Scaling and higher twist in the nucleon Compton amplitude

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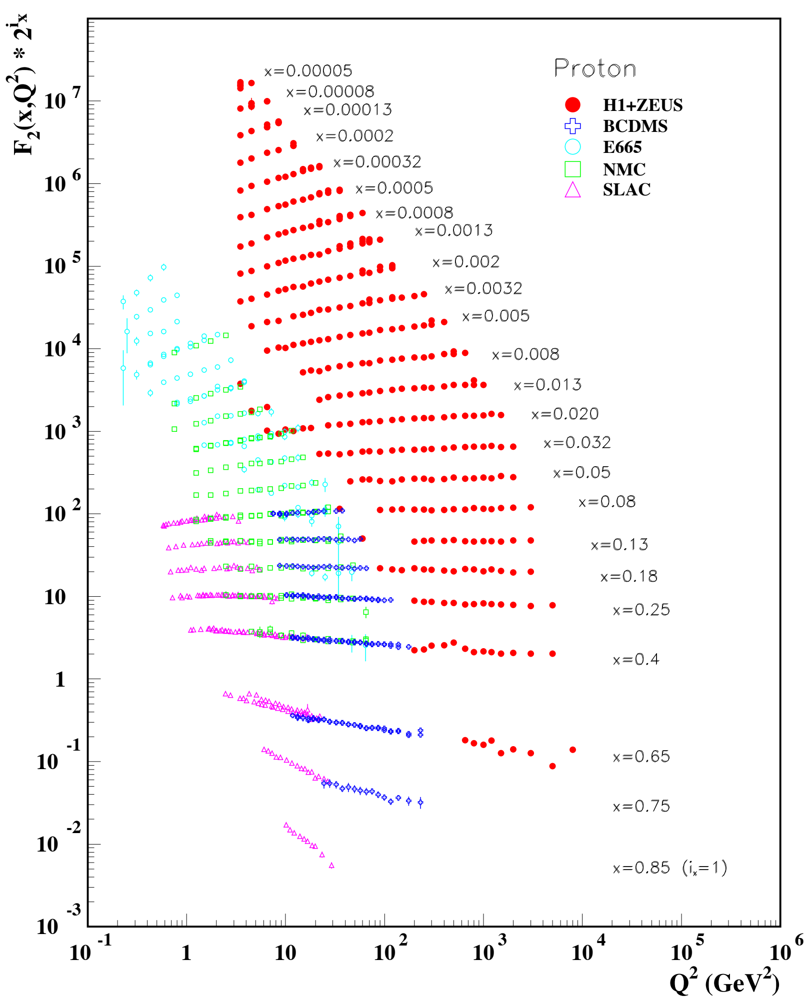
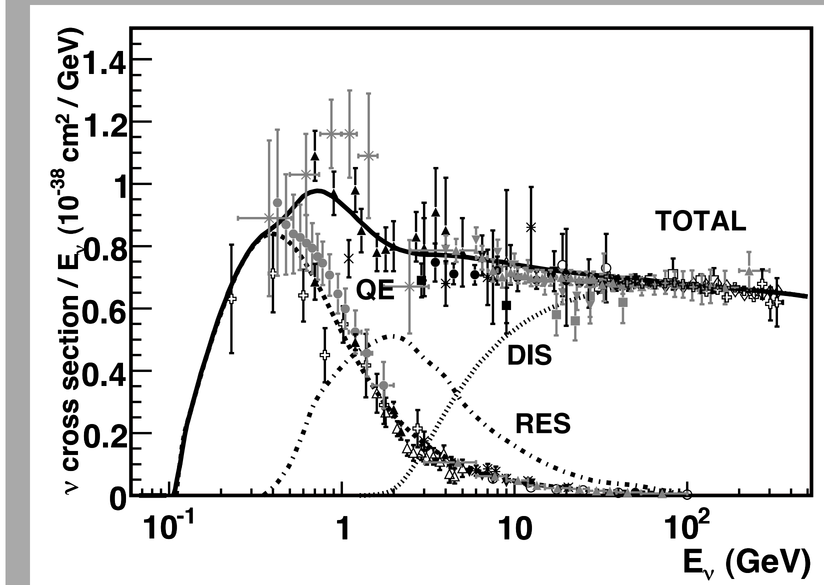
Motivation: Beyond leading twist

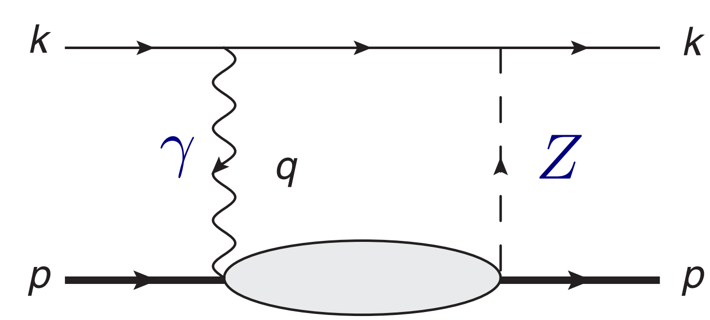
Twist-4 operators
Theoretical foundations to inform Q^2 cuts of empirical parton fits.

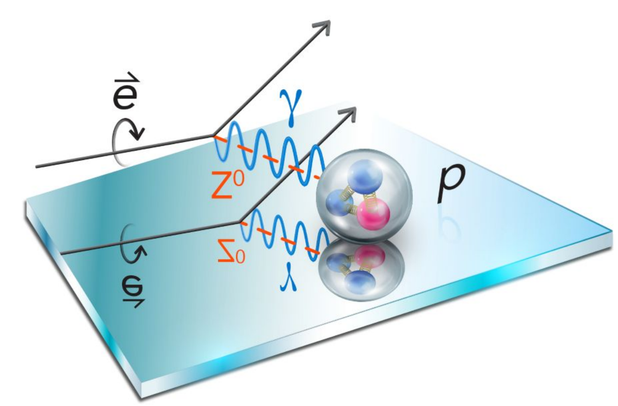




Neutrino-nucleus cross sections
Precise theoretical input required for next-generation neutrino oscillation program

Radiative corrections
Searches for new physics in the proton weak charge.
Require knowledge of gamma-*Z* interference structure functions.



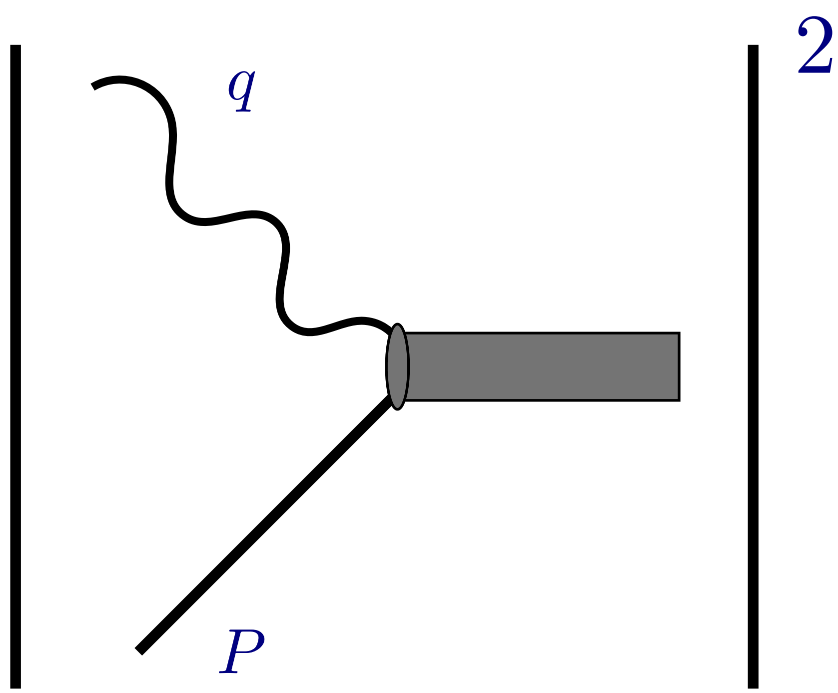


Outline

- Inelastic structure functions and the forward Compton scattering tensor
- Empirical Compton amplitude
- Compton amplitude as an energy shift: Feynman-Hellmann
- Numerical results

Inelastic structure functions and the forward Compton scattering tensor

Inelastic scattering



Cross section ~ Hadron tensor

$$W_{\mu\nu} \sim \int d^4x\, e^{iq.x} \langle p|\, [J_\mu(x), J_\nu(0)]\, |p\rangle$$

Structure functions $F_{1,2}(P.q, Q^2)$

$$F_i = \frac{1}{2\pi} \mathrm{Im}\, T_i$$

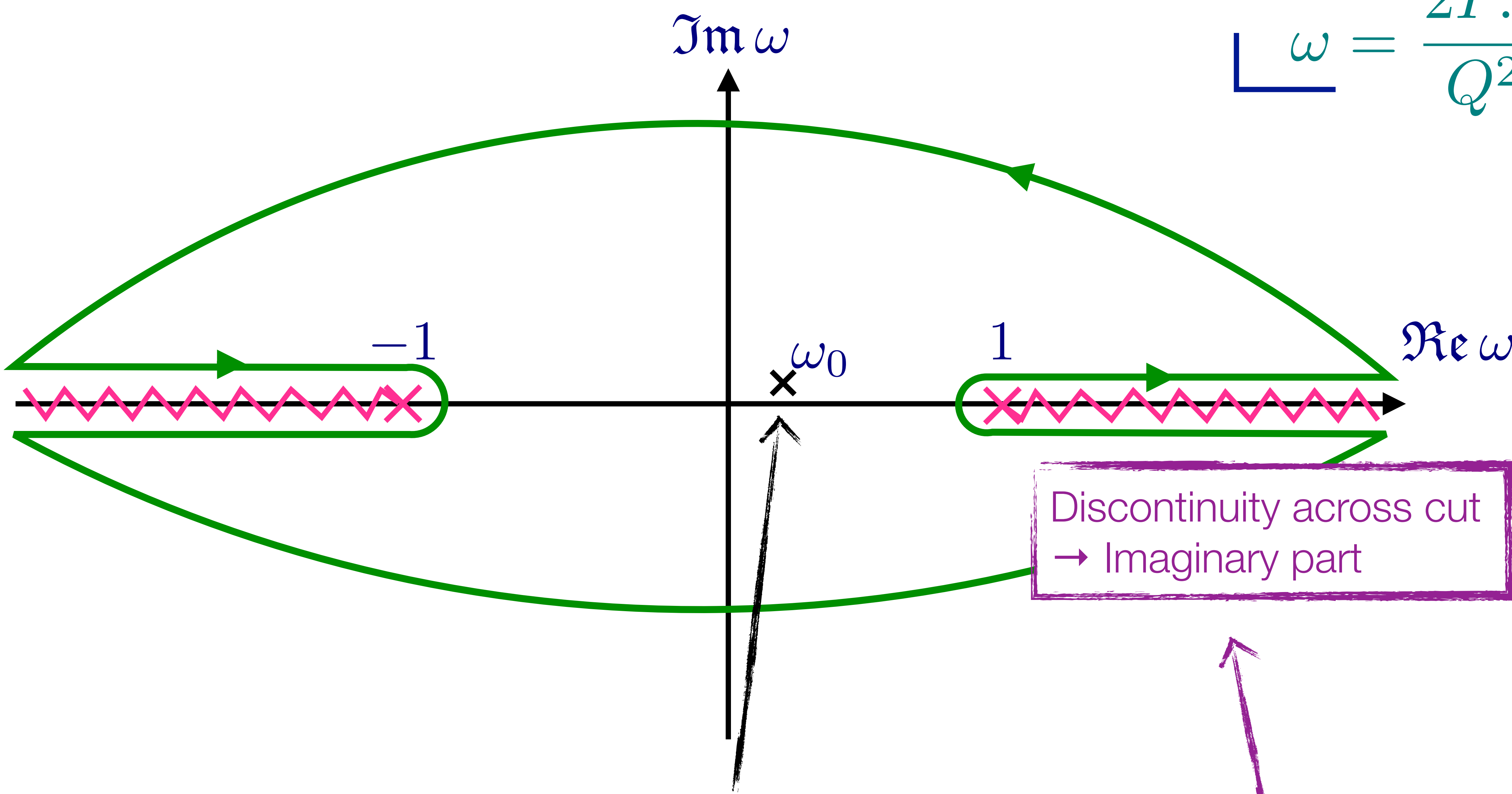
Forward Compton amplitude

$$T_{\mu\nu} \sim \int d^4x\, e^{iq.x} \langle p|\mathrm{T} J_\mu(x) J_\nu(0)|p\rangle$$

