Parton distribution functions of Δ^+ on the lattice

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Motivation

Δ is one of the most interesting baryons:

- Difficult to measure Δ^+ parton distribution function in experiment
- Δ structure important input for phenomenological modelling
- The $\bar u - \bar d$ magnitude in Δ^+ is expected to be twice larger than in proton [1]

LaMET

Quasi-PDFs [2]

$$\tilde q(x, P_z, \mu) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-ixP_z z}\langle P|\bar\psi(z)\Gamma W(z,0)\psi(0)|P\rangle \quad (1)$$

Relation between quasi-PDFs and PDFs

$$\tilde q(x, P_z, \mu) = \int_{-1}^{1}\frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right) \quad (2)$$

matching kernel, perturbatively calculable

Lattice details

Matrix elements

unpolarized case (no mixing [3]):

$$h(\vec p, z) = \langle \Delta(\vec p)|\,\bar\psi(z)\gamma^0 W(z,0)\psi(0)|\Delta(\vec p)\rangle \quad (3)$$

Three point function:

$$C^{3pt}(t,\tau,0;z,\vec p) = \langle \Gamma_{\alpha\beta}\Delta^i_\alpha(\vec p,t)O(z,\tau)\bar\Delta^i_\beta(\vec p,0)\rangle \quad (4)$$

Operator:

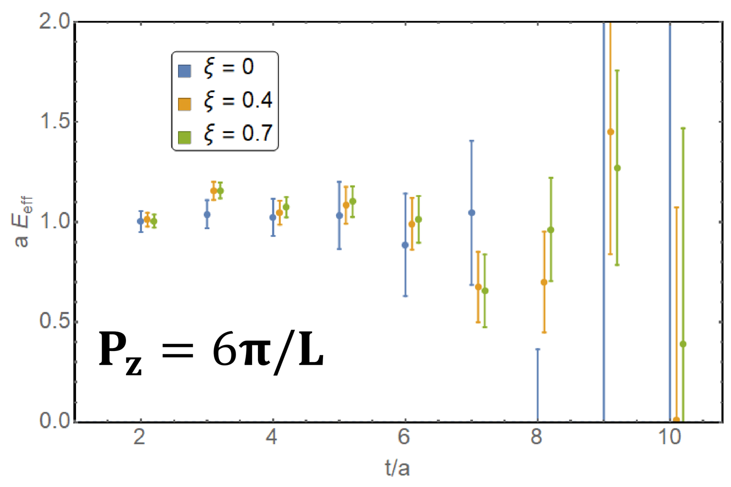
$$O(z,\tau) = \sum_{\vec y}\bar\psi(y+z)\gamma^0 W(y+z,y)\psi(y) \quad (5)$$

Interpolating field for Δ^+

$$\Delta^{\mu}_{\alpha}(\vec p,t) = \sum_{\vec x} e^{-i\vec p\cdot\vec x}\frac{1}{\sqrt{3}}\epsilon^{abc}[2\left(u^{a^T}(x)\,C\,\gamma^\mu d^b(x)\right)u^c_\alpha(x) + \left(u^{a^T}(x)\,C\,\gamma_\mu u^b(x)\right)d^c_\alpha(x)] \quad (6)$$

We use only non-spin-projected interpolating fields here, because the overlap with spin-1/2 state is expected to be small.

Momentum smearing [4]



$$S_{mom} = \frac{1}{1+6\alpha}\left(\psi(x) + \alpha\sum_j U_j(x)\, e^{-i\xi P\cdot j}\,\psi(x+\hat j)\right) \quad (7)$$

error reduces greatly

we choose $\xi = 0.7$

Information of ensembles	
$\beta = 1.726$ $C_{sw} = 1.74$ $a = 0.096\,fm$	
$24^3\times 48$, $L\approx 2.3fm$	$32^3\times 64$, $L\approx 3.1fm$
$a\mu = 0.0053$, $m_\pi L = 3.795$, $m_\pi = 0.330\ GeV$, $m_{\Delta^+} = 1.59(4)\ GeV$, $m_N = 1.21(2) GeV$	$a\mu = 0.003$, $m_\pi L = 3.95$, $m_\pi = 0.255\ GeV$, $m_{\Delta^+} = 1.39(5)\ GeV$, $m_N = 1.08(2)\ GeV$

