Theoretical Development of the LaMET Approach to Parton Physics

Yong Zhao
Massachusetts Institute of Technology
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Outline

- Large-momentum effective theory
  - Physical picture and theoretical framework
  - From the quark PDFs to the gluon PDFs
- Transverse hadron structures from lattice QCD
  - Generalized parton distributions
  - Transverse momentum dependent distributions
- Summary and outlook

Three-dimensional partonic hadron structures

- Longitudinal Parton Distribution Functions (PDFs): $q_{i=q,\bar{q},g}(x)$
- Generalized Parton Distributions (GPDs):

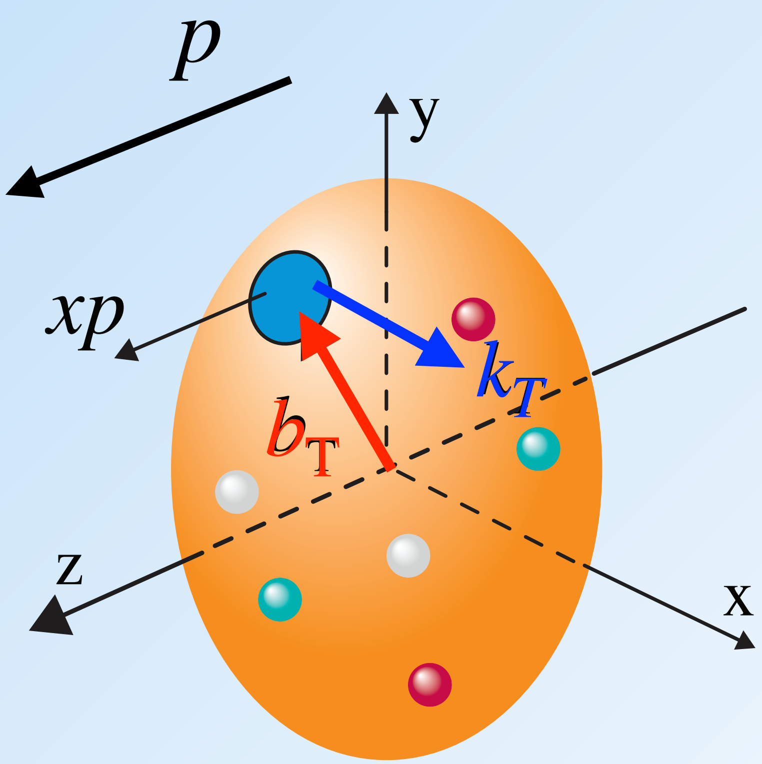
  $$F_i(x, \xi = 0, \vec{b}_T)$$

  - $\vec{b}_T$: transverse position of the parton.
- Transverse momentum dependent (TMD) PDFs

  $$q_i(x, \vec{k}_T)$$

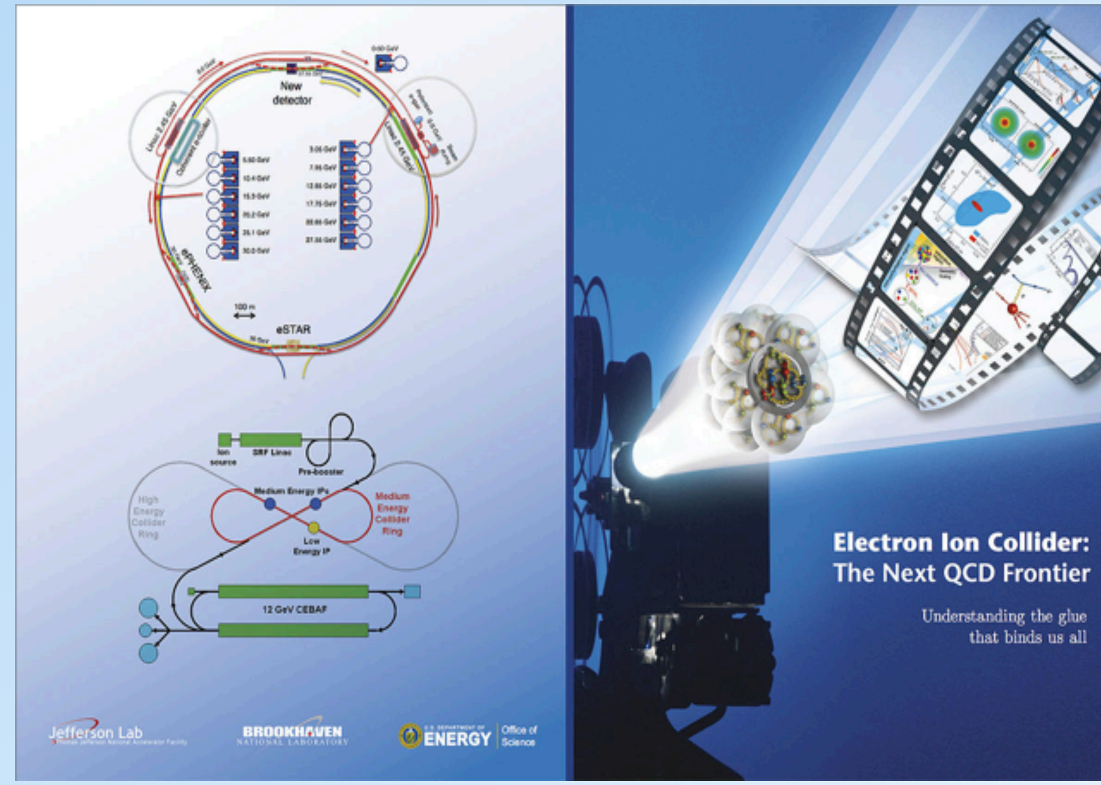
  - $\vec{k}_T$: transverse momentum of the parton.
- Wigner distributions or generalized transverse momentum dependent distributions

  $$W_i(x, \xi = 0, \vec{k}_T, \vec{b}_T)$$

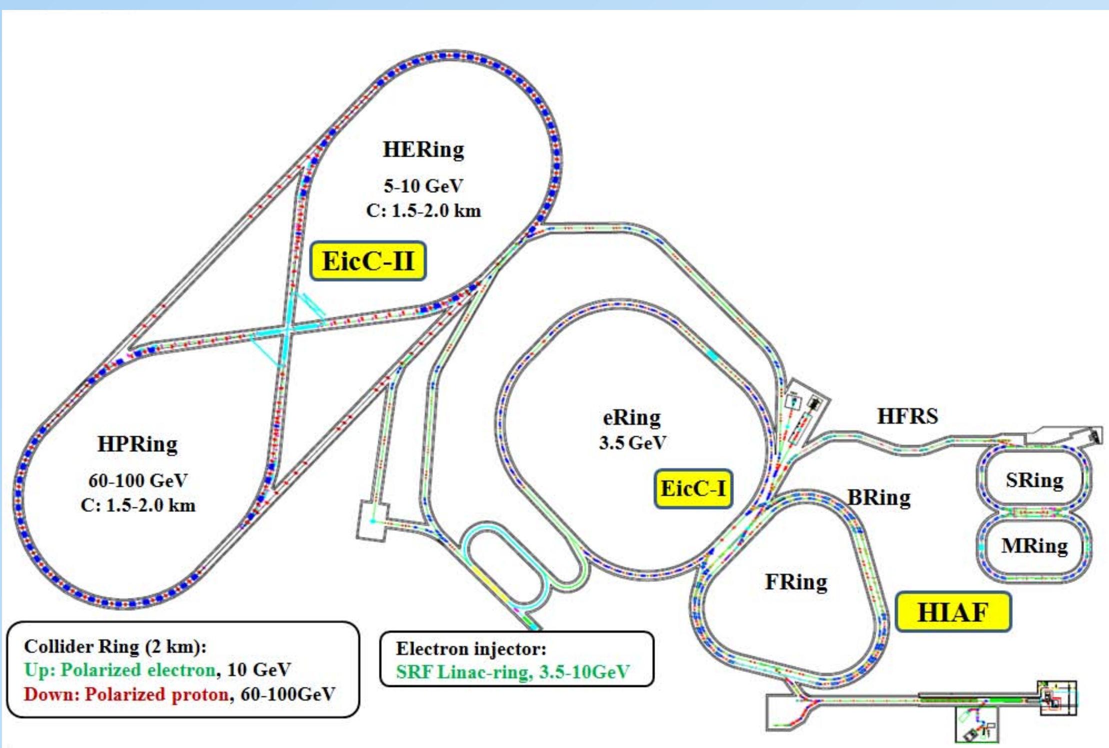


The longitudinal and transverse PDFs provide complete 3D structural information of the proton.

