Lattice “Cross-Sections” - Pion PDFs from Pseudo-PDFs and Pseudo-Structure Functions

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In Collaboration with:
R. Sufian, J. Karpie, K. Orginos, J. Qiu, D. Richards
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A. Radyushkin, S. Zafeiropoulos, T. Khan

WILLIAM & MARY

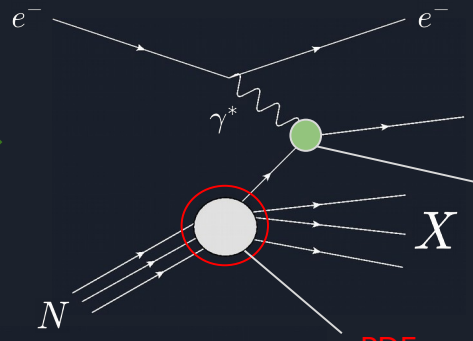
Lattice 19 Wuhan·China June 16-22

Jefferson Lab
Thomas Jefferson National Accelerator Facility

An Essential Window into Hadronic Structure

DIS

Parton Model

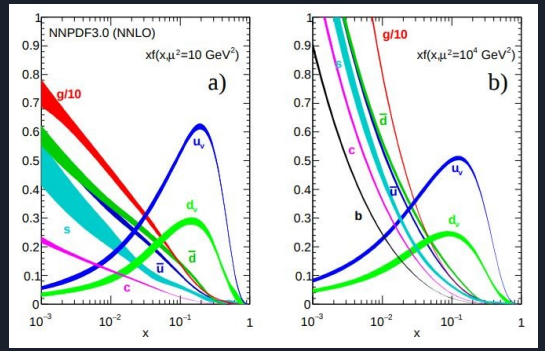


pQCD

PDFs - non-perturbative

$$\sigma^{\mathrm{DIS}}\left(x, Q^2, \sqrt{s}\right) = \sum_{a=q,\bar{q},g} C_a\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_a\left(x, \mu^2\right) + \textit{power corrections}$$

- ➔ Essential for interpretation of high-energy scattering data & BSM searches at energy frontier



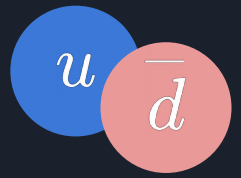
- ➔ Models not a complete picture
- ➔ Global analysis techniques not uniquely defined
- ➔ Not fully known

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

Pion Structure - Input from LQCD

➔ A numerically cheap arena of notable impact

- beyond DCSB, long-range $N-N$ interaction
- nucleon quark sea flavor asymmetry



$$I_{\mathrm{GSR}} = \int_0^1 \frac{\mathrm{d}x}{x}\left[F_2^p(x) - F_2^n(x)\right] = \frac{1}{3} + \frac{2}{3}\int_0^1 \mathrm{d}x\left[\overline{u}(x) - \overline{d}(x)\right] = \frac{1}{3}$$

➡ $\overline{d}(x) \neq \overline{u}(x)$

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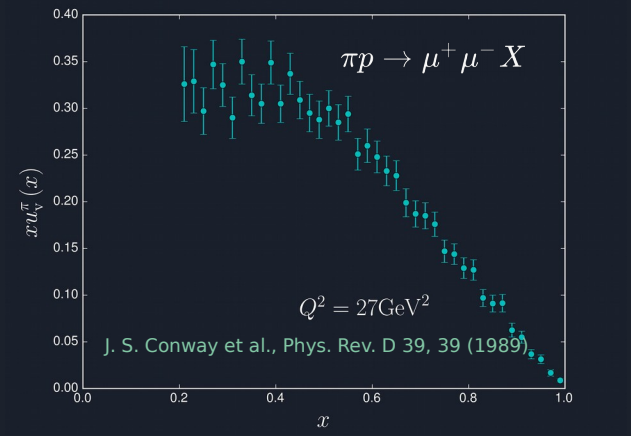
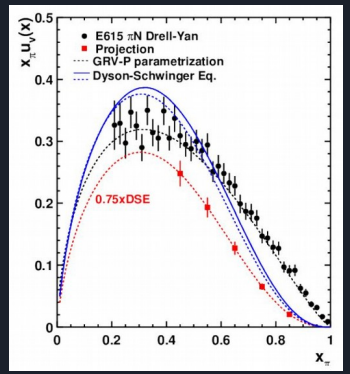
$\Rightarrow$ $\overline{d}(x) \neq \overline{u}(x)$

➔ Chiefly from pionic Drell-Yan

E.g. J. S. Conway et al., Phys. Rev. D 39, 39 (1989)
J. Badier et all., Z. Phys. C18, 281 (1983) B. Betev et al., Z. Phys. C28, 9 (1985)



LO Analysis Conflicts with Expectations



PR12-15-006: TDIS @ JLab
(pion structure via Sullivan process)

Pion Structure - Input from LQCD

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$\lim_{x\to 1} q_{\mathrm{v}}^{\pi}(x)$?

$(1-x)$ $(1-x)^2$

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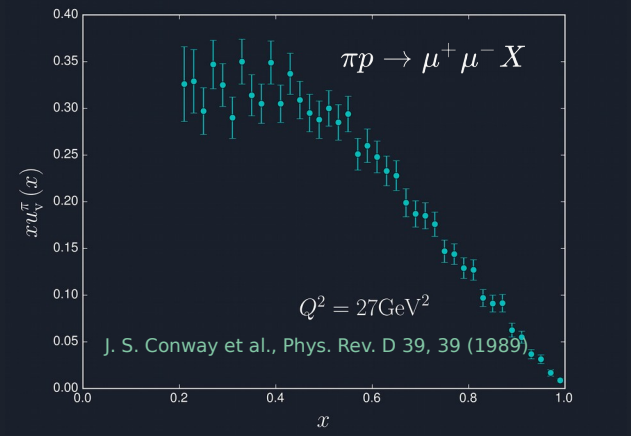
J. Badier et all., Z. Phys. C18, 281 (1983) B. Betev et al., Z. Phys. C28, 9 (1985)



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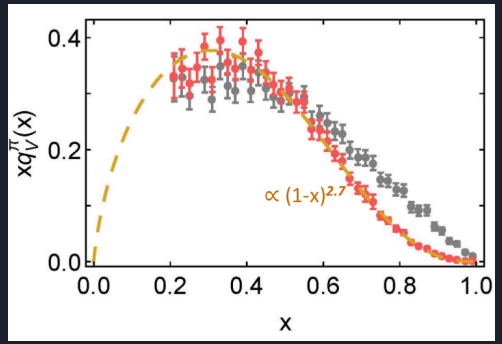

