Accessing flavor-singlet quark and gluon parton distributions from lattice QCD

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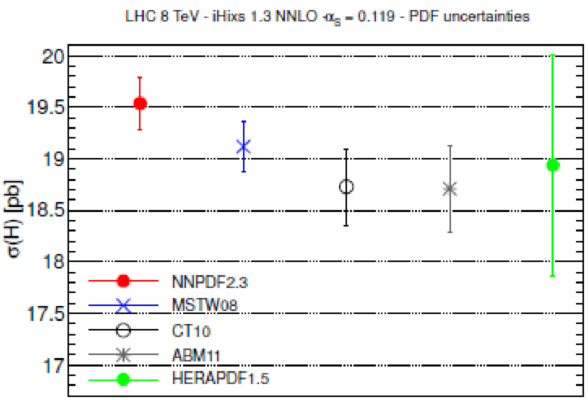
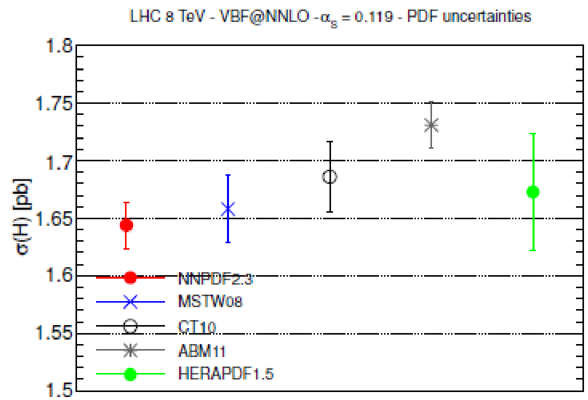
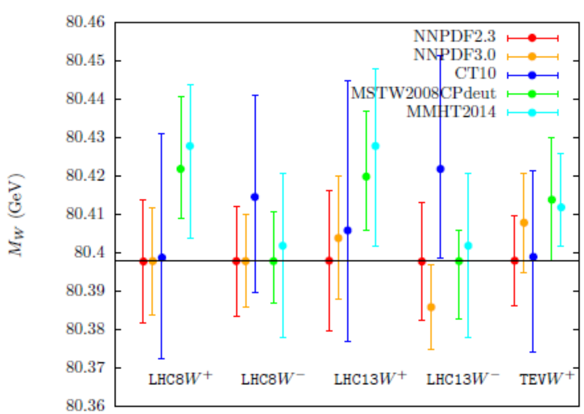
Introduction

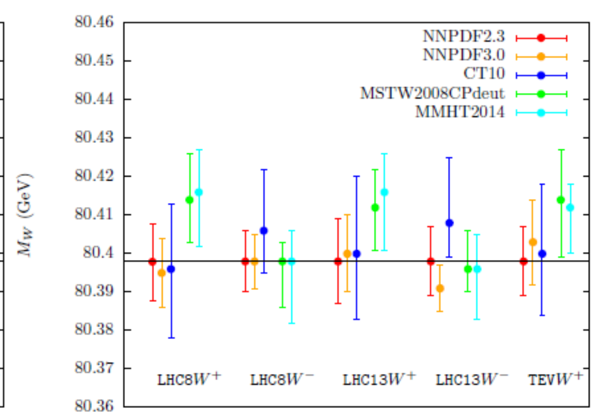
- Hadron structure simplifies at high energy, and can be characterized by certain parton quantities such as the PDFs
- Precise knowledge of PDFs helps
  - better understand the SM and disentangle new physics effects
    - Higgs production and self coupling





    - W mass determination





Introduction

- Hadron structure simplifies at high energy, and can be characterized by certain parton quantities such as the PDFs
- Precise knowledge of PDFs helps
  - three-dim. Imaging of the nucleon

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701

- *Ab initio* determination of PDFs from lattice QCD is desirable and provides complementary information to global analysis program

Introduction

- Parton quantities such as the PDFs are difficult to access on the lattice
  - Defined on the light-cone
  - Example [Collins and Soper, NPB 82'] ($\xi^{\pm} = (t \pm z)/\sqrt{2}$)

