Updates on domain-wall on HISQ elastic nucleon form factors

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Nucleon elastic structure

Scientific purpose
Nucleon form factors and elastic structure in general are relevant to a variety of high energy and nucleon physics experiments.

Single nucleon structure is the first step towards nuclear matrix elements.

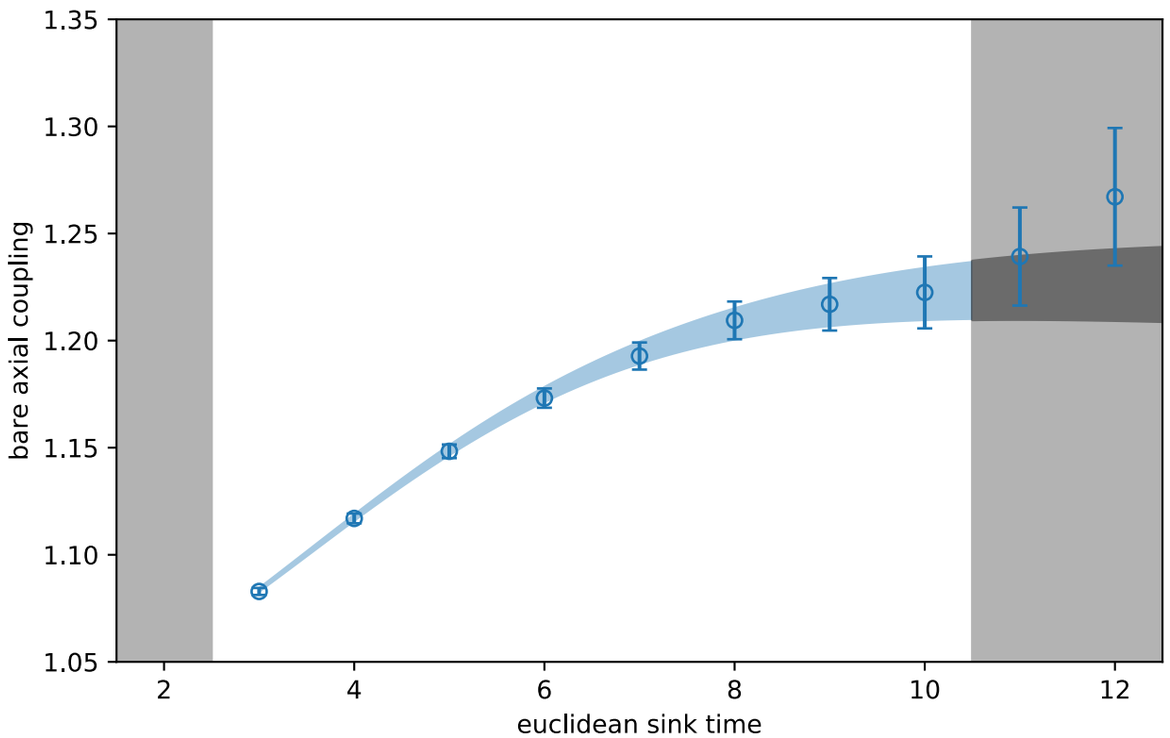
Summary of method from Andre's talk
Using sequential propagators we preserve the 4D dependence of the current.

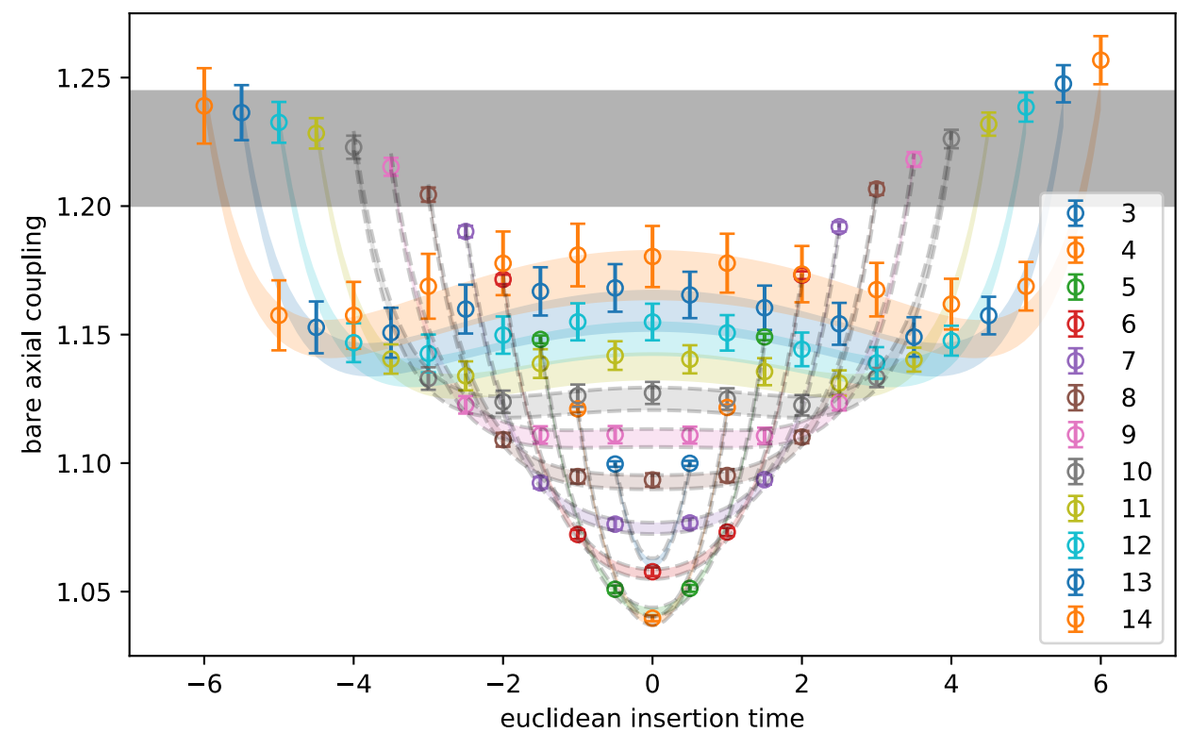
- momentum transfer
- spatial moments of correlation functions

require minimal computational cost to implement. (Lots of disk space however.)