Craig Roberts' talk @ APS GHP 2019

Pion Structure - Input from LQCD

➔ A numerically cheap arena of notable impact

- ◆ beyond DCSB, long-range $N-N$ interaction
- ◆ nucleon quark sea flavor asymmetry

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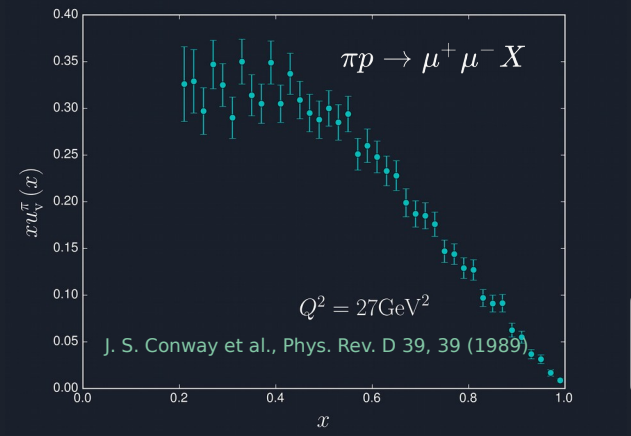
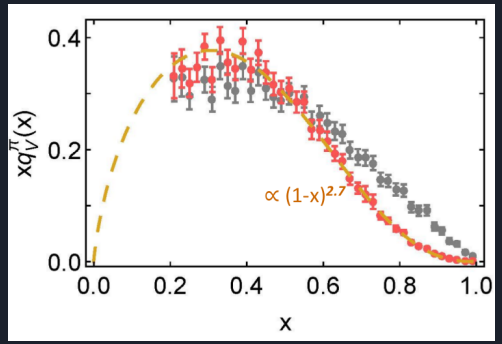


LO Analysis Conflicts with Expectations

Can an LQCD calculation serve as a discriminator?



Craig Roberts' talk @ APS GHP 2019

Lattice "Cross Sections" - Two-Current Correlators

Y. Q. Ma & J. W. Qiu, Phys. Rev. D 98, no. 7, 074021 (2018), arXiv:1404.6860 [hep-ph]
Y. Q. Ma & J. W. Qiu, Phys. Rev. Lett. 120, no. 2, 022003 (2018), arXiv:1709.03018 [hep-ph]

- Single-hadron matrix elements of renormalized non-local ops.

$$\sigma_{ij}^{\mu\nu}(\xi, p) = \langle h(p)| \mathcal{O}_{ij}^{\mu\nu}(\xi) |h(p)\rangle = \xi^4 \langle h(p)| \mathcal{J}_i^{\mu}(\xi/2)\, \mathcal{J}_j^{\nu}(-\xi/2) |h(p)\rangle$$

- Defining properties:
  - calculable in LQCD with Euclidean time
  - well-defined continuum limit; UV finite
  - share same collinear div. w/ PDFs

$p \sim \sqrt{s}$

$\xi^2 \sim \frac{1}{Q^2}$

$$\underbrace{T_i(\omega, \xi^2)}_{\text{Pseudo-structure Functions}} = \sum_{a=q,\bar{q},g} \int_{-1}^{1} \frac{\mathrm{d}x}{x} \underbrace{f_a(x, \mu^2)}_{\text{PDFs}} \underbrace{C_i^a(x\omega, \xi^2, \mu^2)}_{\text{Hard Coefficients}} + \mathcal{O}\left(\xi^2 \Lambda_{\mathrm{QCD}}^2\right)$$

valid for any finite $\{\omega, p^2\xi^2\}$ provided $|\vec{\xi}| \ll \Lambda_{\mathrm{QCD}}^{-1}$

Lattice “Cross Sections” - Pseudo-PDFs

- A single-hadron matrix element of a slightly different character

$$\langle p|\,\overline{\psi}\,(z)\,\gamma^{\alpha}W\,(z)\,\psi\,(0)\,|p\rangle = 2p^{\alpha}\mathcal{M}_{\mathrm{p}}\left(\omega,z^{2}\right)+z^{\alpha}\mathcal{M}_{\mathrm{z}}\left(\omega,z^{2}\right)$$

$$\mathcal{I}\left(\omega,\mu^{2}\right)=\int_{-1}^{1}\mathrm{d}x e^{ix\omega}f\left(x,\mu^{2}\right)$$

- An analogous factorization

$$\mathcal{M}\left(\omega,z^{2}\right)=\sum_{a=q,\overline{q},g}C_{a}\left(z^{2}\mu^{2},\alpha_{s}\right)\otimes I_{a}\left(\omega,\mu^{2}\right)+h.t.$$

Perturbatively computable coefficients
&
Ioffe-Time Distributions

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004, arXiv: 1801.03917 [hep-ph]
A. Radyushkin, Phys.Lett. B781 (2018) 433-442, arXiv: 1710.08813 [hep-ph]
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508, arXiv: 1801.03023 [hep-ph]

Sketch of Pseudo-PDF Lattice Calculations

- Standard 3pt/2pt correlations
- Summation method to improve matrix element extraction

C. Bouchard et.al ,Phys. Rev. D 96, no. 1, 014504 (2017)

$$\frac{\langle \pi(p)| \hat{\mathcal{O}}^{\alpha}_{u-d} |\pi(p)\rangle}{2E(p)} = \lim_{t_{\rm sep}\to\infty} \frac{R(t_{\rm sep}+\delta t) - R(t_{\rm sep})}{\delta t}$$

- Explicit momentum projections at operator insertions

$$C_{3\rm pt}(\vec{p}, t_{\rm sep}) = \sum_{\tau} \boxed{\sum_{\vec{x}}} \langle \Pi(-\vec{p}, t_{\rm sep})\, \hat{\mathcal{O}}^{\alpha}_{u-d}(\vec{x},\tau)\, \overline{\Pi}(\vec{p},0)\rangle$$