Lorentz-scalar functions $T_{1,2}(P.q, Q^2)$

Dispersion relation for Compton amplitude

$$\omega = \frac{2P.q}{Q^2}$$



Compton amplitude in unphysical region as integral over inelastic structure function

Moments of structure functions

- Compton amplitude as integral over inelastic cut:

$$\omega = \frac{2P.q}{Q^2}$$

$$T_1(\omega, Q^2) = \frac{2\omega^2}{\pi}\int_1^\infty d\omega' \frac{\operatorname{Im} T_1(\omega', Q^2)}{\omega'(\omega^2 - \omega'^2)} = 4\omega^2 \int_0^1 dx\, x \frac{F_1(x, Q^2)}{1-(\omega x)^2}$$

subtracted dispersion relation

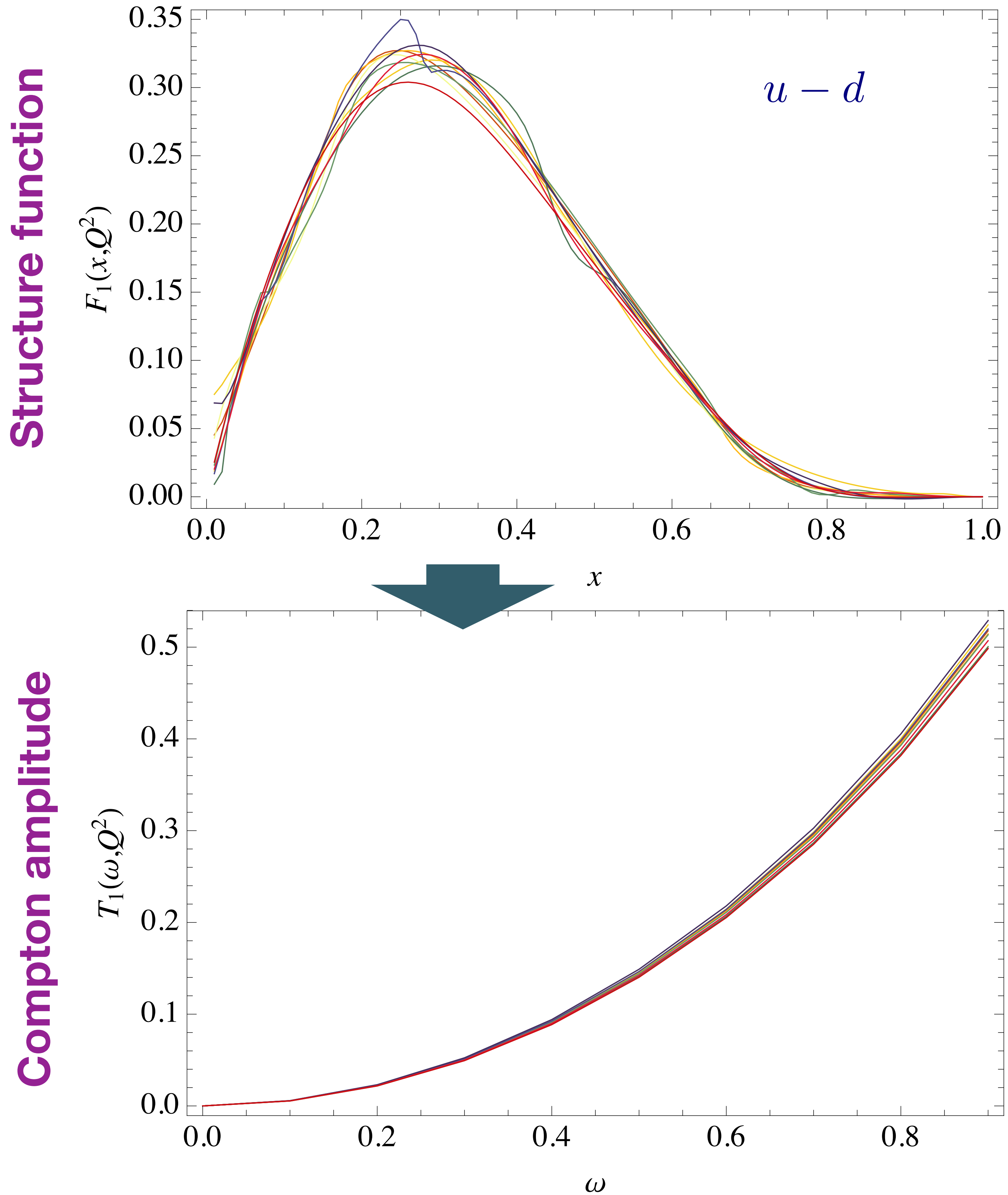
$$x = 1/\omega'$$

Taylor expansion

- **Moments of structure functions**

$$T_1(\omega, Q^2) = \sum_{j=1}^{\infty} 4\omega^{2j} \int_0^1 dx\, x^{2j-1} F_1(x, Q^2) \equiv \sum_{j=1}^{\infty} 4\omega^{2j} f_{1,2j-1}(Q^2)$$

Empirical Compton amplitude



NNPDF3.1NNLO
10 replicas

$$Q^2 = 9\,\mathrm{GeV}^2$$

$$T_1(\omega, Q^2) = \sum_{j=1}^{\infty} 4\omega^{2j}\, f_{1,2j-1}(Q^2)$$

Coefficients of Taylor expansion are moments of structure function

Compton amplitude as an energy shift: Feynman-Hellmann

$$T_1(\omega, Q^2) - T_1(\omega, 0) = 4\omega^2 \int_0^1 dx\, x \frac{F_1(x, Q^2)}{1 - (\omega x)^2}$$

Feynman–Hellmann (2nd order)

- Quantum mechanics: 2nd order perturbation theory

$$E = E_0 + \lambda \langle N|V|N\rangle + \lambda^2 \sum_{X \neq N} \frac{\langle N|V|X\rangle\langle X|V|N\rangle}{E_0 - E_X} + \ldots$$

  - Only get a linear term for elastic case ω=1 [Breit frame]

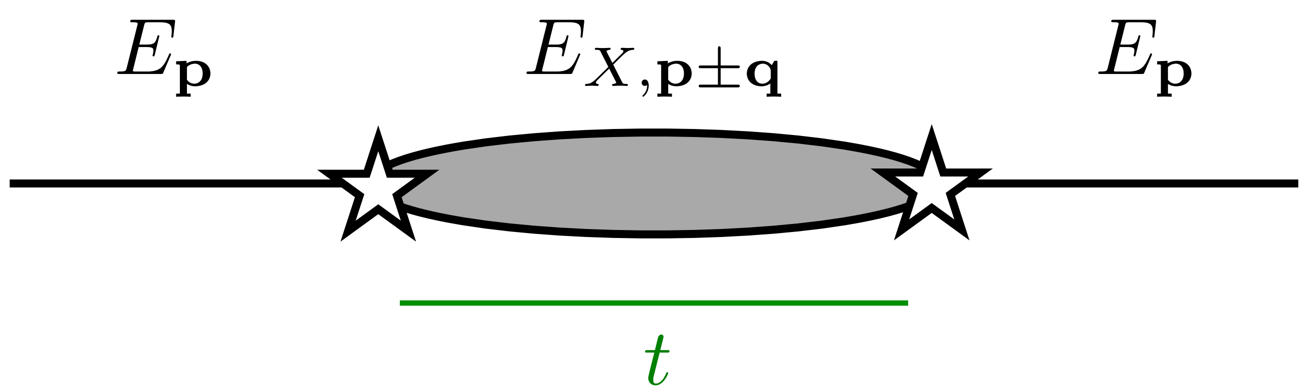
- Insert a weak spatially-varying vector current, e.g.

  - $$S \to S_0 + \lambda \int d^4y \left(e^{i\mathbf{q}\cdot\mathbf{y}} + e^{-i\mathbf{q}\cdot\mathbf{y}}\right) \bar{q}(y)\gamma_3 q(y)$$

- Second-order energy shifts isolate forward Compton amplitude $(\mathbf{q}^2 > |2\mathbf{p}.\mathbf{q}|)$

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial \lambda_{\mathbf{q}}^2} = -\frac{1}{E_{\mathbf{p}}} \int d^4x\, e^{iq.x} \langle \mathbf{p}|\mathrm{T} J(x) J(0)|\mathbf{p}\rangle$$

Two current insertions

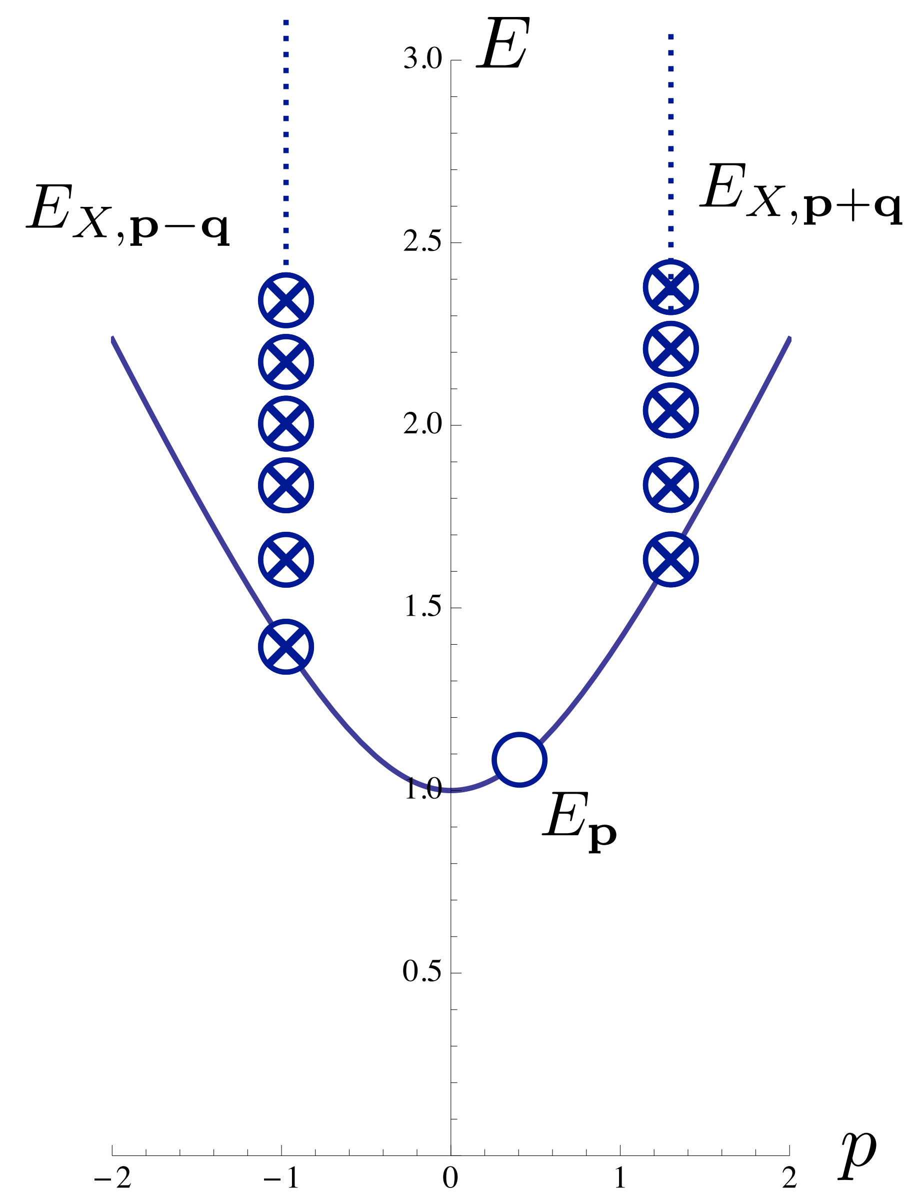


Euclidean decay of intermediate state

FH: integrate over all times

$$\int_0^T dt\, e^{-t(E_{X,\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}})} = \frac{1-e^{-T(E_{X,\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}})}}{E_{X,\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}}}$$