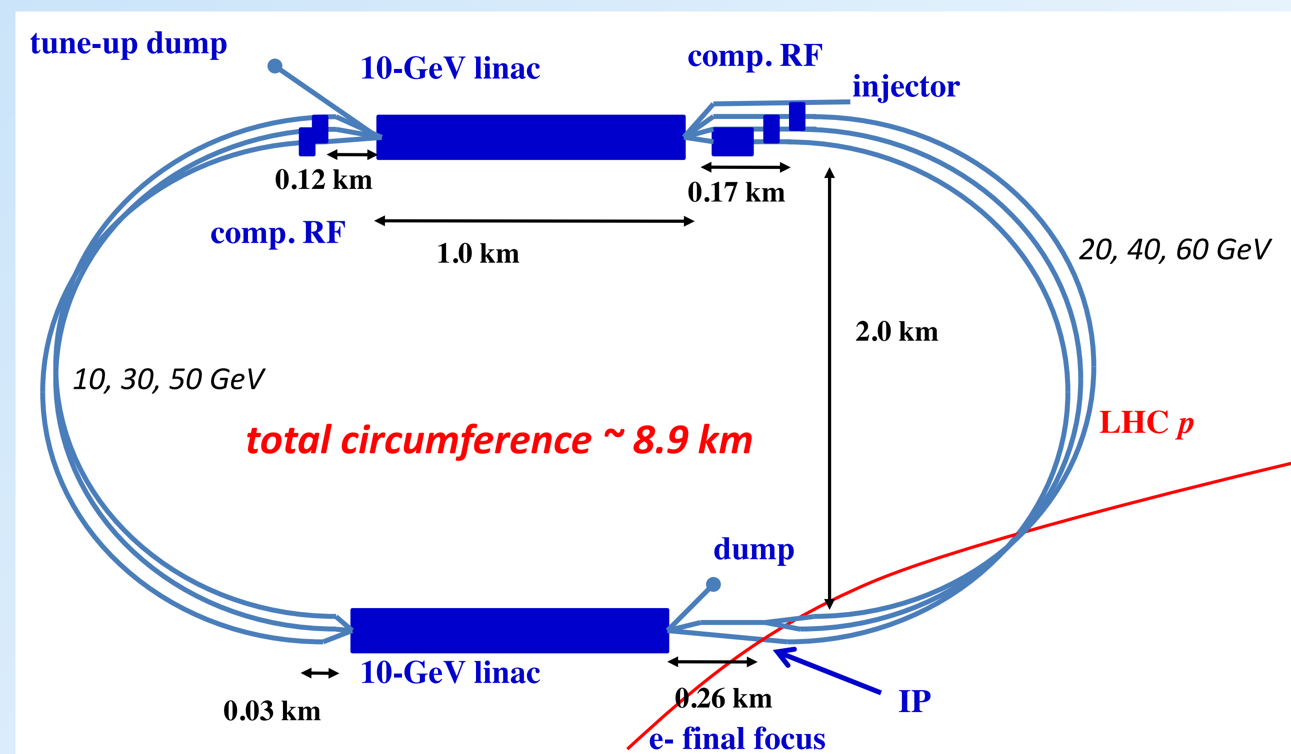
The measurement of partonic hadron structures will enter next level of precision and comprehensiveness with the future generation of colliders.



EIC, A. Accardi et al., Eur.Phys.J. A52 (2016) no.9, 268.

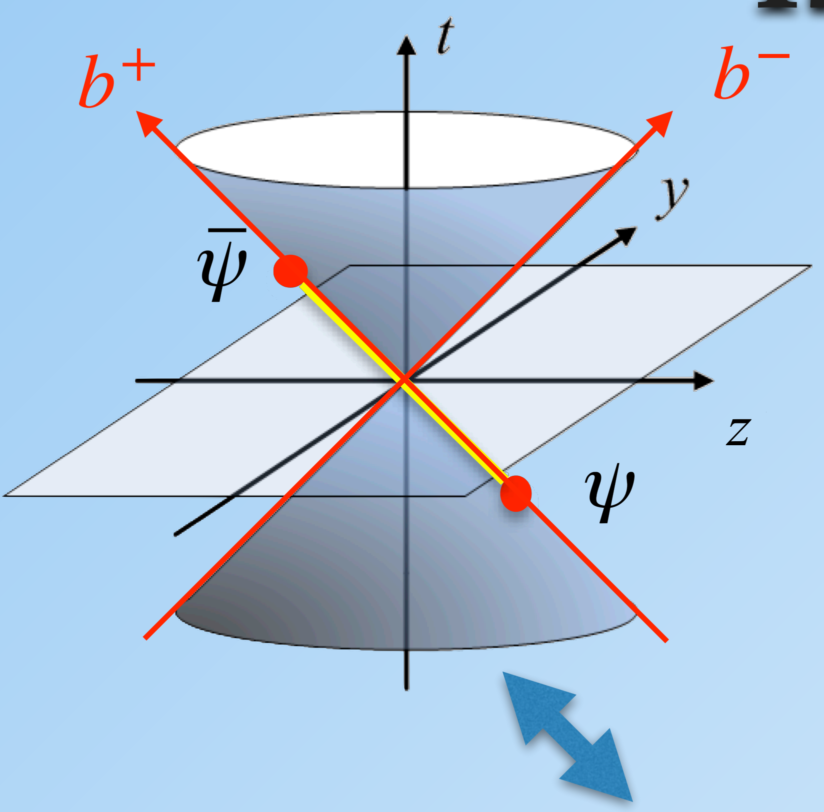


EicC, X.-R. Chen, arXiv:1809.00448



LHeC, LHeC Study Group, J. Phys. G39 (2012)

Lattice QCD calculation of partonic hadron structures?

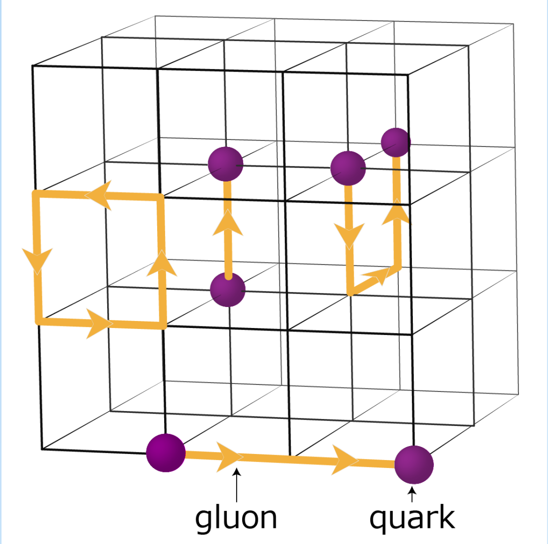


PDF:

$$q(x,\mu) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(b^-) \frac{\gamma^+}{2} W[b^-,0] \psi(0) | P \rangle$$

$$b^\pm = \frac{t \pm z}{\sqrt{2}}$$

- Minkowski space, real time;
- Defined on the light-cone which depends on the real time.



Lattice QCD:

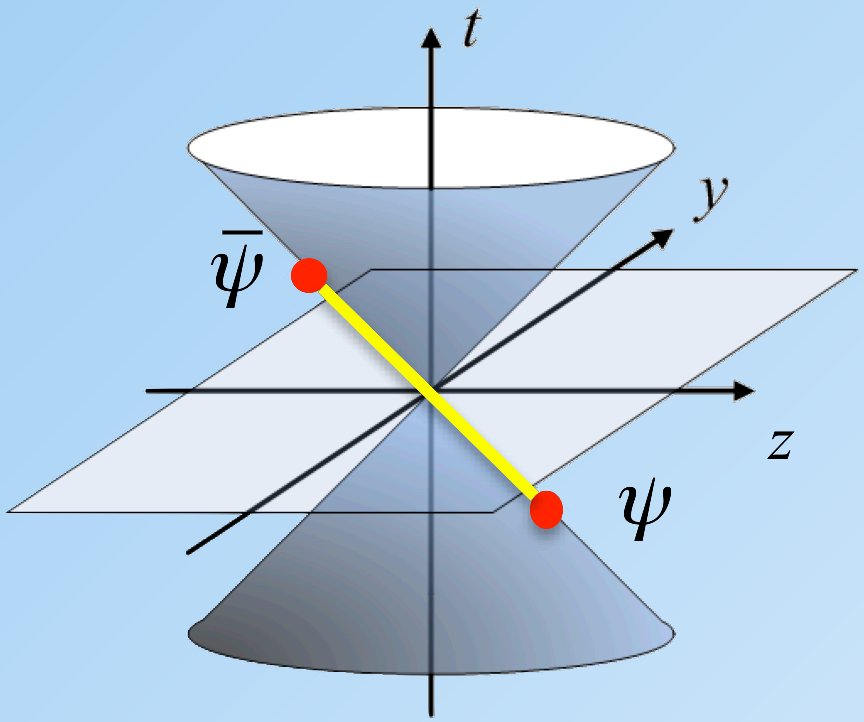
$$t = i\tau, \quad e^{iS} \to e^{-S}, \quad \langle O \rangle = \int D\psi D\bar{\psi} DA \; O(x) e^{-S}$$