Goal of the final analysis
Perform an analysis combining:
two point correlator
slope of the two point overlap factor
gA and gV sequential and FH correlators
slope of the gA and gV FH correlators

Nucleon coupling (zero momentum)





Data on a ~ 0.09fm mpi~310 MeV. Data on same 4D correlator

Quick take away points

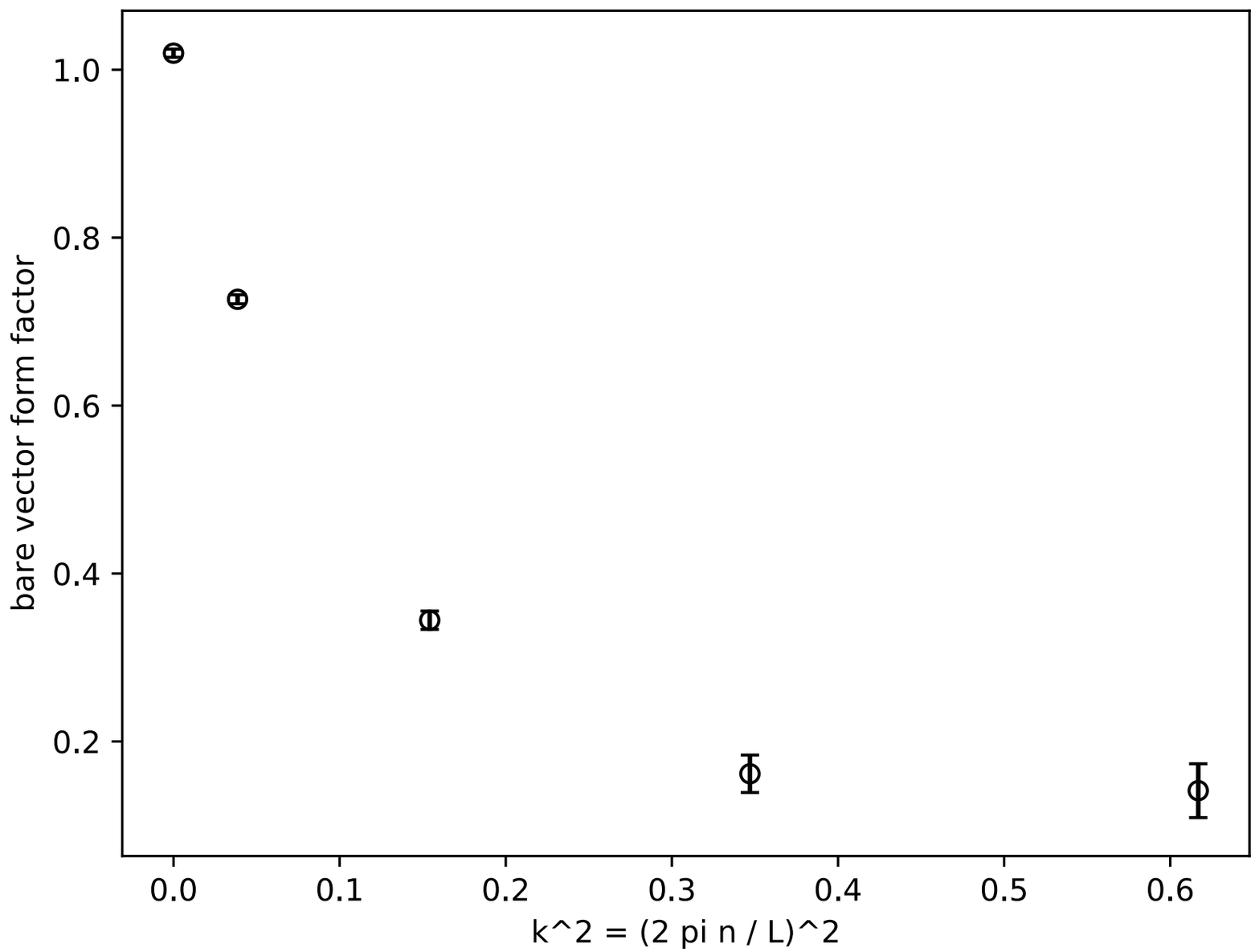
- tsnk = 9 and 14 is approximately 0.81 fm and 0.126 fm source-sink separation

- sequential 3 point still needs extrap. to ground state @ 1.26 fm
- FH correlator is consistent at 1 s.d. below 1 fm source-sink separation

- sequential 3 point has much more information when initial state != final state
  - This is important for non-zero momentum transfer

Different correlators have very different e.s. contributions

Nucleon vector form factor



Andre showed some simultaneous fits of seq. 3pt with FH.
This preliminary result is not performed that way.

This is just two point + sequential 3 point over n = [0, 1, 2, 3, 4] where k =2 pi n /L
Demonstration of very standard sequential 3 point form factor result

Slope of correlation functions

Continuation of UKQCD work: Nucl. Phys. B444 410 (1995)

and work in collaboration with Kostas Orginos, David Richards, Chris Bouchard.