- Removal of pure higher-twist piece

$$p^{\mu} = (p^0, 0, 0, p_3)$$
$$z^{\mu} = (0, 0, 0, z_3)$$

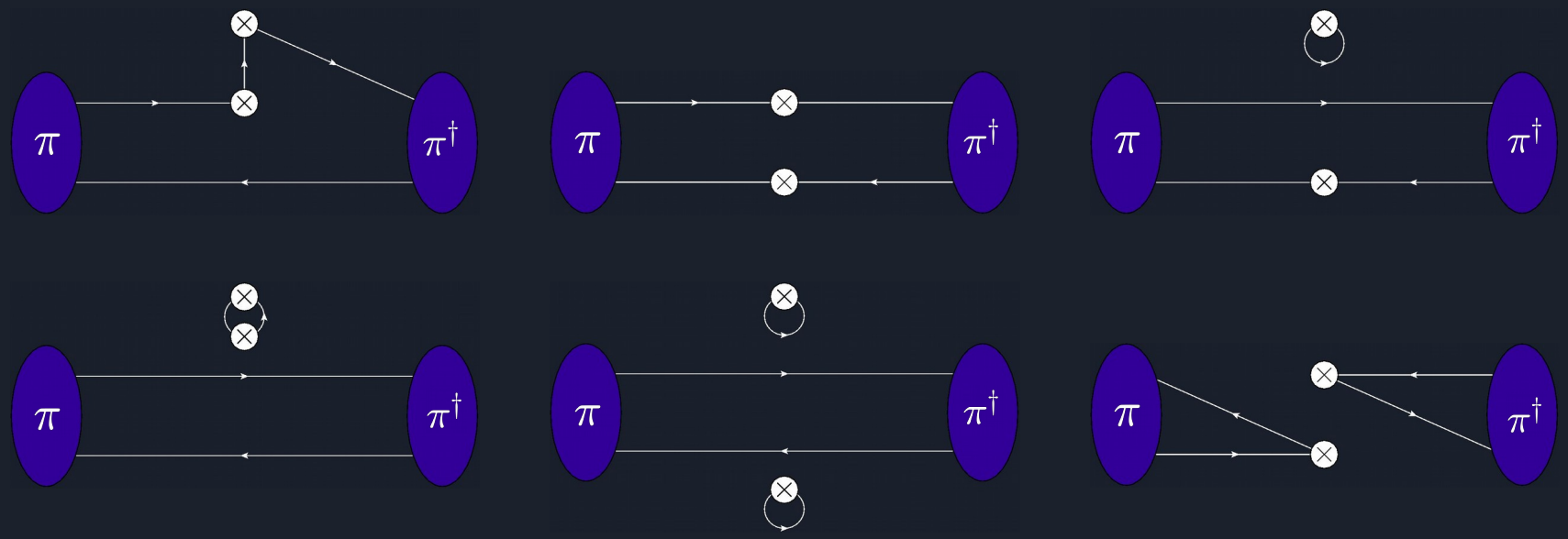
$$\Longrightarrow \quad \cancel{z^{\alpha}\mathcal{M}_{\rm z}(\omega, z^2)}$$

Two-Current Correlators - Wick Contractions

➔ 4pt function with light-quark currents

$$\mathcal{J}_i = \overline{q}\Gamma_i q'$$

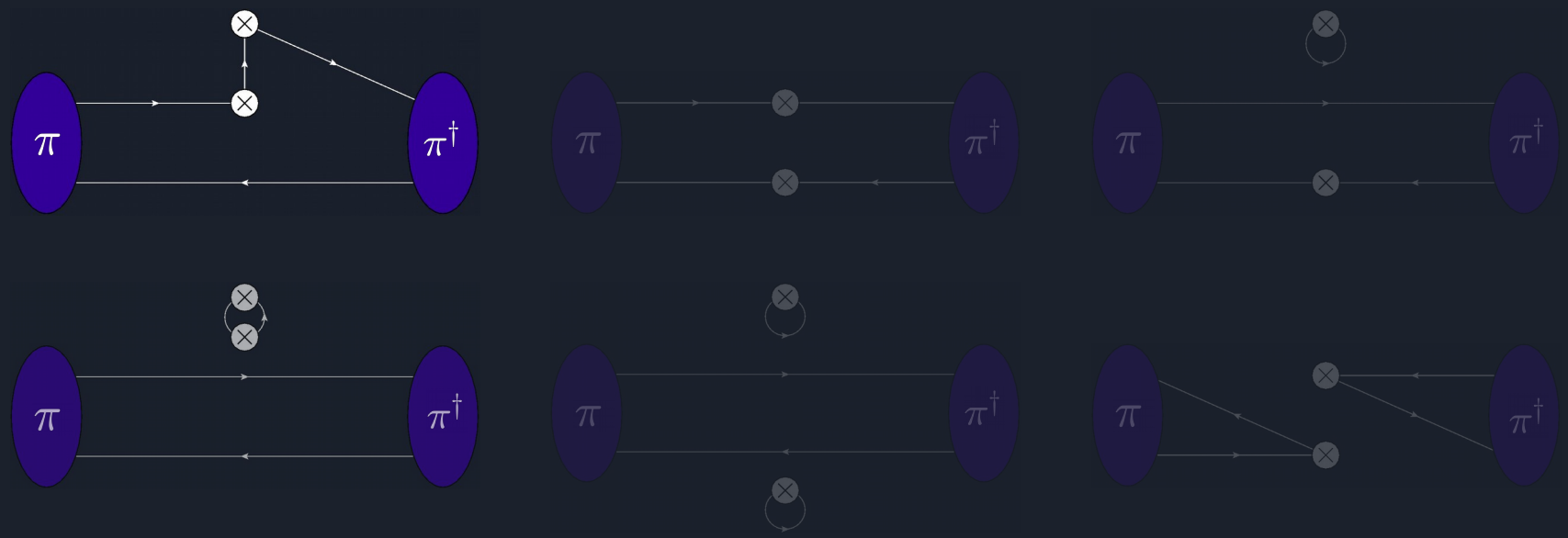
$$C_{4\mathrm{pt}}\left(\xi, p, T, t\right) = \left\langle \Pi_p\left(\vec{z}, T\right) \mathcal{J}_{\Gamma'}\left(x_o + \xi, t\right) \mathcal{J}_{\Gamma}\left(x_0, t\right) \overline{\Pi}_p\left(\vec{y}, 0\right) \right\rangle$$



Two-Current Correlators - Wick Contractions

- Heavy-light currents $\mathcal{J}_i = \{\overline{q}\Gamma_i Q, \overline{Q}\Gamma_i q\}$
  - sufficiently small ξ^2 and leading-twist contribution to PDF

W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007



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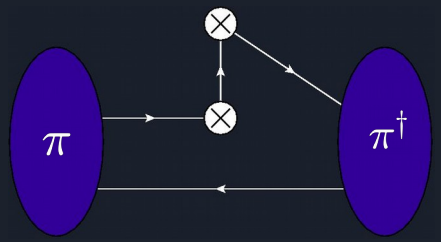
W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007