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

$$t^2 - x^2 = 0 \rightarrow -\tau^2 - x^2 = 0$$

- However, they were originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

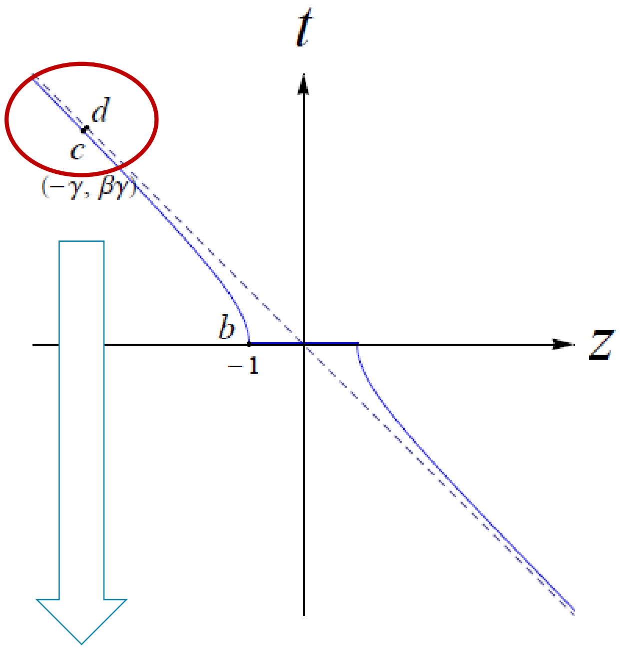
$$q(x) = \lim_{P_z \rightarrow \infty} \tilde{q}(x, P_z)$$

- Boost to infinite momentum leads to light-cone correlations

- If we can construct a $\tilde{q}(x, P_z)$ such that it is calculable on the lattice, and all P_z-dependence can be systematically removed

- Then we can calculate $q(x)$!

Introduction



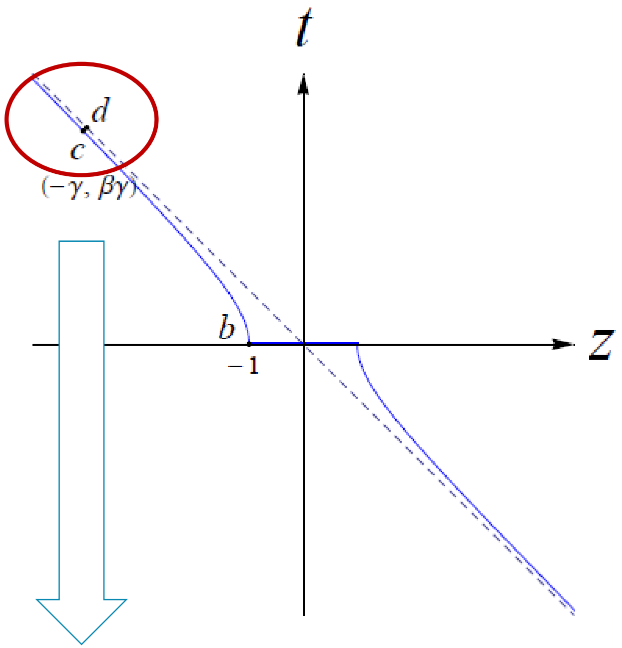
- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
  - Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice, e.g.

$$\tilde{q}(x, \Lambda, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0, 0_\perp, z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0, 0_\perp, z')\right)\psi(0)|P\rangle$$

  - A finite but large P_z already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

$$\tilde{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

Introduction



- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']
- Parton model is an effective theory for the nucleon moving at large momentum

Other proposals

- Current-current correlation functions
- [Liu and Dong, PRL 94']
- [Detmold and Lin, PRD 06']
- [Braun and Müller, EPJC 08']
- [Davoudi and Savage, PRD 12']
- [Chambers et al., PRL 17']

- Lattice cross sections
- [Ma and Qiu, 14' & PRL 17']

- Ioffe-time /pseudo-distribution
- [Radyushkin, PRD 17']

- **They share similar spirit of computing correlations at spacelike separations**