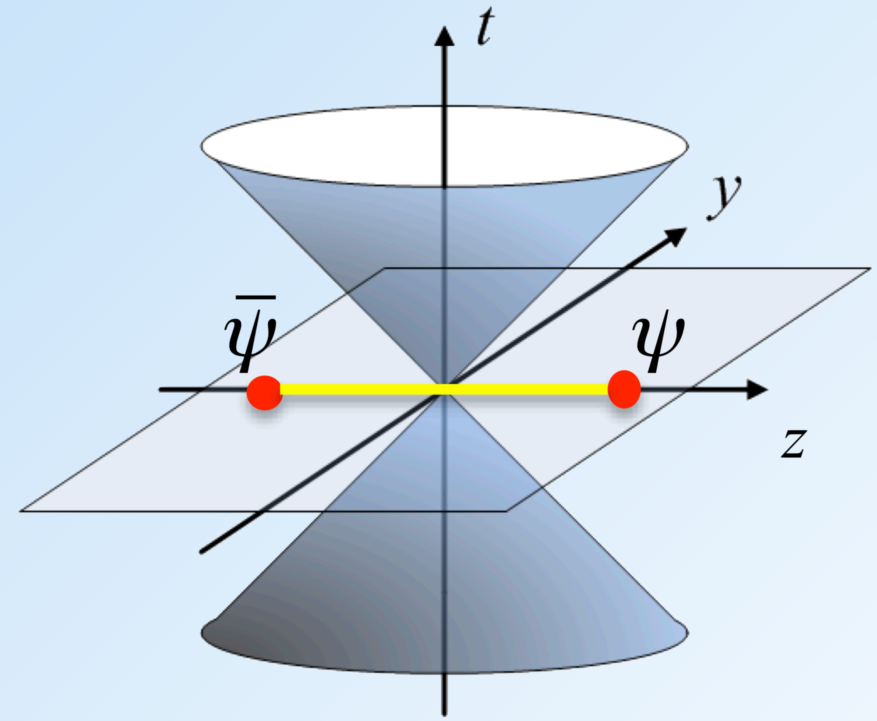
- Euclidean space, imaginary time;
- Sign problem in simulating real-time dynamics.

Light-cone PDFs not directly accessible from the lattice!

Large-momentum effective theory

- Ji, PRL110 (2013);
- Ji, SCPMA57 (2014).





PDF $q(x, \mu)$:
Cannot be calculated on the lattice

Quasi-PDF $\tilde{q}(x, P^z, \mu)$:
Directly calculable on the lattice

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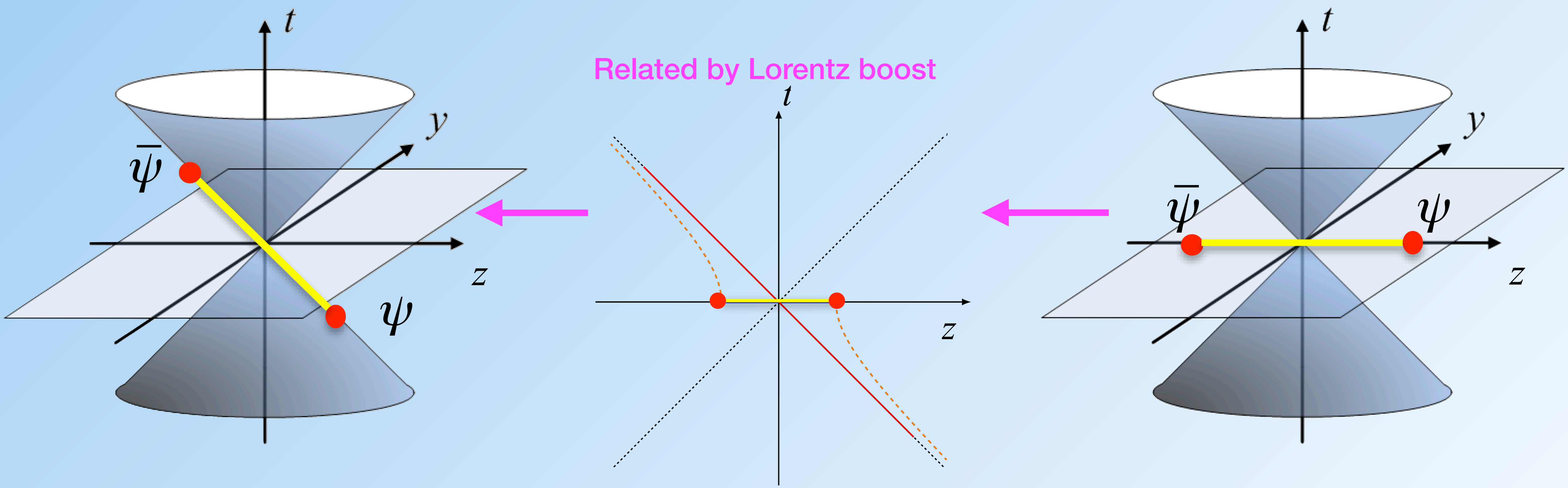

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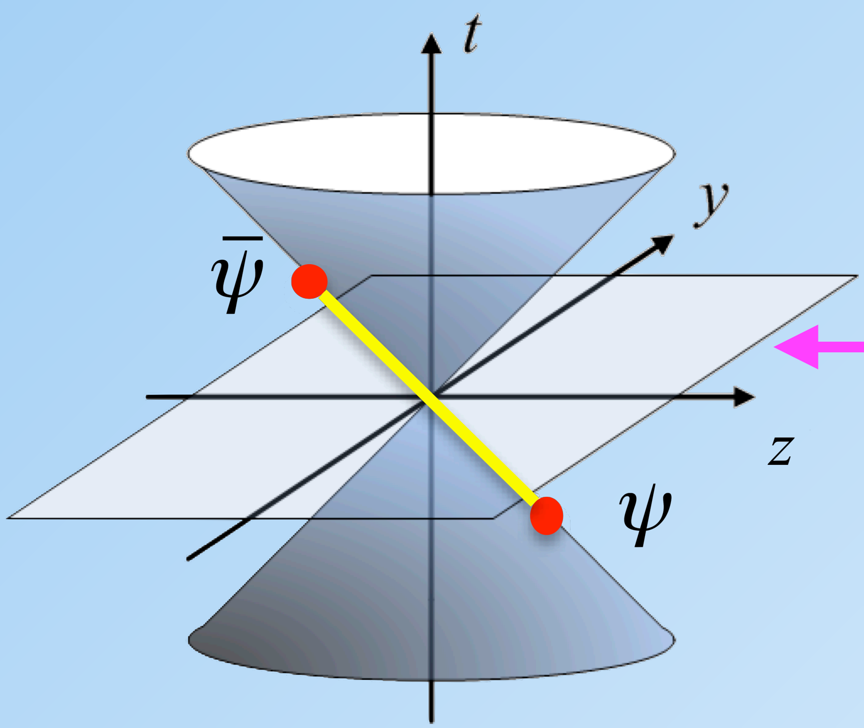
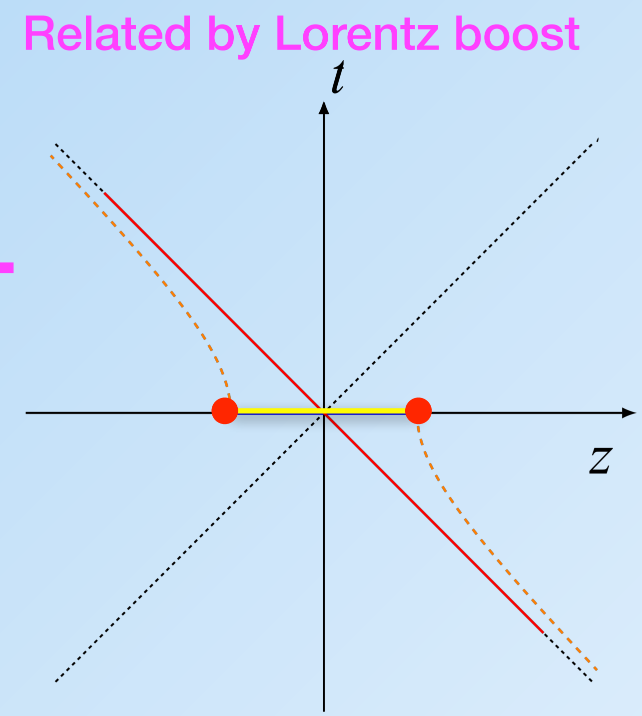
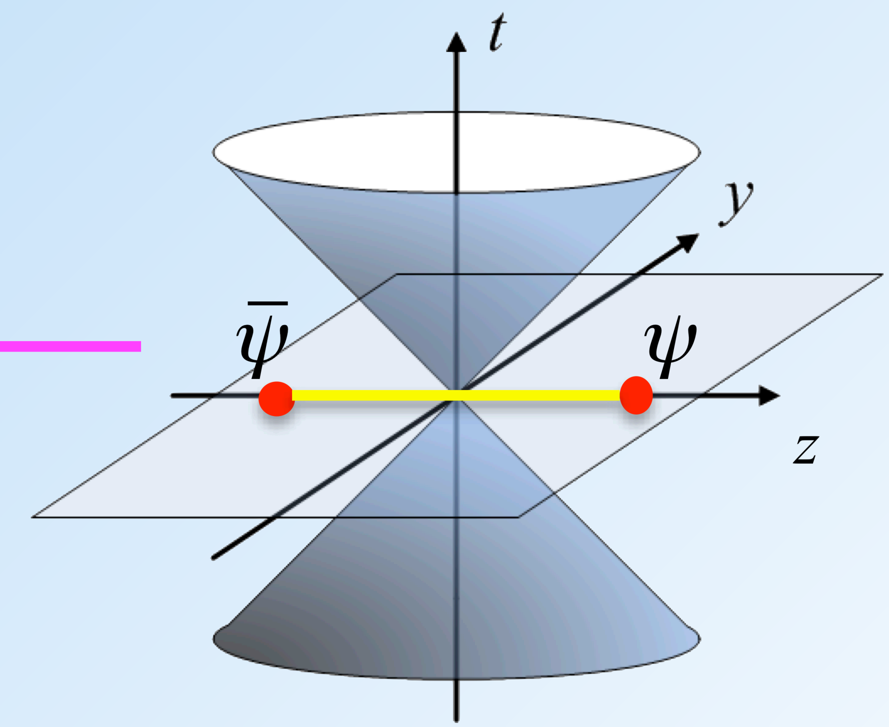






