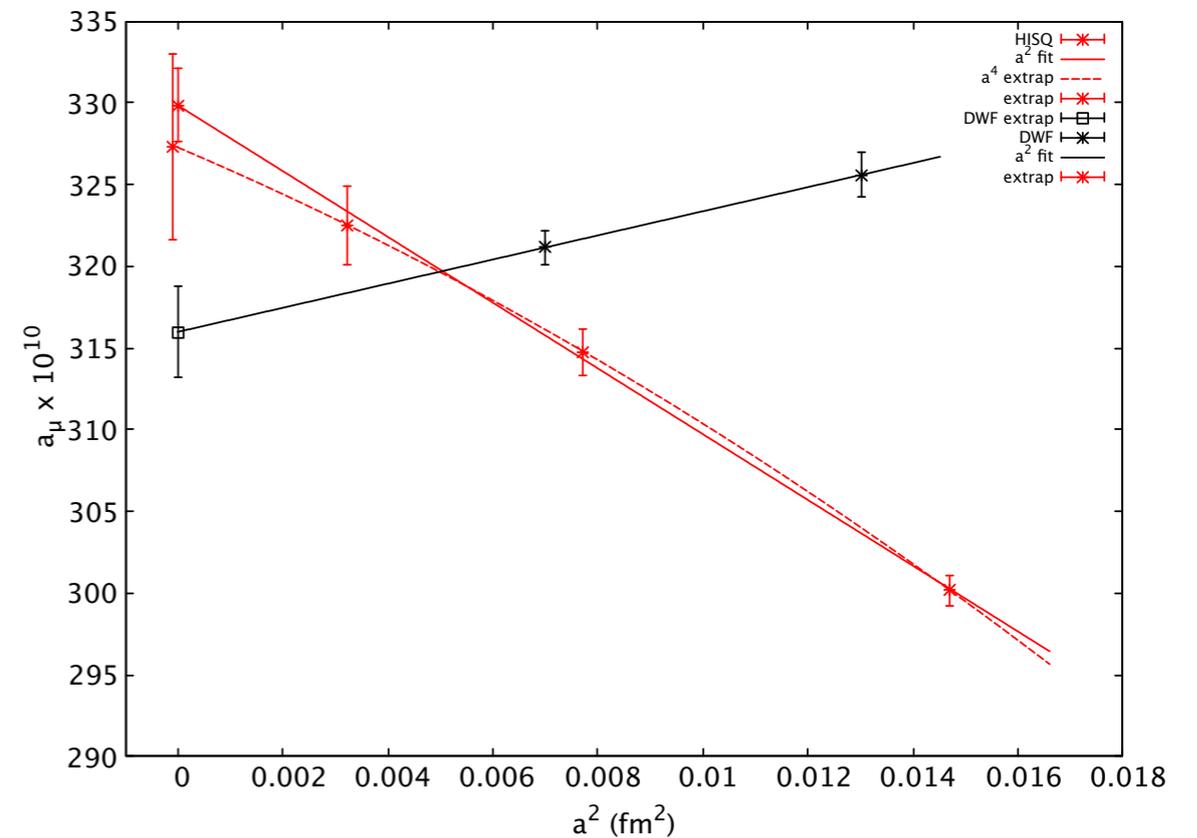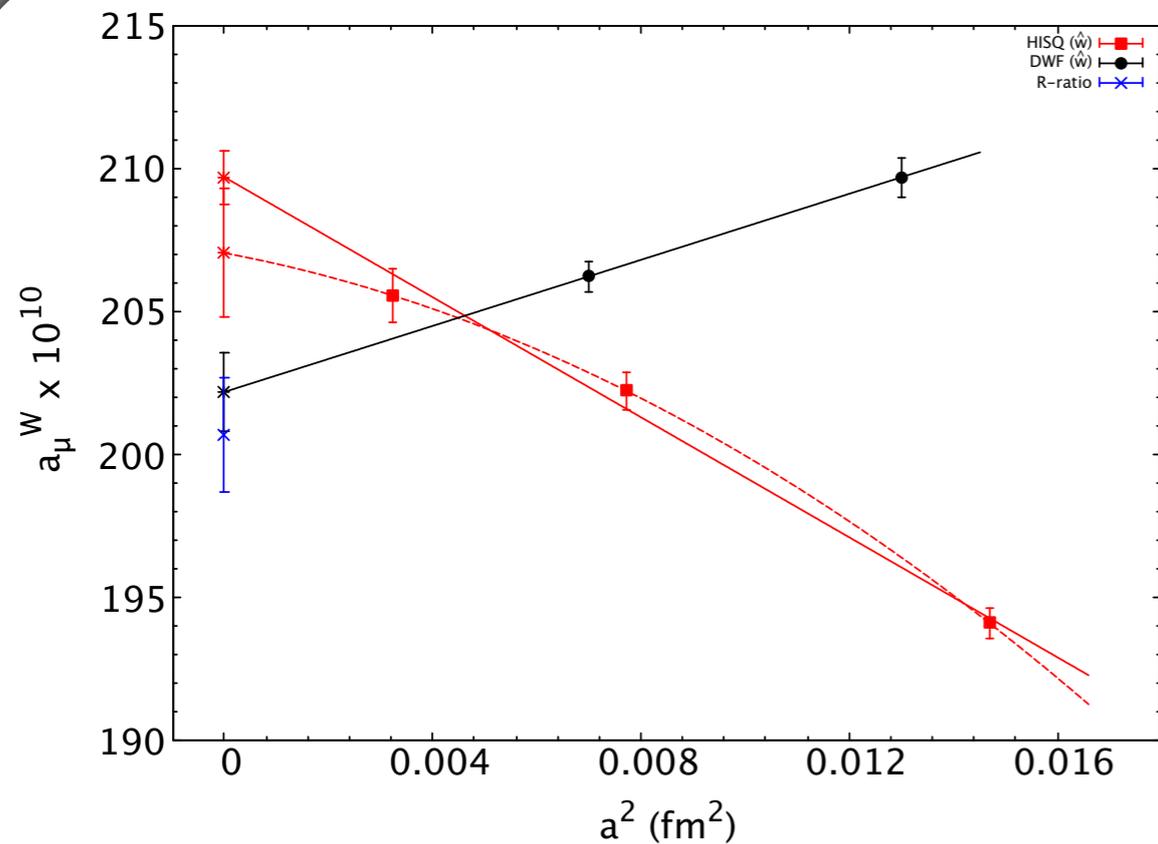
Comparisons
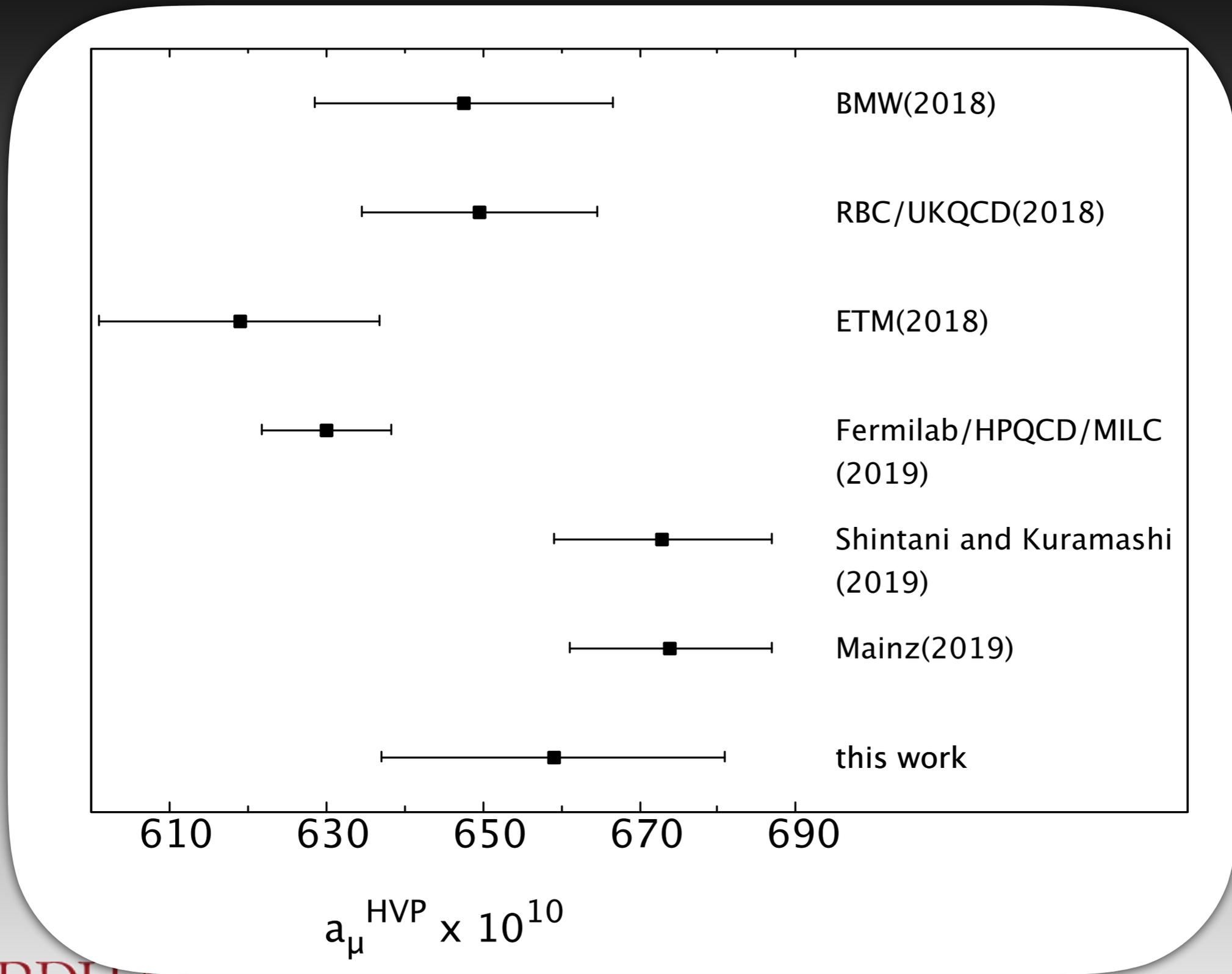
Comparisons

Window method of RBC/UKQCD [Blum et al, PRL 121, 022003]

$$a_\mu^W = 2 \sum_{t=0}^{T/2} C(t) w(t) (\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta))$$

$$\Theta(t, t', \Delta) = \frac{1}{2}(1 + \tanh((t - t')/\Delta))$$

Conclusions

FV ChPT can account for the bulk of the finite volume errors, which are small (NLO effect is 3%, NNLO is 1.5%) – the remaining systematic error is less than 1%

Most needed: Reduce statistical errors (by far the largest error)

Our results are consistent with others (good check!)

$$(659 \pm 20 \pm 5 \pm 5 \pm 4) \times 10^{-10} = 659(22) \times 10^{-10}$$