PDFs from LaMET

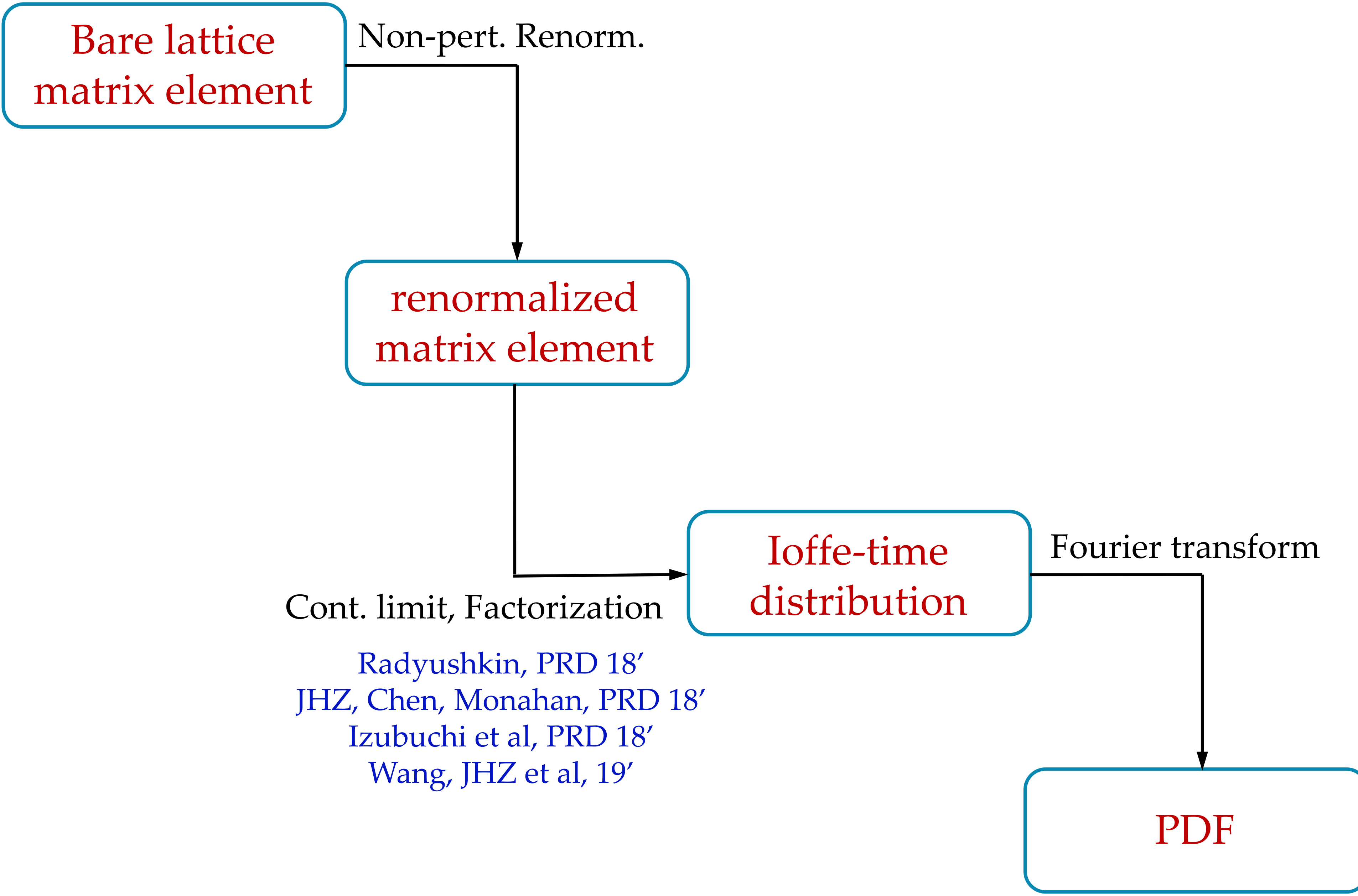
Bare lattice matrix element

Non-pert. Renorm.

renormalized matrix element

Ji, JHZ, Zhao, PRL 18'
Ishikawa et al, PRD 17'
Green et al, PRL 18'
Stewart, Zhao, PRD 18'
Chen, JHZ et al, PRD 18'
Alexandrou et al, NPB 17'
Monahan, Orginos, JHEP 17'
Radyushkin PRD 17' & Orginos et al, PRD 17'
JHZ et al, PRL 19' & Wang, JHZ et al, 19'
Li et al, PRL 19'