PDF $q(x, \mu)$:
Cannot be calculated on the lattice

Calculating the quasi-PDF at hadron momentum P^z is equivalent to boosting it.

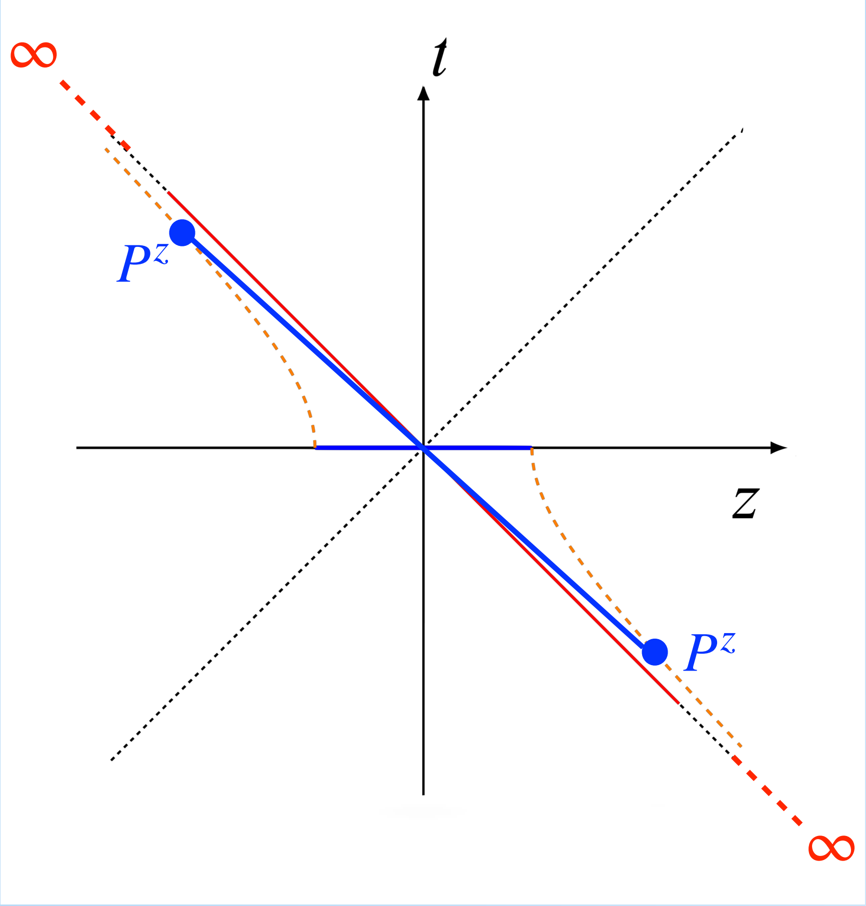
Quasi-PDF $\tilde{q}(x, P^z, \mu)$:
Directly calculable on the lattice

Large-momentum effective theory

$$\lim_{P^z \to \infty} \tilde{q}(x, P^z, \mu) = q(x, \mu)? \quad ✗$$

Large-momentum effective theory

$$\lim_{P^z\to\infty} \tilde{q}(x, P^z, \mu) = q(x, \mu)?$$



Instead of taking $P^z \to \infty$ limit, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Ji, PRL110 (2013);
- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

- $\tilde{q}(x, P^z)$ and $q(x)$ have the same infrared physics (nonperturbative), but different ultraviolet (UV) physics (perturbative);
- Therefore, the matching coefficient C is perturbative, which controls the logarithmic dependences on P^z.

Systematic procedure of calculating the PDFs

1. Simulation of the quasi PDF in lattice QCD

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

Systematic procedure of calculating the PDFs

$$\bar{\psi}_0(z)\frac{\Gamma}{2}W_0[z,0]\psi_0(0)$$
$$= e^{\delta m|z|}\, Z_{j_1}Z_{j_2}Z_{\psi}Z_Q\left[\bar{\psi}(z)\frac{\Gamma}{2}W[z,0]\psi(0)\right]_R$$

Proof of renormalizability:

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);
- T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, PRD96 (2017).

$$\tilde{q}(x,P^z,\mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{yP^z}\right) q(y,\mu) + O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}\right)$$

2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

Nonperturbative renormalization in the regularization invariant momentum subtraction (RI/MOM) scheme:

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).
- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

Systematic procedure of calculating the PDFs

- O Nachtmann, NPB63 (1973);
- J.W. Chen et al. (LP3), NPB911 (2016).

3. Subtraction of power corrections

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y,\mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$q(x) \cdot \frac{\Lambda_{\rm QCD}^2}{x^2(1-x)P_z^2}$$

Renormalon contribution to the power correction:
Braun, Vladimirov, and Zhang, PRD99 (2019).

Systematic procedure of calculating the PDFs

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

4. Matching to the PDF.

- Matching for the quasi-PDF:
  - X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
  - I. Stewart and Y.Z., PRD97 (2018);
  - Y.-S. Liu, Y.Z. et al. (LP3), arXiv:1807.06566;
  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
  - Y.-S. Liu, Y.Z. et al., arXiv:1810.10879;
  - Y.Z., Int.J.Mod.Phys. A33 (2019);
  - C. Alexandrou et al. (ETMC), arXiv: 1902.00587.

Systematic procedure of calculating the PDFs

5. Extract q(y)

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

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  - Y.Z., Int.J.Mod.Phys. A33 (2019);
  - C. Alexandrou et al. (ETMC), arXiv: 1902.00587.

For recent progress on the lattice calculation of quark PDFs, see N. Karthik's plenary talk.

Other proposals in recent years

- **Restoration of rotational symmetry to calculate higher moments**

  Z. Davoudi and M. Savage, PRD86 (2012)
- **Current-current correlation with a fictitious heavy quark**

  D. Lin and W. Detmold, PR73 (2006)
- **Operator product expansion of the Compton amplitude**

  A. J. Chambers et al. (QCDSF), PRL 118 (2017)
- **Hadronic tensor from lattice QCD**

  K.F. Liu (et al.), PRL72 (1994), PRD59 (1999), PRD62 (2000), PRD96 (2017)
- **Smeared Quasi-PDF with Gradient flow**

  C. Monahan and K. Orginos, JHEP 1703 (2017)
- **Pseudo-PDF (alternative to quasi-PDF)**

  A. Radyushkin, PRD96 (2017); K. Orginos et al., PRD96 (2017).
- **Lattice cross section**

  Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018)
- **Factorization of Euclidean correlations in coordinate space**

  V. M. Braun and D. Mueller, EPJ C55 (2008);G. S. Bali, V. M. Braun, A. Schaefer, et al., PRD98 (2018)

Gluon quasi-PDFs

- Gluon light-cone PDF:

$$g(x,\mu) = \int \frac{db^-}{2\pi x P^+} e^{-i(xP^+)b^-} \langle P | F^{+\alpha}(b^-) W[b^-,0] F^{+}_{\alpha}(0) | P \rangle$$

- Gluon quasi-PDF:

$$\tilde{g}(x, P^z, \mu) = \int \frac{dz}{2\pi x P^z} e^{ixP^z z} \frac{1}{N_{n_1,n_2}} \langle P | \tilde{O}^g(z) | P \rangle,$$

$$\tilde{O}^g(z) = g_{\perp,\mu\nu} F^{n_1\mu}(z) W[z,0] F^{n_2\nu}(0)$$

$$N_{n_1,n_2} = \frac{|n_1 \cdot P||n_2 \cdot P|}{P_z^2}$$

$$n_1, n_2 \in \{\hat{z}, \hat{v}\},$$

$$\hat{z}^\mu = (0,0,0,1), \;\; \hat{v}^\mu = (1,0,0,0)$$

- Proof of renormalizability:

$$\tilde{O}_0^g(z) = e^{\delta m|z|} Z_{j_1} Z_{j_2} Z_A Z_Q \left[\tilde{O}^g(z)\right]_R$$

$$+ e^{\delta m|z|} Z_{\mathrm{mix}} \left[g_\perp^{\mu\nu} A_\mu(z) W[z,0] A_\nu(0)\right]_R \delta(z) \delta_{n_1\hat{z}} \delta_{n_2\hat{z}}$$

No mixing with quarks under renormalization!