- Repeat calculation for varying src/snk separations



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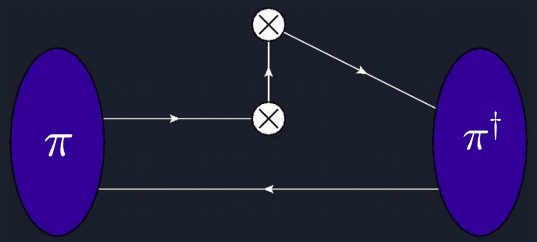

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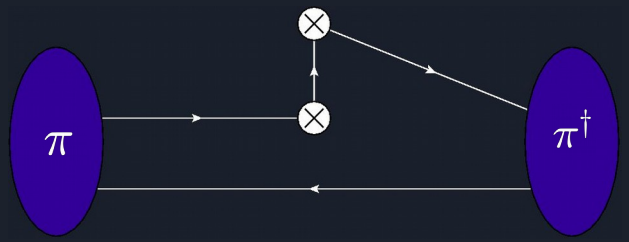


Momentum smearing for high momenta

G. S. Bali et al. Phys. Rev. D93, 094515 (2016), arXiv:1602.05525 [hep-lat]

Choice of Currents

- Anti-symmetric vector-axial currents $\in$ pion

$$\frac{1}{2}\left[\sigma_{\mathrm{VA}}^{\mu\nu}(\xi,p)+\sigma_{\mathrm{AV}}^{\mu\nu}(\xi,p)\right]=\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1\left(\omega,\xi^2\right)+\left(p^\mu\xi^\nu-\xi^\mu p^\nu\right)T_2\left(\omega,\xi^2\right)$$

- Properly chosen kinematics

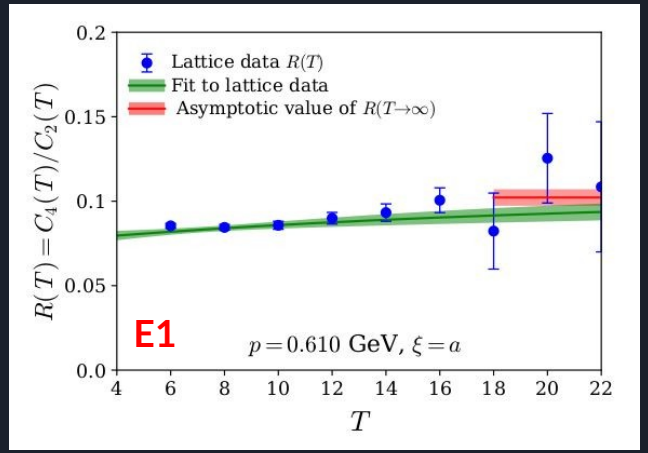
$$\begin{aligned} p^\mu &= \left(p^0,0,0,p^z\right) \\ \xi^\mu &= \left(0,0,0,\xi^z\right) \end{aligned} \quad \Longrightarrow \quad T_1\left(\omega,\xi^2\right)=\frac{1}{p^0\xi^3}\frac{1}{2}\left[\sigma_{\mathrm{VA}}^{12}(\xi,p)+\sigma_{\mathrm{AV}}^{12}(\xi,p)\right]$$

Apply factorization

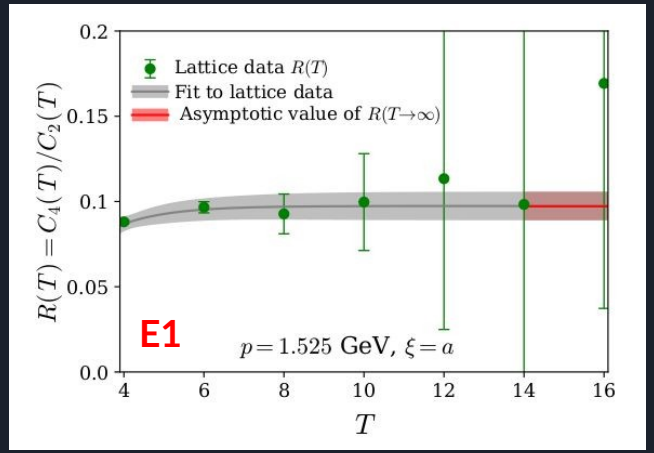
- Why this combination of currents?
  - direct access to q_{v} without $\overline{q}$ contamination

Ensemble & Typical Ratio Fits

Label	Lattice Spacing (a)	Pion Mass (m_π)	Lattice Dimensions
E1	0.127 fm	416 MeV	$32^3 \times 96$
E2	0.127 fm	416 MeV	$24^3 \times 64$
E3	0.09 fm	270 MeV	$32^3 \times 64$



$$\frac{C_{4\text{pt}}(T)}{C_{2\text{pt}}(T)} = A + Be^{-\Delta E_{eff} T}$$



Isolation of Distribution Functions

- Generically proceed from
  - Ioffe-time (Pseudo-) structure fns.
  - Pseudo-structure fns.

$$\mathcal{M}\left(\omega,\xi^2\right) = K\left(x\omega,\xi^2,\mu^2\right)\otimes f\left(x,\mu^2\right)$$

- Inverse transformations grossly ill-posed
  - Supply additional information

$$q_{\mathrm{v}}^{\pi}\left(x\right) = Nx^{\alpha}(1-x)^{\beta}\left(1+\underbrace{\rho\sqrt{x}+\gamma x}_{\mathcal{P}\left(x\right)}\right)$$

Smooth $\forall|x|<1$!

- Numerically perform integration
- More sophisticated analyses
  - Bayesian reconstruction