PDFs from LaMET



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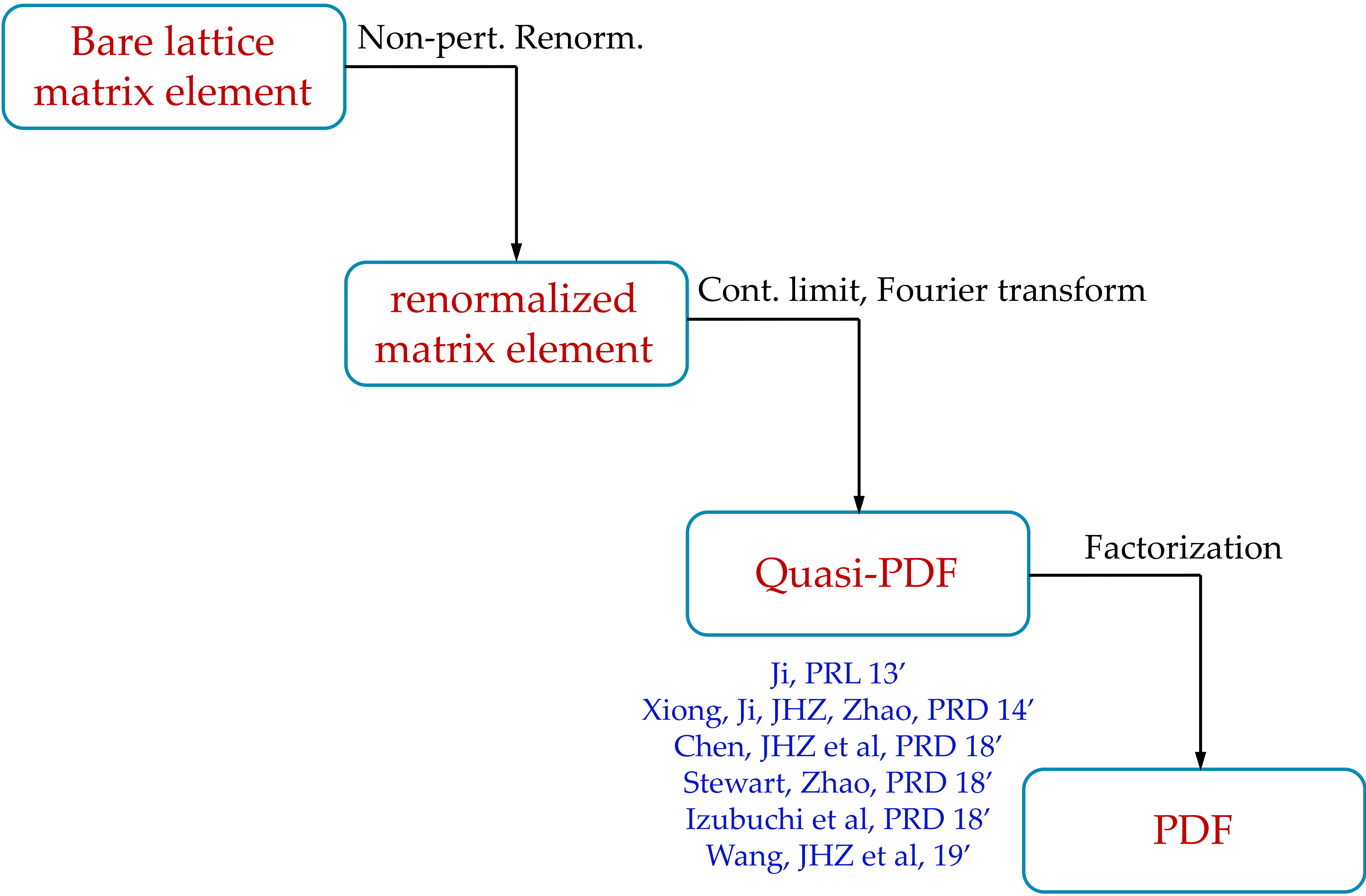

Gluon quasi-PDFs

- Gluon PDF (unpol.) [Collins, Soper, NPB 82′]

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ixP^+\xi^-} \langle P|F_a^{+i}(\xi^-)\mathcal{W}(\xi^-,0)F_a^{+i}(0)|P\rangle$$

- Naively expected gluon quasi-PDF operators

$$O_g^{\mu\nu}(z,0) = F^{\mu\alpha}(z)\mathcal{W}(z,0)F_\alpha^{\ \nu}(0)$$

  - $\{\mu,\nu\} = \{z,t\}$
  - They mix in general with other operators under renormalization

- Auxiliary field approach [Dorn, Fortsch. Phys. 86′, Ji, JHZ, Zhao, PRL 18′]

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \overline{\mathcal{Q}}(x) i n\cdot D \mathcal{Q}(x)$$

  - For a space like n, no dynamical evolution for $\mathcal{Q}$
  - The two-point function of $\mathcal{Q}$ is

$$\int \mathcal{D}\overline{Q}\mathcal{D}Q\, Q(x)\overline{Q}(y) e^{i\int d^4x\mathcal{L}} = S_Q(x,y) e^{i\int d^4x \mathcal{L}_{\text{QCD}}}$$

with

$$n\cdot D\, S_Q(x,y) = \delta^{(4)}(x-y)$$

Gluon quasi-PDFs

- Solution

$$S_Q(x,y) = \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x,y)$$
$$= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x^z,y^z)$$

