Lepton anomalous magnetic moments in Lattice QCD+QED

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Lattice 19

XXXVII International Symposium on Lattice Field Theory

Wuhan

16th – 22nd June

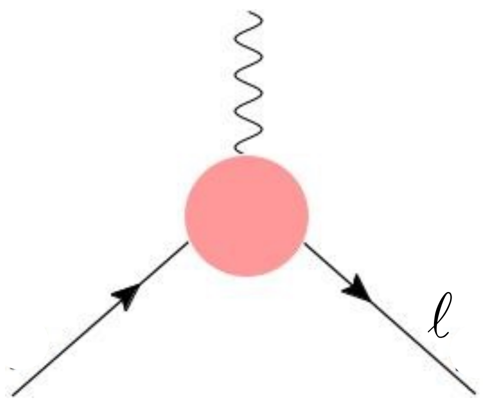
OUTLINE

- Results for the light, strange and charm quark contributions to a_ℓ^{HVP}
- MUonE experiment: lattice contribution to a_μ^{HVP}

In collaboration with:

V. Lubicz, G. Martinelli, F. Sanfilippo and S. Simula

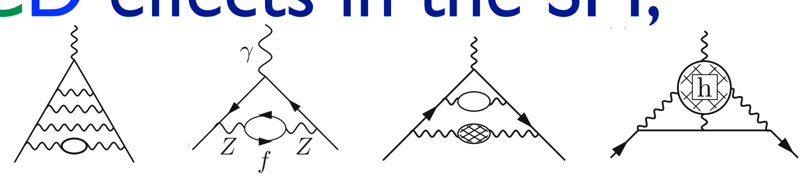
Lepton magnetic anomaly



$$= (-i\, e)\, \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

<u>lepton anomalous magnetic moment:</u> $a_\ell \equiv \frac{g_\ell - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



PDG 2018

$$a_\mu^{SM} = 116\ 591\ 823(1)(34)(26) \cdot 10^{-11}$$

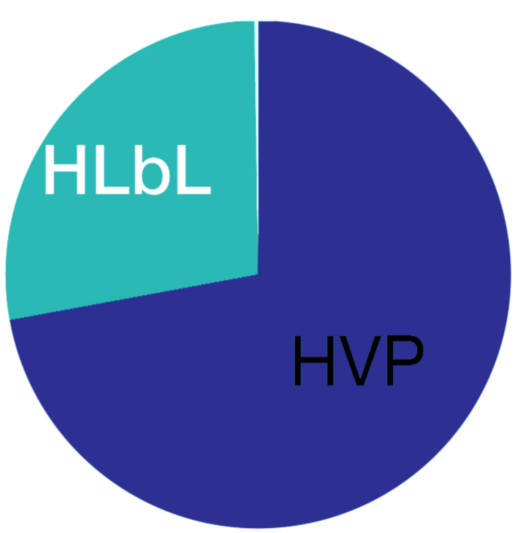
0.4ppm

QED+EW

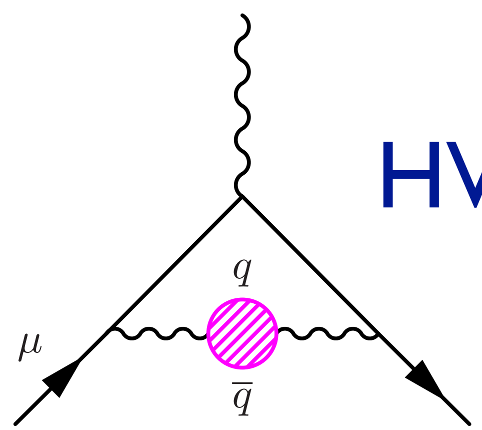
LO Had.

NLO/NNLO Had.

dispersion relations $e^+e^- \rightarrow$ *hadrons*

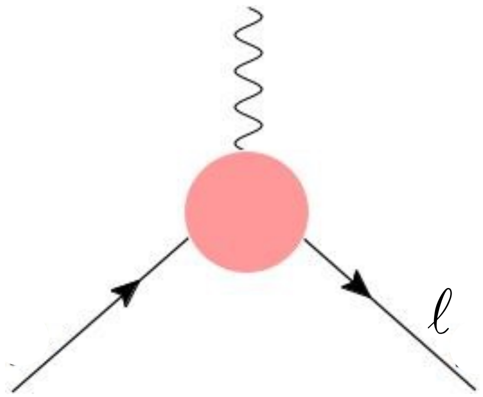


error budget



HVP → *ab-initio* LQCD

Lepton magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

lepton anomalous magnetic moment: $a_\ell \equiv \frac{g_\ell - 2}{2} = F_2(0)$

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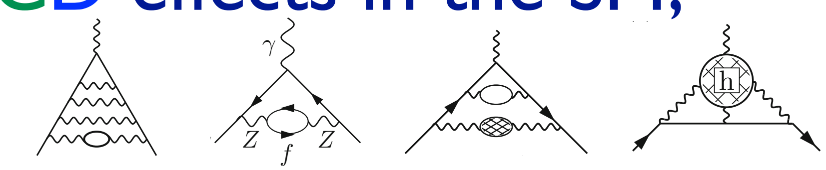


D. Hanneke, 2010; T. Aoyama, 2018; R.H. Parker, 2018

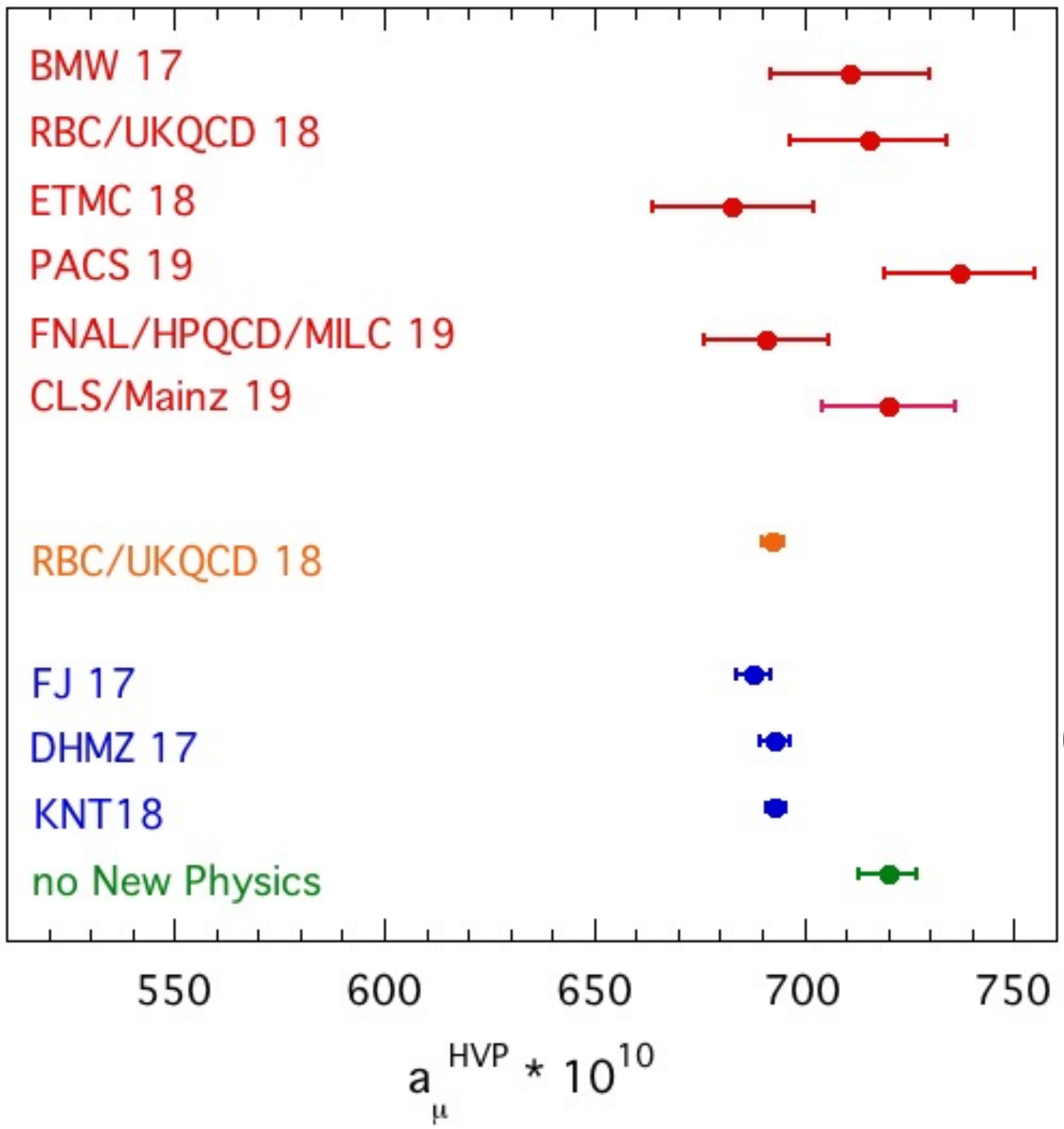
$$a_e^{\exp} = 115\ 965\ 218\ 073(28)\cdot 10^{-14}$$ 0.2ppb

