Parton Distribution Functions from Euclidean-Space Correlation Functions in Ioffe Time

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Introduction

Working definition - calculation of x-dependent PDFs and QDAs (quark distribution amplitudes) from Euclidean-space lattice calculations.

- Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')} \psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

Quasi distributions approach light-cone distributions in limit of large P^z

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

$$\mathcal{M}^\alpha(z,p) = \langle p \mid \bar{\psi}(z)\gamma^\alpha \exp\left(-ig\int_0^z dz' A^z(z')\right) \psi(0) \mid p\rangle$$

A. Radyushkin, PLB767 (2017); Braun et al., Phys.Rev. D51 (1995) 6036

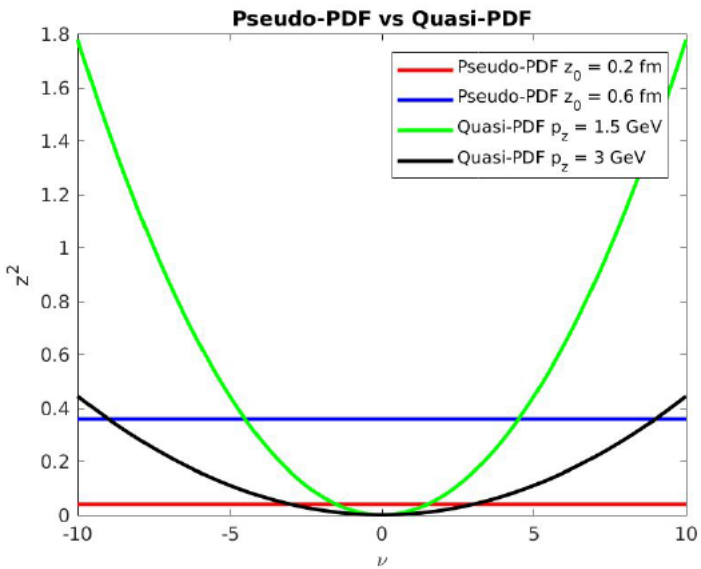
Pseudo-PDF and Quasi-PDF

Relation between qPDF and pPDF approaches

- Both integrals of Ioffe-Time Distribution Function
- Computed matrix elements the same
- Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$



Require sufficiently fine lattice spacings

Good "Lattice Cross Sections" (LCS)

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Short distance scale

"Higher Twist"

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

Factorize in $\boxed{\omega = P \cdot \xi,\ \xi^2 P^2}$ providing $\xi << \frac{1}{\Lambda_{\rm QCD}}$

$P \to \sqrt{s}$ *Collision energy*

$\xi \to \frac{1}{Q}$ *Hard Probe*

Expressed in coordinate space

LCS - II

Choice of Operators

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi) \quad \mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)}{M(0, z^2)}$$

Wilson line

Gauge-Invariant Currents:

Current-current (CC)

$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0)$$

$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0)$$ ← Flavor-changing

$$\mathcal{O}_G(\xi) \propto [F_{\mu\rho} F^{\rho}_{\nu}](\xi)[F_{\mu'\sigma} F^{\sigma}_{\nu'}](0)$$ ← Gluonic