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2} \sim \sum_X \frac{\langle p|J|X,\mathbf{p}+\mathbf{q}\rangle\langle X,\mathbf{p}+\mathbf{q}|J|p\rangle}{E_{X,\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}}}$$
$$+\,(\mathbf{q}\to-\mathbf{q})$$

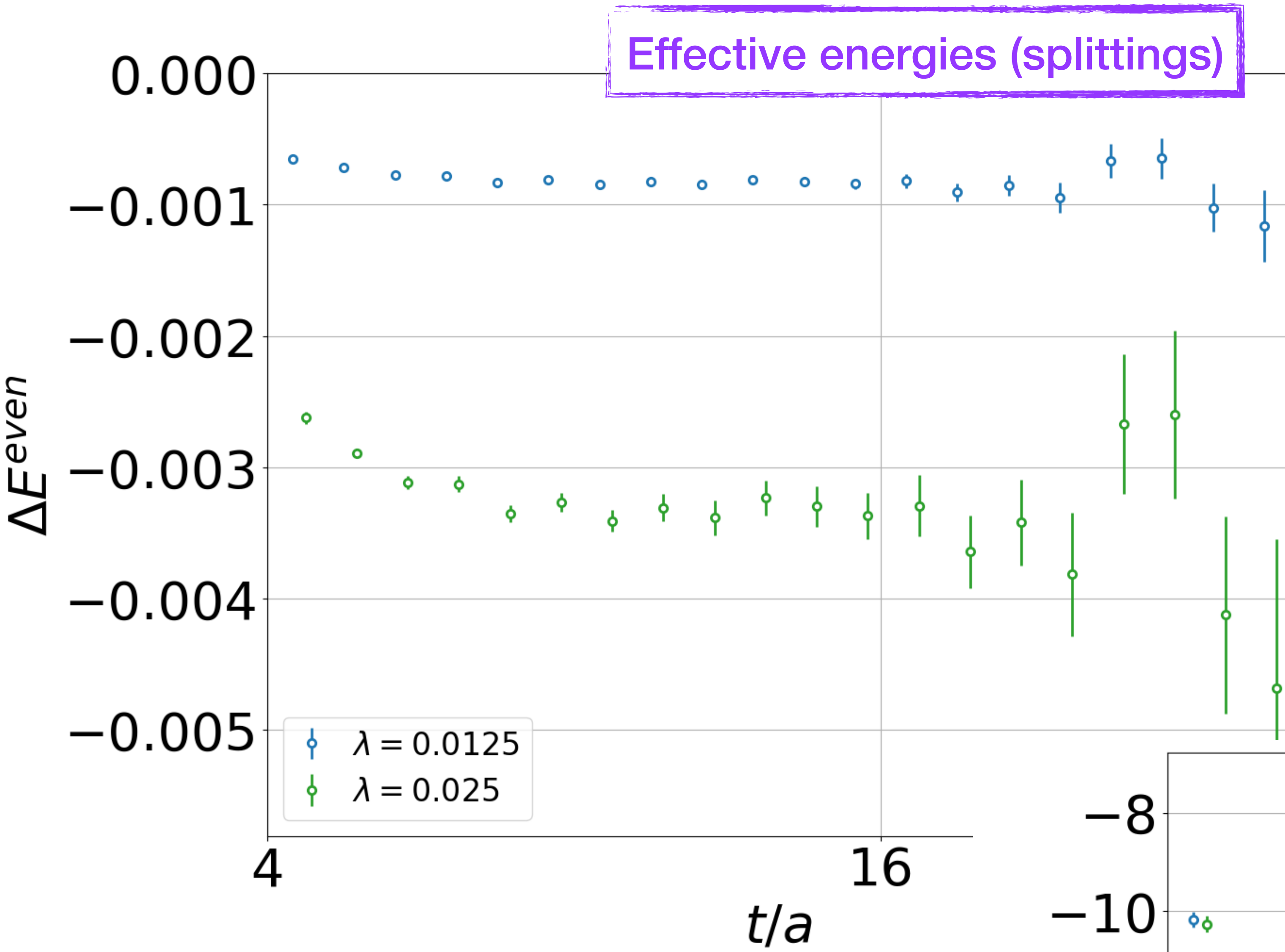


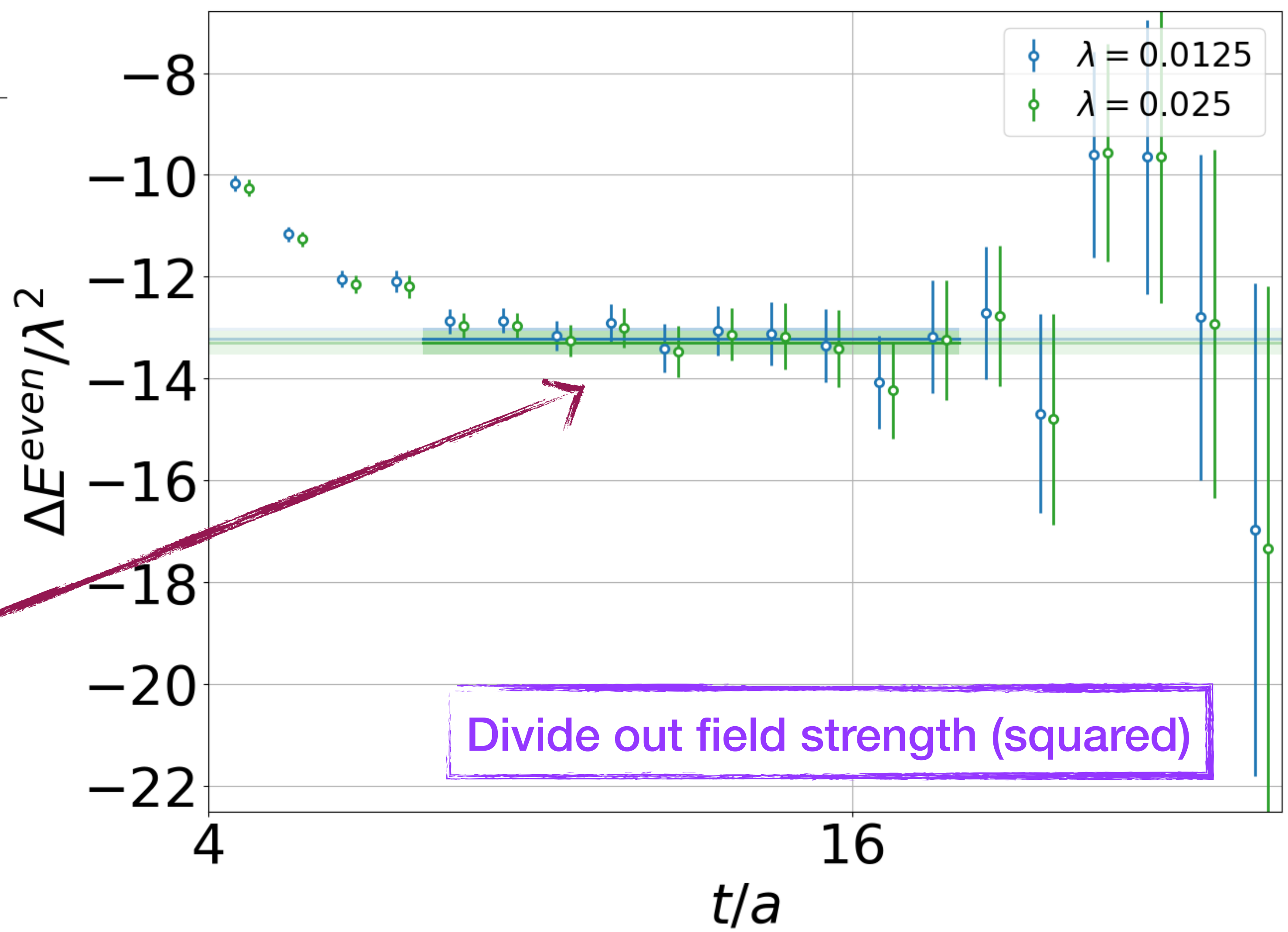
$E_{\mathbf{p}} < E_X$

Intermediate states cannot go on-shell for $\boldsymbol{\omega}<1$

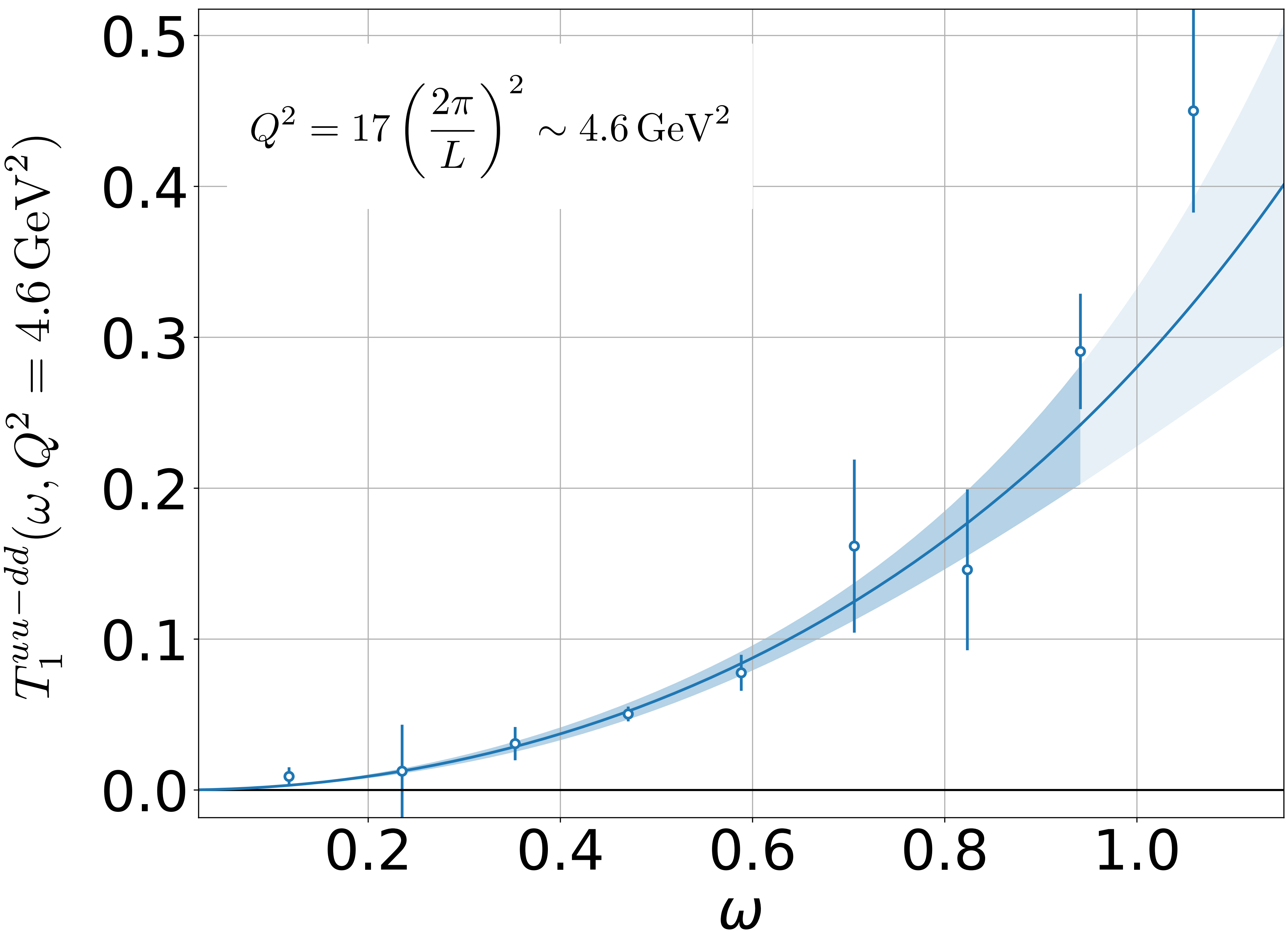
Numerical results

Lattice specs:
NP-improved clover
m_π ~ 470 MeV
SU(3) symmetric
a~0.074 fm
32^3x64
(+ *a couple extras*)