$$a_e^{\mathrm{SM}} = 115\ 965\ 218\ 161(1)(1)(23)\cdot 10^{-14}$$ 0.2ppb

$\sim 2.5\sigma$

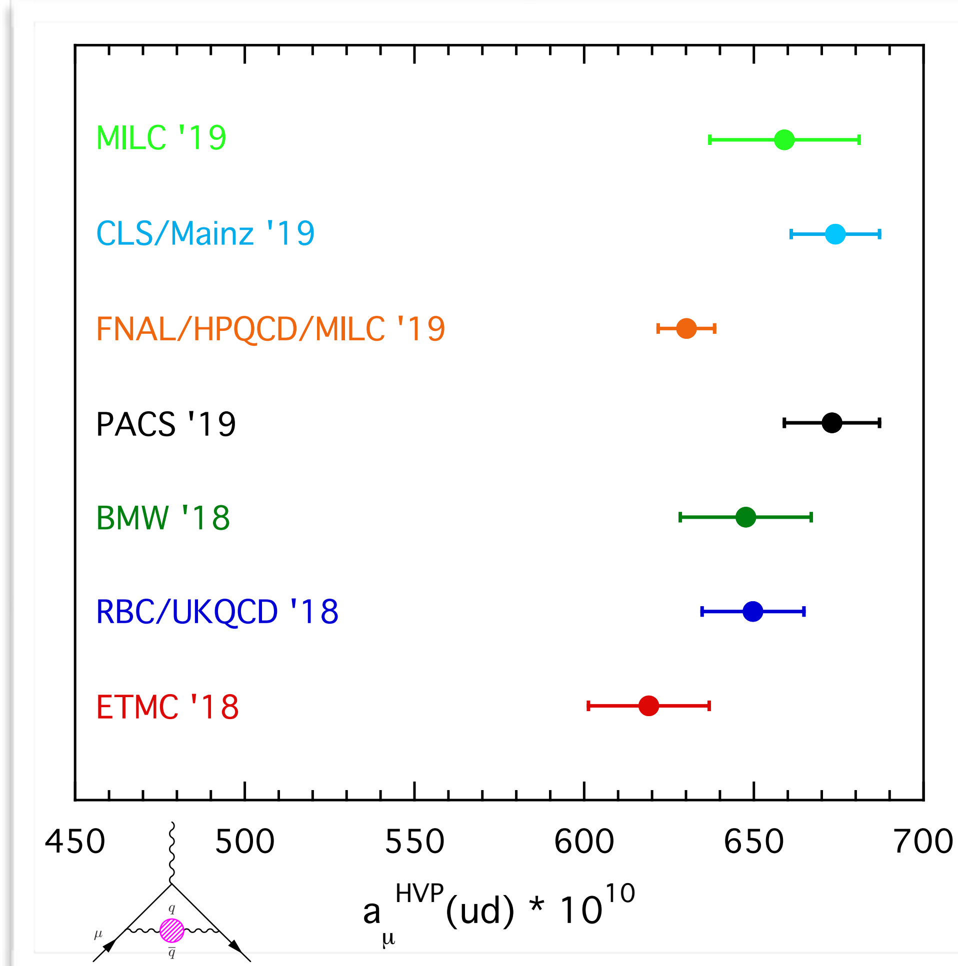
QED (1), Had. + EW (1), α_{em} (23)

Hadronic Vacuum Polarisation

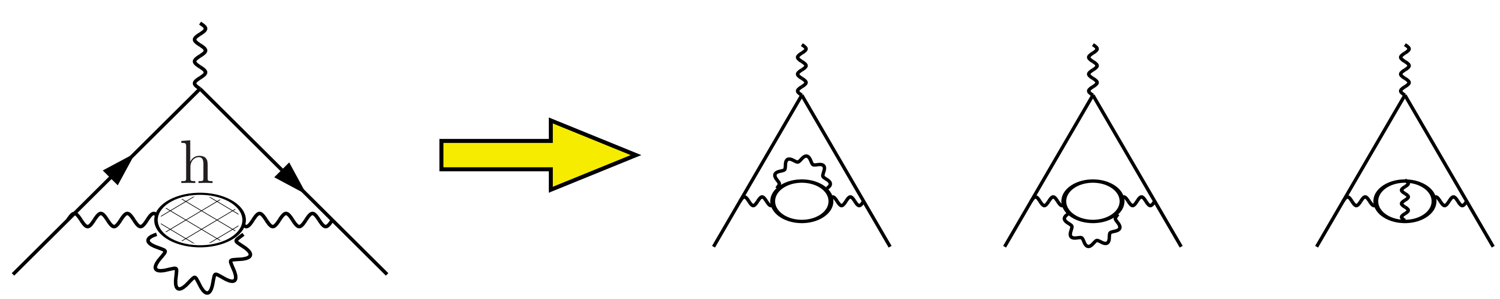


≳ 2%

≳ 0.4%



Given the present exper. and theor. (LQCD) accuracy, an important source of uncertainty are long distance electromagnetic and SU(2) breaking corrections



Lepton anomalous magnetic moments

a_ℓ^{HVP} : Lattice results

Nf=2+1+1
ETMC

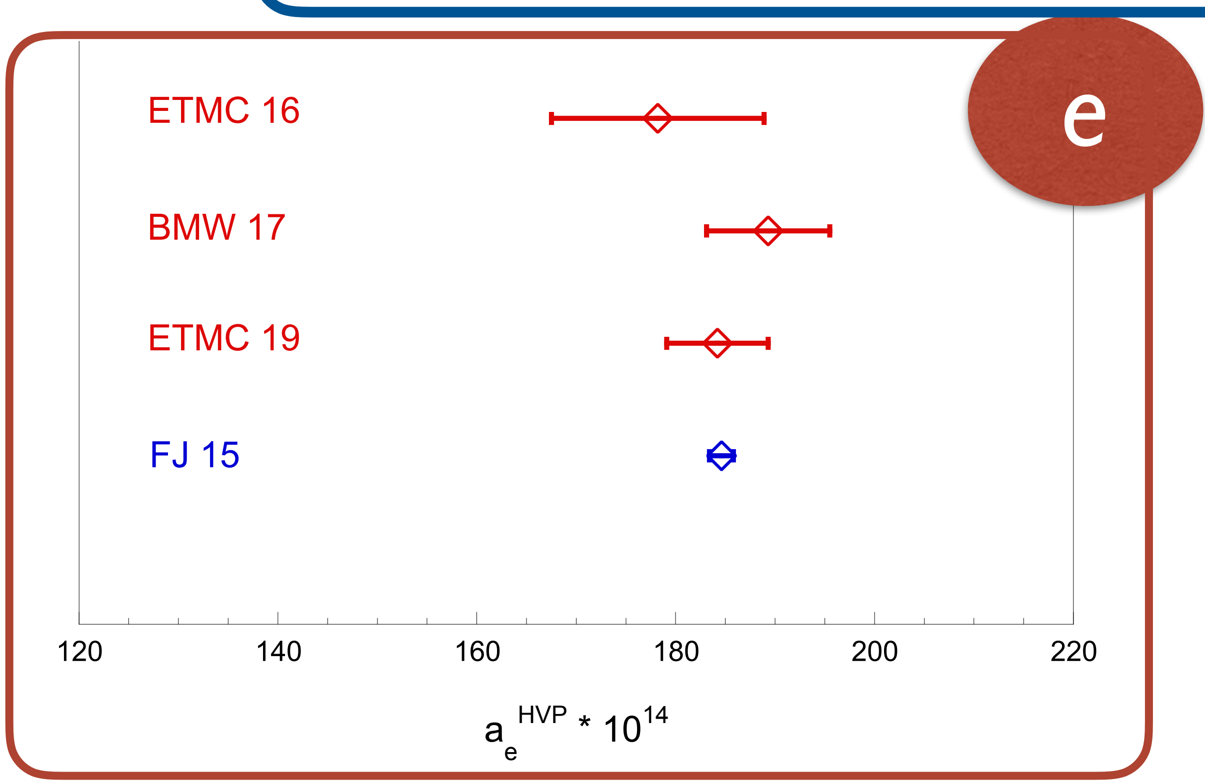
LO

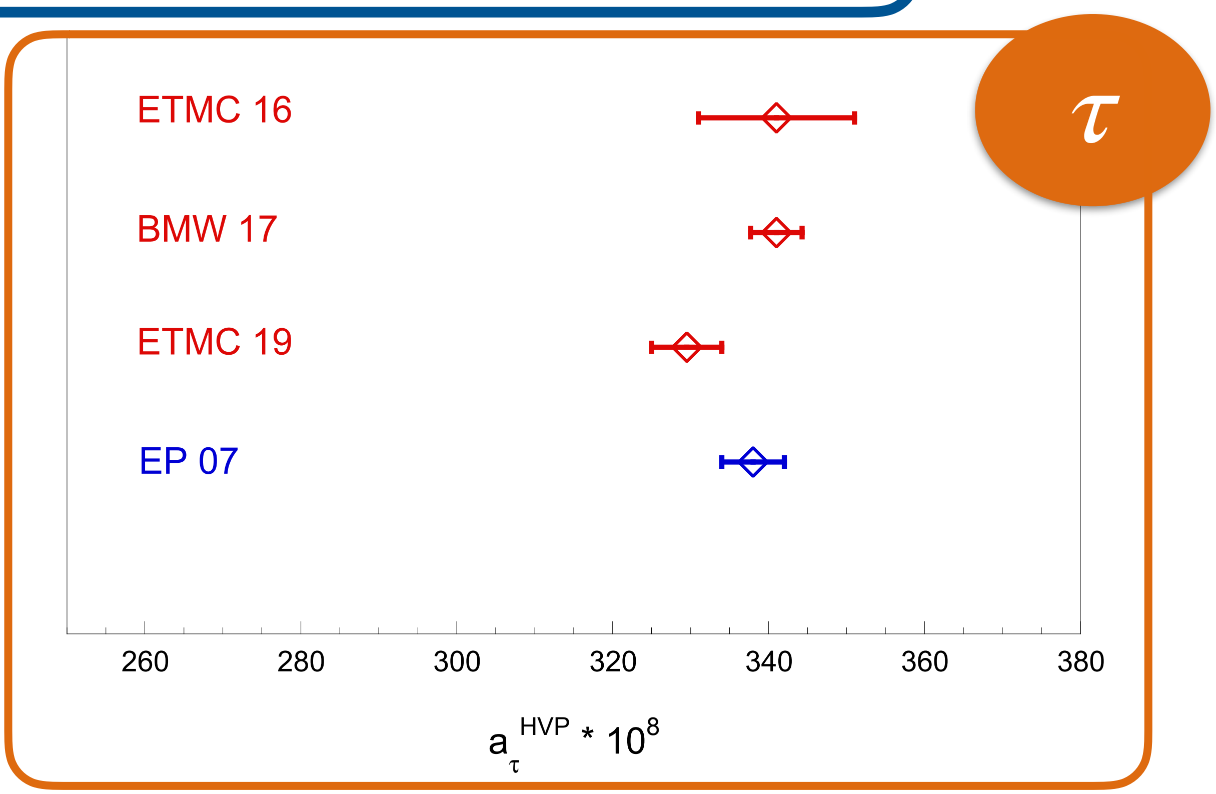
f	$a_e^{\mathrm{LO-HVP}}(f)\cdot 10^{14}$	$a_\mu^{\mathrm{LO-HVP}}(f)\cdot 10^{10}$	$a_\tau^{\mathrm{LO-HVP}}(f)\cdot 10^{8}$
ud	169.1 (4.9)	619.0 (17.8)	266.9 (4.1)
s	13.49 (0.77)	53.1 (2.5)	36.2 (1.1)
c	3.50 (0.16)	14.75 (0.56)	25.8 (0.8)
disc	−3.8 (0.4) BMW17	−12 (4) BMW17 RBC/UKQCD18	−2.4 (0.3) BMW17