Statistics

P_z	2π/L(0.54GeV)				2π/L(0.41GeV)	4π/L(0.82GeV)
T_{sink}	9a	10a	11a	12a	10a	
meas	887	887	768	768	906	1248

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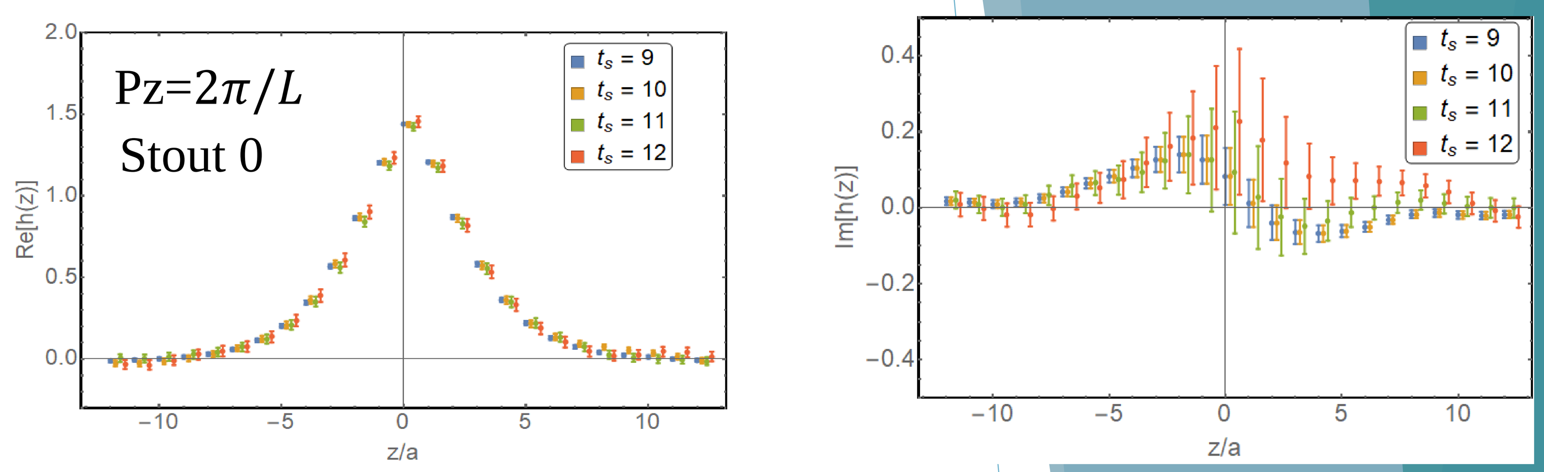
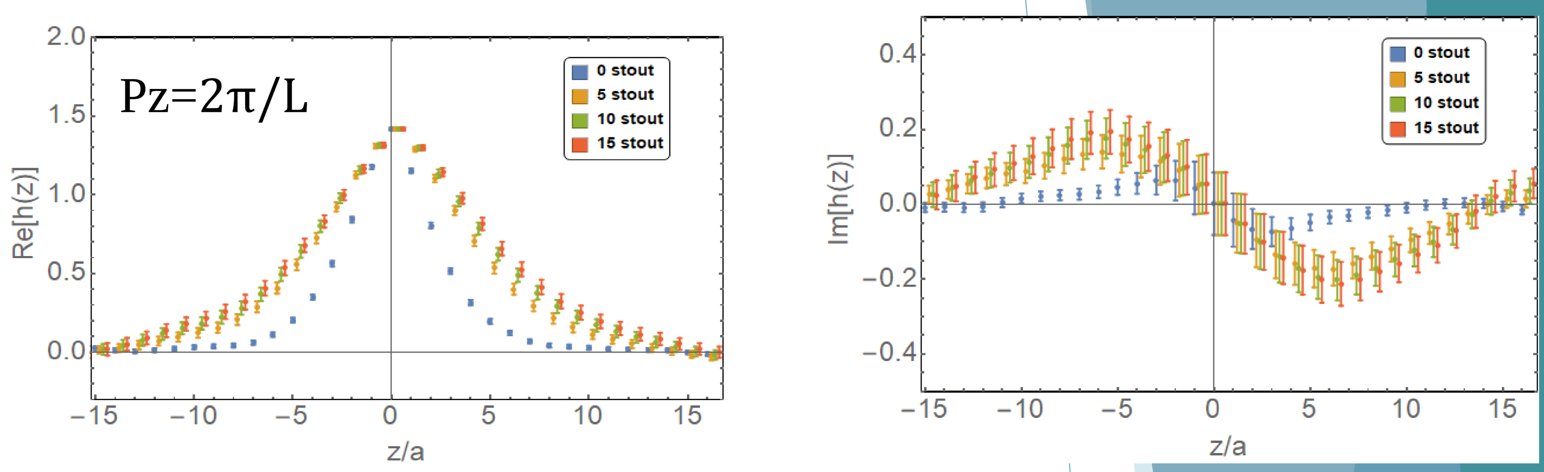
Result