- Zhang, Ji, Schaefer et al., PRL122 (2019);
- Li, Ma, and Qiu, PRL122 (2019).

Aslo see Jianhui Zhang's talk in parallel session.

Gluon and singlet quark PDFs from LaMET

- Factorization formula:

- Wang et al., EPJC78 (2018), JHEP1805 (2018);
- Zhang et al., arXiv: 1904.00978.

$$\tilde{q}_i(x, P_z, \mu) = \int_{-1}^{1} \frac{dy}{|y|} \left[\sum_j C_{q_i q_j}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q_j(y, \mu) + C_{qg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) g(y, \mu) \right] + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2} \right)$$

$$\tilde{g}(x, P_z, \mu) = \int_{-1}^{1} \frac{dy}{|y|} \left[\sum_j C_{gq}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q_j(y, \mu) + C_{gg}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) g(y, \mu) \right] + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2} \right)$$

- Nonperturbative renormalization in the RI/MOM scheme:

$$Z^{-1}(z, p_R^z, \mu_R^2) \frac{\langle F_0^{n_1\mu}(z) W_0[z,0] F_0^{n_2\nu}(0)\ A_a^\rho(p) A_a^\sigma(-p) \rangle\ g_{\perp,\rho\sigma} (N_c^2 - 1)^2}{4 \langle A^\rho(p)_a A^\sigma(-p)_a \rangle g_{\perp,\rho\sigma}} \Bigg|_{p^\mu = p_R^\mu,\ \mu_R^2 = p_R^2}$$

$$= \langle F^{n_1\mu}(z)\ F^{n_2\nu}(0)\ A_a^\rho(p) A_a^\sigma(-p) \rangle_{\text{tree}}\ g_{\perp,\rho\sigma}$$

- $A^\mu(p)$ can be defined from gauge links;
- There is also finite mixings with quark quasi-PDF.

- Yang, Glatzmaier, Liu and Y.Z., arXiv:1612.02855;
- Shanahan and Detmold, PRD99 (2019).

- Matching in the RI/MOM scheme:
  - Zhang et al., arXiv: 1904.00978.

First lattice attempt: Fan et al., PRL121 (2018).

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GPD

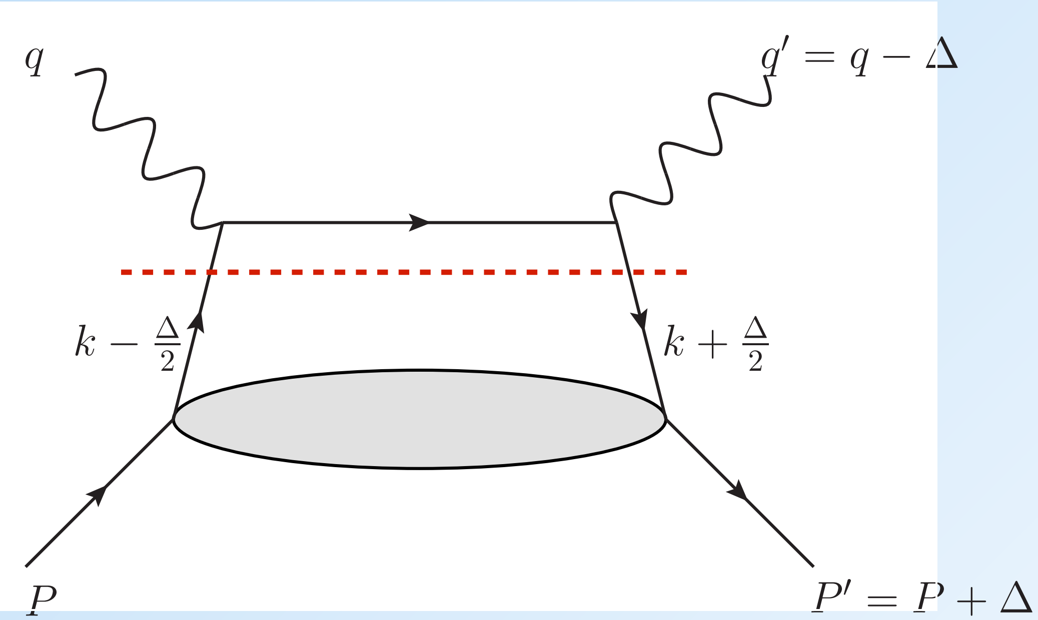
- Light-cone GPD: $\bar{P} = \frac{P+P'}{2} \quad \xi = \frac{P^+ - P'^+}{P^+ + P'^+}, \quad t = (P'-P)^2 \equiv \Delta^2$

$$F_\Gamma(x,\xi,t,\mu) = \int \frac{db^-}{4\pi} e^{-ix\bar{P}^+ b^-} \langle P', S' | \bar{\psi}\left(\frac{b^-}{2}\right) \Gamma U\left(\frac{b^-}{2}, -\frac{b^-}{2}\right) \psi\left(-\frac{b^-}{2}\right) | P, S \rangle$$

Color charge density in impact parameter space: $F(x, \xi = 0, \vec{b}_T) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} e^{i\vec{\Delta}_T \cdot \vec{b}_T} F(x, \xi = 0, t)$

- M. Burkardt, PRD62 (2000), IJMPA18 (2003).

- Measurable in hard exclusive processes such as deeply virtual Compton scattering: $l + p \to l' + \gamma + p'$



$$\sim \int dx \; C(x,\xi) F(x,\xi,t)$$

Quasi-GPD

$$\tilde{\xi} = \frac{P^z - P'^z}{P^z + P'^z} = \xi + O\left(\frac{M^2}{P_z^2}\right)$$

- Definition:

$$\tilde{F}_{\tilde{\Gamma}}(x, \tilde{\xi}, t, \mu) = \int \frac{dz}{4\pi} e^{-ix\bar{P}^z z} \langle P', S' | \bar{\psi}\left(\frac{z}{2}\right) \tilde{\Gamma} U\left(\frac{z}{2}, -\frac{z}{2}\right) \psi\left(-\frac{z}{2}\right) | P, S \rangle$$

- Renormalization:
  - Same operator as the quasi-PDF, can be renormalized by the same factors!

- Factorization formula:

- Y.-S. Liu, Y.Z. et al., arXiv:1902.00307.

Aslo see Yu-Sheng Liu's talk in parallel session.

$$\tilde{F}_{\tilde{\gamma}^z}(x, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} C\left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

$$= \int_{-1}^{1} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

- Perturbative Matching in the RI/MOM Scheme: Y.-S. Liu, Y.Z. et al., arXiv:1902.00307.
- Mass correction and $O(t/P_z^2)$ corrections: Y.-S. Liu, Y.Z. et al., in progress.

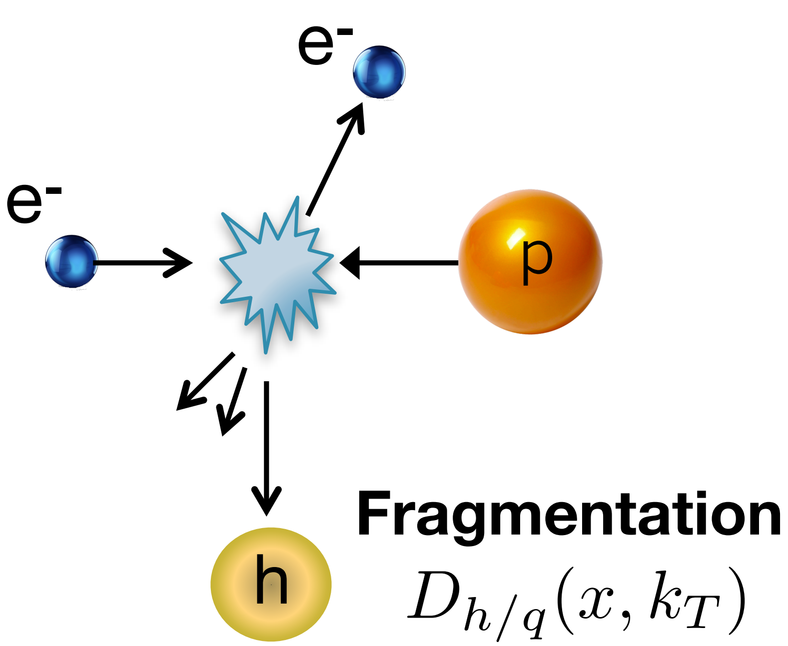
- First lattice calculation of pion GPD, Chen, Lin and Zhang, arXiv: 1904.12376.
- Preliminary results for quasi-GPDs (ETMC), M. Constantinou's talk at QCD Evolution 2019.