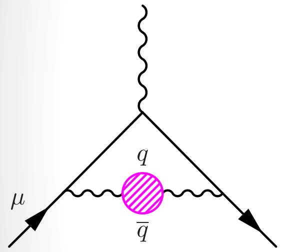
preliminary

IB

f	$\delta a_e^{\mathrm{HVP}}(f)\cdot 10^{14}$	$\delta a_\mu^{\mathrm{HVP}}(f)\cdot 10^{10}$	$\delta a_\tau^{\mathrm{HVP}}(f)\cdot 10^{8}$
ud	1.9 (0.8)	7.1 (2.5)	3.0 (1.1)
s	−0.002 (0.001)	−0.0053 (0.0033)	0.001 (0.002)
c	0.004 (0.001)	0.0182 (0.0036)	0.032 (0.006)
total	1.9 (1.0)	7.1 (2.9)	3.0 (1.3)





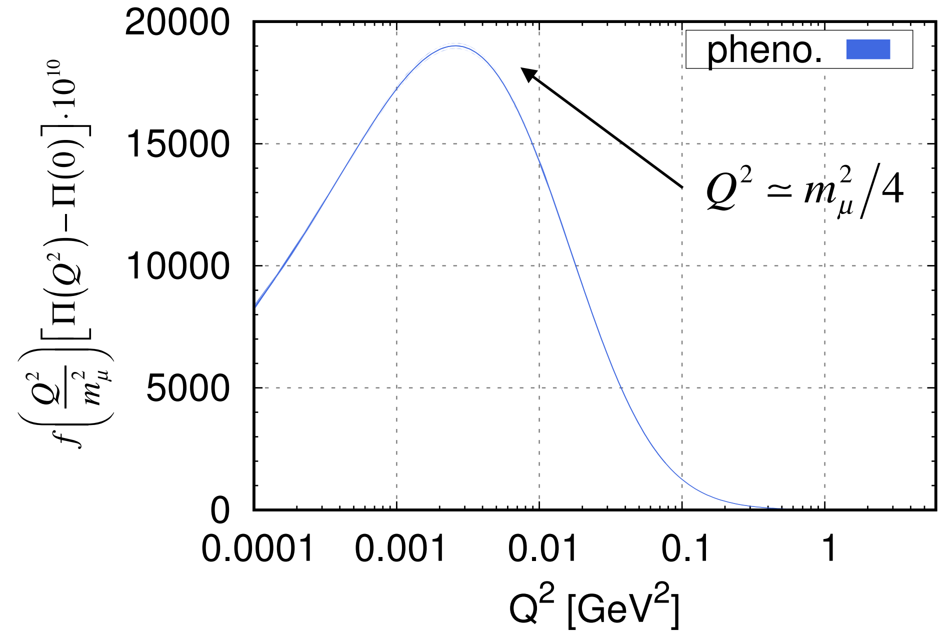


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu \right] \Pi(Q^2)$$

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right) \left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972; T. Blum, 2002



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}(t)\, V(t)$$

D. Bernecker and H. B. Meyer, 2011

$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \, \langle J_i(\vec{x},t) J_i(0) \rangle$$

$$a_\ell^{\mathrm{HVP}} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \tilde{f}(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} \tilde{f}(t) \frac{G_V^f}{2M_V^f} e^{-M_V^f t} \right\}$$

$t \leq T_{data} < T/2$ (avoid bw signals)

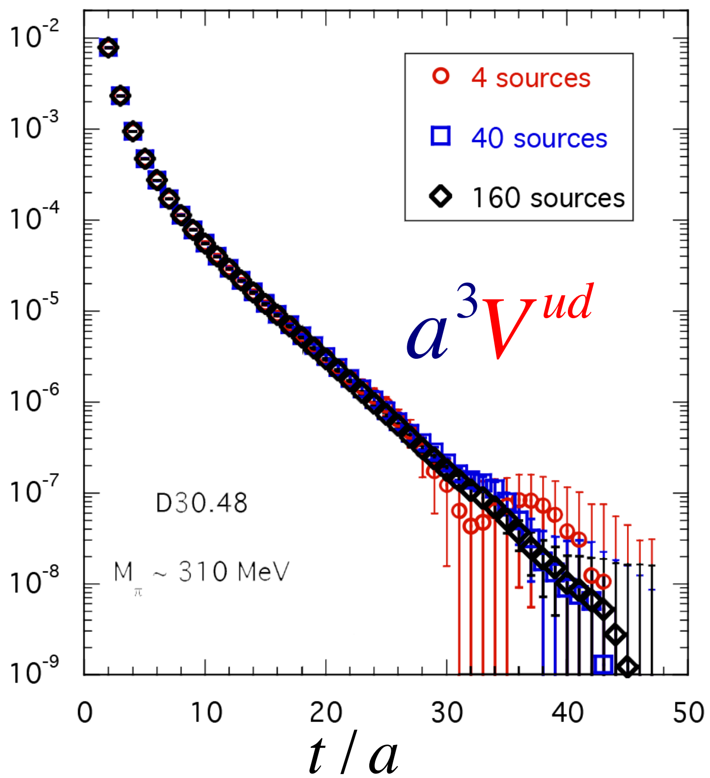
$t > T_{data} > t_{min}$ (ground-state dom.)

