Lattice 19

Wuhan · China | June 16-22

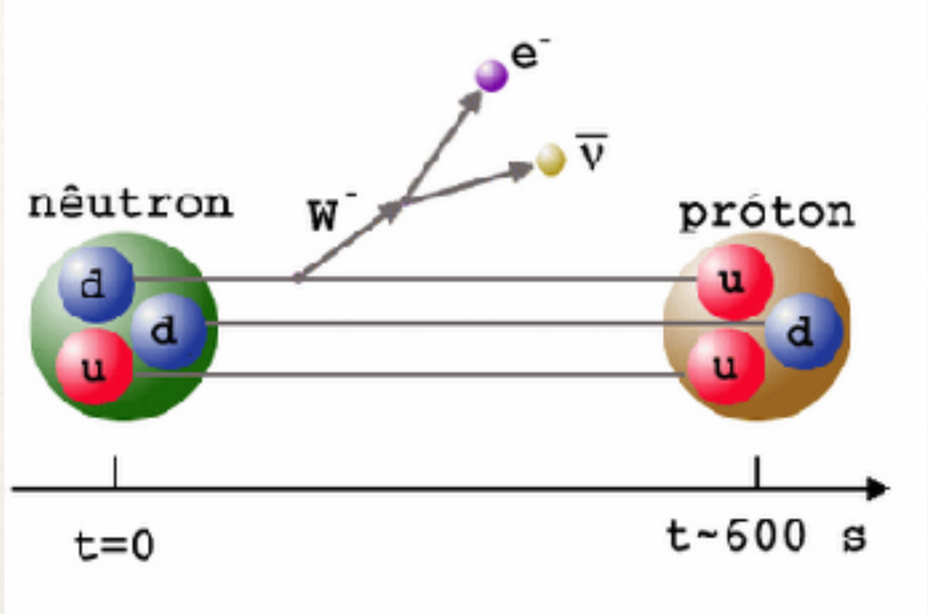
California Lattice Collaboration

Elastic Nucleon Structure, 1

André Walker-Loud

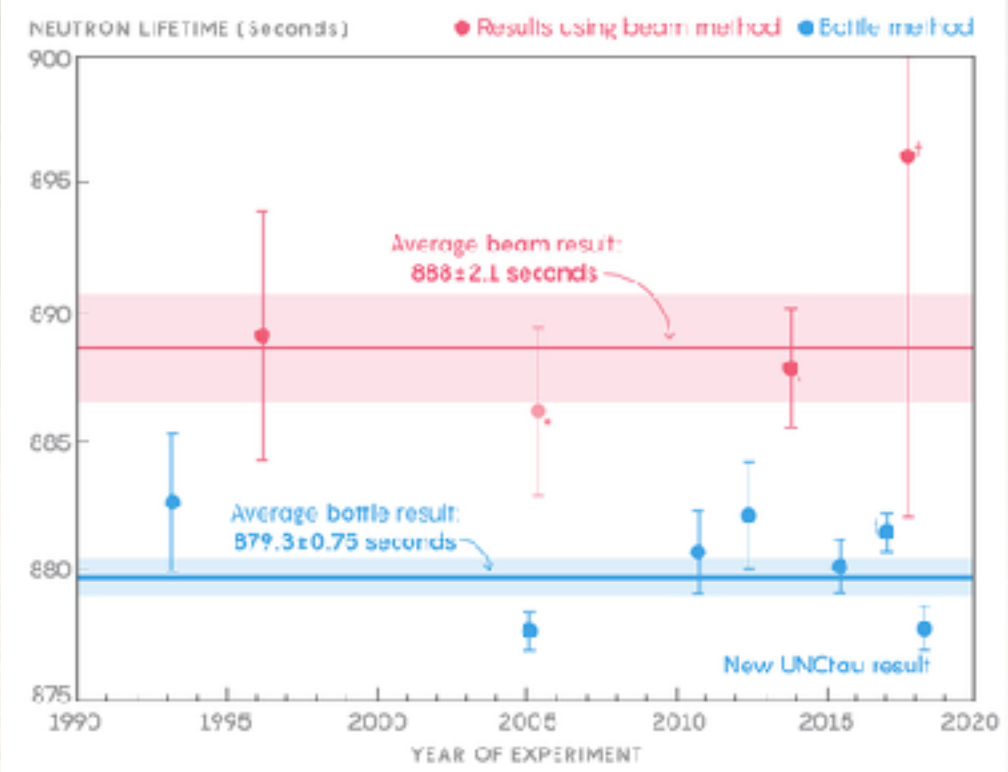
Berkeley Lab

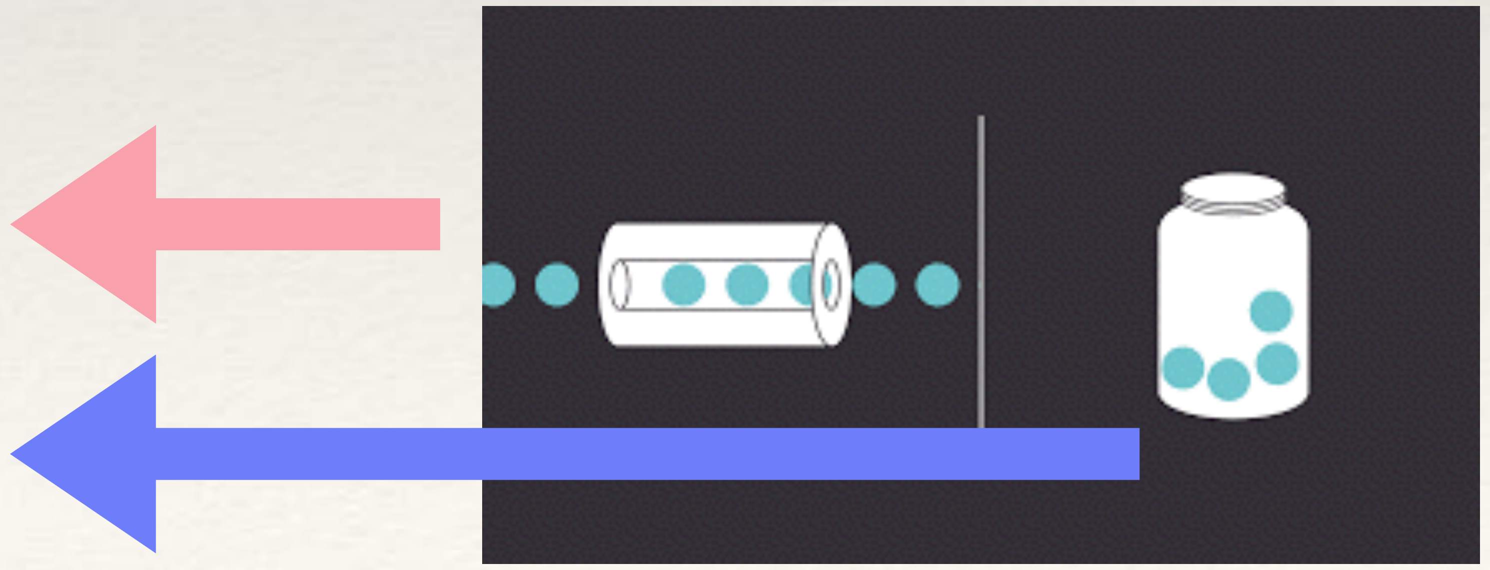
Neutron lifetime and the axial coupling



$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{ud}|^2}{2\pi^3} m_e^5 (1 + 3g_A^2)(1 + RC) f_{V,A}$$

- ☐ The neutron lifetime and g_A (neutron decay) are used to probe the limits of the Standard Model
- ☐ We should have a (meaningful) Standard Model prediction for g_A - LQCD (lattice QCD)