TMDPDF

- TMD processes:

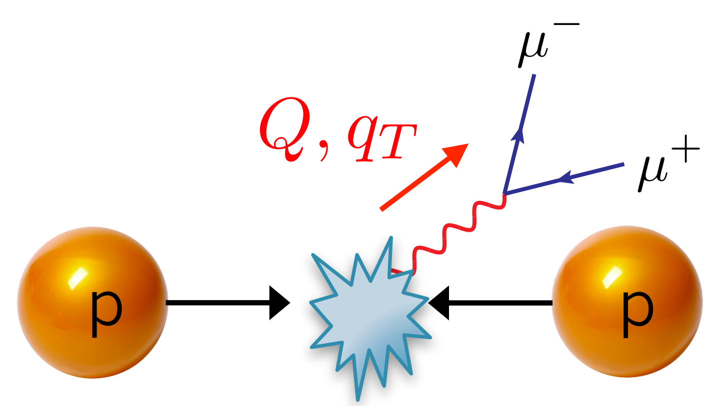
Semi-Inclusive DIS

$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$



Drell-Yan

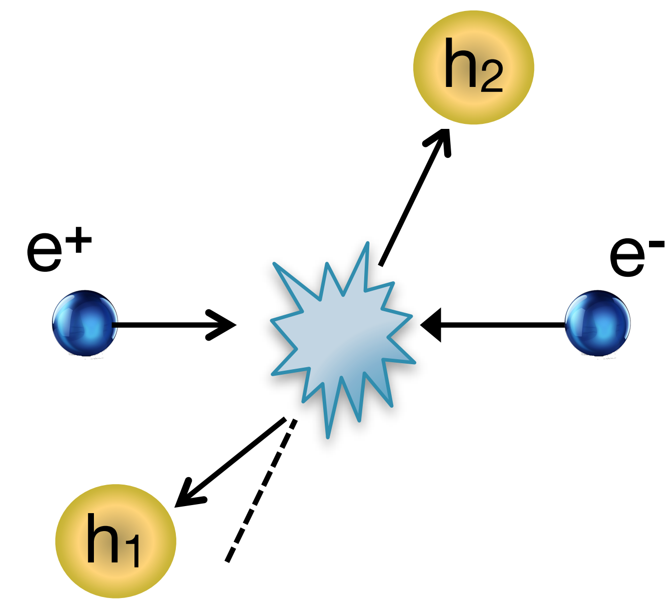
$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$



$q_T \ll Q$

Dihadron in e+e-

$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$

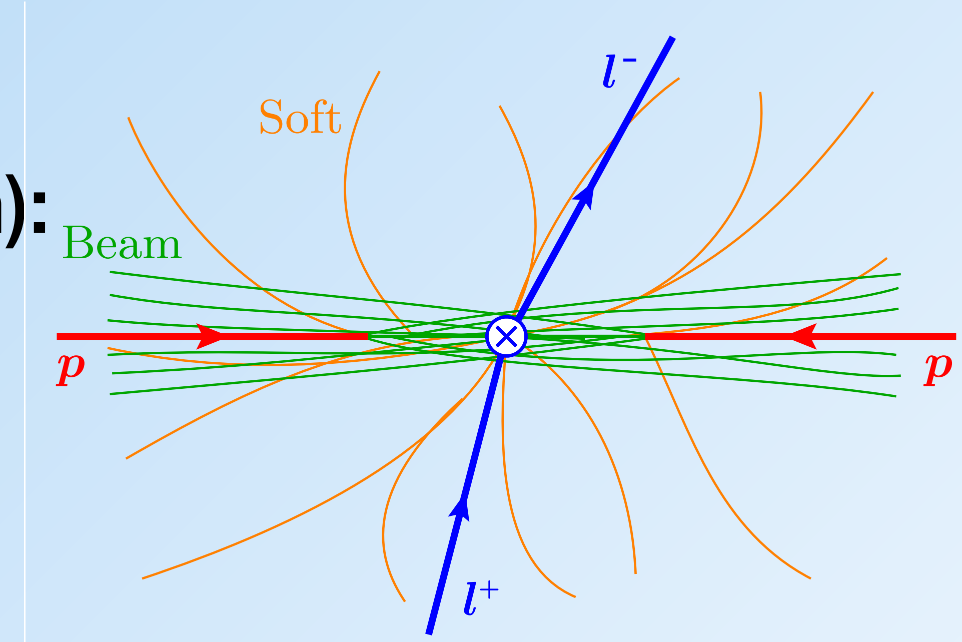


Picture credit, Iain Stewart

Many different schemes for TMD factorization in literature:

- Collins, Soper and Sterman, NPB250 (1985); Collins, 2011;
- Becher and Neubert, EPJC71 (2011);
- Echevarria, Idilbi and Scimemi, JHEP07 (2012), PLB726 (2013);
- Chiu, Jain, Neil and Rothstein, JHEP05 (2012), PRL108 (2012);
- Li, Neil and Zhu, arXiv: 1604.00392.

TMDPDF



- Collinear factorization (e.g., for Drell-Yan):

$$\frac{d\sigma}{dQdY} = \sum_{a,b} \sigma_{ab}(Q, \mu, Y)\, f_a(x_1, \mu)\, f_b(x_2, \mu)$$

- TMDPDF factorization:

$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{i,j} H_{ij}(Q, \mu) \int d^2b_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_i^{\mathrm{TMD}}(x_a, \vec{b}_T, \mu, \zeta_a) f_j^{\mathrm{TMD}}(x_b, \vec{b}_T, \mu, \zeta_b)$$

q_T: Net transverse momentum of the color-singlet final state, and $q_T \ll Q$;

ζ: Collins-Soper Scale. $\zeta_a\zeta_b = Q^4$

- The definition of TMDPDF involves a collinear beam function (or un-subtracted TMD) and soft function:

Rapidity divergence regulator

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\mathrm{UV}}(\epsilon, \mu, xP^+) B_i(x, \vec{b}_T, \epsilon, \tau, xP^+) \Delta S^i(b_T, \epsilon, \tau)$$

UV divergence regulator

Evolution of TMDPDF

- Evolution of TMDPDF:

$$f_i^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu, \zeta) = f_i^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu_0, \zeta_0)\ \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right]\ \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln \frac{\zeta}{\zeta_0}\right]$$

  - $\mu \sim Q$, $\zeta \sim Q^2 >> \Lambda_{\mathrm{QCD}}^2$;
  - μ_0, ζ_0: initial or reference scales, measured in experiments or determined from lattice (~2 GeV).

$\gamma_\mu^i(\mu', \zeta_0)$ Anomalous dimension for μ evolution, perturbatively calculable;

$\gamma_\zeta^i(\mu, b_T)$ Collins-Soper kernel, becomes nonperturbative when $b_T \sim 1/\Lambda_{\mathrm{QCD}}$.

Both Initial-scale TMDPDF and the Collins-Soper kernel must be modeled in global fits of TMDPDF from experimental data.