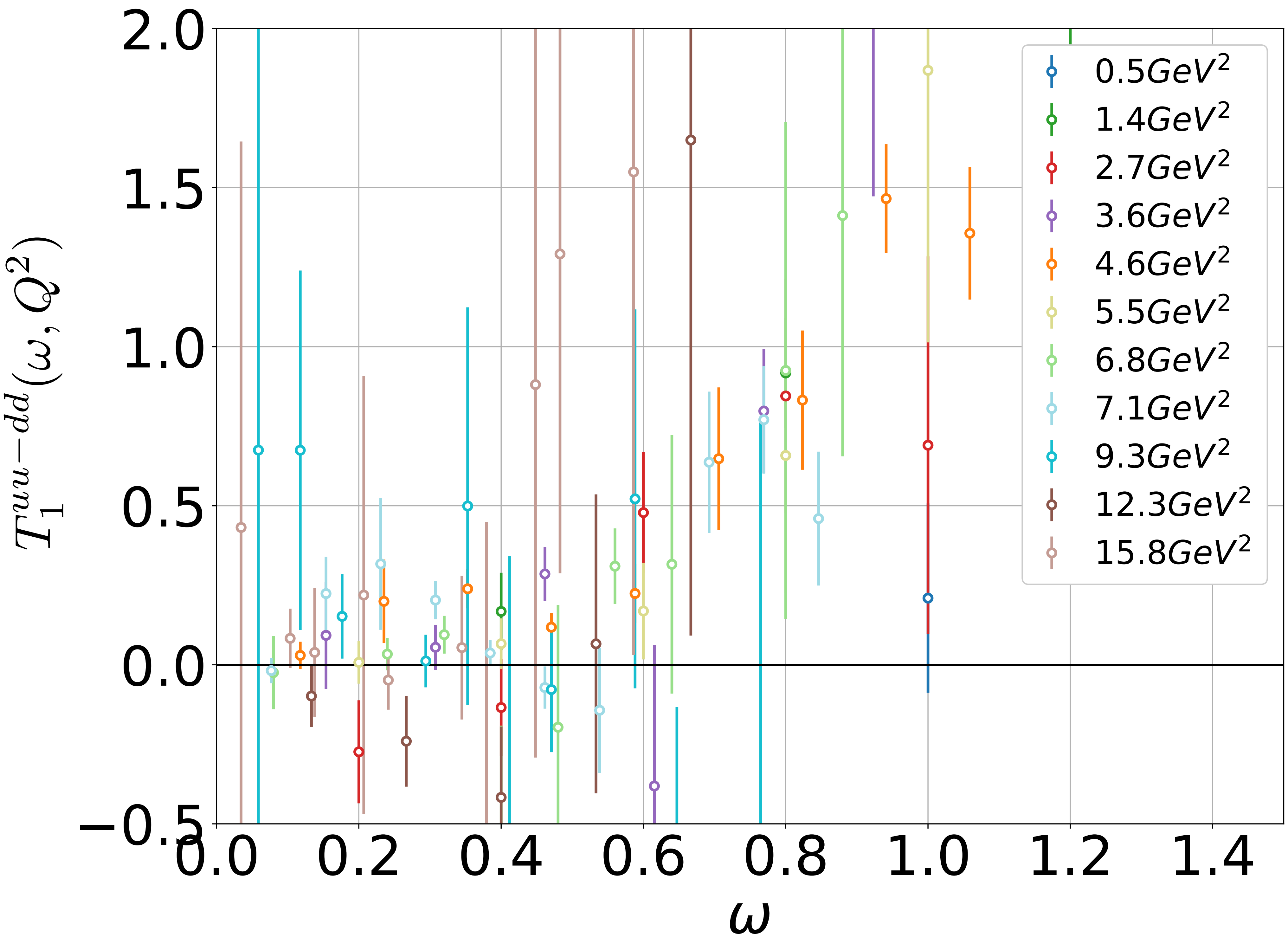
Quadratic energy shift realised (almost) exactly point-by-point in effective mass