$$\int_0^1 \mathrm{d}x q_{\mathrm{v}}^{\pi}\left(x\right) = 1$$

J. Karpie, et al. JHEP 1904 (2019) 057 arXiv:1901.05408 [hep-lat] JLAB-THY-19-2898

Lattice "Cross Sections" and $q_{\mathrm{v}}(x)/q_{+}(x)$

- 490 configuration ensemble
  - increasing numbers of sources per momenta

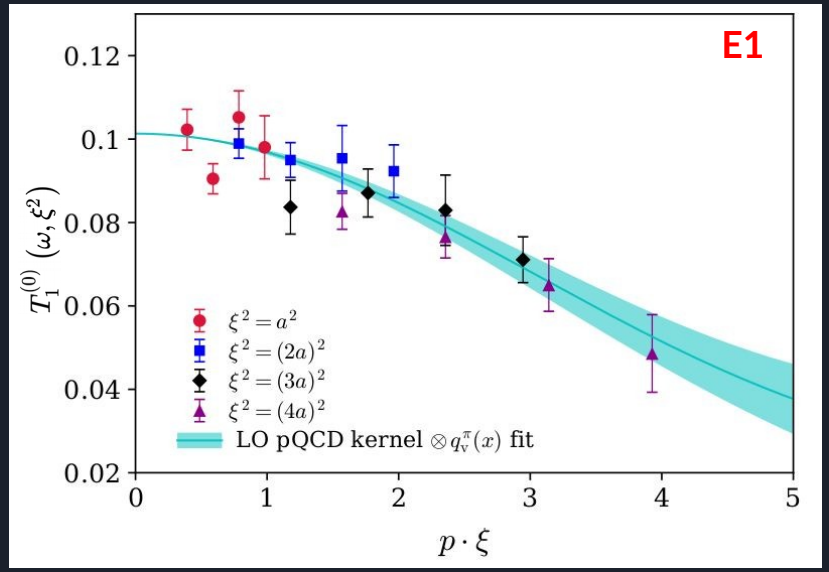
$$\vec{p} = [0,0,2] \rightarrow 2 \text{ srcs}$$
$$\vec{p} = [0,0,5] \rightarrow 7 \text{ srcs}$$

- Single determination

$$\langle \pi(p) | \mathcal{J}_2^{\mu}(\xi) \mathcal{J}_1^{\nu}(0) | \pi(p) \rangle$$

No volume average!

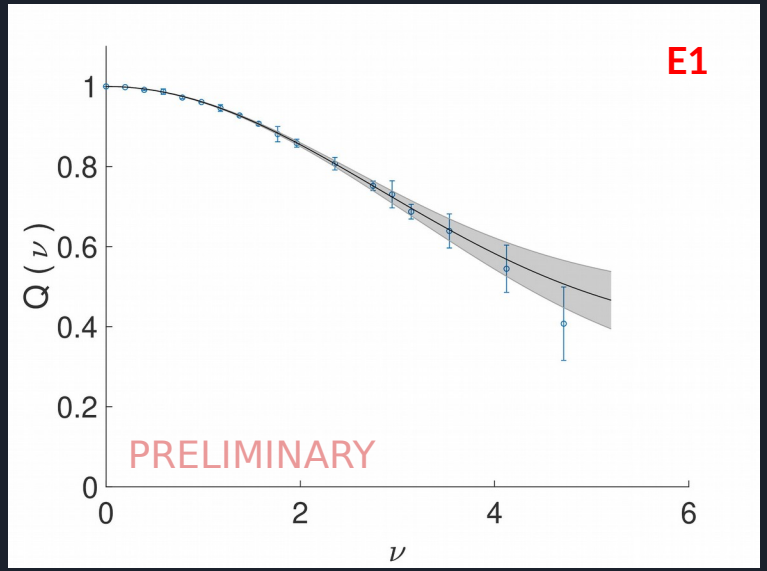
- $\mathcal{O}(\xi^2)$ not yet quantified



R. Sufian, J. Karpie, ***CE*** et al., Phys. Rev. D99 (2019) no.7, 074507, arXiv:1901.03921 [hep-lat]

$$T_1^{(0)}(\omega, \xi^2) = \int_0^1 \mathrm{d}x \frac{1}{\pi^2} \cos(x\omega)\, q_{\mathrm{V}}^{\pi}(x)$$

Lattice "Cross Sections" and $q_{\mathrm{v}}(x)/q_{+}(x)$



- 2560 configuration ensemble
  - 10 statistically indep. streams
  - 4 sources per cfg
- Numerous determinations $\sim \mathcal{O}(10^5)$

$$\langle p|\overline{\psi}(z)\gamma^0 W(z)\psi(0)|p\rangle = 2p^0\mathcal{M}_{\mathrm{p}}(\omega, z^2)$$

Volume average!

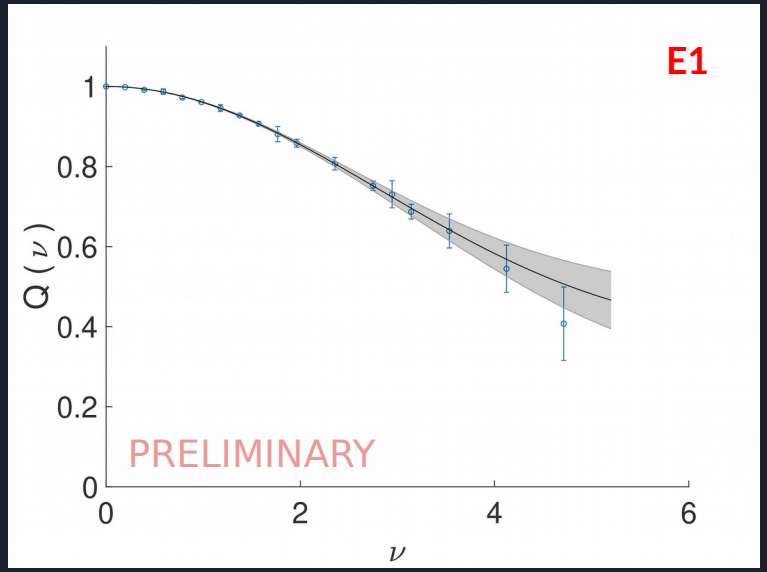
- Reduced matrix elements $\mathfrak{M}(\omega, z^2) = \dfrac{\mathcal{M}(\omega, z^2)}{\mathcal{M}(0, z^2)}$

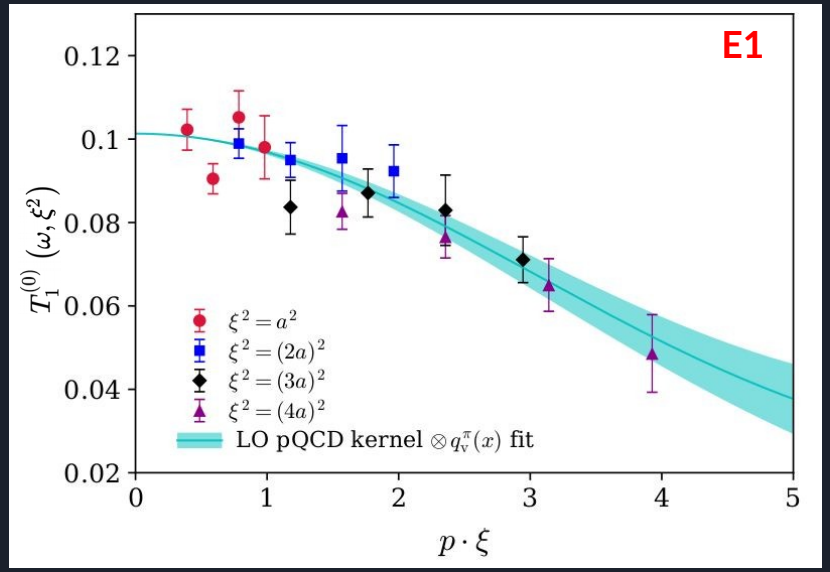
K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017)