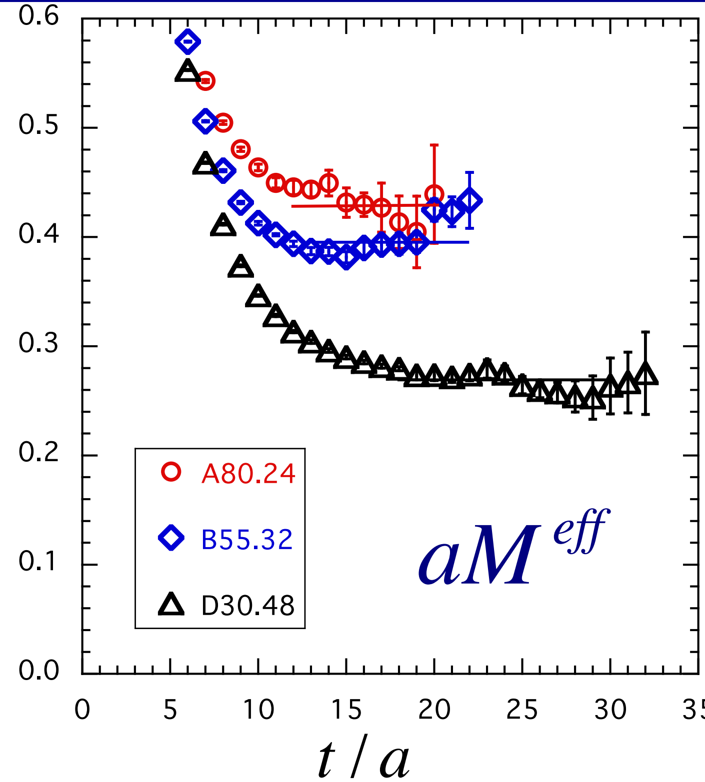
quark-connected terms only

lattice data
local vector currents

analytic representation

Light quark contribution



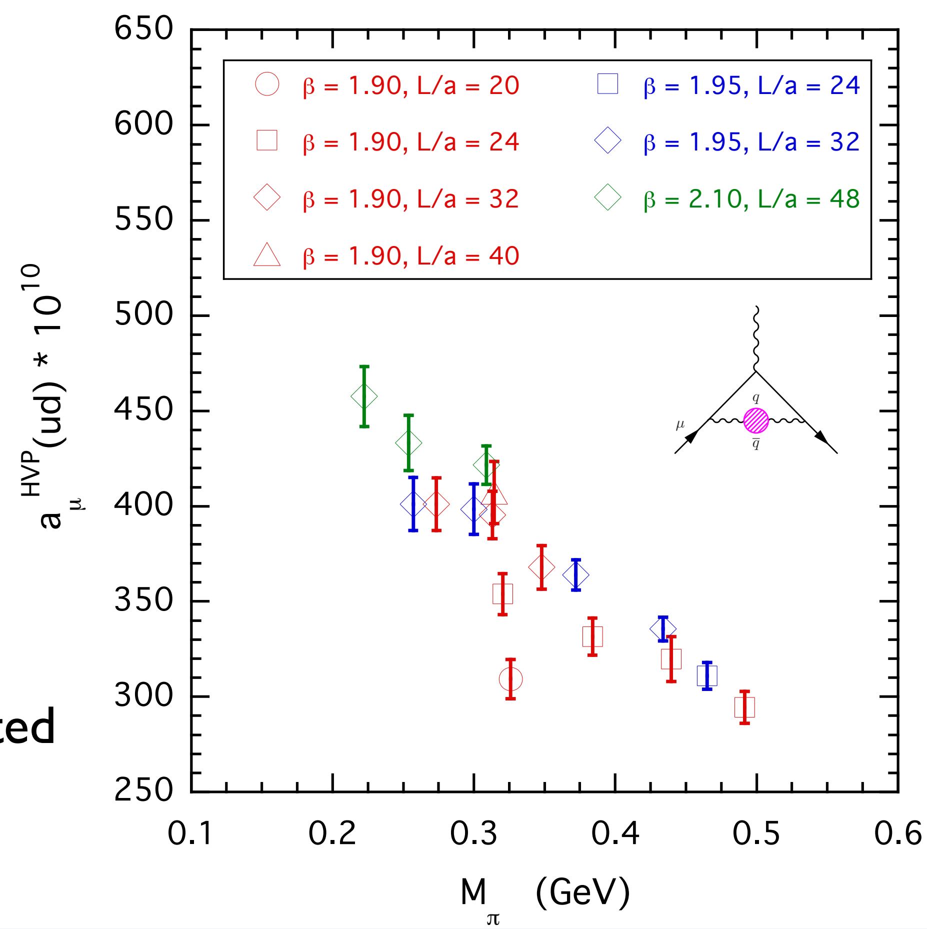


StN: $\propto e^{-(M_\rho - M_\pi)t}$

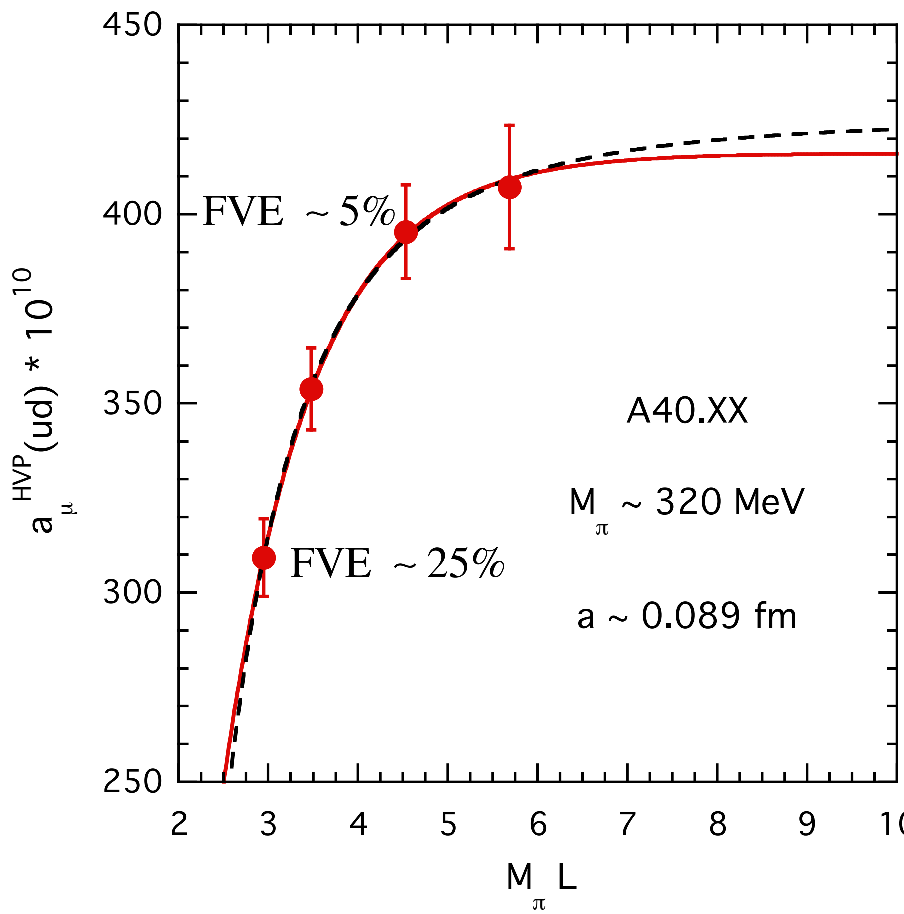
G. Parisi, 1984;
G. P. Lepage, 1989

160 stoch. sources / gauge conf.

DG *et al.*, 2018
PRD98(2018)114504



quark-connected terms only



s-quark contribution
FVE ~ few %
DG *et al.*, 2017

Correlator representation

$$V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)$$

low and intermediate time distances

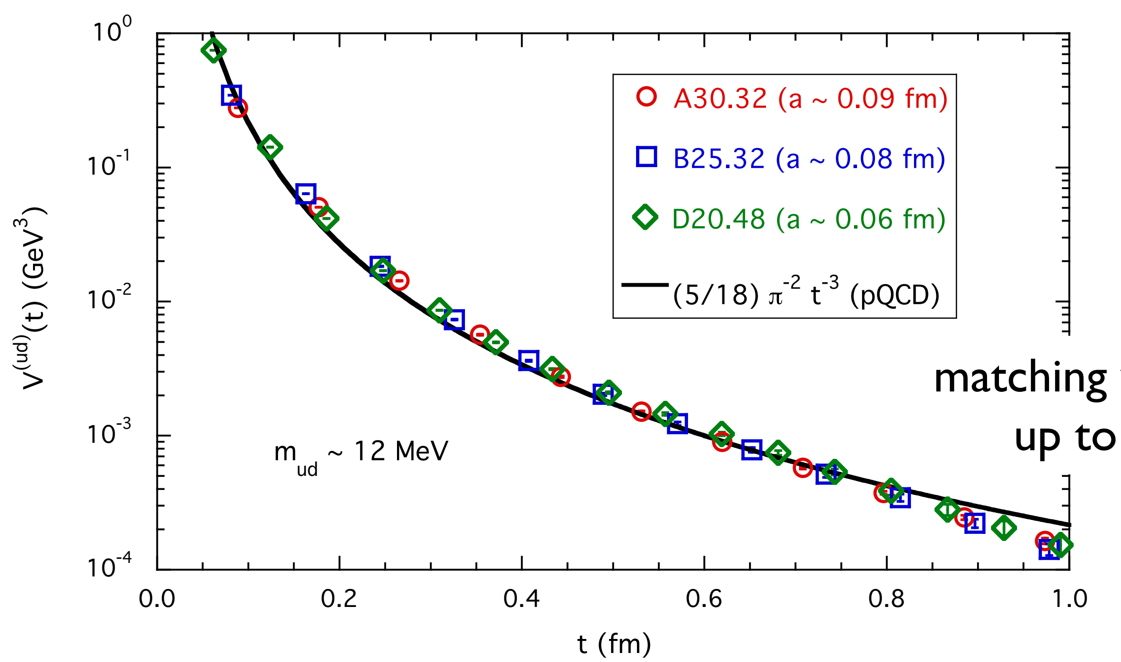
$$V_{dual}(t) \equiv \frac{1}{24\pi^2}\int_{s_{dual}}^{\infty} ds\sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

$$s_{dual} = \left(M_\rho + E_{dual}\right)^2 \qquad R_{dual} = 1 + O\left(\frac{m_{ud}^4}{s_{dual}^2}\right) + O(\alpha_s) + O(a)$$

$$V_{dual}(t) = \frac{5}{18\pi^2}\frac{R_{dual}}{t^3} e^{-(M_\rho + E_{dual})t}\left[1 + \left(M_\rho + E_{dual}\right)t + \frac{1}{2}\left(M_\rho + E_{dual}\right)^2 t^2\right]$$