Compton amplitude

Single external momentum

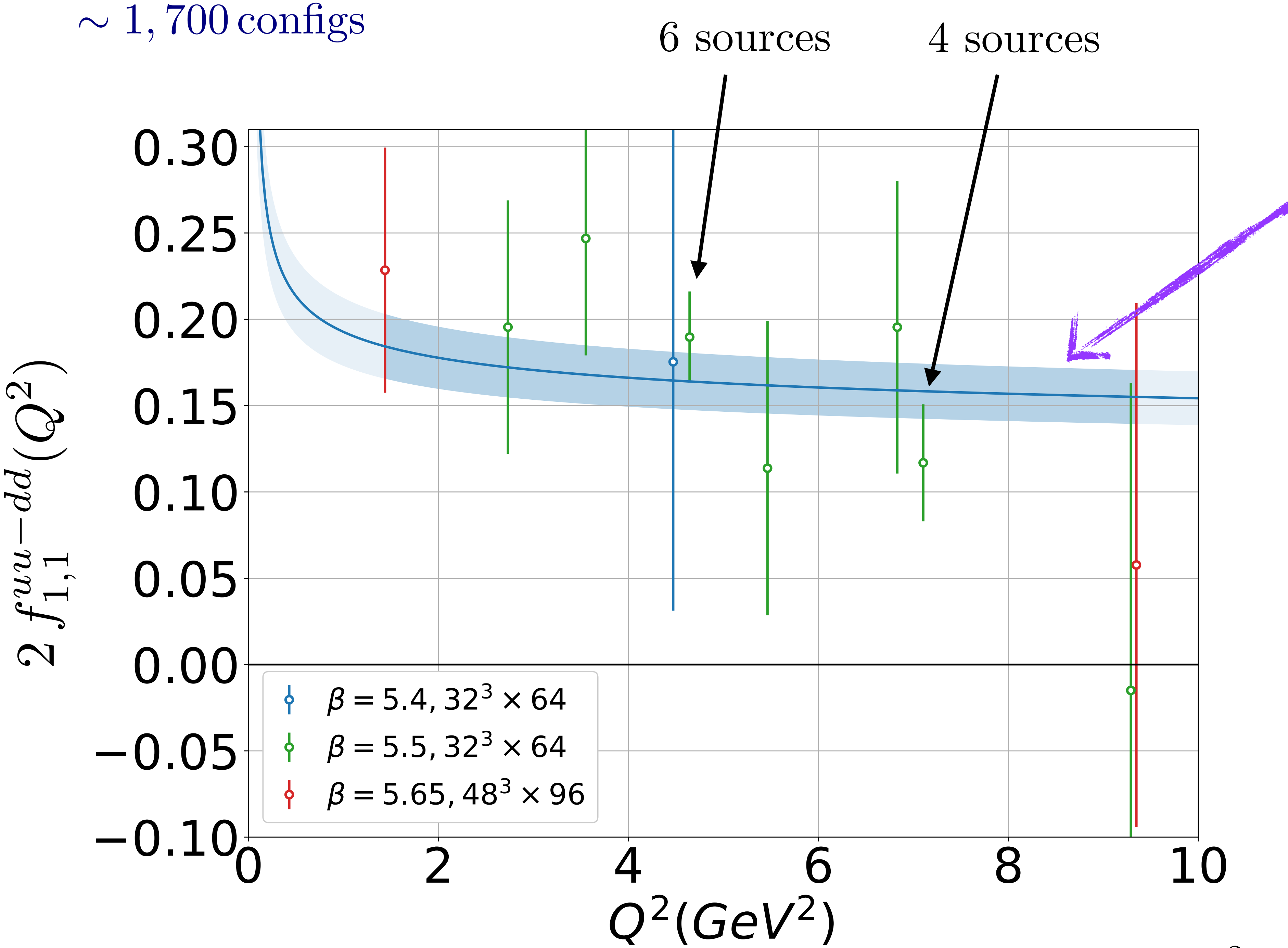
$$\frac{\vec{q}L}{2\pi} = (4, 1, 0)$$

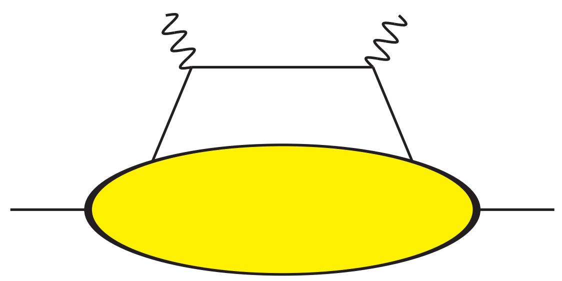


Compton amplitude

Lots of external momenta

$\sim 1,700$ configs



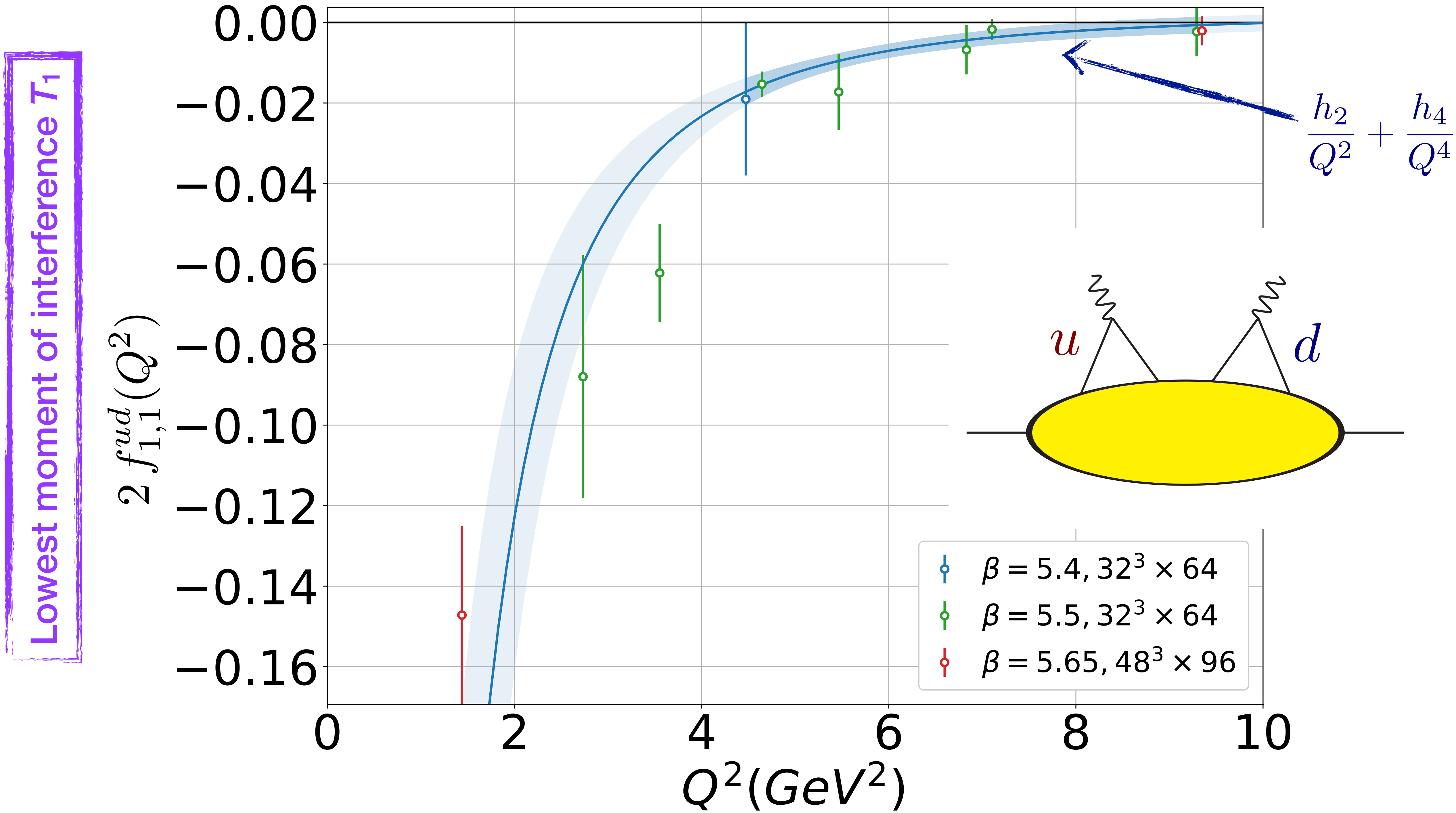


$$f_{1,1}(Q^2) \sim \frac{1}{2}\langle x \rangle\,(1 + \log)$$

$$T_1(\omega, Q^2) = \sum_{j=1}^{\infty} 4\omega^{2j}\, f_{1,2j-1}(Q^2)$$

Scaling: Lowest moment

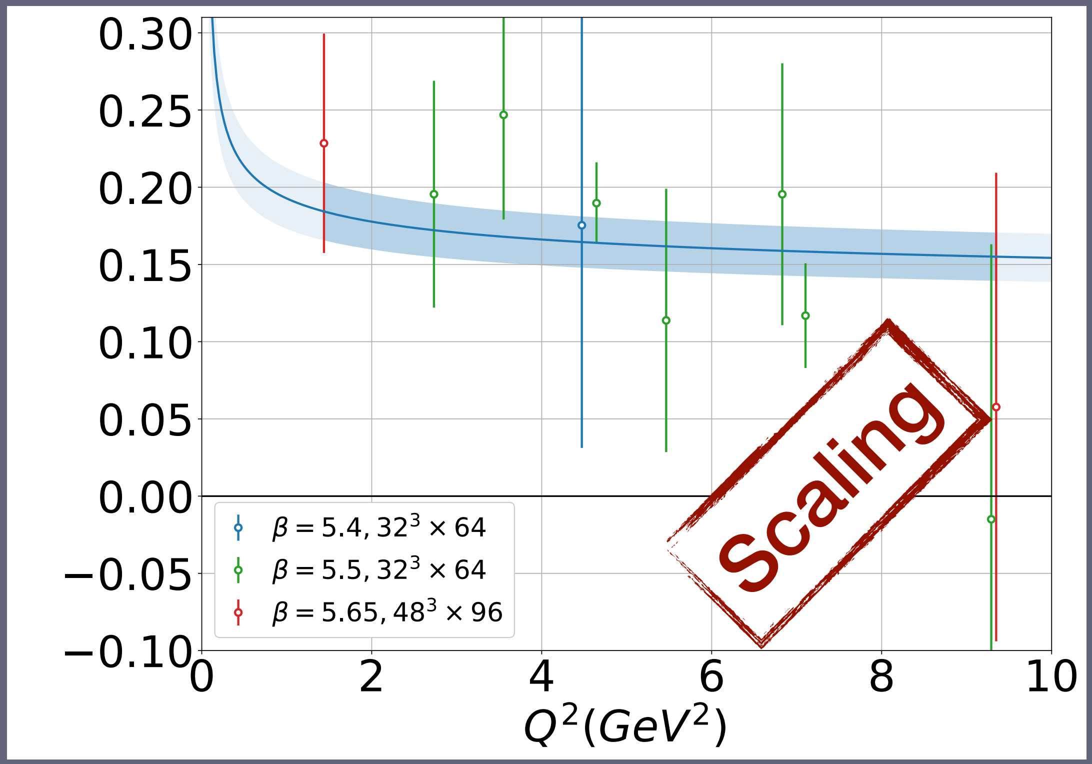
- Compatible with scaling
- Trend *not in*consistent with pQCD
- too early to assess higher twist…

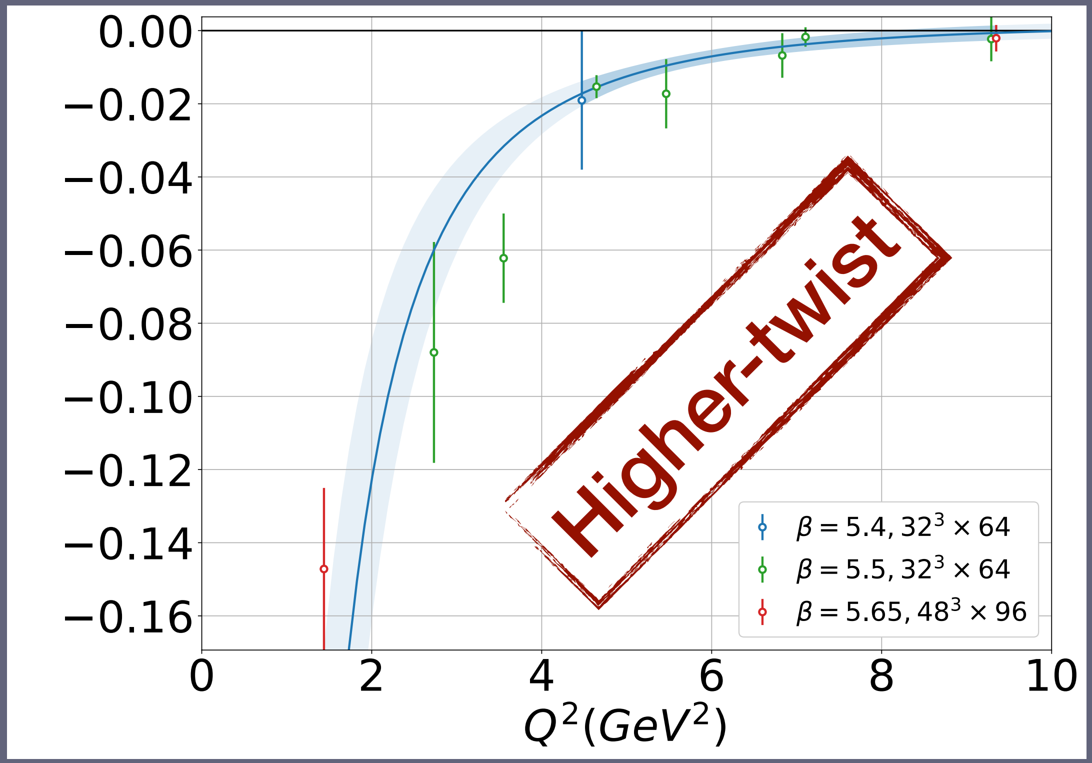


ud Interference: Higher twist

Pure higher-twist effect
Vanishes asymptotically, as expected

Concluding remarks





Feynman-Hellmann reduces computation to analysis of 2-point functions

Still some work to do:

- Inversion problem [*see poster* H. Perlt]
- Subtraction term not yet understood
- Understanding continuum limit
- Better statistics
- New insight into twist expansion

…

Physical Compton amplitude, can vary kinematics directly

Back-up slides

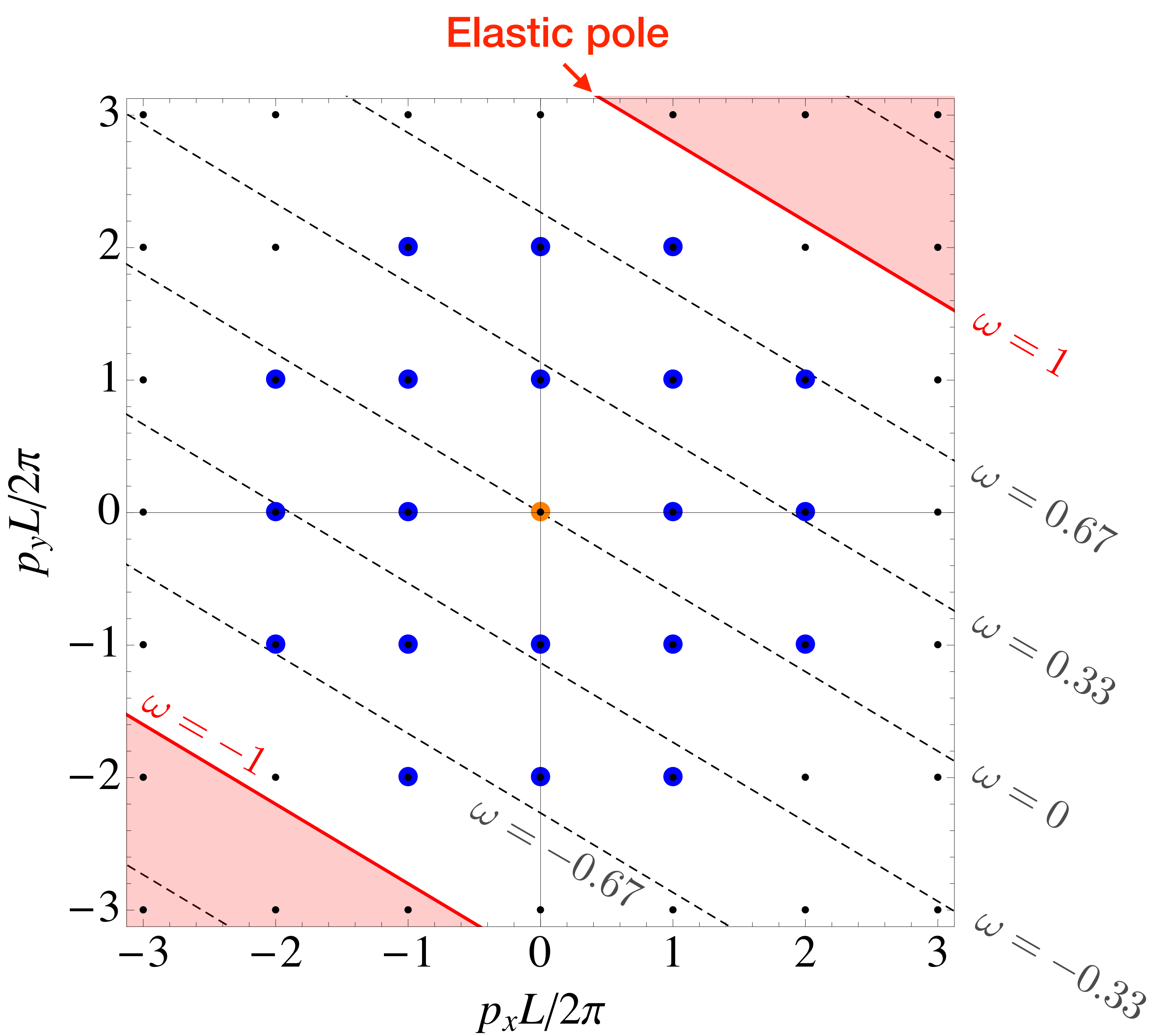
Numerical example

Single external momenta

$$\vec{q} = (3, 5, 0)\,\frac{2\pi}{L}$$

$$\omega = \frac{2P.q}{Q^2} = \frac{2\vec{P}.\vec{q}}{\vec{q}^2}$$

$q_4 = 0$



Blue dots: different nucleon Fourier momenta

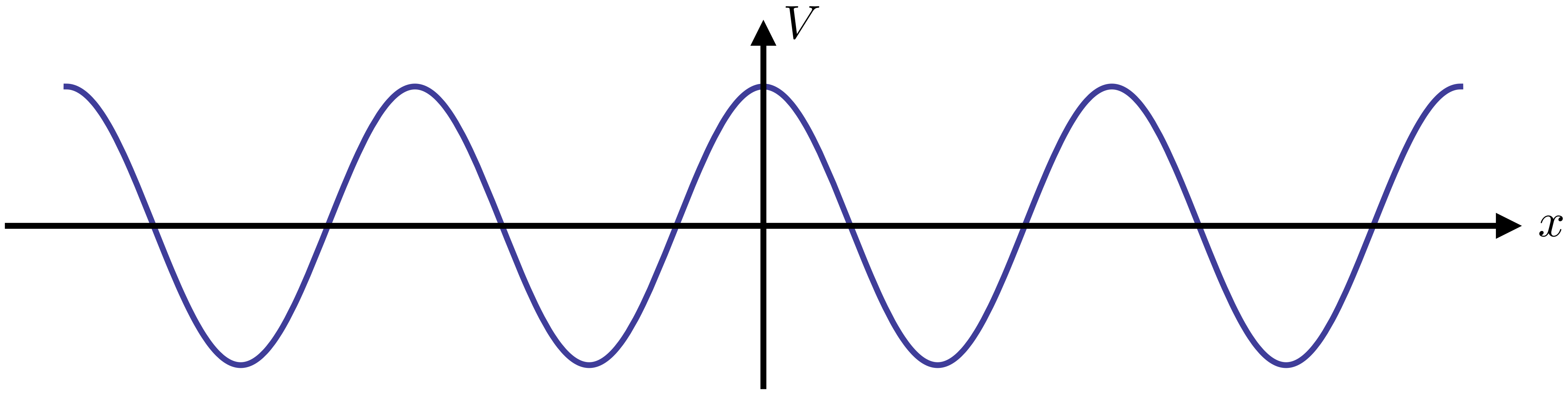
Lattice kinematics

Broad coverage of ω from single calculation (computationally "cheap")