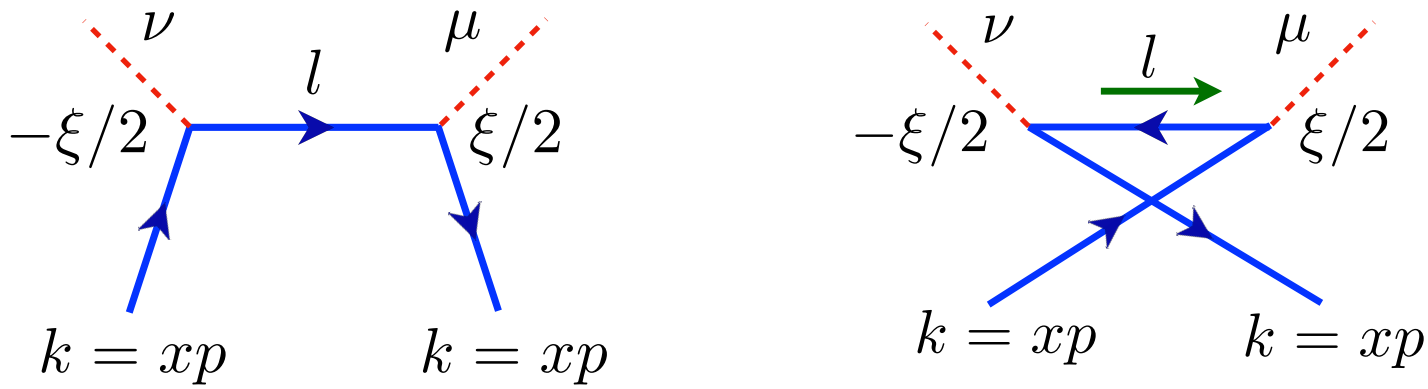
- Straight-forward operator renormalization - *matching* + *continuum Pert. Th.*
- ξ can be *off-axis*

c.f. Pion QDA, Braun *et al*, Phys. Rev. D 98, 094507 (2018)

Perturbative Kernel

$$\sigma_{ij}^{\mu\nu}(\xi,p) = \langle \pi(p) \mid \mathcal{O}_{ij}^{\mu\nu}(\xi) \mid \pi(p) \rangle$$
$$= \xi^4 \langle \pi(p) \mid \mathcal{J}_i^{\mu}(\xi/2)\mathcal{J}^{\nu} + j(-\xi/2) \mid \pi(p) \rangle$$

Calculate *K* at tree-level between quark states



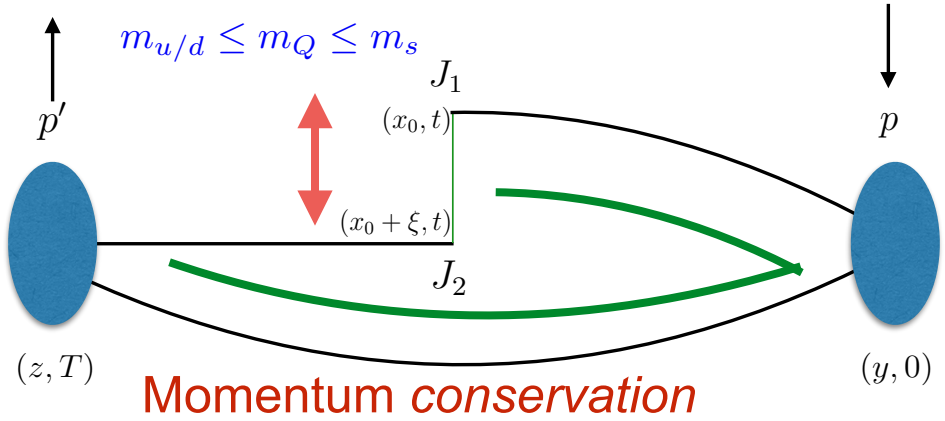
Process, i.e. current, dependent

$$\frac{1}{2}[\sigma_{V,A}^{\mu\nu}(\xi,p) + \sigma_{A,V}^{\mu\nu}(\xi,p)]$$

$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\omega,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)\, T_2(\omega,\xi^2)$$

Computational Setup - Pion

Momentum projection



Momentum projection

$C_4(T)$

$$\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle =$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi,t)\, \bar{q}\, J_1\, q(x_0,t)\, \bar{q}\,\Gamma_\Pi\, q(y,0)\rangle$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi,t;x_0,t)\, J_1\, G(x_0,t;y,0)\, \Gamma_\Pi G(y,0;z,T)\Gamma_\Pi\, G(z,T;x_0+\xi,t)]$$

