Valence Pion PDF from quasi-PDF and Ioffe-Time Distributions

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Motivation

- Parton Distribution Functions: Partonic structure of hadrons embedded within hadronic light-cone correlation functions
- Techniques developed to map nonlocal euclidian correlators to parton distribution functions
  - quasi-pdf
  - pseudo-pdf
  - etc...
- Theoretical framework, implemenatation and systematic uncertianties different across these different methods
- Here we compare quasi-pdf method and pseudo-pdf method using the same lattice data

Lattice Details

- Mixed Action
  - Sea: HotQCD HISQ gauge ensemble [arXiv:1407.6387]
  - Valence: Wilson Clover with 1HYP
- Lattice Spacing: 0.06 fm
- Lattice Size: $48^3 \times 64$
- Pion Mass: 300 MeV
- Configuration Number
  - N_{CFG}: 48; P_z =0, 0.43 GeV
  - N_{CFG}: 216; P_z =0.86, 1.29, 1.72 GeV

Quasi-PDF Analysis

T. Izubichi et al [arXiv:1905.06349]

- Remove Excited State Contamination to project to Pion ground state
- Renormalize quasi-PDF matrix element in RI-MOM scheme
- Match RI-MOM quasi-PDF to $\overline{\text{MS}}$ PDF via matching formula

$$q(x, P^z, \tilde{\mu}) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{|y|P^z}\right) f(y, \mu) + \mathcal{O}\left(\frac{m_h^2}{P_z^2}, \frac{\Lambda_{QCD}^2}{P_z^2}\right) \quad (1)$$

How we implement this matching formula?

- Parameterize PDF via

$$f(x;a,b) = N(a,b)x^a(1-x)^b;\ N(a,b) = \frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} \quad (2)$$

- Define qPDF ansatz as

$$\tilde{h}(z;a,b) = \int_{-\infty}^{\infty} dx e^{ixP^z z} \int \frac{dy}{|y|} C(\frac{x}{y}, \frac{\tilde{\mu}}{P^z}, \frac{\mu}{|y|P^z}) f(y;a,b) \quad (3)$$

- Fit to lattice qPDF data across z-data to obtain (a, b)

$$\text{Minimize } \chi^2 = \sum_i \frac{(h(z_i) - \tilde{h}(z_i;a,b))^2}{\sigma_i^2} \quad (4)$$

Quasi-PDF Results

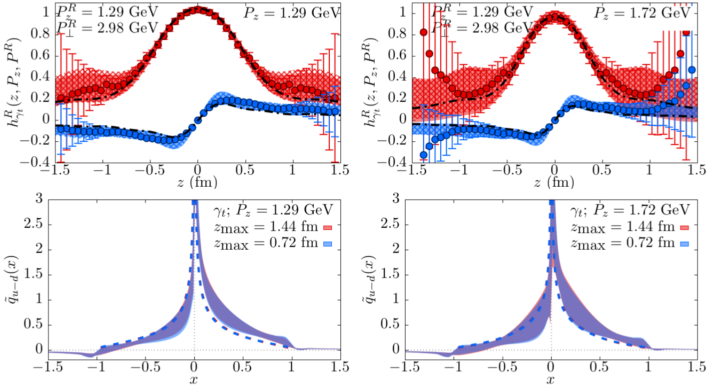
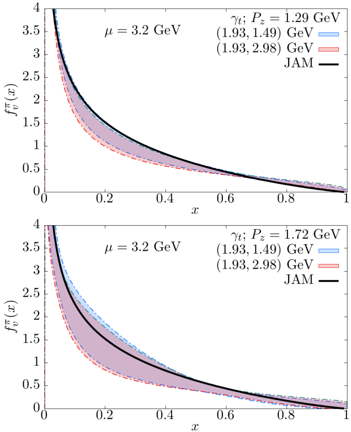
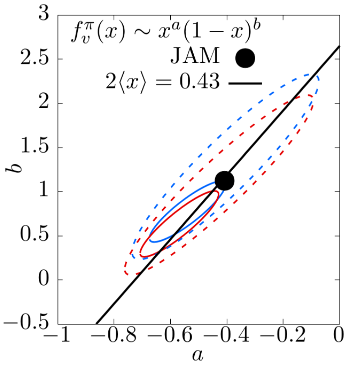


Figure: Fit results.
LHS: Matching at $P^z = 1.29$ GeV. RHS: Matching at $P^z = 1.72$ GeV.
Top: Fit to Real Space qPDF. Bottom: Fit to corresponding PDF in x-space.
Solid: Fit to $z_{\max} = 1.44$ fm. Patterned: Fit to $z_{\max} = 0.72$ fm.