Feynman–Hellman with momentum transfer

Warm up: Periodic potential, 1-D QM

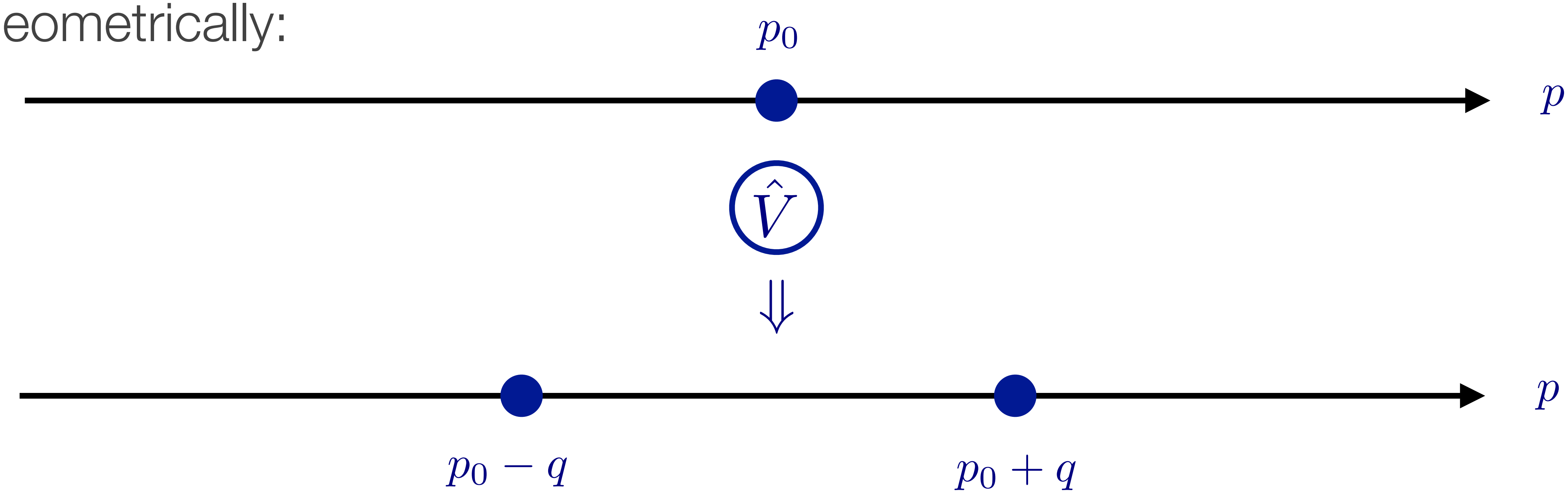
- Almost free particle $H_0|p\rangle = \frac{p^2}{2m}|p\rangle$
- Subject to weak external periodic potential $V(x) = 2\lambda V_0 \cos(qx)$



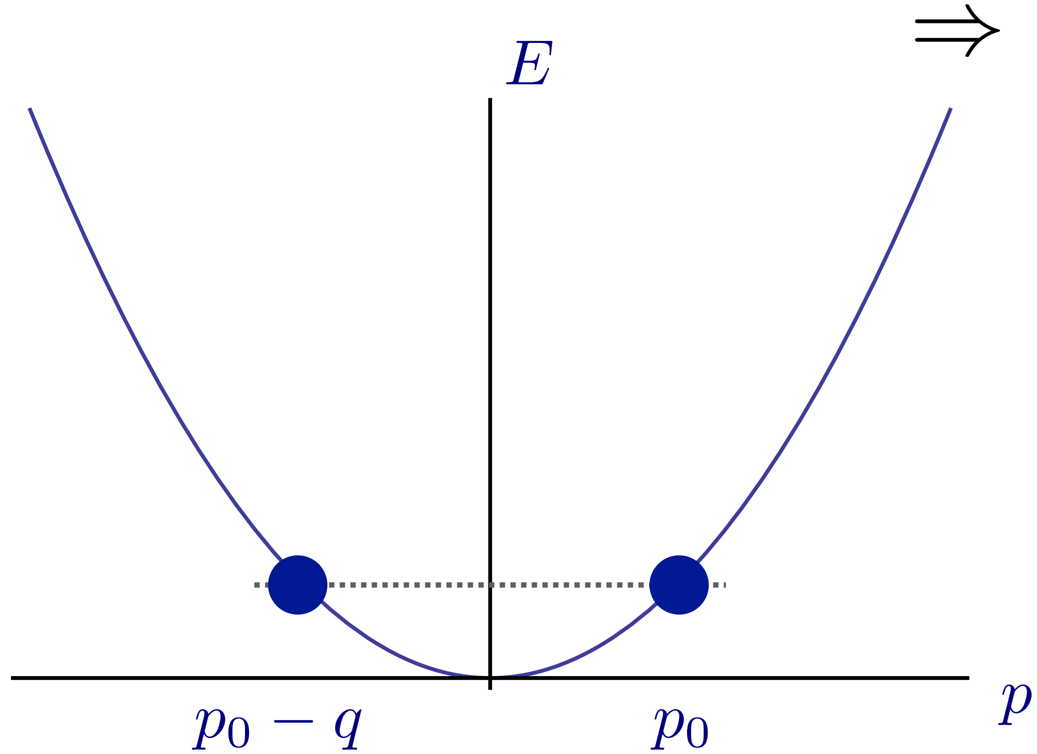
$$\hat{V}|p\rangle = \lambda V_0|p+q\rangle + \lambda V_0|p-q\rangle$$

Warm up: Periodic potential, 1-D QM

- Geometrically:



$$\Rightarrow \langle p|\hat{V}|p\rangle = 0$$



No first order energy shifts?

If $p_0 = \pm q/2$
$\Rightarrow$ transition between degenerate states

Degenerate perturbation theory

- Exact degeneracy: $p = q/2$

$$H = \begin{pmatrix} \frac{p^2}{2m} & \lambda V_0 \\ \lambda V_0 & \frac{p^2}{2m} \end{pmatrix}$$

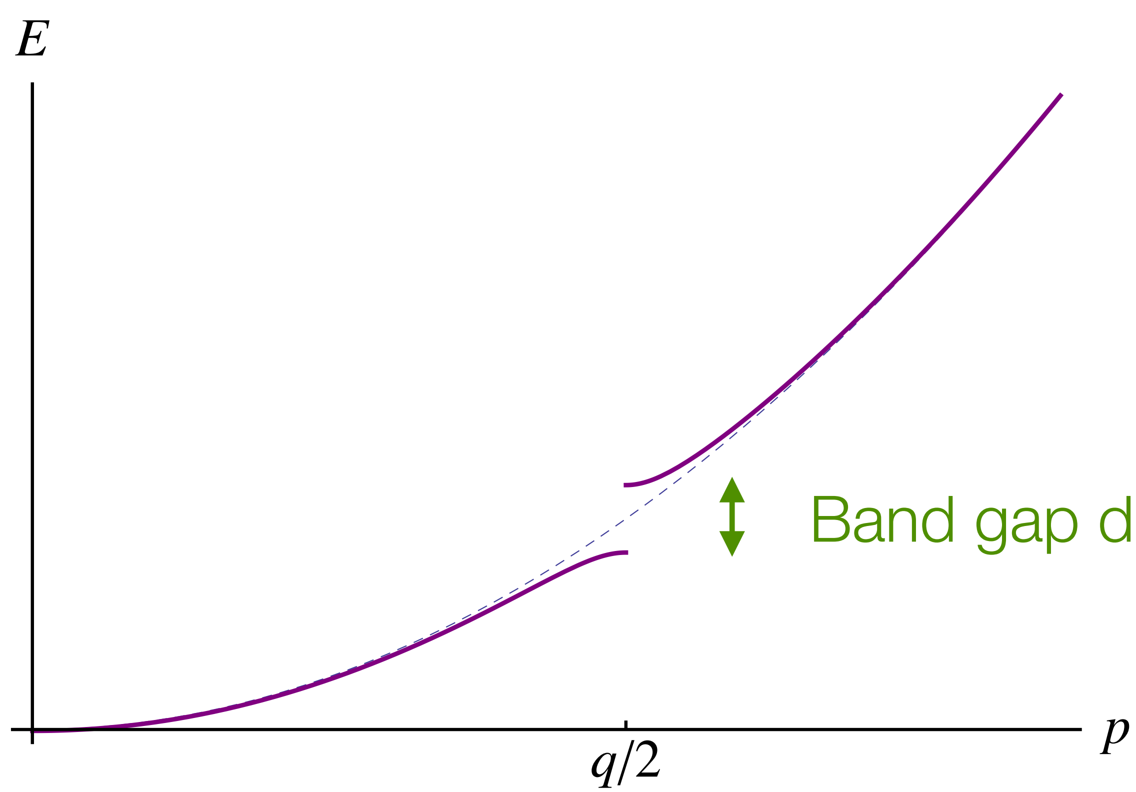
$$H \left\{ |q/2\rangle \pm |-q/2\rangle \right\} = \left(E_{q/2} \pm \lambda V_0 \right) \left\{ |q/2\rangle \pm |-q/2\rangle \right\}$$

- Consider mixing on almost-degenerate states $p \sim q/2$

$$H = \begin{pmatrix} \frac{p^2}{2m} & \lambda V_0 \\ \lambda V_0 & \frac{(p-q)^2}{2m} \end{pmatrix}$$

Eigenvalues

$$\frac{p^2 + (p-q)^2}{4m} \pm \sqrt{\frac{q^2(q-2p)^2}{16m^2} + \lambda^2 V_0^2}$$



Band gap determined by strength of perturbation

$$\Delta E = 2\lambda V_0$$

External momentum field on the lattice

- Modify Lagrangian with external field containing a spatial Fourier transform [constant in time]

$$\mathcal{L}(y) \to \mathcal{L}_0(y) + \lambda 2\cos(\vec{q}.\vec{y})\overline{q}(y)\gamma_\mu q(y)$$

- Project onto "back-to-back" momentum state: $|\vec{q}/2\rangle + |-\vec{q}/2\rangle$