Straightforward computational setup using sequential-source method:

$$D(Z,T;w)H(w;x_0,t) = \sum_y e^{-ip\cdot y}\Gamma_\Pi G(y,0;x_0,t)$$

$$D(s;w)\tilde{H}(w;x_0,t) = \sum_z e^{ip\cdot z}\Gamma_\Pi H(z,T;x_0,t)$$

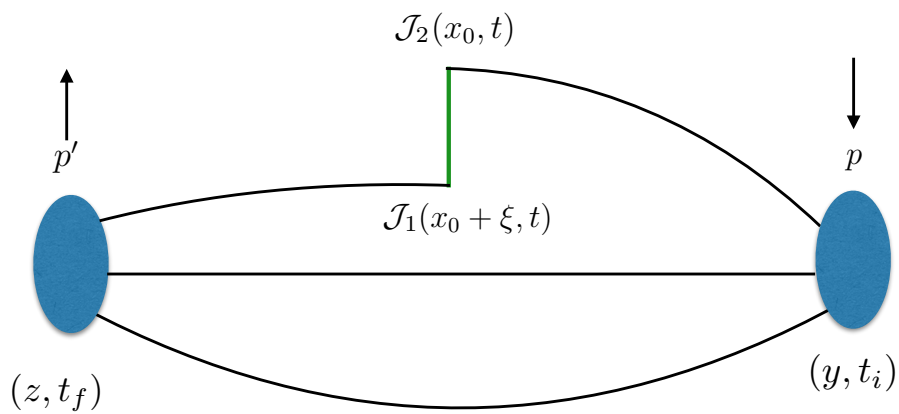
Computational Setup - II

- Can compute both Wilson-line pPDF, and current-current correlators
  - CC allows both on- and off-axis separations
  - Wilson-line corrrelator allows more complete sampling of lattice - *momentum projection at currents*

Feynman-Hellman

Colin Egerer, next talk

Nucleon



Christian Zimmermann, next session

- Wilson-line calculation straightforward
- CC: No *straightforward* implementation using Sequential Sources

Inverse Problem

"Inverse Problem" - ill-posed inverse Fourier transform.

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Calculate on Lattice Extract PDF? Calculate in PQCD

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} f(x, \mu^2)$$

Similar challenge to that of global fitting community!

Require additional information:

- *Bayesian Prior*
- *Backus-Gilbert approach*
- *<u>Phenomenologically motivated parametrization</u>*

Or neural-network approach

Computational Details

- Programme is to use 2+1 isotropic clover lattices
  - Tadpole tree-level improved gauge
  - 1 iteration stout smearing
  - Used for many projects within USQCD

e.g. nucleon charges - matching Zs calculated for many lattices

Yoon et al, Phys. Rev. D 95, 074508 (2017)

Ensemble ID	a (fm)	M_π (MeV)	β	$C_{\rm SW}$	am_{ud}	am_s	$L^3 \times T$	$M_\pi L$
a127m285	0.127(2)	285(3)	6.1	1.24930971	-0.2850	-0.2450	$32^3 \times 96$	5.85
a094m280	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	4.11
a091m170	0.091(1)	166(2)	6.3	1.20536588	-0.2416	-0.2050	$48^3 \times 96$	3.7
a091m170L	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	5.08