Quasi-PDF Results



(a) Pion Valence PDF from quasi-PDF.
Top: Results for $P^z = 1.29$ GeV. Bottom: Results for $P^z = 1.72$ GeV.
Parenthesis indicates renormalization scale: (p_T^R, p_z^R)



(b) Constraints on parameters (a, b) in Ansatz

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Fine Lattice

- Lattice Spacing: 0.04 fm
- Lattice Size: $64^3 \times 64$
- Pion Mass: 300 MeV
- Configuration Number
  - N_{CFG}: 89; P_z =0, 0.48, 0.96, 1.45, 1.93 GeV

Reduced Ioffe-Time Distribution Approach

- Define pseudo-Ioffe-Time Distribution as
$$\mathcal{M}(\nu, z^2) = \langle h(P)|\, \bar{\psi}(z)\gamma^t W_L(z,0)\psi(0)\,|h(P)\rangle\,;\ \nu = z\cdot P \tag{5}$$

- Define Reduced Ioffe-Time Distribution as
$$\mathscr{M}(\nu, z^2) = \mathcal{M}(\nu, z^2)/\mathcal{M}(0, z^2) \tag{6}$$

- At $z^2 \to 0$ limit we have
$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-i\nu)^n}{n!} a_{n+1}(\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2) \tag{7}$$

- $a_{n+1}(\mu) = \int_{-1}^{1} dx\ x^n f(x,\mu)$

- $C_n(\mu^2 z^2)$ Wilson Coefficient of twist-2 operator

Reduced Ioffe-Time Data

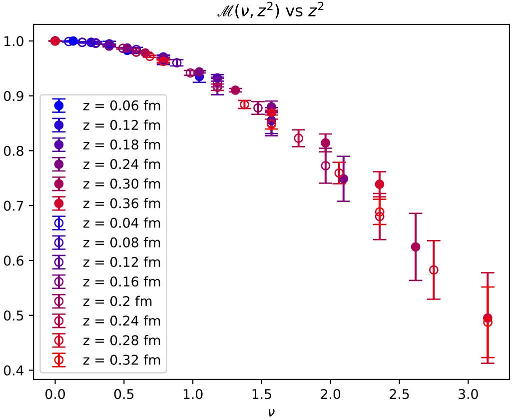


Figure: Reduced Ioffe-Time Distribution for a = 0.06 fm (closed circles) and a = 0.04 fm (open circles). Little to no dependence in lattice spacing observed.

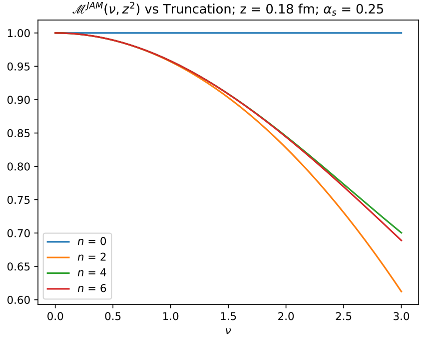
Sensitivity to moments a_{n+1}

$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-i\nu)^n}{n!} a_{n+1}(\mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

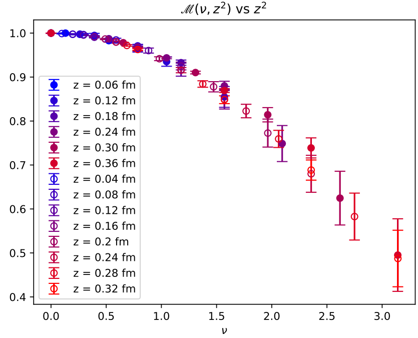
- Limited ν means only certian moments contribute to lattice data

Take JAM data for valence pdf of Pion, transform it to isovector Pion PDF, and study the moment dependence; P.C. Barry et al [arXiv:1804.01965]

- Isovector Pion PDF is real and symmetric about x
- Only even moments contribute to matching formula



(a) Reduced Ioffe-Time distribution reconstructed from JAM data. Up to $\nu = 3.2$, only relevant moments are $\langle x^2 \rangle$ and $\langle x^4 \rangle$