"Breit frame" kinematics

- E.g. pion form factor

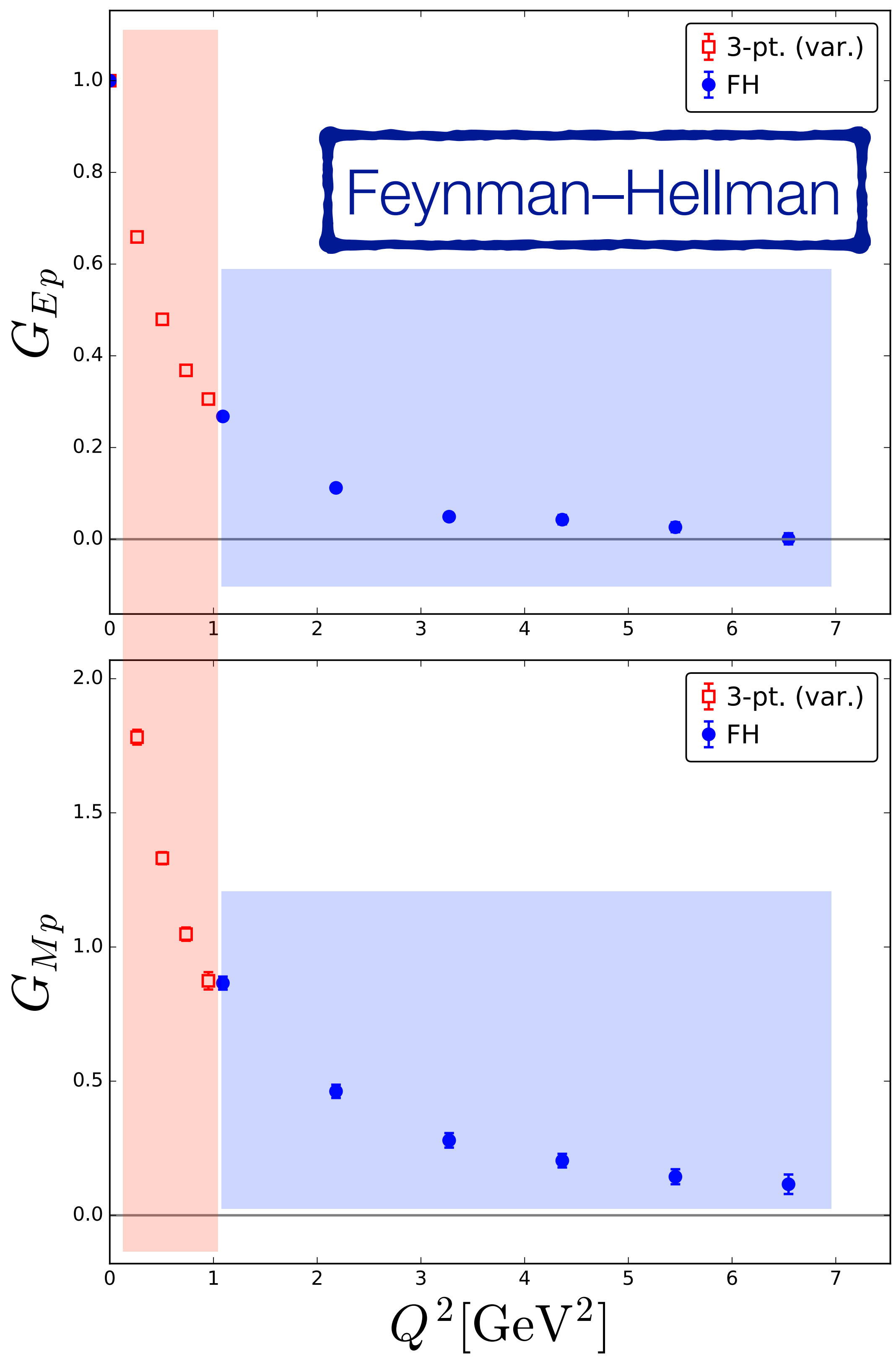
$$\langle\pi(\vec{p}')|\overline{q}(0)\gamma_\mu q(0)|\pi(\vec{p})\rangle = (p+p')_\mu F_\pi(q^2)$$

- "Feynman-Hellmann"

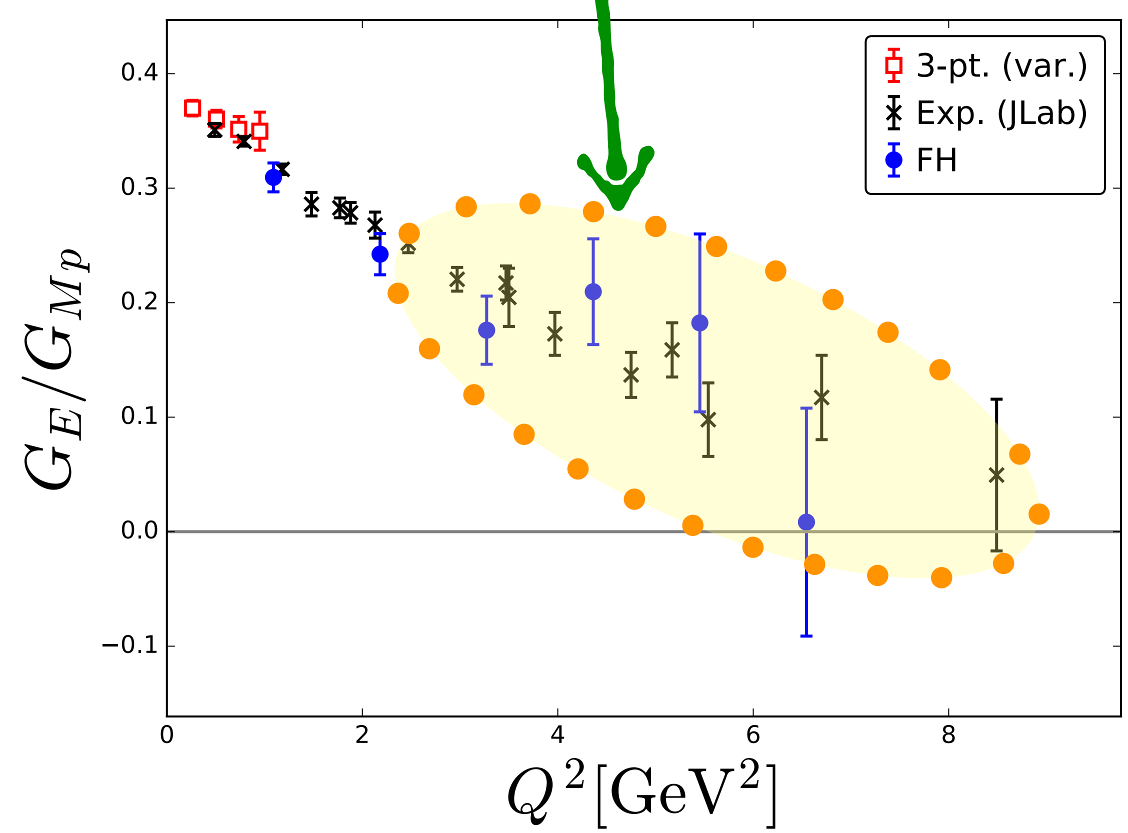
$$\left.\frac{\partial E}{\partial\lambda}\right|_{\lambda=0} = \frac{(p+p')_\mu}{2E} F_\pi(q^2) \quad \xrightarrow{\mu=4} \quad \left.\frac{\partial E}{\partial\lambda}\right|_{\lambda=0} = F_\pi(q^2)$$

Proton Form Factors

3-pt functions



Phenomenologically-interesting region.
Domain dominated by model calculations…
previously prohibitive to study in lattice QCD.

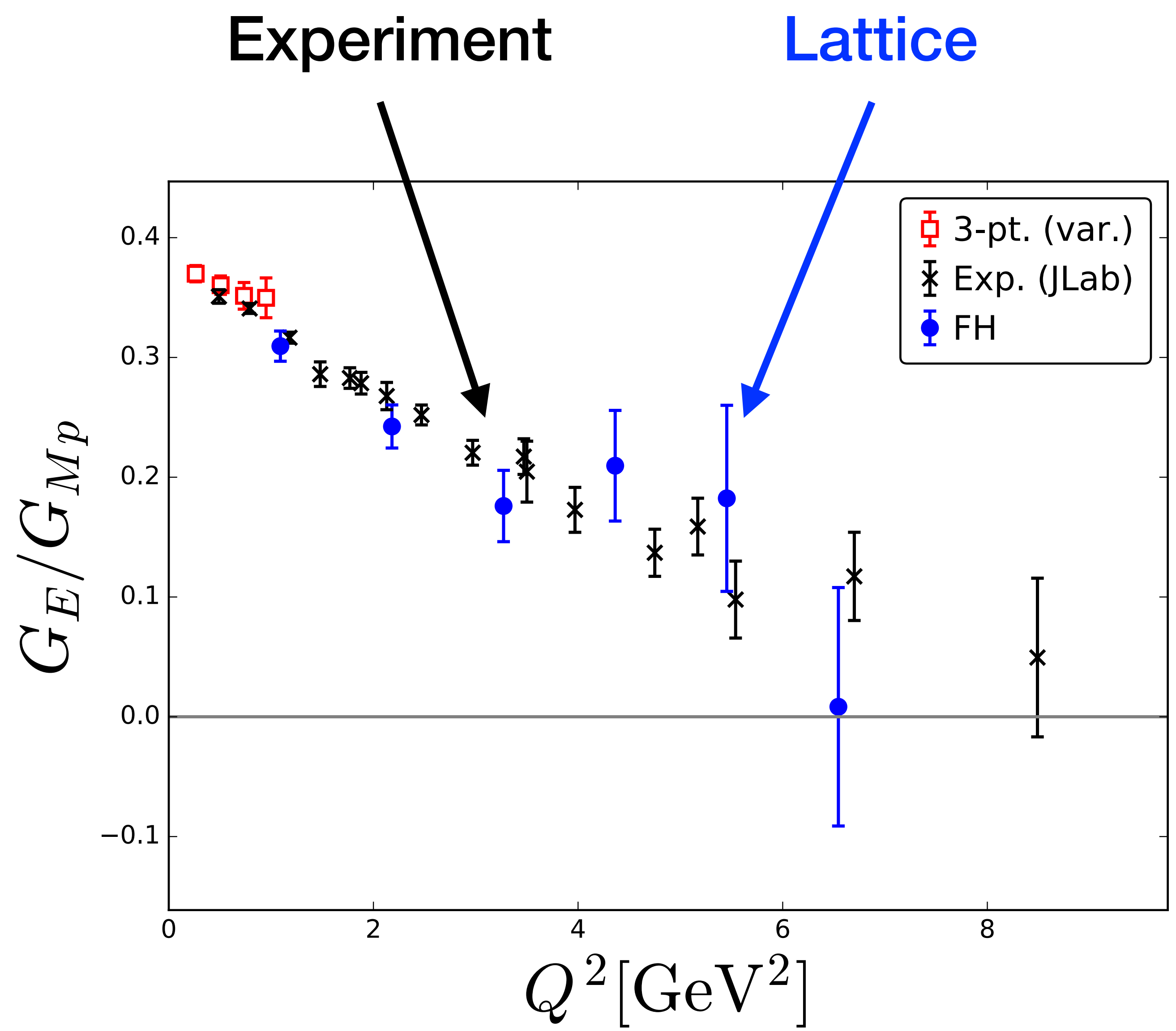


[Chambers *et al*. arXiv:1702.01513]

Proton form factors

[my comments]

- One volume
  - Not worried (yet)
- One quark mass
  - Surprised that we see a similar trend as experiment
- One lattice spacing
  - We should investigate further



[Chambers *et al*. arXiv:1702.01513]

Second-order “Feynman-Hellmann”
(with external momentum)

Feynman–Hellmann (2nd order)

- Two-point correlator

$$\int d^3x\, e^{-i\mathbf{p}.\mathbf{x}} \frac{1}{\mathcal{Z}(\lambda)} \int \mathcal{D}\phi\, \chi(x)\chi^\dagger(0) e^{-S(\lambda)} = \sum_N \frac{|{}_\lambda\langle\Omega|\chi|N,\mathbf{p}\rangle_\lambda|^2}{2E_{N,\mathbf{p}}(\lambda)} e^{-E_{N,\mathbf{p}}(\lambda)x_0}$$