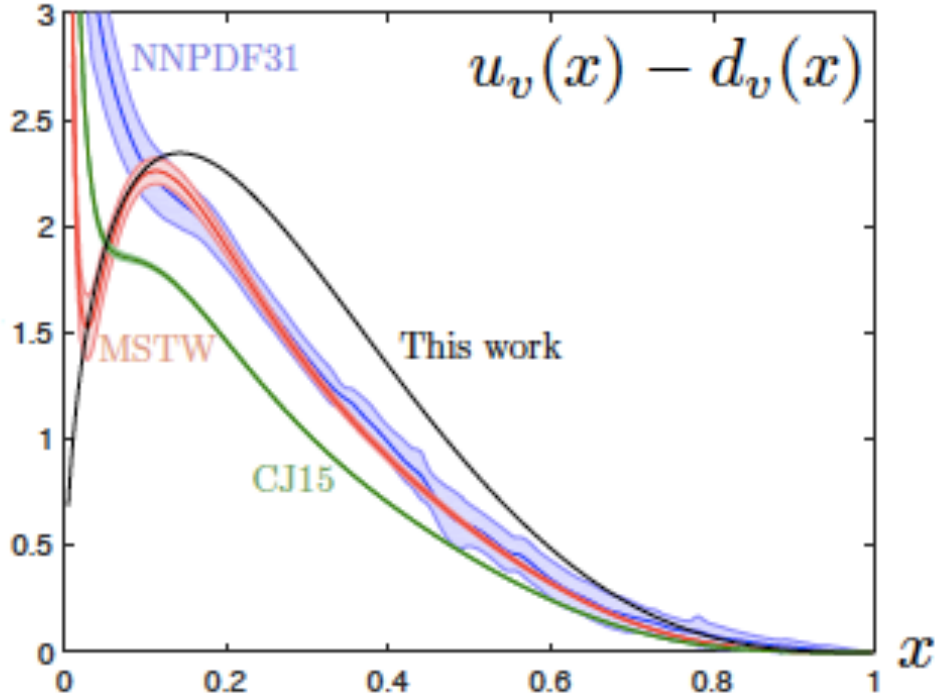
For today…

- **Lattice spacing a ~ 0.127 fm**
- **Pion mass 460 MeV**
- **32(24)^3 x 96 lattice**

Pseudo-PDF of Nucleon - I

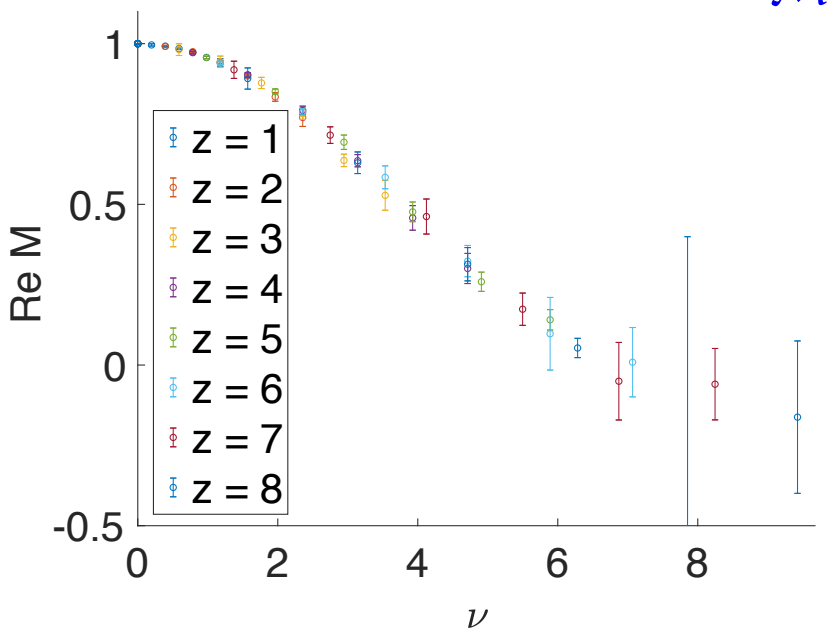
First implementation in quenched approximation to QCD

K.Orginos, A.Radyushkin, J.Karpie, S.Zafeiropoulos, PRD96 (2017), 094503



Pseudo-PDF of Nucleon - II

J Karpie, PhD Thesis + K.Orginos,S. Zafeiropoulos



Form *reduced matrix element*

$$\mathcal{M}(\nu, z^2) = \left(\frac{M(\nu, z^2)}{M(\nu, z=0)}\right) / \left(\frac{M(0, z^2)}{M(0, z=0)}\right)$$

Quark-field renormalization

Wilson line

Preliminary

ITD of Nucleon

Obtaining ITD can be considered two-step process:

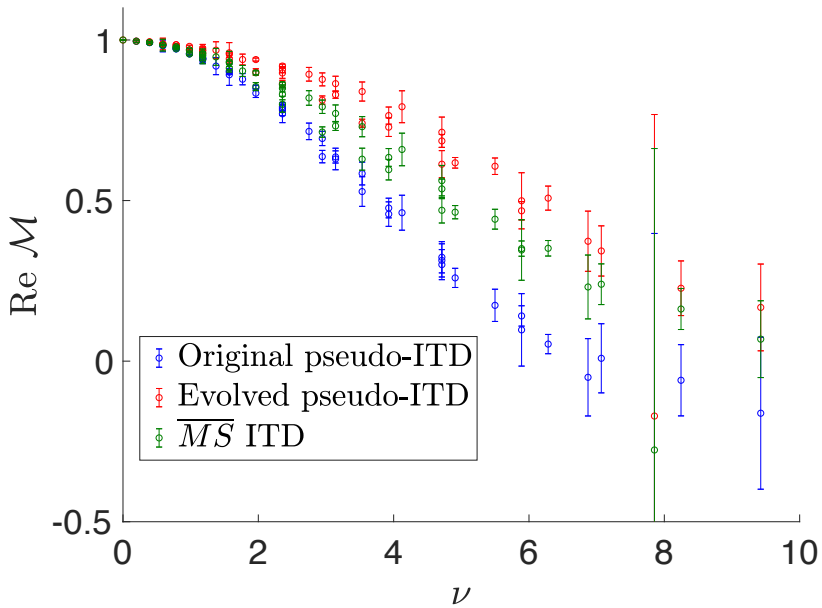
- Match to common separation z_0:

DGLAP kernel

$$\mathcal{M}(\nu, z_0^2) = \mathcal{M}(\nu, z^2) + \ln\left(\frac{z^2}{z_0^2}\right)\frac{\alpha_s C_f}{2\pi} B \otimes \mathcal{M}(\nu, z^2)$$

- Evolve to $\overline{MS}$

$$Q(\nu, \mu^2) = \mathcal{M}(\nu, z_0^2) - \frac{\alpha_s C_f}{2\pi}\int_0^1 du \left[\ln\left(z_0^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)B(u) + L(u)\right]\mathcal{M}(u\nu, z_0^2)$$