Quick summary:
Two-point moments give slope of the overlap factor
Three-point moments give slope of matrix element

Some relevant formulas for constructing the correlators
Key Idea: momentum space derivative = position space moments

$$\frac{\partial}{\partial k^2} C_{2\mathrm{pt}}(t) = \int d^3x \langle N_{t,x} | N^\dagger_{0,0} \rangle \frac{-ix_i}{2k} e^{-ikx_i}$$

$$\lim_{k^2 \to 0} C'_{2\mathrm{pt}}(t) = \int d^3x \langle N_{t,x} | N^\dagger_{0,0} \rangle \frac{-x_i^2}{2}$$

$$\frac{\partial}{\partial k^2} C_{3\mathrm{pt}}(T,t') = \int d^3x d^3x' \langle N_{T,x} | \Gamma_{t'x'} | N^\dagger_{0,0} \rangle \frac{-ix'}{2k} e^{-ikx'_i}$$

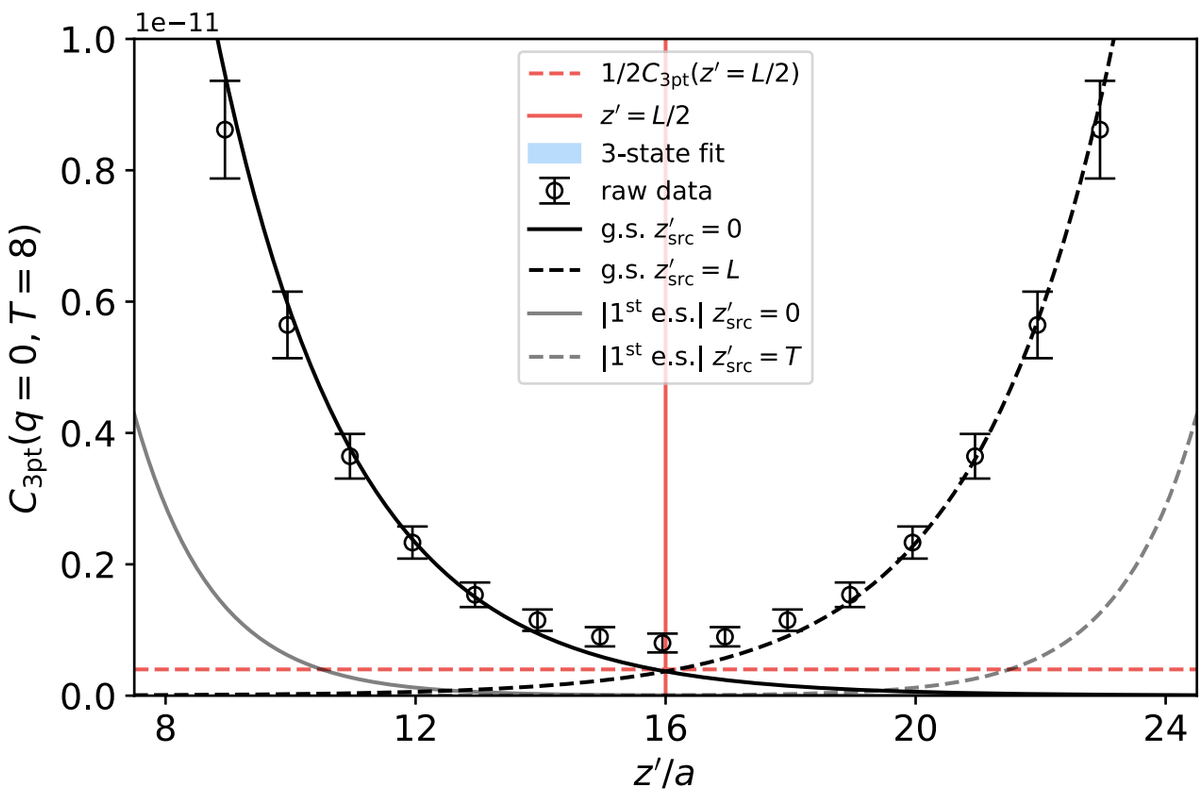
$$\lim_{k^2 \to 0} C'_{3\mathrm{pt}}(T,t') = \int d^3x d^3x' \langle N_{T,x} | \Gamma_{t'x'} | N^\dagger_{0,0} \rangle \frac{-x'^2}{2}$$