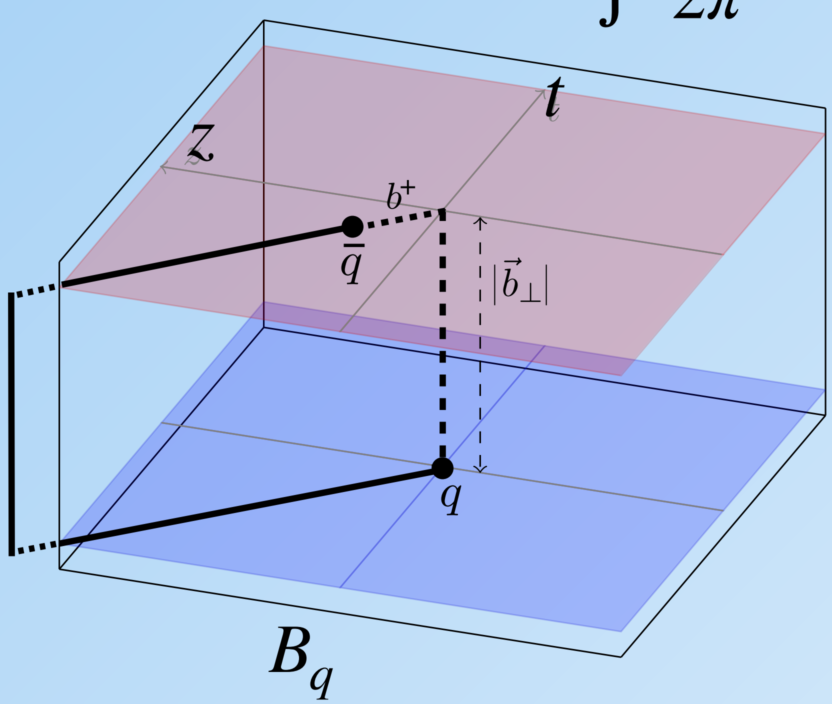
- Bachetta et al., JHEP 1706 (2017);
- Scimemi and Vladimirov, EPJC78 (2018);
- Bertone, Scimemi and Vladimirov, JHEP 1906 (2019).

Towards a lattice calculation of the TMDPDF

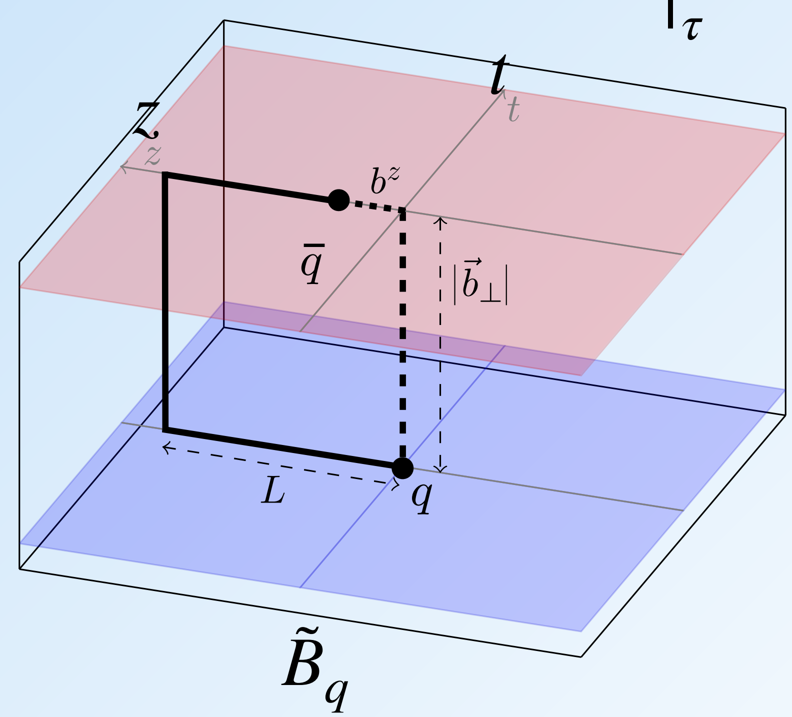
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

- Beam function:

$$B_q(x, \vec{b}_T, \epsilon, \tau) = \int \frac{db^+}{2\pi} e^{-i(xP^+)b^-} \langle P \,|\, \bar{q}(b^\mu) W(b^\mu) \frac{\gamma^+}{2} W_T(-\infty \bar{n}; \vec{b}_T, \vec{0}_T) W^\dagger(0) q(0) \Big|_\tau \,|\, P \rangle$$



Lorentz boost and $L \to \infty$



- Quasi-beam function on lattice:

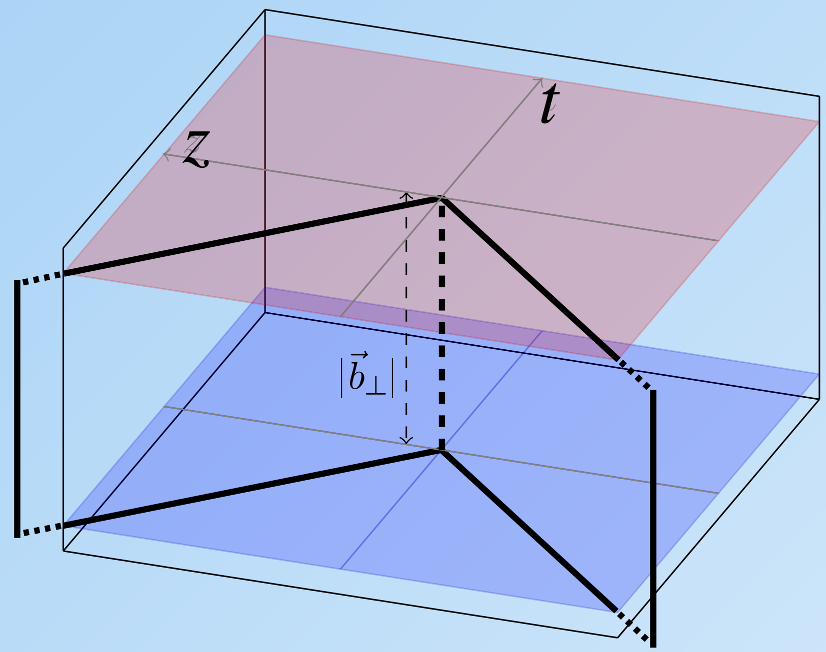
$$\tilde{B}_q(x, \vec{b}_T, a, L, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$$

$$= \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \langle P \,|\, \bar{q}(b^\mu) W_{\hat{z}}(b^\mu; L - b^z) \frac{\Gamma}{2} W_T(L\hat{z}; \vec{b}_T, \vec{0}_T) W_{\hat{z}}^\dagger(0) q(0) \,|\, P \rangle$$

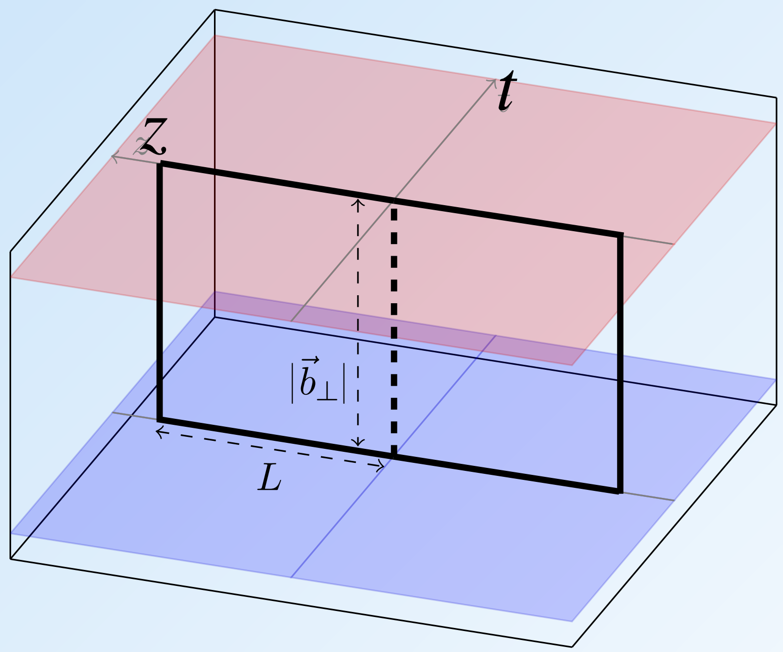
Towards a lattice calculation of the TMDPDF

- Soft function:

$$S_q(b_T, \epsilon, \tau) = \frac{1}{N_c}\langle 0 | \mathrm{Tr}\left[S_n^\dagger(\vec{b}_T)S_{\bar{n}}(\vec{b}_T)S_T(-\infty\bar{n}; \vec{b}_T, \vec{0}_T)S_{\bar{n}}^\dagger(\vec{0}_T)S_n(\vec{0}_T)S_T^\dagger(-\infty n; \vec{b}_T, \vec{0}_T)\right]\Big|_\tau |0\rangle$$