- Evolved to common scale (4 GeV $\overline{\mathrm{MS}}$)

$$Q(\omega,\mu^2) = \mathcal{M}(\omega, z_0^2) + \frac{\alpha_s C_F}{2\pi}\int_0^1 \mathrm{d}u \left[B(u)\left(\ln\left(z_0^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)+1\right) + \left(\frac{4\ln(1-u)}{1-u} - 2(1-u)\right)_+\right]\mathcal{M}(u\omega, z_0^2)$$

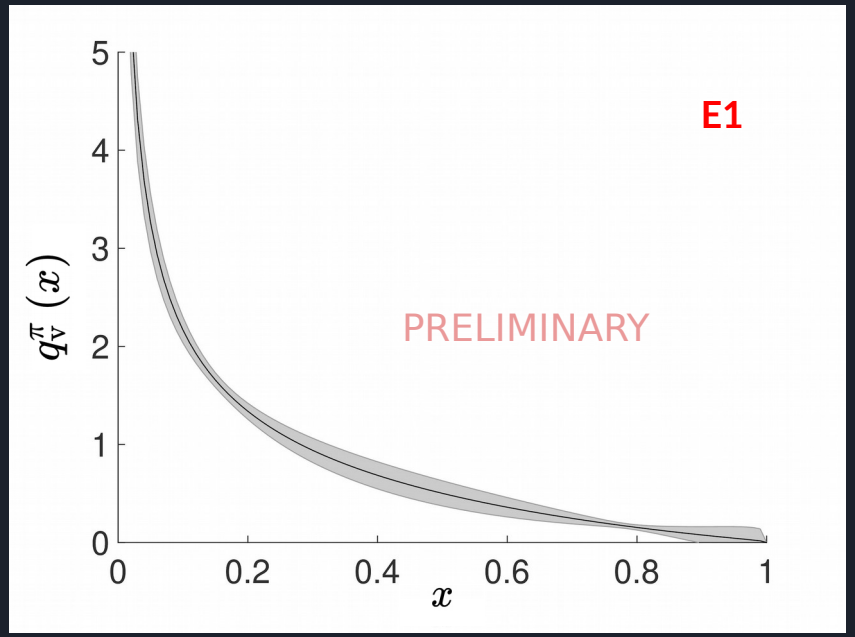
Lattice "Cross Sections" and $q_{\mathrm{v}}(x)/q_{+}(x)$

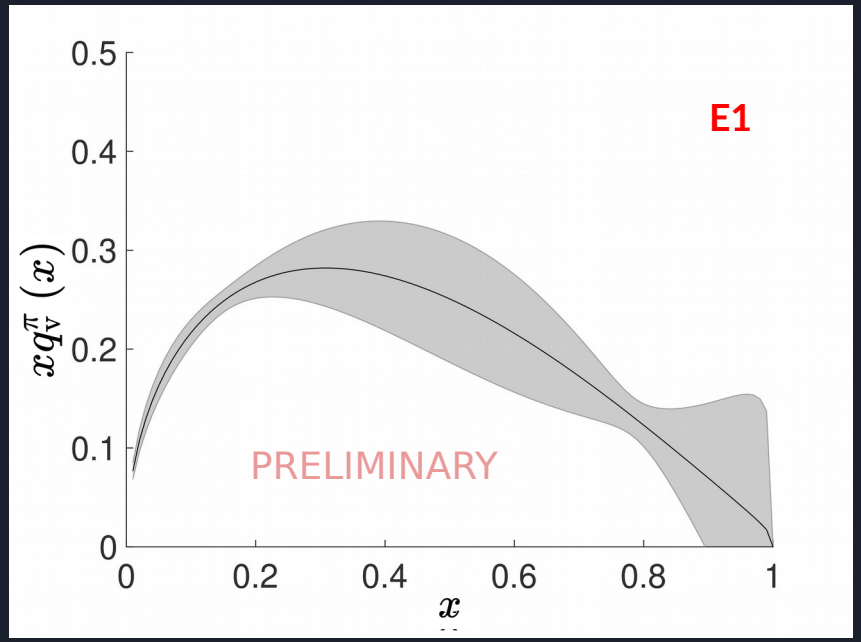




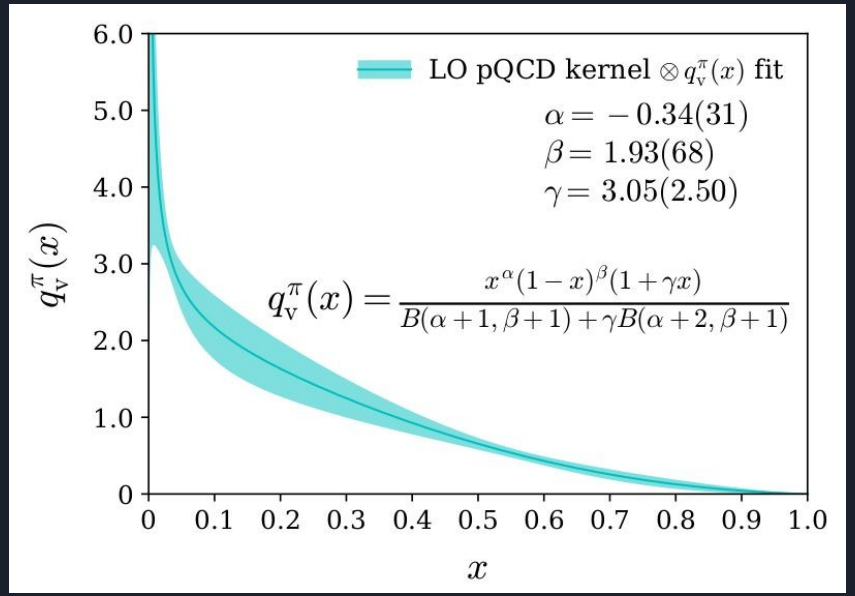
R. Sufian, J. Karpie, ***CE*** et al., Phys. Rev. D99 (2019) no.7, 074507, arXiv:1901.03921 [hep-lat]