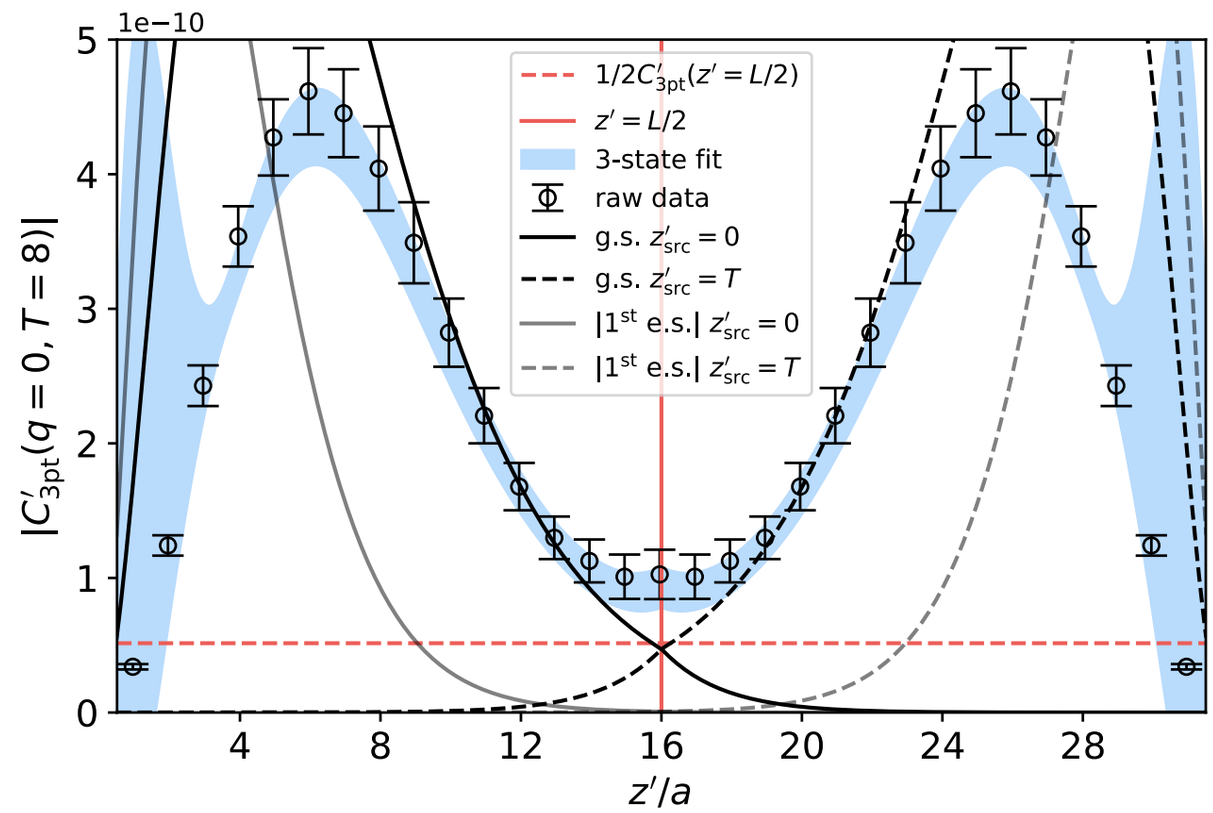
Finite volume contributions

Main Issue: Spatial moments are NOT periodic

- Regular correlators are by construction (anti-)symmetric and periodic.
- Non-zero momentum projections are periodic (discrete momenta only)

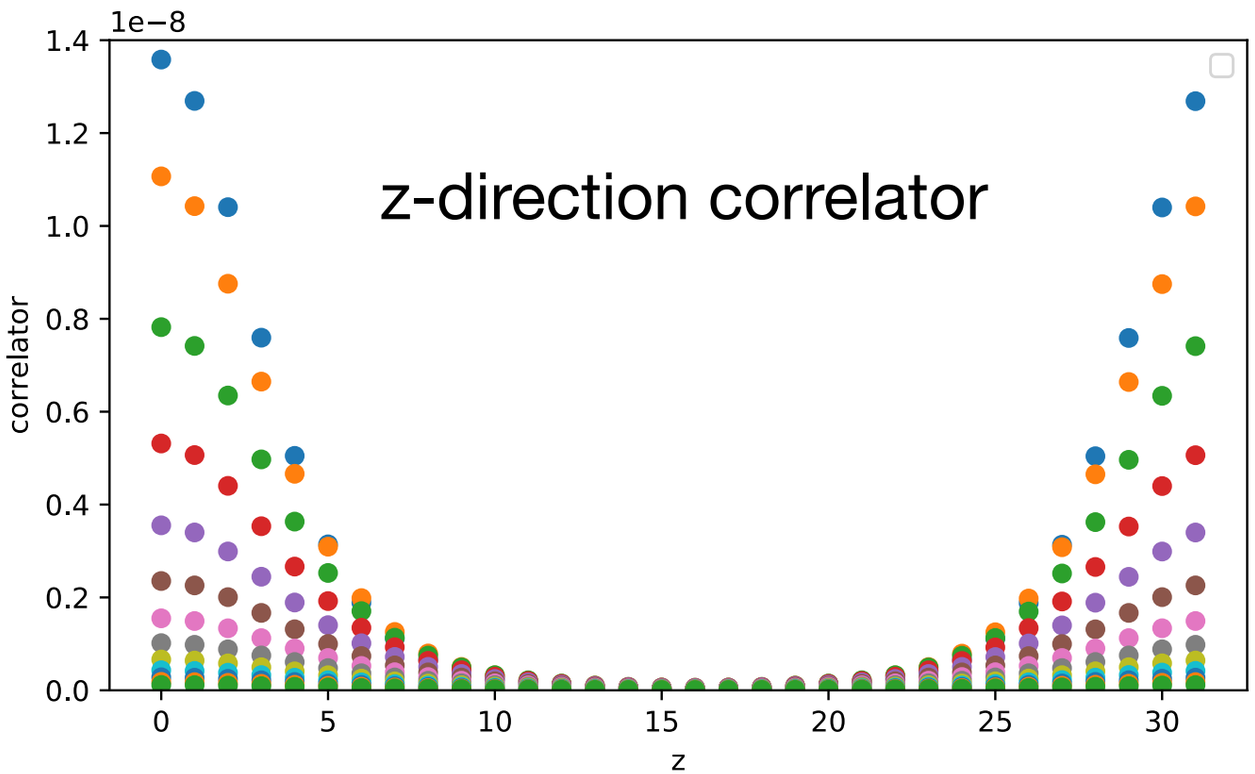
Picture of what's going on (these plots are on JLab ensembles):
(Left) Correlator after projecting current insertion to x, y, t
(Right) Same correlator but with z^2 applied symmetrically (cut off |z| > L/2)