quark-hadron duality *à la SVZ*

SVZ, 1979



matching with pQCD up to t ~1 fm

long time distances

$$V_{\pi\pi}(t) = \sum_n \nu_n \left|A_n\right|^2 e^{-\omega_n t}$$

M. Lüscher 1991

$$\omega_n = 2\sqrt{M_\pi^2 + k_n^2}$$

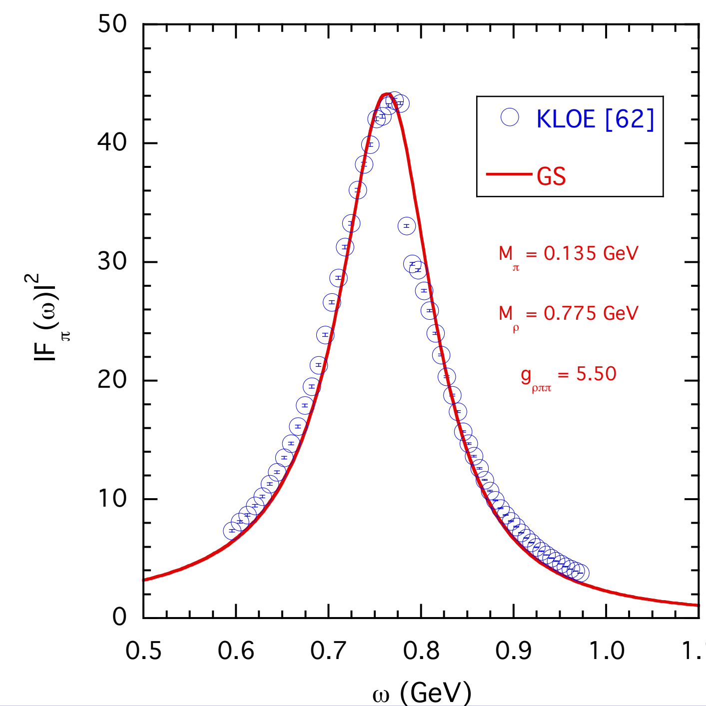
↘ Lüscher condition

L. Lellouch and M. Lüscher, 2001

$$\left|A_n\right|^2 \;\rightarrow\; \left|F_\pi(\omega_n)\right|^2$$

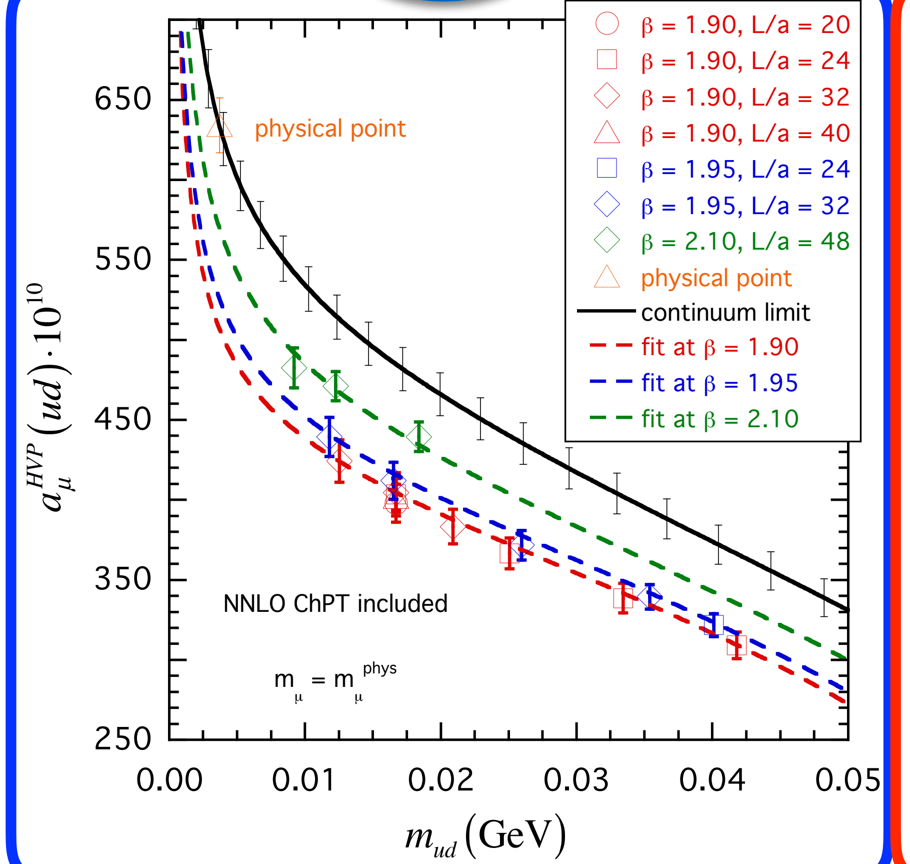
Gounaris-Sakurai parameterization

$M_\rho \quad g_{\rho\pi\pi}$ GS, 1968

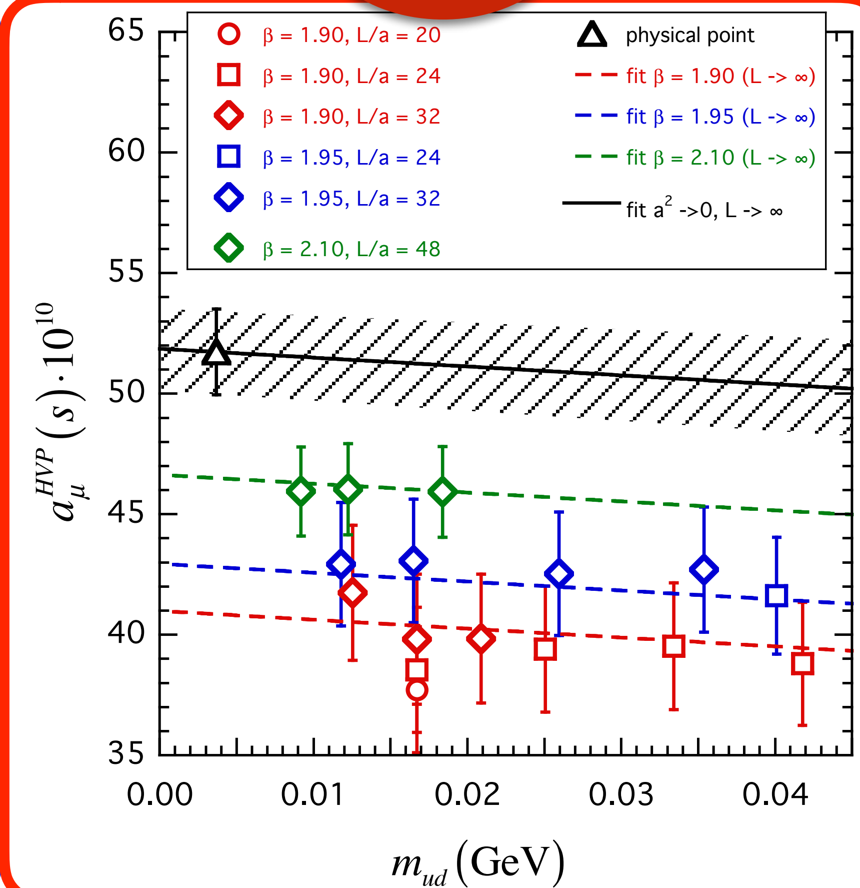


udsc-quark contributions

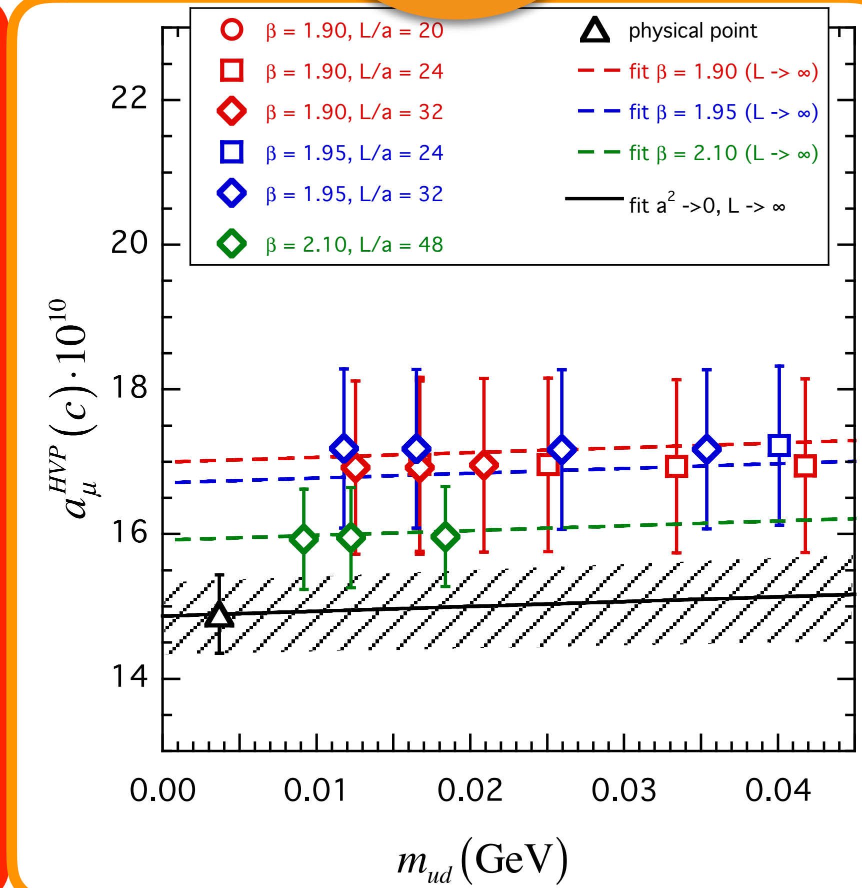
ud



s



c



$$a_\mu^{\text{LO-HVP}}(ud) = 619.0(17.8)\cdot 10^{-10}$$

$$a_\mu^{\text{LO-HVP}}(s) = 53.1(2.5)\cdot 10^{-10}$$

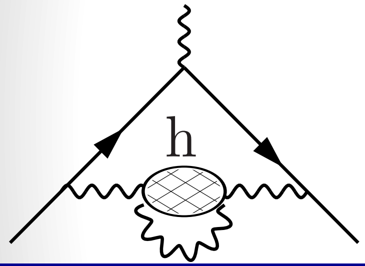
$$a_\mu^{\text{LO-HVP}}(c) = 14.75(0.56)\cdot 10^{-10}$$

DG *et al.*, 2018
PRD98(2018)114504

DG *et al.*, 2017
JHEP1710(2017)157

quark-connected terms only

Isospin-breaking corrections to a_ℓ^{HVP}



LIB corrections

quark-connected terms only

$$\delta a_\ell^{\rm HVP} = \delta a_\ell^{\rm HVP}(QCD) + \delta a_\ell^{\rm HVP}(QED)$$

photon zero-mode: QED_L

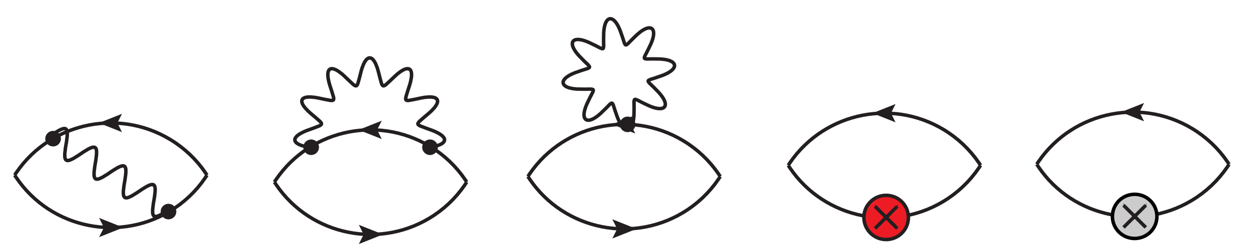
M. Hayakawa and S. Uno, 2008

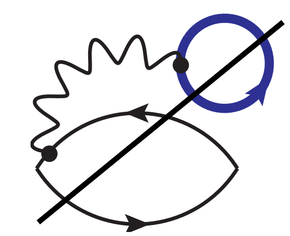
$$\delta a_\ell^{\rm HVP}(QCD) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QCD}(t)$$

$$\delta a_\ell^{\rm HVP}(QED) = 4\alpha_{em}^2 \sum_{t=0}^{\infty} \tilde{f}(t)\,\delta V^{QED}(t)$$