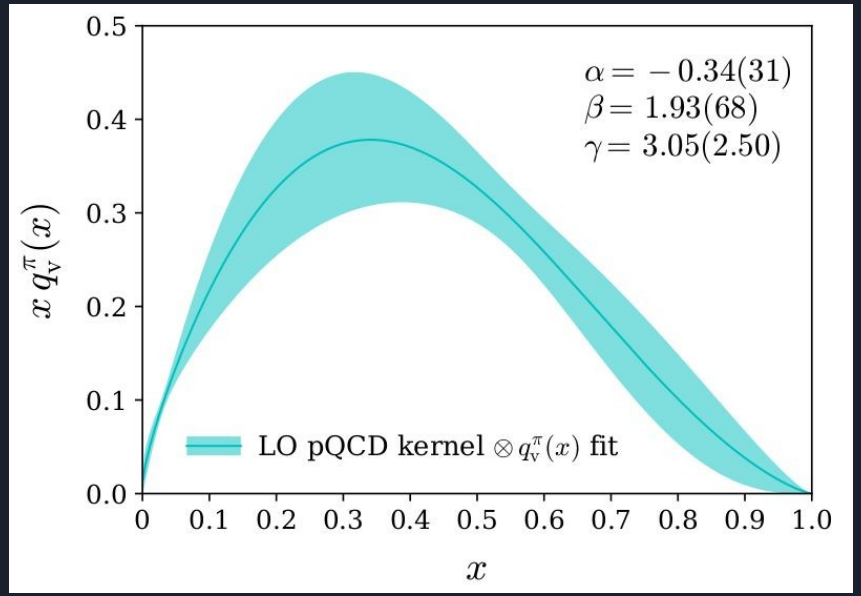
Extracted Valence PDF via Pseudo-PDF



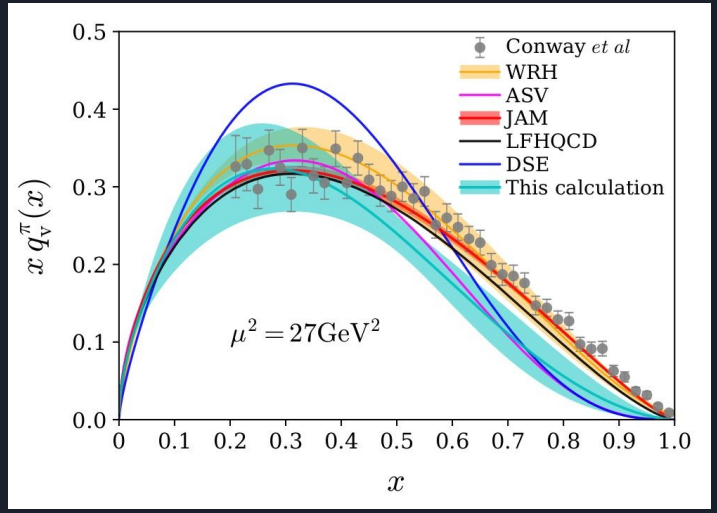


Extracted Valence PDF via Two-Currents

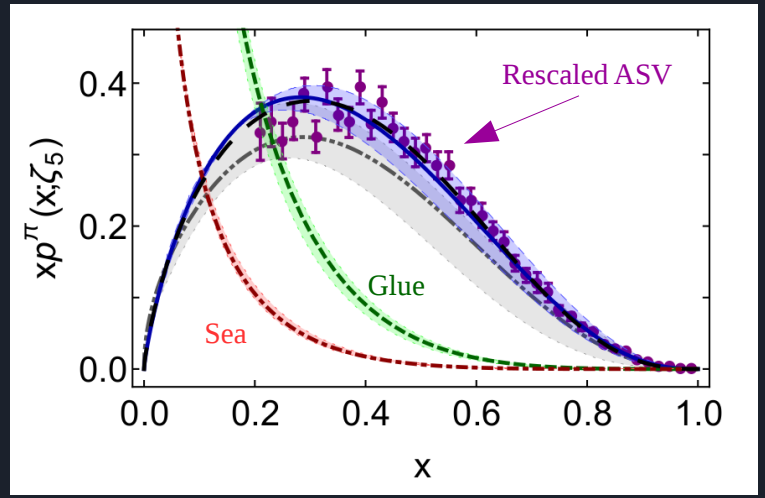




Comparison with the Literature



R. Sufian, J. Karpie, ***CE*** et al., Phys. Rev. D99 (2019) no.7, 074507



M. Ding, K. Raya, D. Binosi, L. Chang, C. Roberts, S. Schmidt, arXiv:1905.05208 [nucl-th]

- ➔ Favoring $\lim_{x\to 1}(1-x)^2$
- ➔ Consistent w/ NLL threshold soft-gluon resummation effects (ASV)
- ➔ Appear to be capturing importance of gluons

P. C. Barry et al., Phys. Rev. Lett. 121, 152001 (2018), arXiv:1804.01965 [hep-ph]

- ➔ Parameter free cont. approach to two valence-body bound-state problem
- ➔ Calculation/prediction of 6 Mellin moments
- ➔ $\beta = 2.66(12)\ [2.45(58)]$

Outlook

- PDFs determined experimentally via a global analysis of numerous observables
- Calculations of lattice calculable and factorizable matrix elements are encouraging
  - some systematics unquantified

Discretization effects...

G. S. Bali et al., Phys. Rev. D 98, no. 9, 094507 (2018), arXiv:1807.06671 [hep-lat]
G. S. Bali et al., Eur. Phys. J. C78 (2018) no. 3, 217, arXiv:1709.04325 [hep-lat]

Finite volume effects...

R. Briceño et al., Phys. Rev. D 98, no. 1, 014511 (2018), arXiv:1805.01034 [hep-lat]

Log. corrections & $\mathcal{O}(\xi^2)$

- Statistics limited
- PDFs ought to be extracted via a simultaneous analysis of matrix elements
  - e.g. pPDF, SS, SP, VV, AA, etc.

THANK YOU!