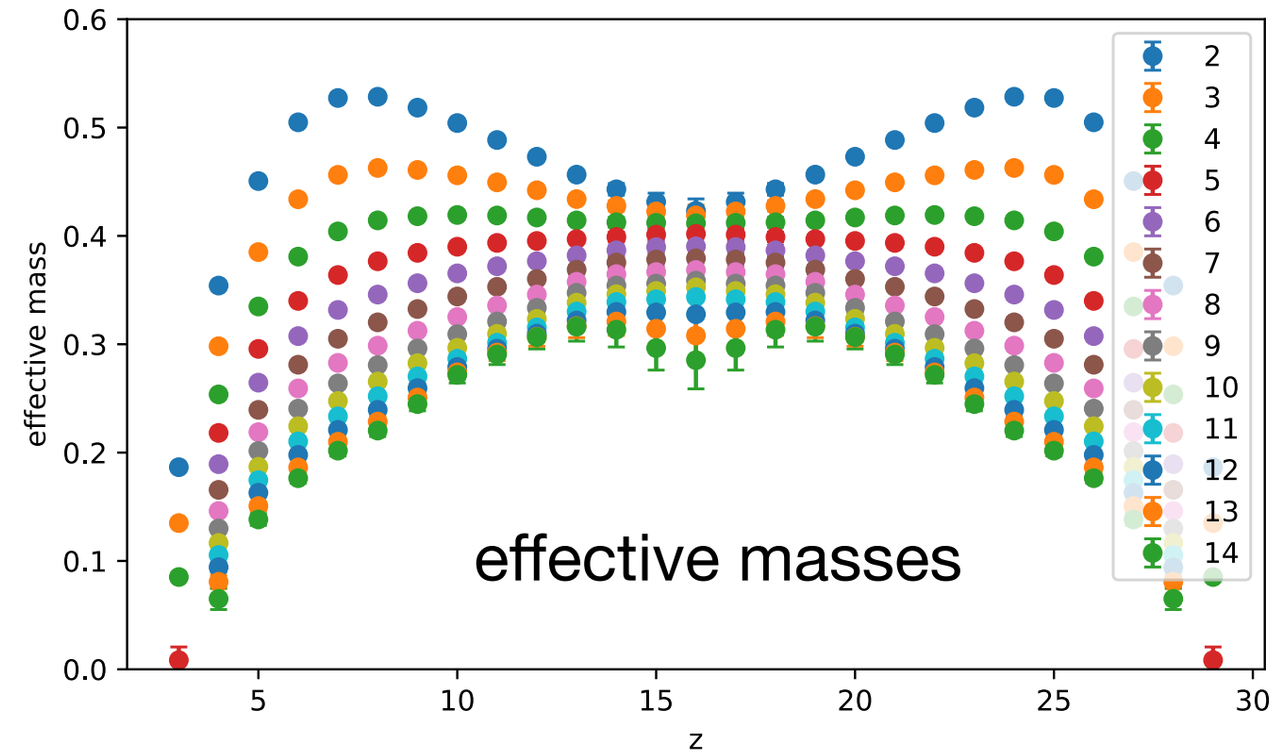




Take away: Need to account for |z| > L/2 that was dropped.
Fit z-direction correlator, and fix this.

Finite volume contributions cont.





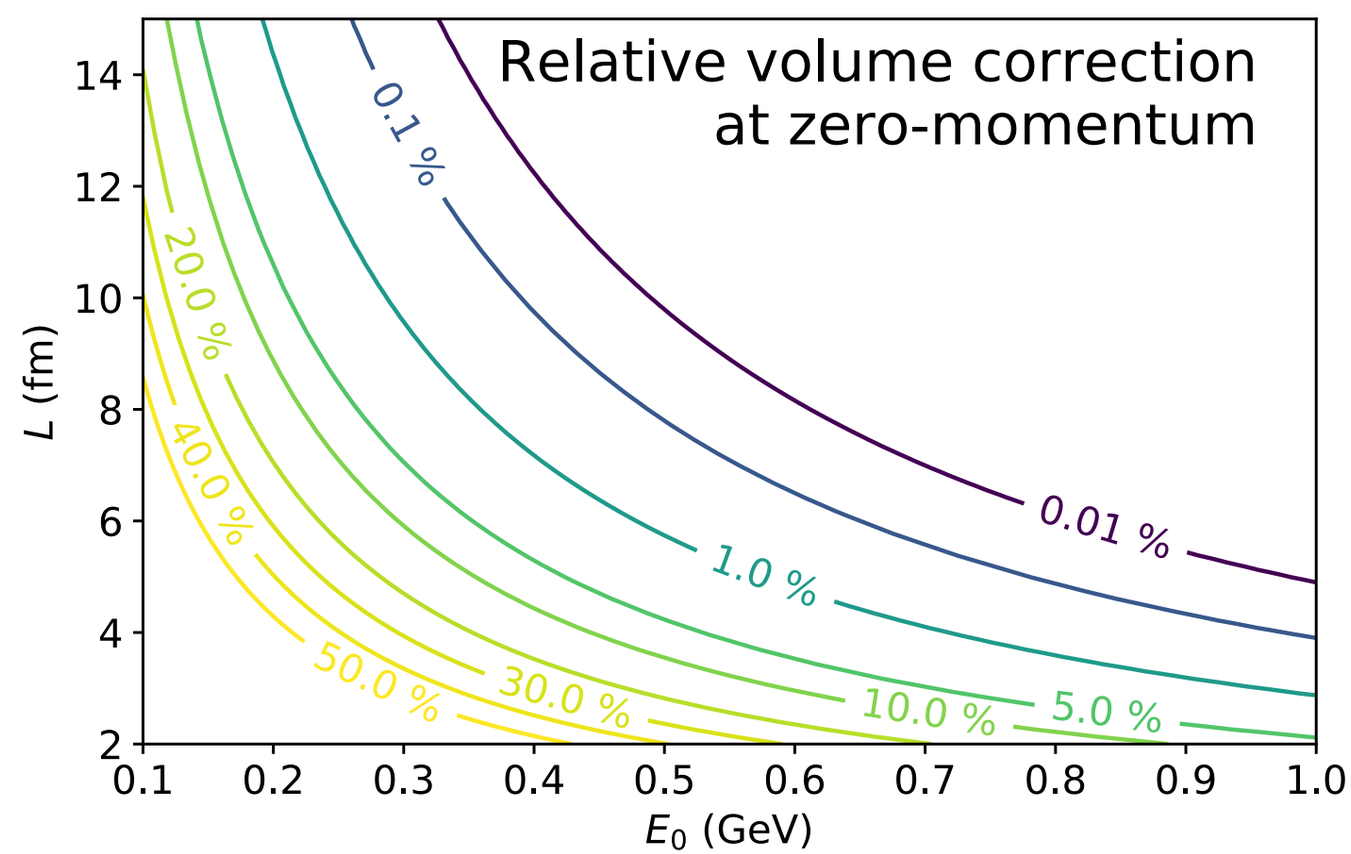
Some DWF-HISQ data with a lot more source-sink separations