  - δ-function ensures that the time and transverse components are equal, and therefore generates a spacelike Wilson line

  - The non-local gluon quasi-PDF operator can be replaced by a product of two local composite operators

$$\mathcal{O}_g^{(3)}(z_2,z_1) = J_1^{ti}(z_2)\overline{J}_{1,i}{}^{z}(z_1)$$

$$J_1^{ti}(z_2) = F_a^{ti}(z_2)\mathcal{Q}_a(z_2), \quad \overline{J}_{1,i}{}^{z}(z_1) = \overline{\mathcal{Q}}_b(z_1)F_{b,i}{}^{z}(z_1)$$

  - After integrating out $\mathcal{Q}$, the gluon quasi-PDF is recovered

Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76′, Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - For $J_1^{\mu\nu}$, the operators allowed to mix are

$$J_2^{\mu\nu} = n_\rho (F_a^{\mu\rho} n^\nu - F_a^{\nu\rho} n^\mu) \mathcal{Q}_a / n^2,$$
$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in \cdot D - m)\mathcal{Q})_a / n^2,$$

  - The mass term might be absent in DR, but can be generated by radiative corrections in a cutoff regularization such as lattice regularization

  - General mixing pattern

$$\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix},$$

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  - Renormalization constants are not all independent

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, \qquad J_{1,R}^{ti} = Z_{11} J_1^{ti}, \quad J_{1,R}^{ij} = Z_{11} J_1^{ij}$$

  - Different components have different renormalization due to Lorentz symmetry breaking

Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - We can identify building blocks that can be used to construct multiplicatively renormalizable gluon quasi-PDFs, e.g.

$$\mathcal{O}_R^1(z_2, z_1) \equiv J^{ti}_{1,R}(z_2)\overline{J}^{ti}_{1,R}(z_1)$$

  - After integrating out $\mathcal{Q}$,

$$O_R^1(z_2, z_1) = (F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1))_R = Z_{11}^2 e^{\overline{\delta m}|z_2-z_1|}F^{ti}(z_2)L(z_2, z_1)F^{ti}(z_1)$$

    - The only linear divergence comes from the Wilson line self energy

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    - The only linear divergence comes from the Wilson line self energy

  - Four such operators have been identified [JHZ et al, PRL 19']

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i{}^{t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i{}^{z}(0),$$
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i{}^{z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu{}^{z}(0),$$

Gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76', Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - In principle, any linear combination of the operators

$$O_g^{(1)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ t}(0), \quad O_g^{(2)}(z,0) \equiv F^{zi}(z)\mathcal{W}(z,0)F_i^{\ z}(0),$$
$$O_g^{(3)}(z,0) \equiv F^{ti}(z)\mathcal{W}(z,0)F_i^{\ z}(0), \quad O_g^{(4)}(z,0) \equiv F^{z\mu}(z)\mathcal{W}(z,0)F_\mu^{\ z}(0),$$

can be used to study gluon quasi-PDFs, but usually they are not multiplicatively renormalizable

  - For example [Fan et al, PRL 18']

$$O_{g,R}^{(5)}(z_2,z_1) \equiv (F^{t\mu}(z_2)\mathcal{W}(z_2,z_1)F^t{}_\mu(z_1))_R = -O_{g,R}^{(1)}(z_2,z_1) - O_{g,R}^{(2)}(z_2,z_1) - O_{g,R}^{(4)}(z_2,z_1)$$

is not multiplicatively renormalizable

RI/MOM scheme

- Nonlocal quasi-PDF operators at different *z* do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each *z* (RI/MOM)

- Inserting gluon (quark) quasi-PDF operators into a quark (gluon) state yields finite mixing

- Taking it into account in RI/MOM renormalization helps improve convergence in the implementation of the matching

$$\begin{pmatrix} O_g^{(n)}(z,0) \\ O_q^s(z,0) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O_{g,R}^{(n)}(z,0) \\ O_{q,R}^s(z,0) \end{pmatrix},$$

with

$$O_q^s(z_1,z_2) = 1/2[\bar{q}_i(z_1)\Gamma W(z_1,z_2)q_i(z_2) - (z_1 \leftrightarrow z_2)]$$

RI/MOM scheme

- Nonlocal quasi-PDF operators at different z do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each z (RI/MOM)