Integral over all fields

only interested in perturbative shift of ground-state energy

$$\simeq A_{\mathbf{p}}(\lambda) e^{-E_{\mathbf{p}}(\lambda)x_0}$$

"Momentum" quantum# at finite field

$$|N,\mathbf{p}\rangle_\lambda$$

$$\mathbf{p} \equiv \mathbf{p} + n\mathbf{q}, \;\; n \in \mathbb{Z}$$

Feynman–Hellmann (2nd order)

- Differentiate spectral sum

$$\frac{\partial}{\partial\lambda}\sum_N \frac{|{}_\lambda\langle\Omega|\chi|N,\mathbf{p}\rangle_\lambda|^2}{2E_N(\mathbf{p},\lambda)}e^{-E_{N,\mathbf{p}}(\lambda)x_4} = \sum_N\left[\frac{\partial A_{N,\mathbf{p}}(\lambda)}{\partial\lambda} - A_{N,\mathbf{p}}(\lambda)x_4\frac{\partial E_{N,\mathbf{p}}}{\partial\lambda}\right]e^{-E_{N,\mathbf{p}}(\lambda)x_4}$$

$$\to\left[\frac{\partial A_{\mathbf{p}}(\lambda)}{\partial\lambda} - A_{\mathbf{p}}(\lambda)x_4\frac{\partial E_{\mathbf{p}}}{\partial\lambda}\right]e^{-E_{\mathbf{p}}(\lambda)x_4}$$

- And again

Not Breit frame, ω<1 ⇒ 0

$$\frac{\partial^2}{\partial\lambda^2}\left[\cdots\right] = \sum_N\left[\frac{\partial^2 A_{N,\mathbf{p}}(\lambda)}{\partial\lambda^2} - 2\frac{\partial A_{N,\mathbf{p}}(\lambda)}{\partial\lambda}x_4\frac{\partial E_{N,\mathbf{p}}(\lambda)}{\partial\lambda} - A_{N,\mathbf{p}}(\lambda)x_4\frac{\partial^2 E_{N,\mathbf{p}}(\lambda)}{\partial\lambda^2} + A_{N,\mathbf{p}}(\lambda)x_4^2\left(\frac{\partial E_{N,\mathbf{p}}(\lambda)}{\partial\lambda}\right)^2\right]$$

$$\to\left[\frac{\partial^2 A_{\mathbf{p}}(\lambda)}{\partial\lambda^2} - A_{\mathbf{p}}(\lambda)x_4\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2}\right]e^{-E_{\mathbf{p}}(\lambda)x_4}$$

Quadratic energy shift

Watch for temporal enhancement $\sim x_4\, e^{-E_{\mathbf{p}}x_4}$

Feynman–Hellmann (2nd order)

- **Differentiate path integral**

$$\frac{\partial}{\partial\lambda}\int d^3x\, e^{-i\mathbf{p}.\mathbf{x}}\frac{1}{\mathcal{Z}(\lambda)}\int \mathcal{D}\phi\,\chi(x)\chi^\dagger(0)e^{-S(\lambda)}$$

$$= \int d^3x\, e^{-i\mathbf{p}.\mathbf{x}}\frac{1}{\mathcal{Z}(\lambda)}\int \mathcal{D}\phi\,\chi(x)\chi^\dagger(0)\left[-\frac{\partial S}{\partial\lambda}-\frac{1}{\mathcal{Z}(\lambda)}\frac{\partial\mathcal{Z}}{\partial\lambda}\right]e^{-S(\lambda)},$$

"Disconnected" operator insertions; drop for simplicity

- Differentiate again, take zero-field limit and note: $\frac{\partial^2 S}{\partial\lambda^2}=0$

$$\frac{\partial^2}{\partial\lambda^2}[\cdots]\bigg|_{\lambda=0} = \int d^3x\, e^{-i\mathbf{p}.\mathbf{x}}\frac{1}{\mathcal{Z}_0}\int \mathcal{D}\phi\,\chi(x)\chi^\dagger(0)\left(\frac{\partial S}{\partial\lambda}\right)^2 e^{-S_0}$$

Current insertions integrated over 4-volume

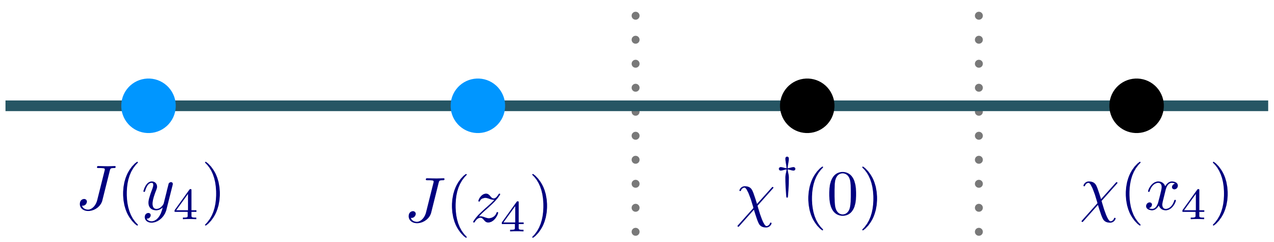
$$\frac{\partial S}{\partial\lambda} = \int d^4y\, 2\cos(\mathbf{q}.\mathbf{y})\overline{q}(y)\gamma_\mu q(y)$$

Field time orderings

ignore finite T

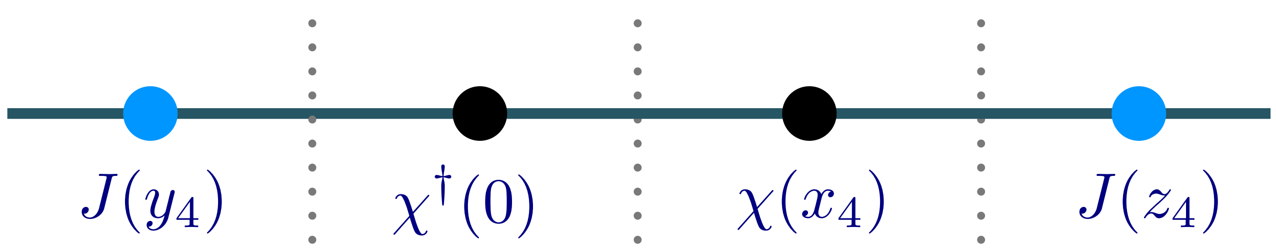
- Current insertion possibilities

  - Both currents “outside” (together)



$$\langle \chi(x)\chi^\dagger(0)\mathrm{T}(J(y)J(z))\rangle, \quad y_4, z_4 < 0 < x_4$$
$$\sim e^{-E_X x_4}, \quad E_X \gtrsim E_{\mathbf{p}}$$

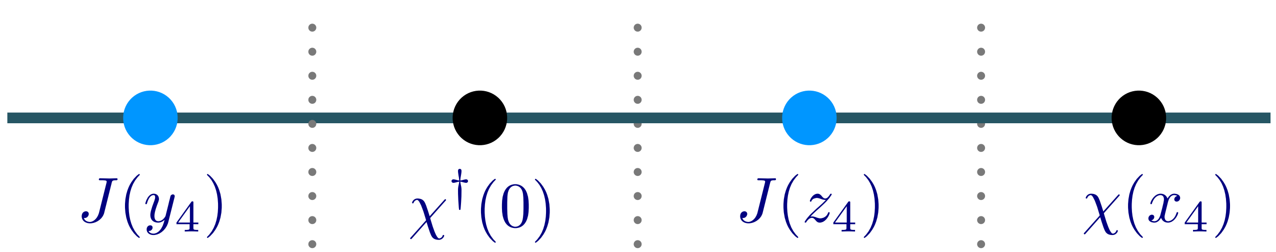
  - Both currents “outside” (opposite)



$$\langle J(z)\chi(x)\chi^\dagger(0)J(y)\rangle, \quad y_4 < 0 < x_4 < z_4$$
$$\sim e^{-E_X x_4}, \quad E_X \gtrsim E_{\mathbf{p}}$$

$E_X = E_{\mathbf{p}} \Rightarrow$ changes amplitudes

  - One current “inside”

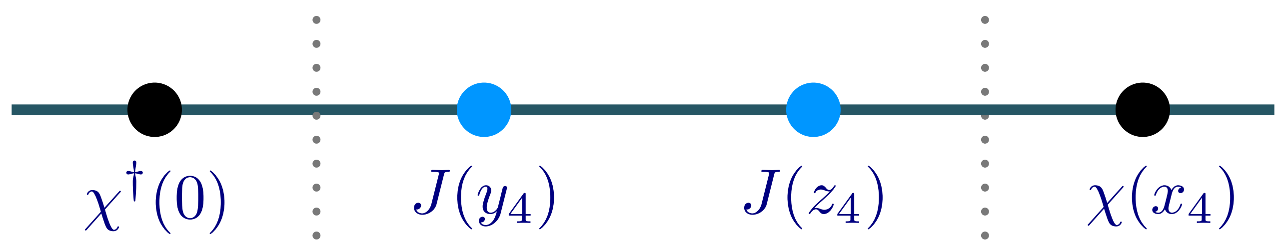


$$\langle \chi(x)J(z)\chi^\dagger(0)J(y)\rangle, \quad y_4 < 0 < z_4 < x_4$$
$$\sim \frac{\partial E_{\mathbf{p}}}{\partial \lambda} x_4 \, e^{-E_{\mathbf{p}} x_4} \to 0$$

linear energy shift
(and changed amplitude)

Field time orderings

- Both currents between creation/annihilation



$$\int d^3x\, e^{-i\mathbf{p}.\mathbf{x}} \frac{1}{\mathcal{Z}_0} \int \mathcal{D}\phi\, \chi(x)\chi^\dagger(0) \left(\frac{\partial S}{\partial \lambda}\right)^2 e^{-S_0}$$

$$= \sum_{N,N'} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_{N,\mathbf{k}}} \int \frac{d^3k'}{(2\pi)^3} \frac{1}{2E_{N',\mathbf{k}'}} \int d^3x \int d^4z \int d^4y\, e^{-i\mathbf{p}.\mathbf{x}} \left(e^{i\mathbf{q}.\mathbf{z}} + e^{-i\mathbf{q}.\mathbf{z}}\right) \left(e^{i\mathbf{q}.\mathbf{y}} + e^{-i\mathbf{q}.\mathbf{y}}\right)$$

$$\times \langle\Omega|\chi(x)|N,\mathbf{k}\rangle\langle\mathbf{k}|\mathrm{T}J(z)J(y)|\mathbf{k}'\rangle\langle N',\mathbf{k}'|\chi^\dagger(0)|\Omega\rangle,$$

$$\vdots$$

$$\to \frac{A_{\mathbf{p}}}{2E_{\mathbf{p}}} x_4 e^{-E_{\mathbf{p}}x_4} \int d^4\xi \left(e^{iq.\xi} + e^{-iq.\xi}\right) \langle\mathbf{p}|\mathrm{T}J(\xi)J(0)|\mathbf{p}\rangle$$

Note $q_4 = 0 \quad \Rightarrow \mathbf{q}.\boldsymbol{\xi} = q.\xi$

Final steps

- Equate spectral sum and path integral representation
  - Asymptotically, we have

$$-A_{\mathbf{p}}\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2}x_4 e^{-E_{\mathbf{p}}x_4} = \frac{A_{\mathbf{p}}}{2E_{\mathbf{p}}}x_4 e^{-E_{\mathbf{p}}x_4}\int d^4\xi\,\left(e^{iq.\xi}+e^{-iq.\xi}\right)\langle\mathbf{p}|\mathrm{T}J(\xi)J(0)|\mathbf{p}\rangle$$

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial\lambda^2} = -\frac{1}{2E_{\mathbf{p}}}\int d^4\xi\,\left(e^{iq.\xi}+e^{-iq.\xi}\right)\langle\mathbf{p}|\mathrm{T}J(\xi)J(0)|\mathbf{p}\rangle$$