More on quasi/pseudo-distributions

$$\tilde{q}\left(x, P_3\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{-izxP_3} \left\langle P \middle| \overline{\psi}\left(z\right) \gamma^0 W\left(z\right) \psi\left(0\right) \middle| P \right\rangle$$

Fourier transform over length of Wilson line

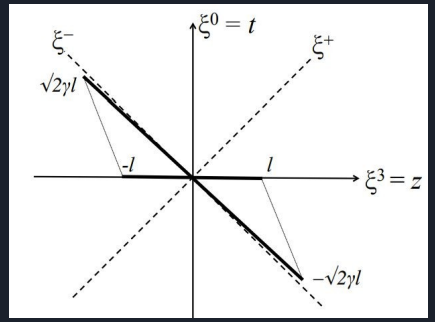
$$\tilde{q}\left(x, \mu^2, P_3\right) = \int_{-1}^{1} \frac{\mathrm{d}y}{|\, y \,|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q\left(y, \mu^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 / P_3^2, M_N^2 / P_3^2\right)$$

X. Ji, Phys. Rev. Lett. 110, 262002 (2013), arXiv:1305.1539 [hep-ph]

$$\mathcal{P}\left(x, z^2\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega e^{-ix\omega} \mathcal{M}\left(\omega, z^2\right)$$

Fourier transform Ioffe time - fixed length Wilson line

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017), arXiv:1706.05373 [hep-ph]



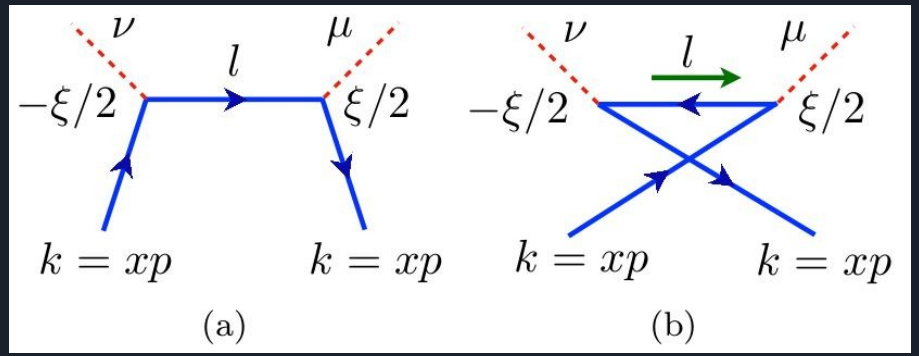
K. Cichy & M. Constantinou, arXiv:1811.07248v1 [hep-lat]

Leading Order Coefficient Functions - Coordinate Space

$$\mathcal{M}_{ij}^{(a)} = \frac{\xi^4}{2}\sum_s \langle 0 \mid \overline{u}_s(k)\, e^{ik\cdot\xi/2}\Gamma_i^\mu \psi(\xi/2)\, \overline{\psi}(-\xi/2)\, \Gamma_j^\nu e^{ik\cdot\xi/2} u_s(k) \mid 0\rangle$$

$$= \frac{\xi^4}{2} e^{ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_i^\mu \int \frac{d^4 l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon} e^{-il\cdot\xi}\ \Gamma_j^\nu\right]$$

$$\mathcal{M}_{ji}^{(b)} = \frac{\xi^4}{2} e^{-ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_j^\nu \int \frac{d^4 l}{(2\pi)^4}\frac{-i\gamma\cdot l}{l^2+i\epsilon} e^{-il\cdot\xi}\ \Gamma_i^\mu\right]$$



$$\sigma_{\mathrm{VA}}^{\mu\nu(0)} + \sigma_{\mathrm{AV}}^{\mu\nu(0)} = \frac{1}{\pi^2} x\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta \left(e^{ix\omega} + e^{-ix\omega}\right)$$

$$T_1^{(0)}\left(\omega, \xi^2\right) \propto \left(e^{ix\omega} + e^{-ix\omega}\right)$$

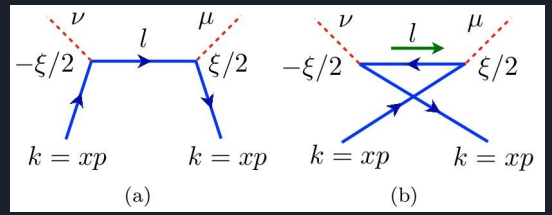
$$T_2^{(0)}\left(\omega, \xi^2\right) = 0$$

Matching Kernels & the Pion Valence Distribution

➔ Factorized relation → asymptotic parton state → order-by-order in α_s

$$T_i^q\left(\omega,\xi^2\right)=\sum_a\int_{-1}^{1}\frac{\mathrm{d}x}{x}f_a^q\left(x,\mu^2\right)C_i^a\left(x\omega,\xi^2,\mu^2\right)$$

$$f_a^{q(0)}\left(x,\mu^2\right)=\delta(1-x)\delta^{qa}$$



$$\mathcal{M}_{ij}^{(a)}=\frac{\xi^4}{2}\sum\langle 0\mid\overline{u}_s\left(k\right)e^{ik\cdot\xi/2}\Gamma_i^\mu\psi\left(\xi/2\right)\overline{\psi}\left(-\xi/2\right)\Gamma_j^\nu e^{ik\cdot\xi/2}u_s\left(k\right)\mid 0\rangle$$

$$=\frac{\xi^4}{2}e^{ik\cdot\xi}\mathrm{Tr}\left[\left(\gamma\cdot k\right)\Gamma_i^\mu\int\frac{d^4l}{\left(2\pi\right)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\,\Gamma_j^\nu\right]$$

$$\sigma_{\mathrm{VA}}^{\mu\nu(0)}+\sigma_{\mathrm{AV}}^{\mu\nu(0)}=\frac{1}{\pi^2}x\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta\left(e^{ix\omega}+e^{-ix\omega}\right)$$

$$T_i^{q(0)}\left(\omega,\xi^2\right)=K_i^{q(0)}\left(\omega,\xi^2\right)$$

$$T_1^{(0)}\left(\omega,\xi^2\right)\propto\left(e^{ix\omega}+e^{-ix\omega}\right)\qquad T_2^{(0)}\left(\omega,\xi^2\right)=0$$

➔ To ensure access to valence distribution

- momentum space pseudo-structure fns.

$$\tilde{T}_1\left(\tilde{x},\xi^2\right)\equiv\int\frac{\mathrm{d}\omega}{2\pi}e^{-i\tilde{x}\omega}T_1\left(\omega,\xi^2\right)\propto\int\frac{\mathrm{d}\omega}{2\pi}e^{-i\tilde{x}\omega}\int_0^1\frac{\mathrm{d}x}{x}q\left(x\right)\underbrace{\left(e^{ix\omega}+e^{-ix\omega}\right)}_{\text{L.O.}}$$

$$\propto\left[q\left(\tilde{x}\right)+q\left(-\tilde{x}\right)\right]=q_{\mathrm{v}}\left(\tilde{x}\right)$$

Y. Q. Ma & J. W. Qiu, Phys. Rev. Lett. 120, no. 2, 022003 (2018), arXiv:1709.03018 [hep-ph]

Higher-Twist Effects?