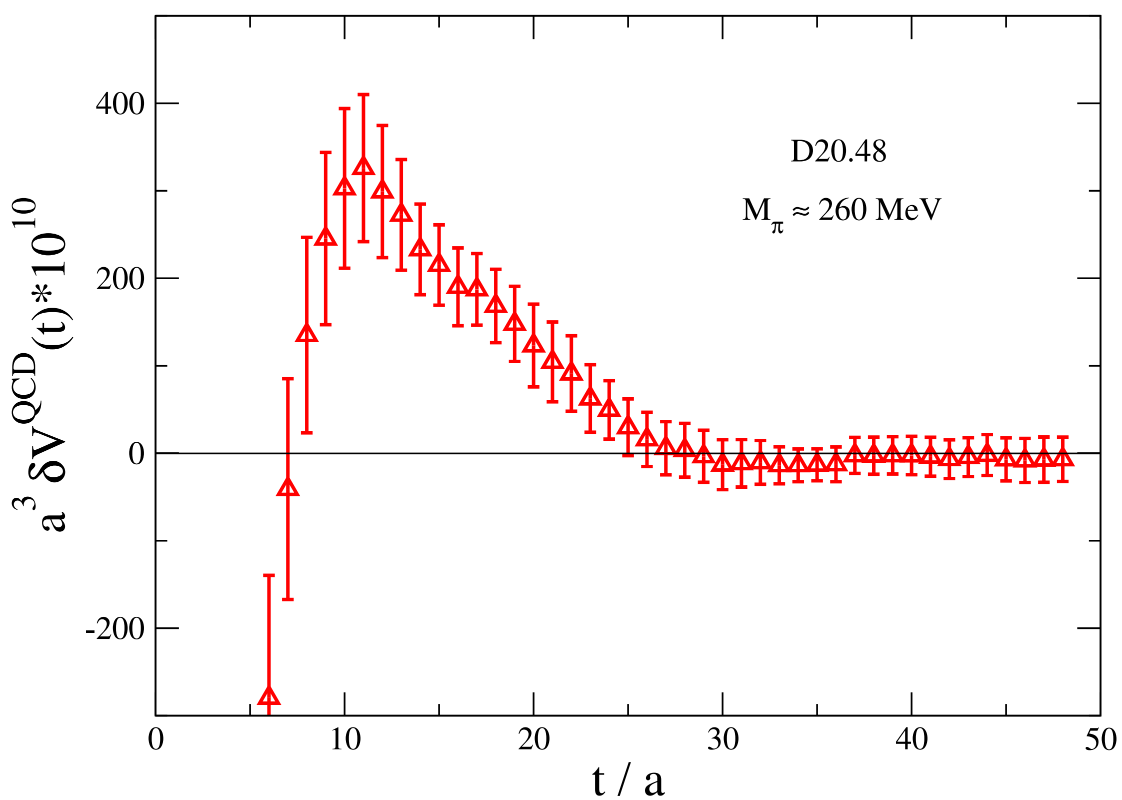
<u>RM123 method</u>

G. M. de Divitiis *et al.*, 2012; 2013



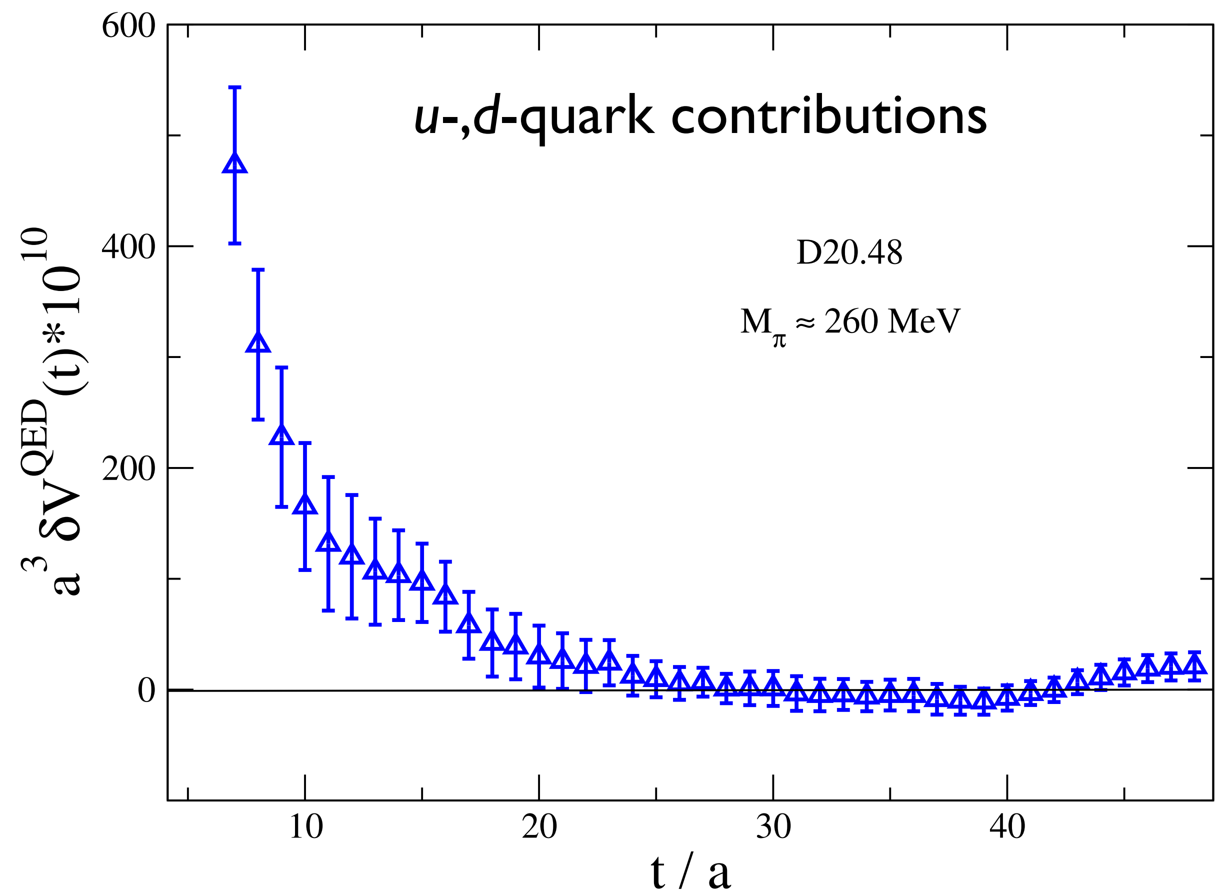


qQED approximation

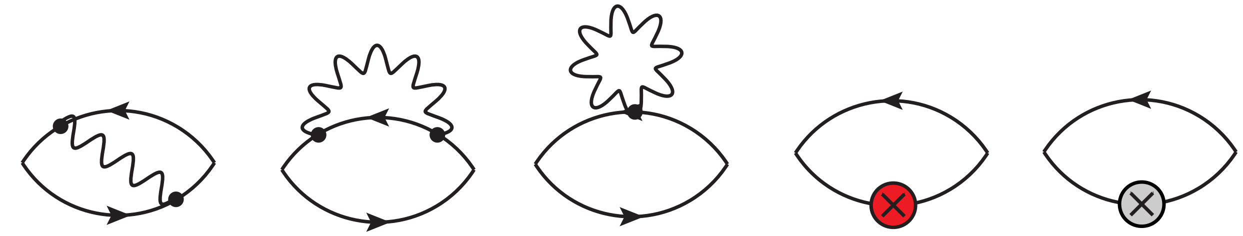


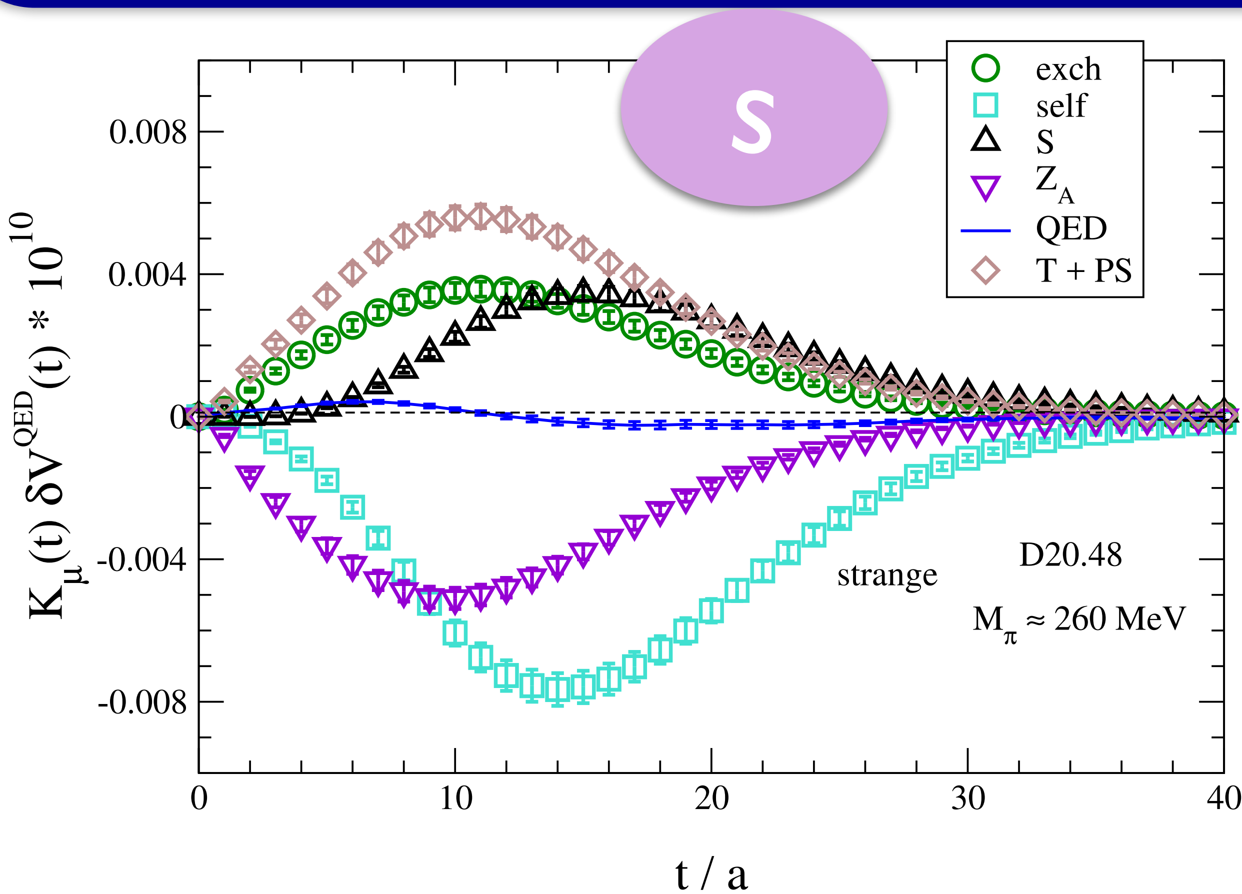
$$[m_d - m_u](\overline{MS}, 2\ \text{GeV}) = 2.38(0.18)\ \text{MeV}$$