Excited state contamination

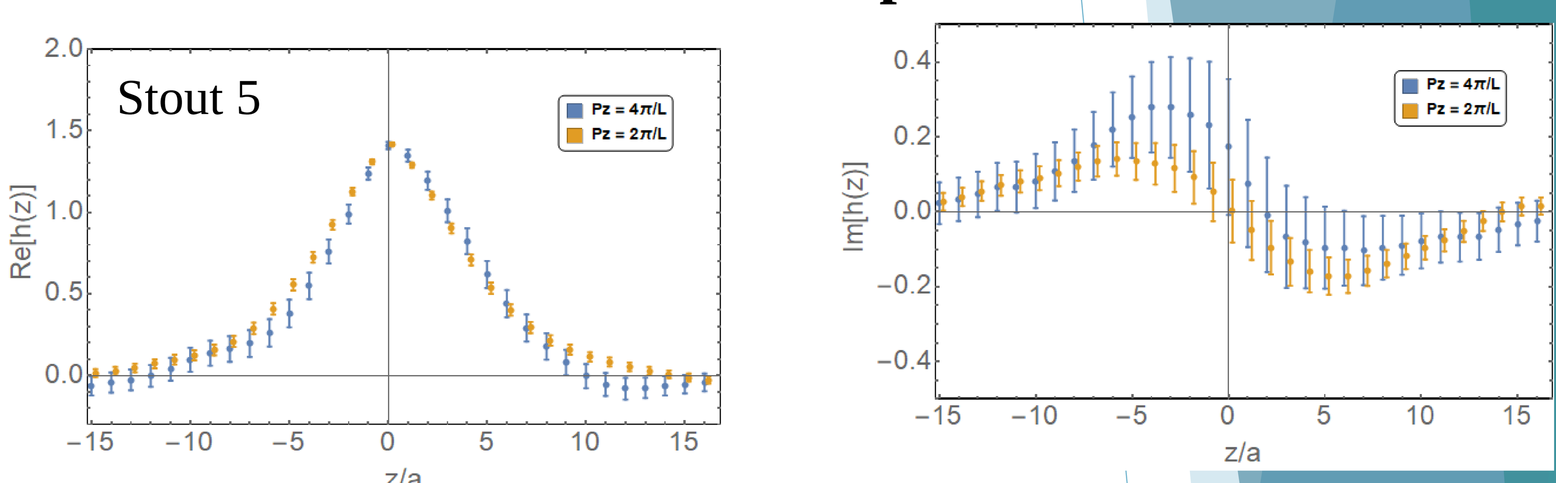


$t_s = 10a \approx 0.96fm$ later

Stout smearing

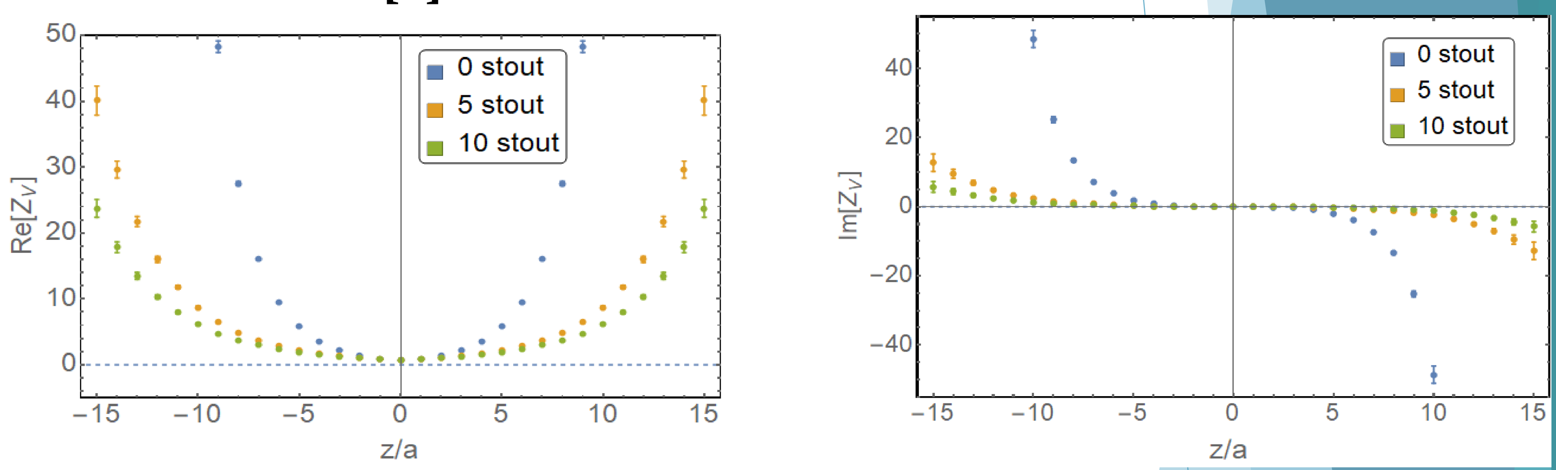


Momentum dependence

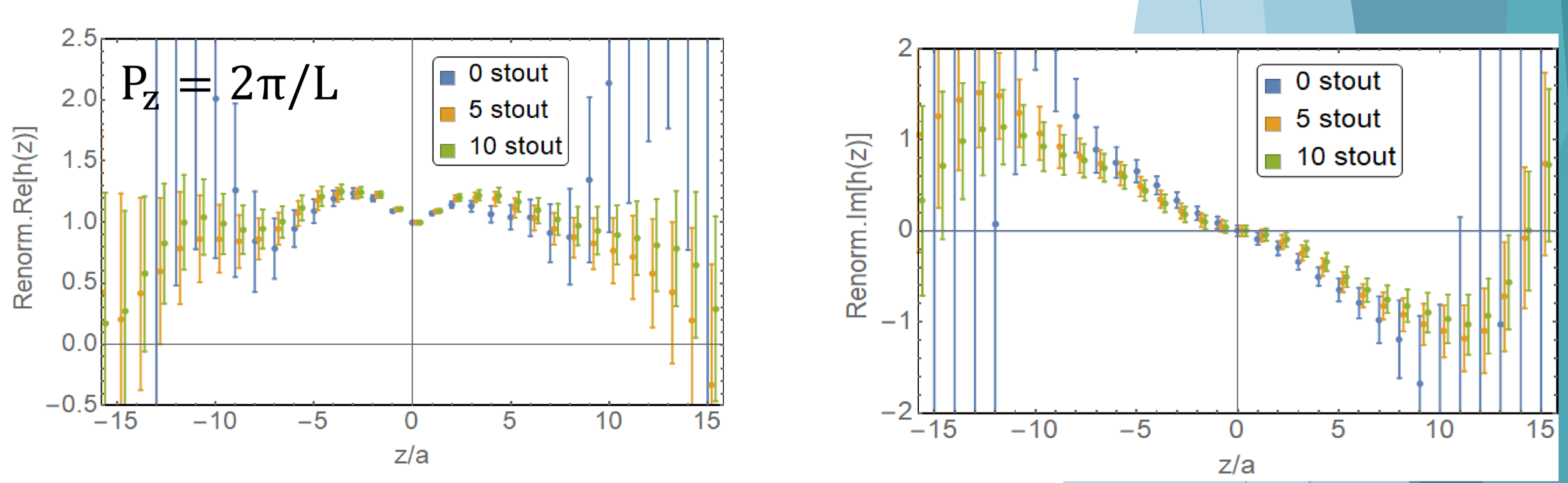


Renormalization factor

bare -> $\overline{\text{MS}}$ [5]



Renormalized matrix element



Conclusion

We obtain first result of matrix element for Δ^+ unpolarized PDFs

Future work

- Larger momenta
- Increase statistics
- Fourier transform
- Matching to light-cone PDFs

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Reference

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