The effective mass is approx. a rho meson slow going into 2 pions.

Small box size (L < 3 fm) doesn't help with seeing a proper plateau.

The FV formula is a bit of a mess (shown in lattice 2018 slides)

Take away: L > 5 fm preferred

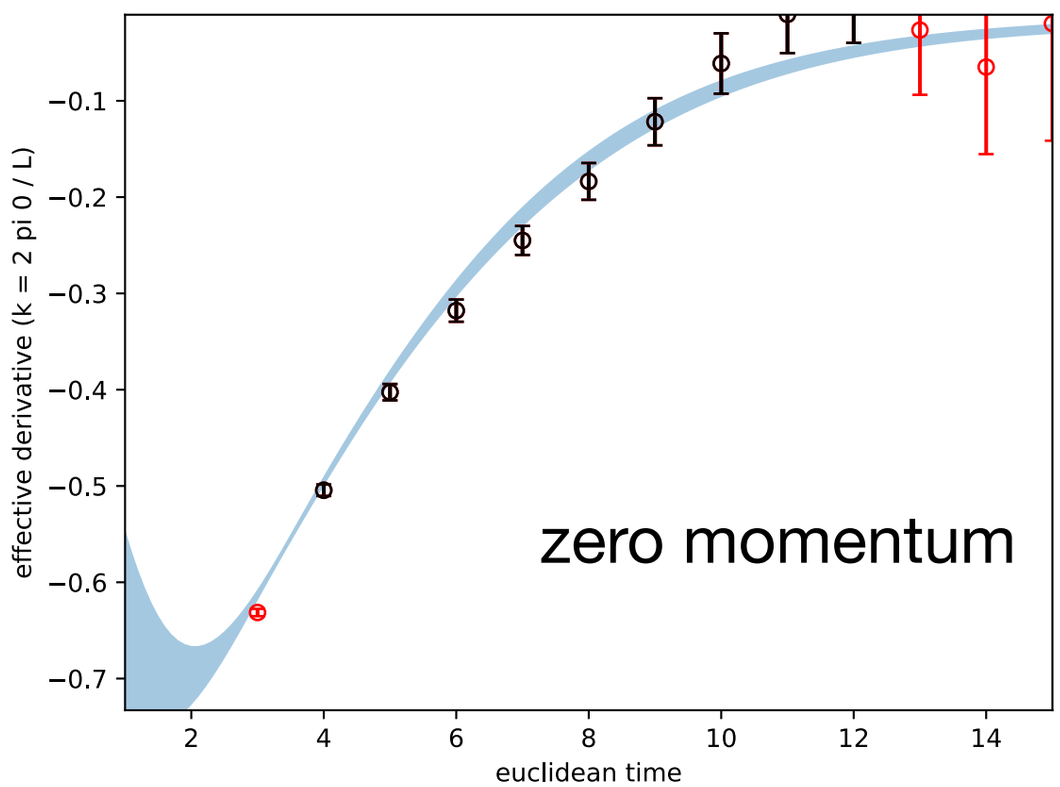


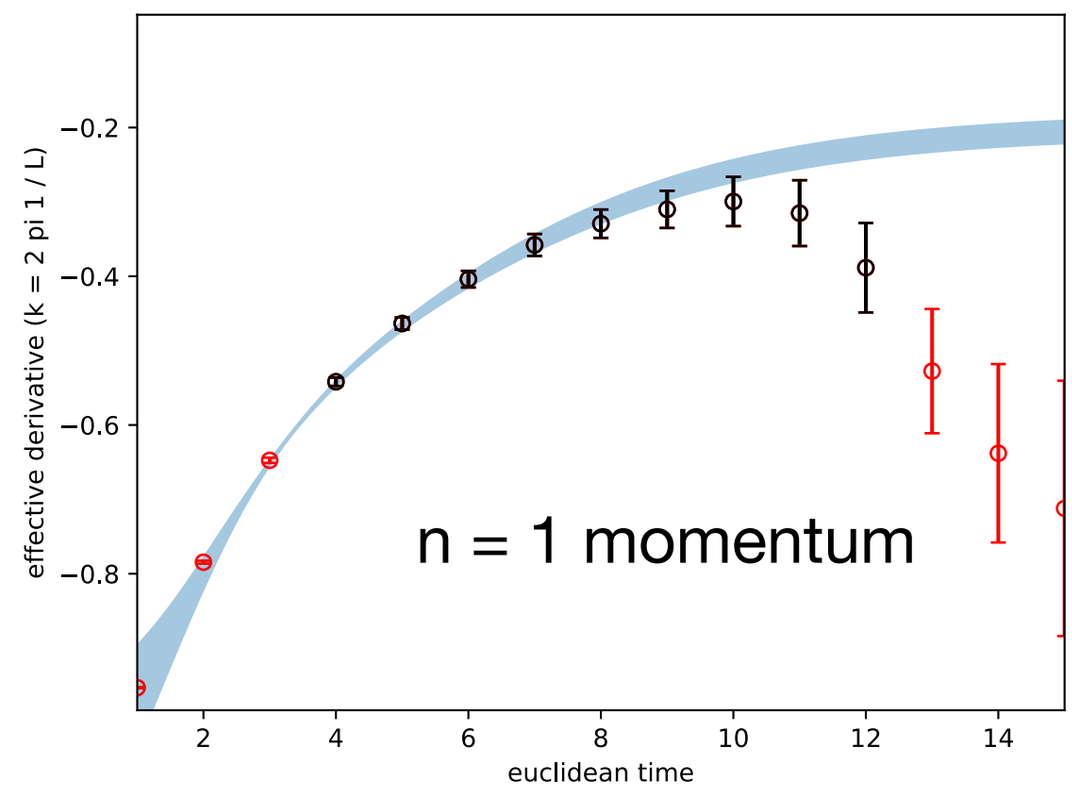
Two point fit function for slopes

$$C_{2\mathrm{pt}}(t) = \sum_n \frac{Z_n Z_n^\dagger}{2E_n} e^{-E_n t}$$

$$C'_{2\mathrm{pt}}(t) = \sum_n C^n_{2\mathrm{pt}}(t) \left(\frac{2Z'_n}{Z_n} - \frac{1}{2E_n^2} - \frac{t}{2E_n} \right)$$

Data plotted has leading time dependence removed





Fits of the two point moment correlators on top of data

Observation: Moment correlators are more sensitive to fluctuations.

Let's see the three-point for more evidence.

Feynman-Hellmann slopes! :)

Reminder of CalLat's favorite ratio

1) Make summation method correlator
2) R(t) = summation correlator / two point correlator
3) FH correlator = R(t+1) - R(t)

Repeating this on the moment correlators changes

$$C'_{3\mathrm{pt}}(t,t') = \sum_{n,m} C^{nm}_{3\mathrm{pt}}(t,t') \left(\frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z'_m}{Z_m} - \frac{1}{2E_m^2} - \frac{t'}{2E_m} \right)$$

insertion time

to this in the large source-sink t-limit

$$\frac{\Gamma'_{mn}}{\Gamma_{mn}} + \frac{Z'_m}{Z_m} - \frac{1}{2E_m^2} - \frac{t}{4E_m}$$