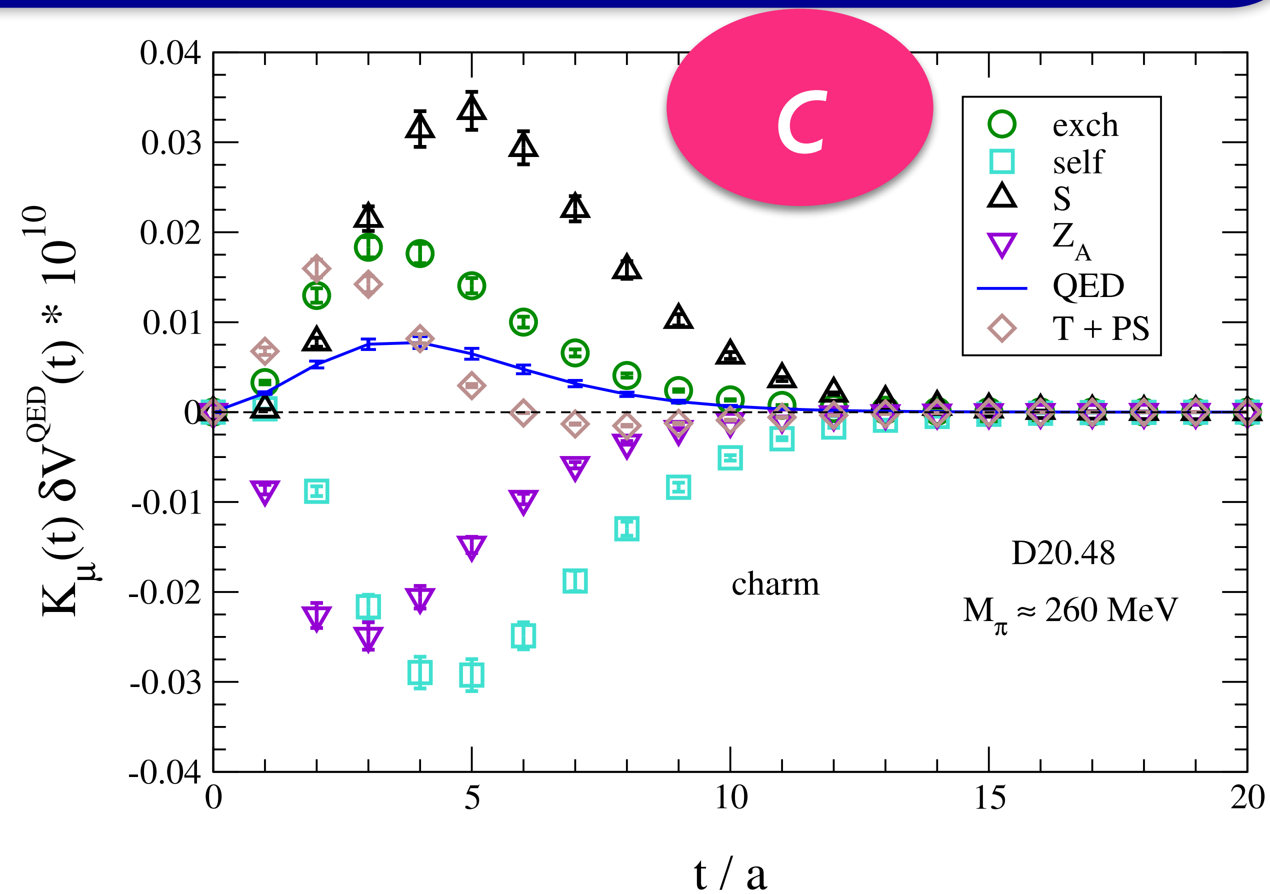
DG *et al.*, 2017



LIB corr.:







$$Z_A = Z_A^{(0)}\left(1+\frac{\alpha_{em}}{4\pi}\delta Z_A^{QED} Z_A^{fact}\right)+O\left(\alpha_{em}^m \alpha_s^n\right)$$

$$\delta Z_A^{QED} = -15.7963\ q_f^2$$

perturbative estimate at LO

G. Martinelli and Y.-C. Zhang, 1982

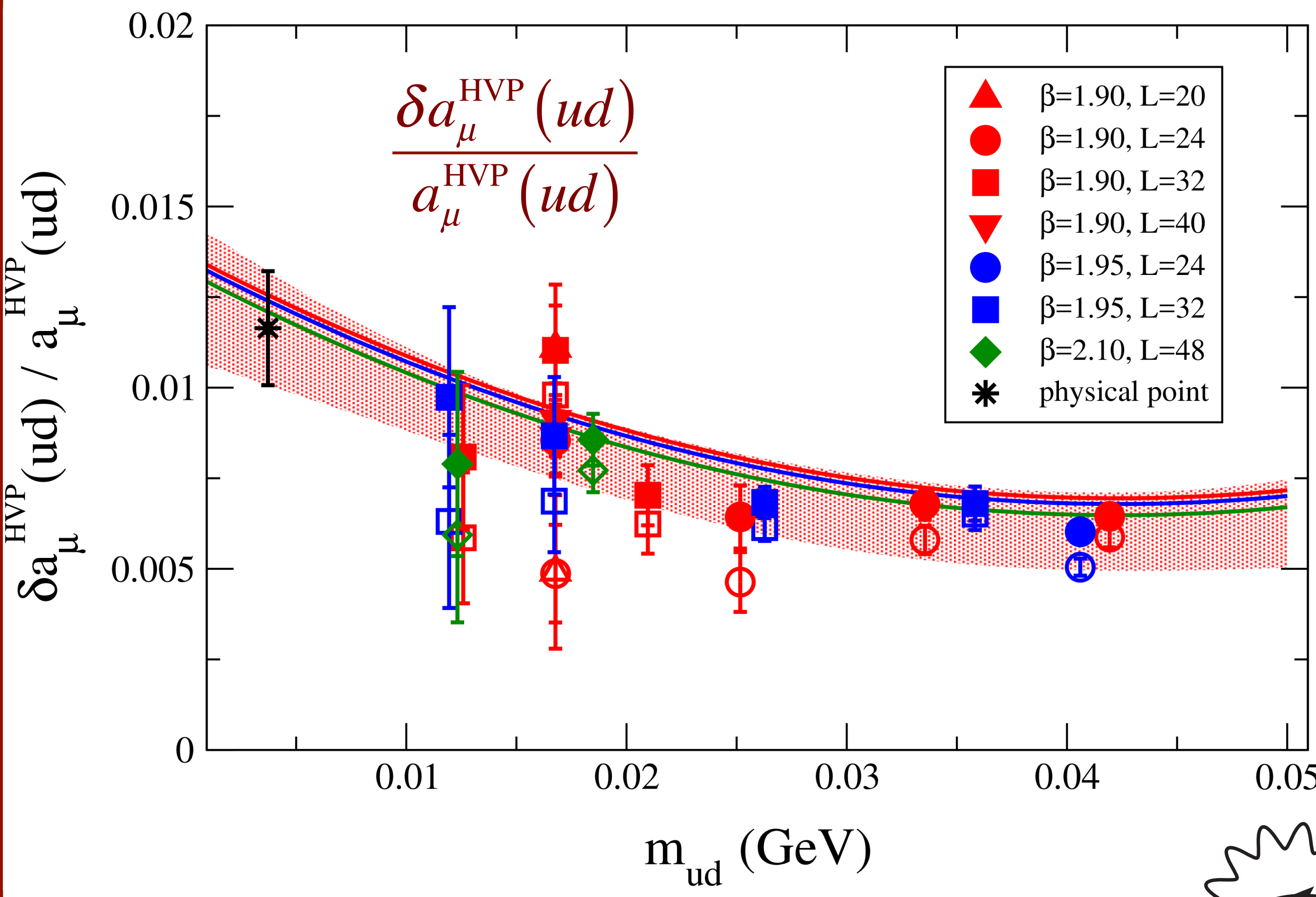
$$\delta V_f^{Z_A}(t) = \frac{\alpha_{em}}{4\pi}\delta Z_A^{QED} Z_A^{fact}\ V^f(t)$$

β	Z_m^{fact} (M1)	Z_A^{fact} (M1)	Z_m^{fact} (M2)	Z_A^{fact} (M2)
1.90	1.629 (41)	0.859 (15)	1.637 (14)	0.990 (9)
1.95	1.514 (33)	0.873 (13)	1.585 (12)	0.980 (8)
2.10	1.459 (17)	0.909 (6)	1.462 (6)	0.958 (3)

RI'-MOM @ $O\left(\alpha_{em}\alpha_s^n\right)$

see talk by M. Di Carlo, Tue 17.30

LIB corr.: *udsc*-quark contributions

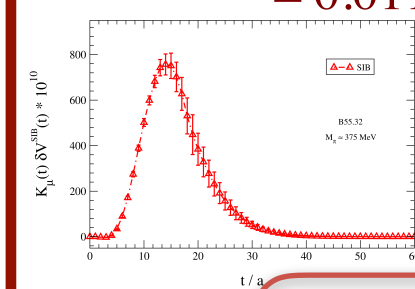


DG *et al.*, 2019; PRD99(2019)114502

in the ratio various systematics cancel out

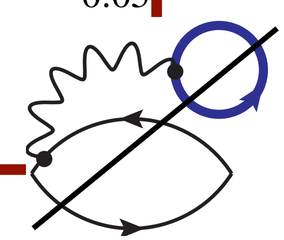
$$\frac{\delta a_\mu^{HVP}(ud)}{a_\mu^{HVP}(ud)} = 0.0115(18)_{stat+fit}(21)_{input}(20)_{chir}(19)_{FVE}(9)_{cont}$$

$$= 0.0115(40)$$



$$a_\mu^{HVP}(ud) = 619.0(17.8)\cdot 10^{-10}$$

$$\delta a_\mu^{HVP}(ud) = 7.1(2.5)\cdot 10^{-10}$$

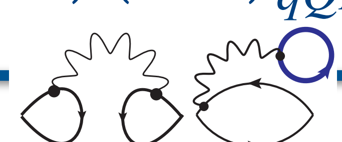


quark-connected terms only

$$\delta a_\mu^{HVP}(s) = -0.0053(33)\cdot 10^{-10}$$

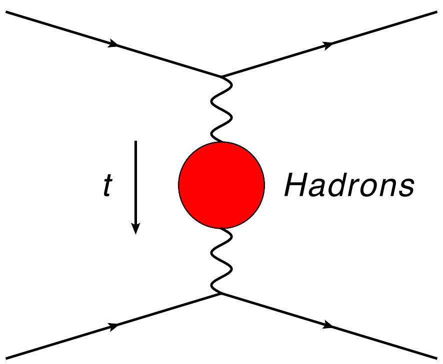
$$\delta a_\mu^{HVP}(c) = 0.0182(36)\cdot 10^{-10}$$

$$\delta a_\mu^{HVP} = 7.1(2.6)(1.2)_{qQED+disc}\cdot 10^{-10} = 7.1(2.9)\cdot 10^{-10}$$