A. Radyushkin, Phys. Rev. D 98, 014019 (2018)

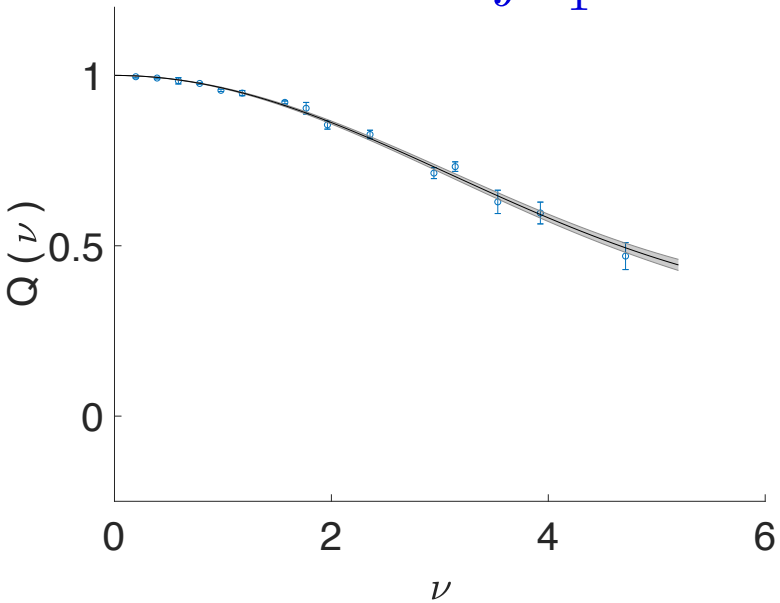
$$z_0^{-2} = 4e^{2\gamma_E+1}$$

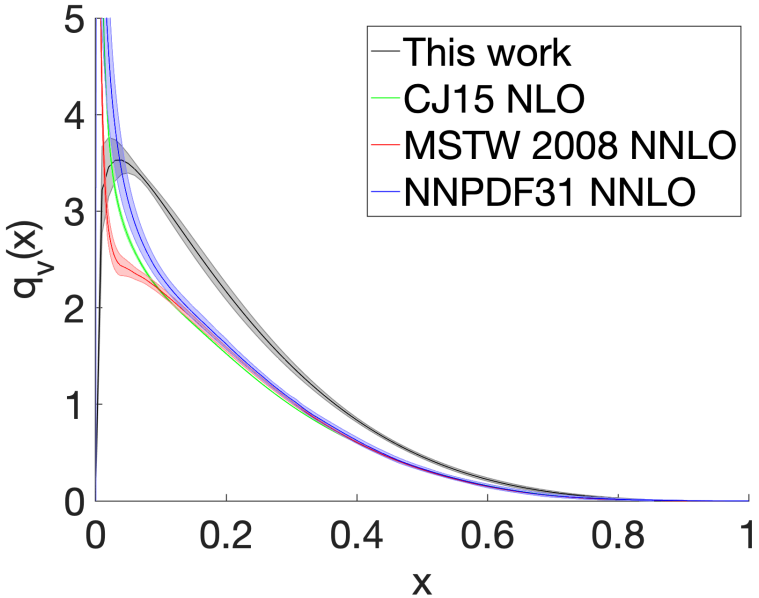
$$\mu = 4\,GeV$$

Nucleon PDF

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} f(x, \mu^2)$$

J Karpie, PhD Thesis

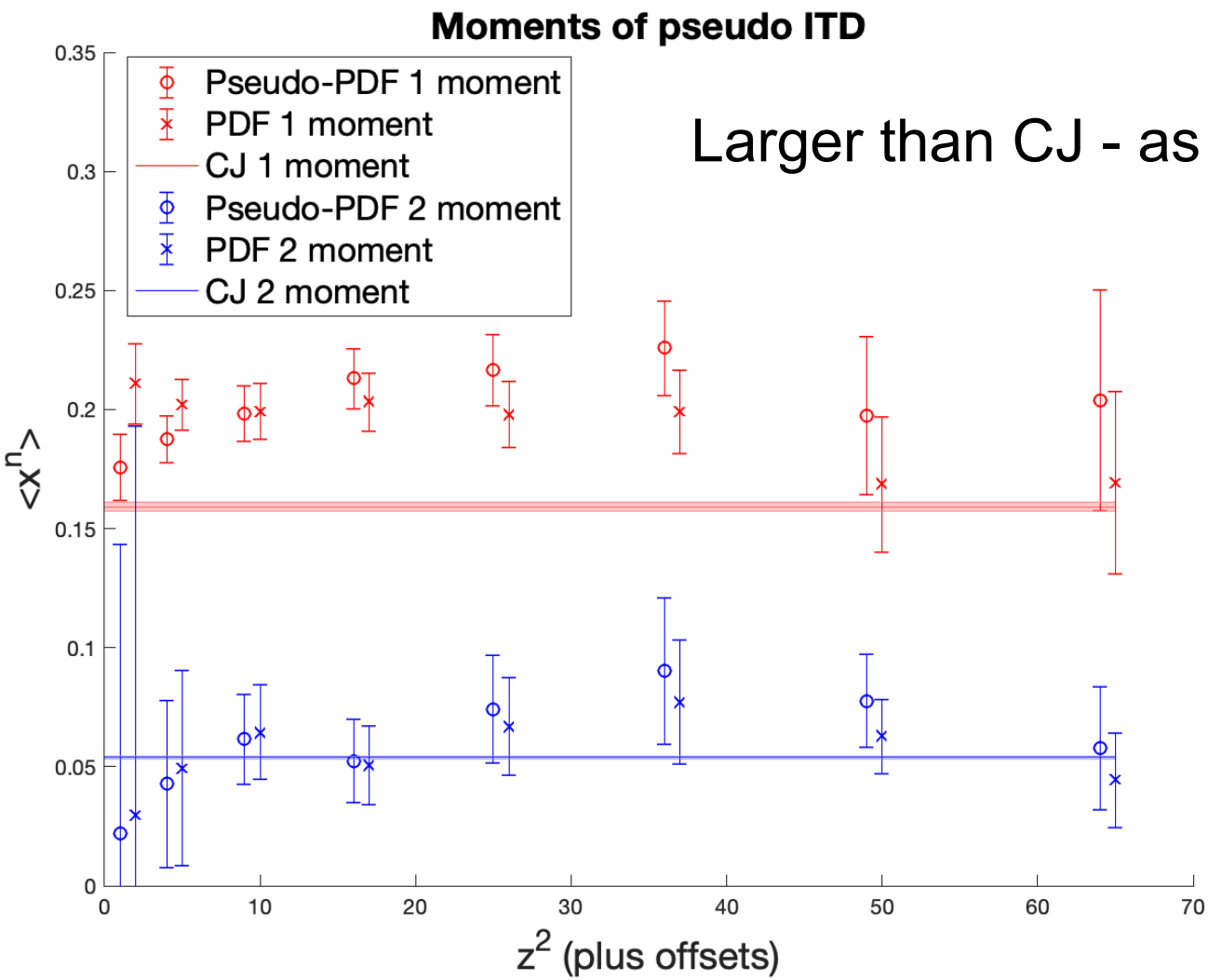




JAM Collaboration:

$$f(x) = N(\alpha, \beta, c, d) x^{\alpha} (1 - x)^{\beta} (1 + c\sqrt{x} + dx)$$

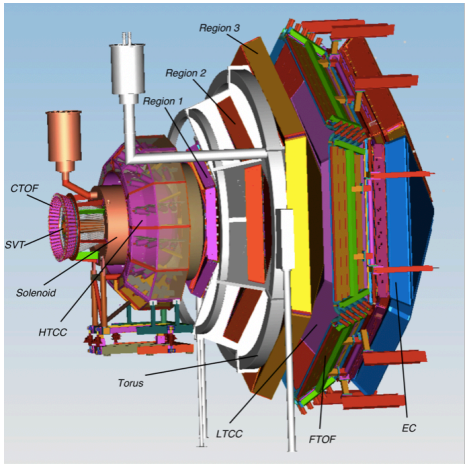
Moments of ITD + PDF



Larger than CJ - as expected!

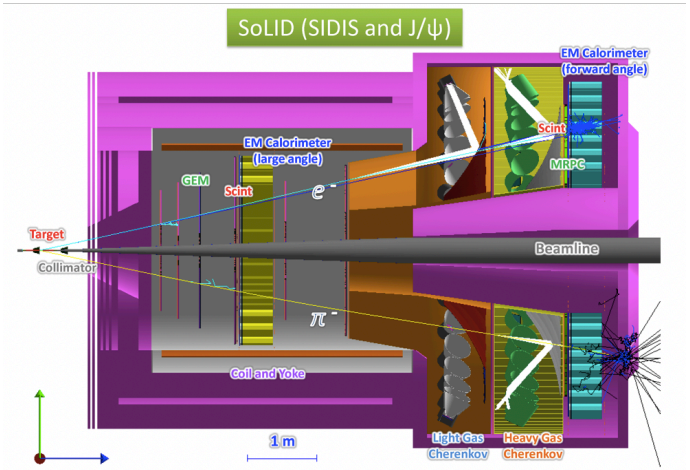
Summary

- Calculation of ITD distributions provides an first-principles way of determining PDFs
- Formulated in coordinate-space - factorization into PDF and perturbative kernel.
- Encompasses both Wilson-line and gauge-invariant current-current correlators.
- Calculation for lattice currents: VA + *Wilson line* for pion: *Colin Egerer*
- Exploring methods for current-current correlators for nucleon
- *Program to study systematics: lattice spacing, volume, quark mass, higher-twist….*
- ***Method admits calculation of three-dimensional picture of hadrons:***
  - *GPDs should be straightforward…*
  - *Operator renormalization independent of external states*



GPDs (DVCS)