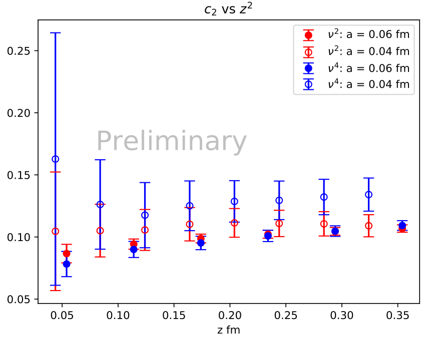
(b) Reduced Ioffe-Time data for coarse and fine lattice up to z = 0.32 fm

Determining Pion Moments J. Karpie et al[arXiv:1807.10933]

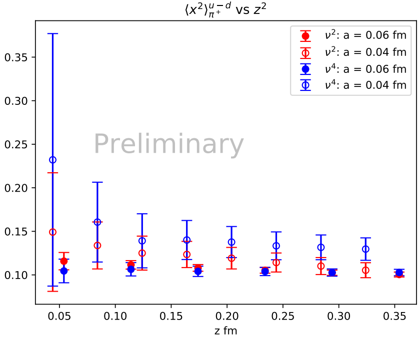
$$\mathscr{M}(\nu, z^2) = \sum_{n=0}^{\infty} \frac{C_n(\mu^2 z^2)}{C_0(\mu^2 z^2)} \frac{(-i\nu)^n}{n!} a_{n+1}(\mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

- Fit to polynomial of degree m at fixed z^2; $\quad 1 - \frac{\nu^2}{2!} c_2 + \ldots \frac{\nu^m}{m!} c_m$
- $\langle x^j \rangle_{u-d} = \frac{C_0(\mu^2 z^2)}{C_j(\mu^2 z^2)} c_j$
- Check if divergent logs from fit cancel

Fit Results; $\langle x^2 \rangle_{\pi^+}^{u-d}$



(a) Coefficient to quadradic term in polynomial fit



(b) Second moment of pion PDF. Extracted from divinding coefficient by Wilson

Constraints on (a, b) assuming ansatz

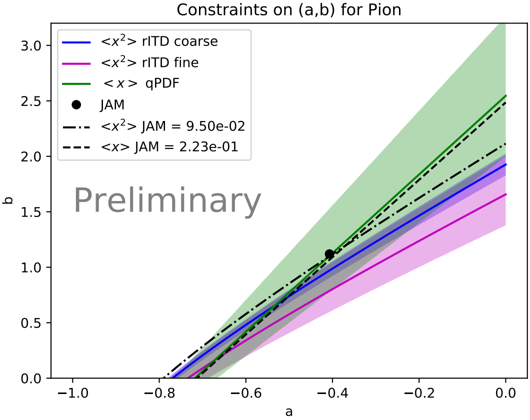
Assuming $f(x; a, b) = N(a, b)x^a(1 - x)^b$ we have

$$\langle x^n \rangle(a, b) = \frac{\Gamma(a + b + 2)\Gamma(a + n + 1)}{\Gamma(a + 1)\Gamma(a + n + b + 2)} \tag{8}$$

Therefore we can constrain (a, b) along curves of constant

$$\langle x^2 \rangle = \frac{(a + 2)(a + 1)}{(a + b + 3)(a + b + 2)}$$

Constraints on (a,b) from this study and qPDF



Conclusions and Outlook

Conclusions

- Results of quasi-PDF analysis presented
  - Pion valence PDF computed at $P^z = 1.29$ and 1.72 GeV for 300 MeV Pion mass at lattice spacing a $= 0.06$ fm
  - Constraints of parameters (a, b) from quasi-PDF consistent with those determined by JAM collaboration
- Preliminary results from Reduced Ioffe-Time Distribution presented
  - Calculation done across all momomenta and at lattice spacings 0.06 fm and 0.04 fm.
  - Little dependence on lattice spacing observed
  - Possible to constrain the second moment

Currently in Progress

- Evolve all Reduced Ioffe-Time data to the same z^2 to increase Ioffe-Time extent
- Constraint on $\langle x^4 \rangle$ possible?
- Fit evolved Reduced Ioffe-Time distribution to moments parameterized by $f(x; a, b) \sim x^a(1-x)^b$.

Backup: Defining $C_n(\mu^2 z^2)$

$$C_n(\mu^2 z^2) = 1 + \frac{\alpha_s C_F}{2\pi}\Big[\Big(\frac{3+2n}{2+3n+n^2} + 2H_n\Big)\log\big(\mu^2 z^2 \frac{e^{2\gamma_E}}{4}\big) + \frac{5+2n}{2+3n+n^2} + 2(1-H_n)H_n - 2H_n^{(2)}\Big]$$