MUonE experiment

MUonE



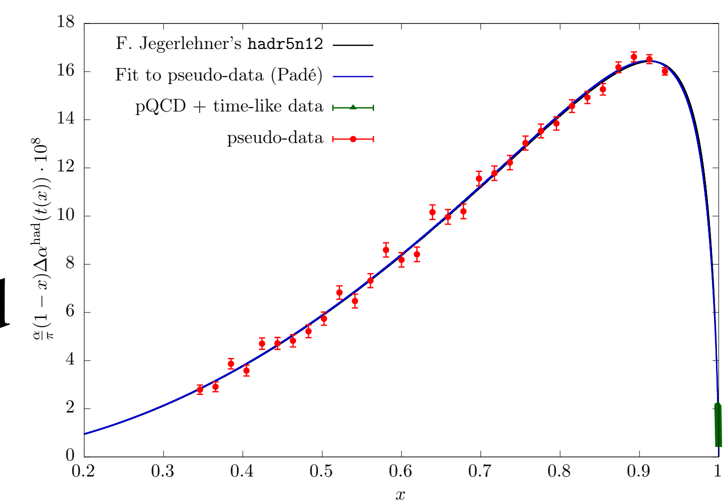
B. E. Lautrup *et al.*, 1972

$$a_\mu^{\rm HVP} = \frac{\alpha_{em}}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{em}^{\rm HVP}\left[t(x)\right]$$

$$\sigma(\mu e \to \mu e)$$

$$t(x) \equiv -\frac{x^2}{1-x}m_\mu^2$$

$x \in [0.93,\ 1]$ not experimentally reached



LQCD

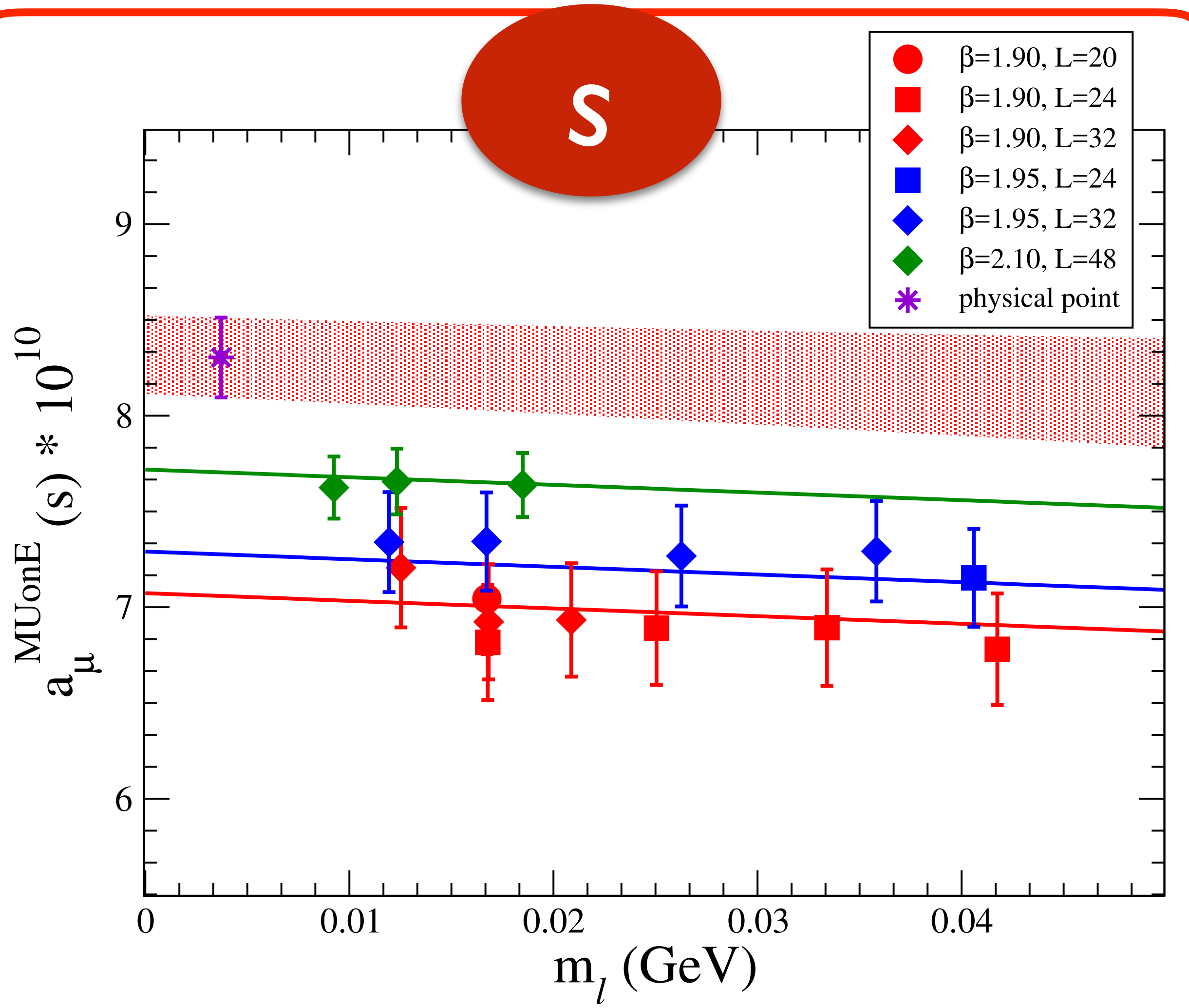
Using the (dual + π-π) repr.

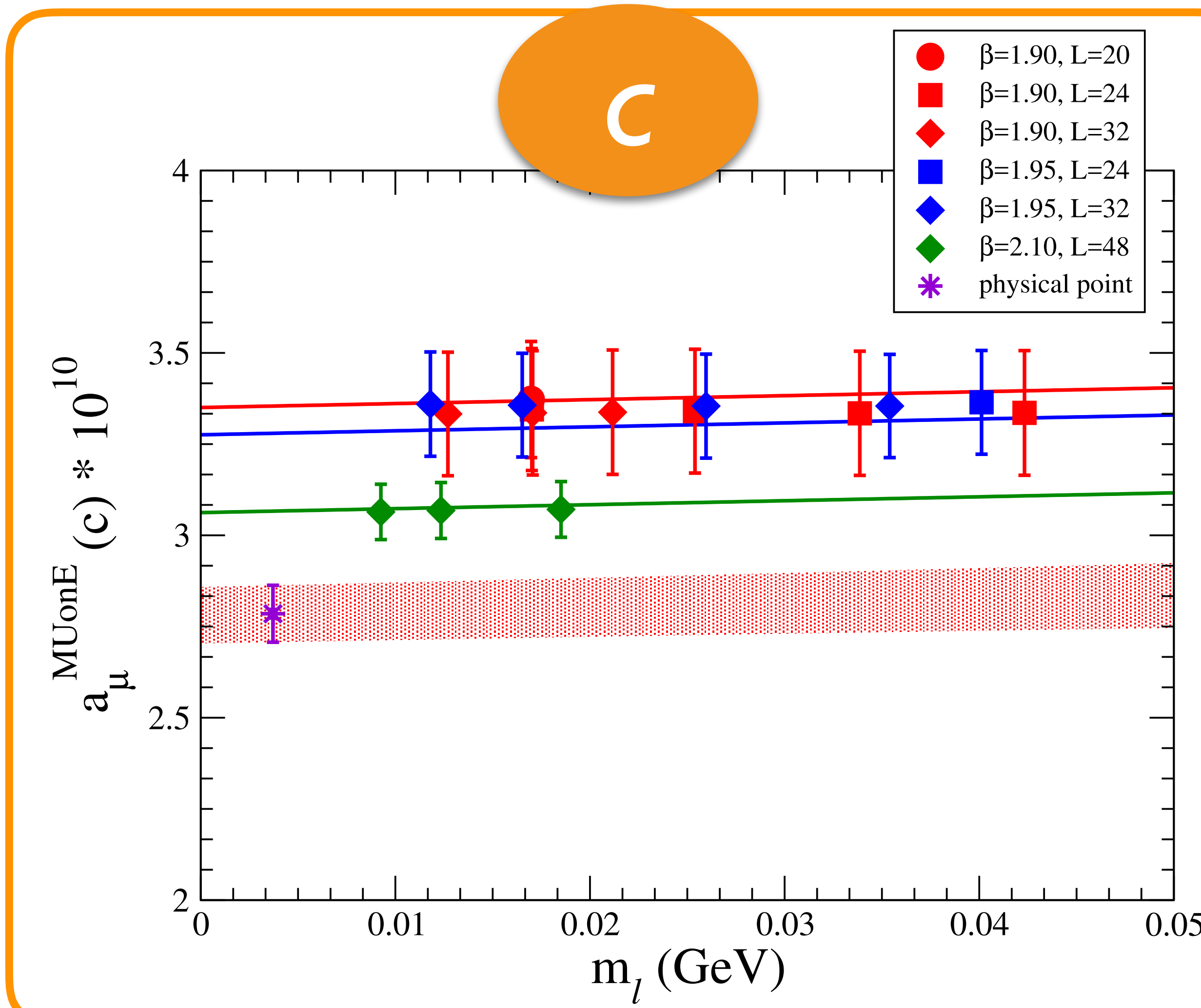
$$\left[a_\mu^{\rm HVP}\right]_> = 4\alpha_{em}^2\int_0^\infty dt\, f_>(t) V(t) \quad\rightarrow\quad \left[a_\mu^{\rm LO\text{-}HVP}\right]_>(ud) = 81.2(1.7)\cdot 10^{-10}$$

quark-connected terms only

Uncertainty $(\simeq 2\cdot 10^{-10})$ close to the experimental statistical target $(\simeq 0.3\%)$ of $\left[a_\mu^{\rm HVP}\right]_<$

MUonE





$$\left[a_\mu^{\text{LO-HVP}}\right]_>(s) = 8.30(21)_{stat}(32)_{syst} \cdot 10^{-10}$$

$$\left[a_\mu^{\text{LO-HVP}}\right]_>(c) = 2.785(78)_{stat}(68)_{syst} \cdot 10^{-10}$$

f	$[\delta a_\mu^{\text{HVP}}]_>(f) \cdot 10^{10}$
ud	0.9 (0.3)
s	−0.0005 (0.0004)
c	0.0034 (0.0007)
total	0.9 (0.3)

$$\left[a_\mu^{\text{HVP}}\right]_> = 92(2) \cdot 10^{-10}$$

preliminary

Conclusions

- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) ➔ LQCD
- We have performed a first-principles lattice QCD+QED calculation of a_ℓ^{HVP}. Our result agrees with recent determinations based on dispersive analyses.

RM123 method

$$a_e^{\mathrm{HVP}} = 184(5)\cdot 10^{-14} \qquad a_\mu^{\mathrm{HVP}} = 682(19)\cdot 10^{-10} \qquad a_\tau^{\mathrm{HVP}} = 330(5)\cdot 10^{-8}$$

preliminary

$$\left[a_\mu^{\mathrm{HVP}}\right]_> = 92(2)\cdot 10^{-10} \qquad \Pi_1^{tot} = 0.100(3)\ \mathrm{GeV}^{-2}$$

FHM19

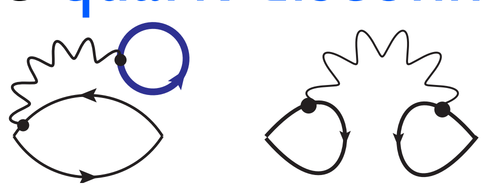
$$\Pi_1^{tot} = 0.1000(23)\ \mathrm{GeV}^{-2}$$

$$\Pi_1^{tot} = 0.1000(30)\ \mathrm{GeV}^{-2}$$

BMW17

In progress...

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation



- development of an analytic representation of the correlator for *s*- and *c*-quark contributions
- use of the new ETMC lattice setup @ the physical pion point