LQCD

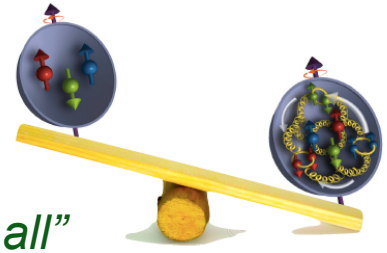


TMDs (SIDIS/DY)

Imaging the Quarks

EicC | EIC

CENTER for NUCLEAR FEMTOGRAPHY

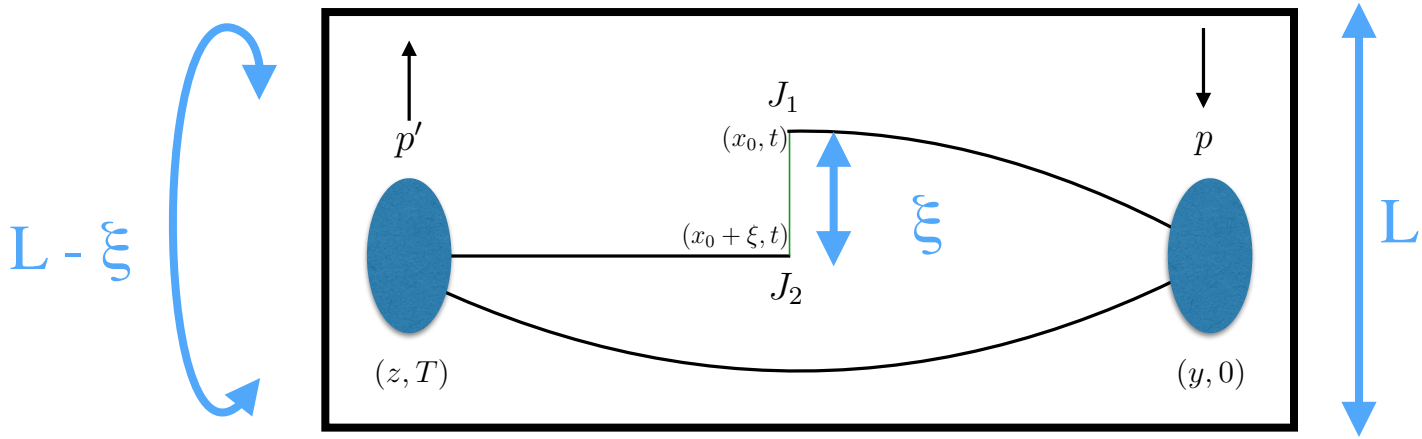


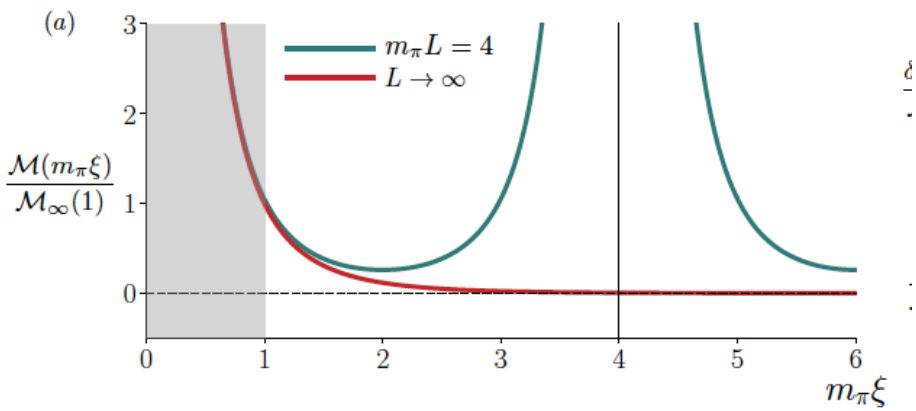
"Understanding the glue that binds us all"

BACKUP

Finite Volume Effects

Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304





Typically $m_\pi L \simeq 4$

Future? $\begin{cases} \xi \text{ shortdistance} \\ m_\pi \to m_\pi^{\text{phys}} \end{cases}$