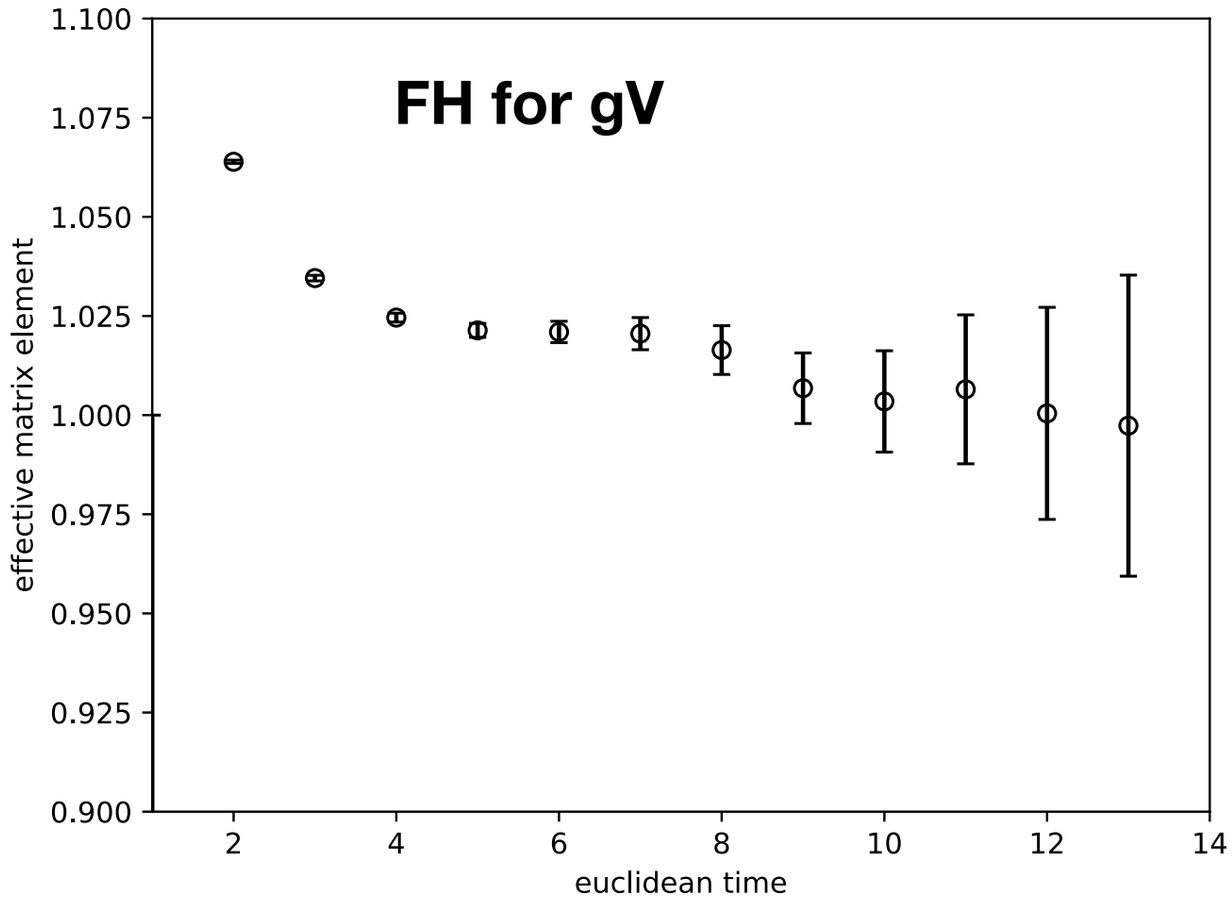
sink time

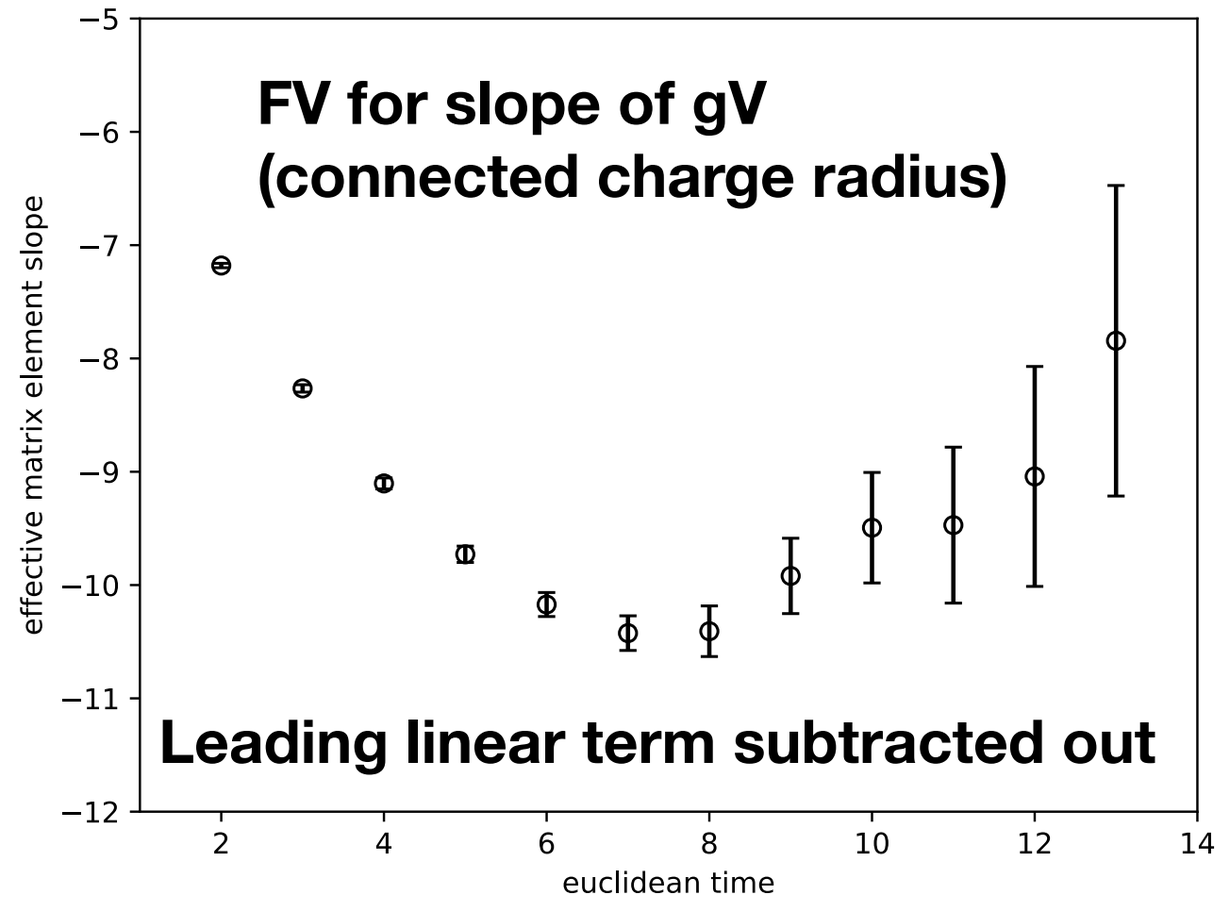
(the e.s. contributions is similar to FH otherwise)

(embrace the name)

(embrace the name)

Some FH plots





Similar to FH for zero-momentum matrix elements, the slope plateaus @ < 1 fm.
Main point: We see a plateau! (We will fit all the data… but this is comforting.)

Details:
We think (from experience) at t = 9, there is some statistical fluctuation for gV.
The same statistical fluctuation is in the slope, but MUCH more enhanced.

Ending remarks

Saving the 4-dimensional correlators we can analyze (almost) everything we want.

The standard 3 point come out nicely even at very small source-sink separations.
The Feynman-Hellmann correlators are completely consistent with standard 3 point.
The two methods illuminate **different** excited state contributions. <- very useful.

Large volume lattices will help keep finite volume contributions under control.
Throwing FH at the slopes (out of necessity to fix FV) looks promising.

The final analysis will probably be very involved, with many possibilities to explore.

Thank you! Enjoy Wuhan! Sorry I could not be there in person.
But you are welcome to grill Andre.