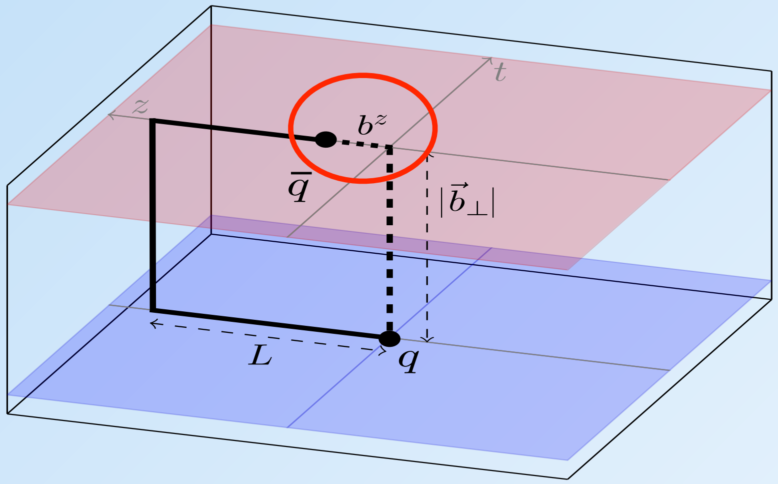
Cannot be related by Lorentz boost



- Quasi-soft function on lattice (naive definition):

$$\tilde{S}_q(b_T, a, L) = \frac{1}{N_c}\langle 0 | \mathrm{Tr}\left[S_{\hat{z}}^\dagger(\vec{b}_T; L)S_{-\hat{z}}(\vec{b}_T; L)S_T(L\hat{z}; \vec{b}_T, \vec{0}_T)S_{-\hat{z}}^\dagger(\vec{0}_T; L)S_n(\vec{0}_T; L)S_T^\dagger(-L\hat{z}; \vec{b}_T, \vec{0}_T)\right] |0\rangle$$

Quasi-TMDPDF and its relation to TMDPDF



- Quasi-TMDPDF (in the MSbar scheme):

$$\tilde{f}_q^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = \lim_{L\to\infty} \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \mu, a) \frac{\tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)}{\sqrt{\tilde{S}_q(b_T, a, L)}}$$

- Relation to TMDPDF:

- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

$$\tilde{f}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right) g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right]$$

$$\times f_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

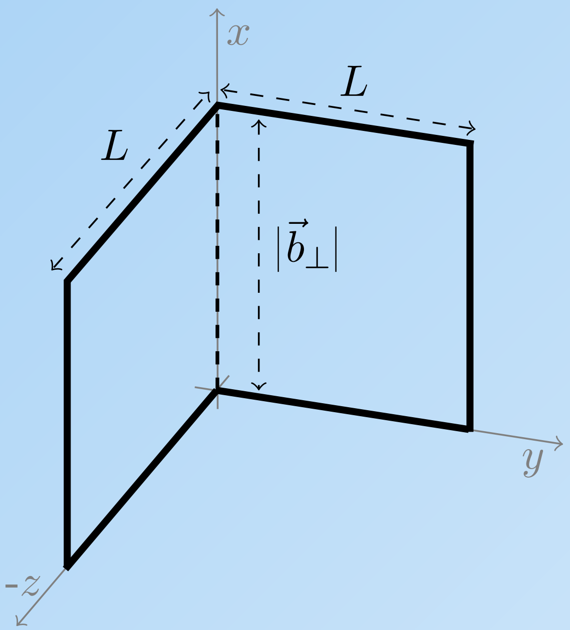
Hierarchy of scales: $b^z \sim \frac{1}{P^z} \ll b_T \ll L$

For $b_T \ll \Lambda_{\rm QCD}^{-1}$ $\quad g_q^{S_{\rm naive}}(b_T, \mu) = 1 + \frac{\alpha_s C_F}{2\pi} \ln\frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} + O(\alpha_s^2)$

- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

Perturbatively matchable quasi-TMDPDF

- A bent quasi-soft function?



- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

$$g_q^{S_{\rm bent}}(b_T,\mu) = 1 + O(\alpha_s^2)$$

However, this should be checked to all orders of perturbation theory.

- An ultimate solution relies on having

$$g_q^{S}(b_T,\mu) = 1$$

Collins-Soper kernel from lattice

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\tilde{f}_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\tilde{f}_{\rm ns}^{\rm TMD}(x, \overrightarrow{b}_T, \mu, P_2^z)}$$

Ebert, Stewart and Y.Z., PRD99 (2019).

$$= \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z \ e^{ib^z xP_1^z}\tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z \ e^{ib^z xP_2^z}\tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_2^z)}$$

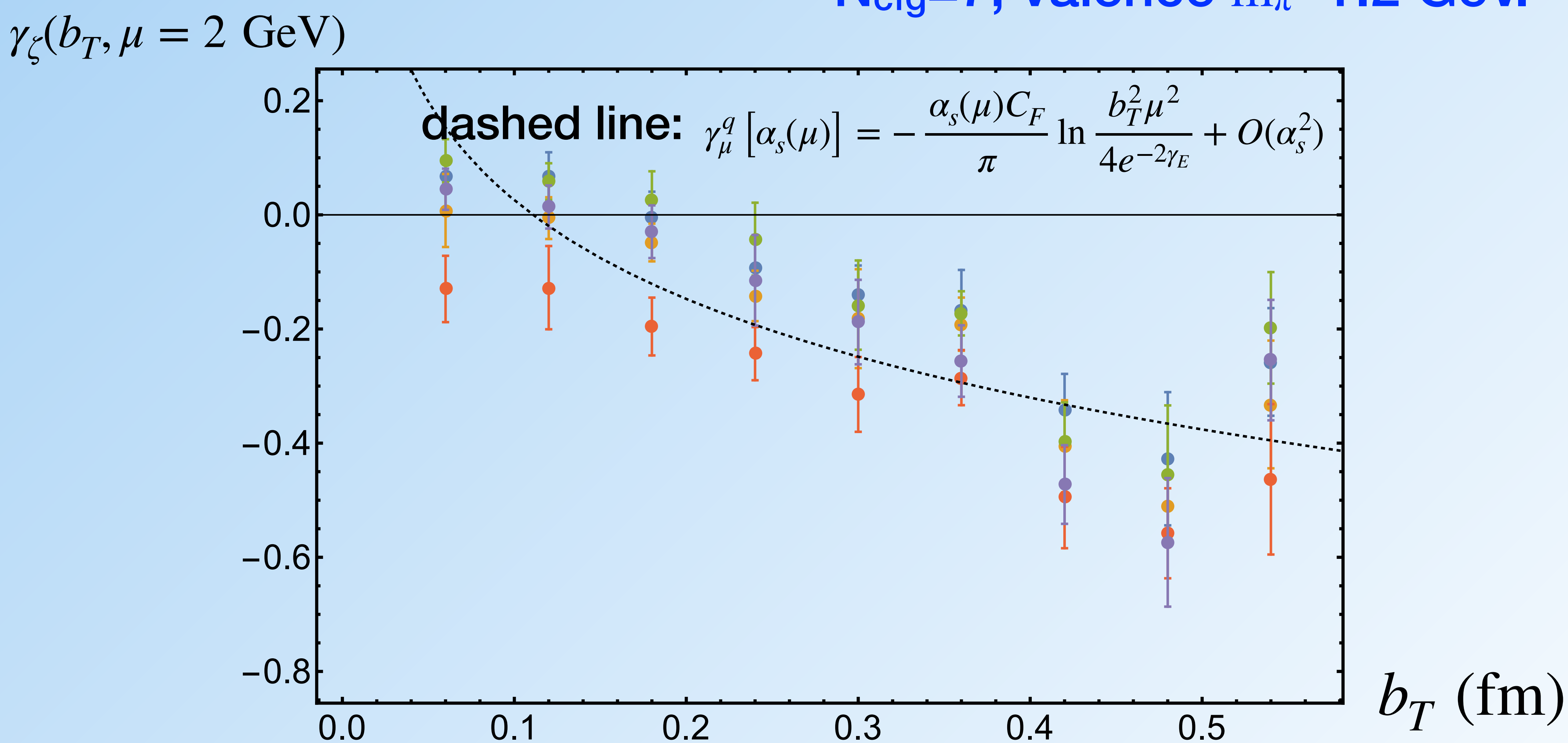
- g^S as well as the quasi-soft function are canceled in the ratio;
- Does not depend on the external state hadron, could be calculated with pion for simplicity;
- Independent of the choice of *x* and P^z, which provides a window to control systematic uncertainties;
- One can also calculate ratios of TMDPDFs with different spin structures.

The idea of forming ratios has been used in the calculation of *x*-moments of TMDPDFs:
Hagler, Musch, Engelhardt, Yoon, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

A first look at the Collin-Soper kernel

Work in progress with Phiala Shanahan and Michael Wagman

Caveat: quenched approximation, N_{cfg}=7, valence m_π~1.2 GeV.



Different colored points correspond to the Collins Soper kernel calculated at *x* = 0.4, 0.45, 0.5, 0.55, 0.6.

Summary and outlook

- The complete renormalization and matching for the quark and gluon PDFs have been derived;
- The nonperturbative renormalization and matching for the GPDs have been derived;
- The Collins-Soper kernel can be calculated with ratios of quasi-beam functions. A first look shows encouraging results with present-day resources.
- Future work will include (much) larger statistics, different lattice spacings, and more systematic treatment than the naive Fourier transform;
- The ultimate solution to calculating TMDPDF on the lattice relies on the soft sector, which will be pursued vigorously in the future.