- RI/MOM renormalization condition [Wang, JHZ et al, 19']

$$\left.\frac{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_R}{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1, \qquad \left.\frac{[P^{ab}_{ij}\Lambda_{11}^{ab,ij}(p,z)]_R}{[P^{ab}_{ij}\Lambda_{11}^{ab,ij}(p,z)]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1,$$

$$\left.\mathrm{Tr}[\Lambda_{12}(p,z)\mathcal{P}]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0, \qquad \left.[P^{ab}_{ij}\Lambda_{21}^{ab,ij}(p,z)]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0,$$

$$h_{g,R}^{(n)}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{11}(z,\mu_R,p_z^R,1/a)h_g^{(n)}(z,P^z,1/a)+\bar{Z}_{12}(z,\mu_R,p_z^R,1/a)/z\ h_q^s(z,P^z,1/a),$$
$$h_{q,R}^{s}(z,P^z,\mu_R,p_z^R) = \bar{Z}_{22}(z,\mu_R,p_z^R,1/a)h_q^s(z,P^z,1/a)+z\bar{Z}_{21}(z,\mu_R,p_z^R,1/a)\ h_g^{(n)}(z,P^z,1/a).$$

with

$$\bar{\mathcal{Z}}=\begin{pmatrix}\bar{Z}_{11}(z) & \bar{Z}_{12}(z)/z\\ z\bar{Z}_{21}(z) & \bar{Z}_{22}(z)\end{pmatrix}=\begin{pmatrix}Z_{11}(z) & Z_{12}(z)/z\\ zZ_{21}(z) & Z_{22}(z)\end{pmatrix}^{-1}$$

Factorization and matching

- Coordinate space

$$\tilde{h}_{q_i,R}(z,P^z,\mu)=\int_{-1}^{1}du\,\mathcal{C}_{q_iq_j}(u,\mu^2z^2)h_{q_j}(u\nu,\mu)+\int_{-1}^{1}du\,\mathcal{C}_{qg}(u,\mu^2z^2)h_g(u\nu,\mu).$$

$$\tilde{h}_{g,R}(z,P^z,\mu)=\int_{-1}^{1}du\,\frac{\mathcal{C}_{gg}(u,\mu^2z^2)}{\nu}h_g(u\nu,\mu)+\int_{-1}^{1}du\frac{\mathcal{C}_{gq}(u,\mu^2z^2)}{\nu}h_{q_i}(u\nu,\mu).$$

- Momentum space

$$\begin{aligned}\tilde{f}^{(n)}_{g/H}(x,P^z,p^R_z,\mu_R)&=\int_{-1}^{1}\frac{dy}{|y|}\Big[C_{gg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu)+C_{gq}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu)\Big]\\&\quad+\mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),\\ \tilde{f}_{q_i/H}(x,P^z,p^R_z,\mu_R)&=\int_{-1}^{1}\frac{dy}{|y|}\Big[C_{q_iq_j}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{q_j/H}(y,\mu)+C_{qg}\Big(\frac{x}{y},\frac{\mu_R}{p^R_z},\frac{yP^z}{\mu},\frac{yP^z}{p^R_z}\Big)f_{g/H}(y,\mu)\Big]\\&\quad+\mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),\end{aligned}\tag{2.53}$$

- Perturbative matching coefficients have been available at one-loop

Polarized gluon PDF

- For

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- x P^+} \langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+, 0; L_{n_+})F^{j+}(0)|P\rangle$$

- We have identified three multiplicatively renormalizable quasi-PDF operators [JHZ et al, PRL 19']

$$\Delta O_g^1(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{tj}(z_1),$$
$$\Delta O_g^2(z,0) = i\epsilon_{\perp,ij}F^{zi}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$
$$\Delta O_g^3(z,0) = i\epsilon_{\perp,ij}F^{ti}(z_2)\mathcal{W}(z_2,z_1)F^{zj}(z_1),$$

- Renormalization, factorization and matching are similar to the unpolarized case [Wang, JHZ et al, 19']

- Perturbative matching coefficients also available at one-loop

Summary and outlook

- Rapid progress has been achieved in the past few years on direct computations of x-dependence of hadron structure from lattice QCD

- Applications to nucleon PDFs have yielded encouraging results, but so far most of them are for isovector quark combinations which do not mix with gluons

- Gluon PDF and flavor-singlet quark PDF
  - Appropriate gluon quasi-PDF operators identified
  - Renormalization and factorization understood
  - Perturbative matching available at 1-loop
  - Can be appropriately studied on the lattice

- Generalization to GPDs …