- ☐ To gain confidence in the application of LQCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest, such as g_A - **done - see previous talk**
- ☐ In order for the theoretical uncertainty on g_A to match the larger uncertainty in the neutron lifetime measurements, we must determine g_A with < 0.2% uncertainty - **is this crazy?**

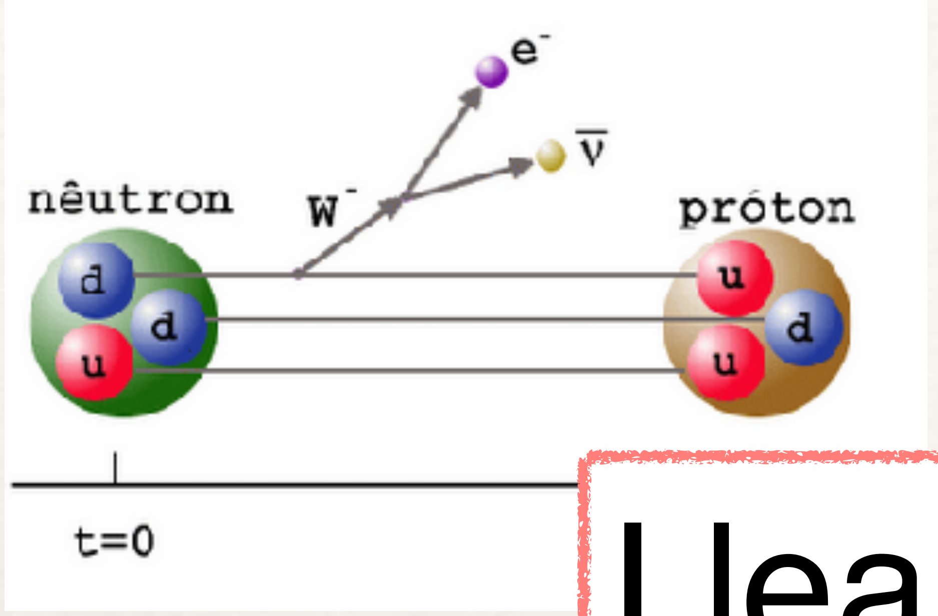




$$\tau_n^{\text{beam}} = 888.0(2.0)s$$

$$\tau_n^{\text{bottle}} = 879.4(0.6)s$$

Neutron lifetime and the axial coupling



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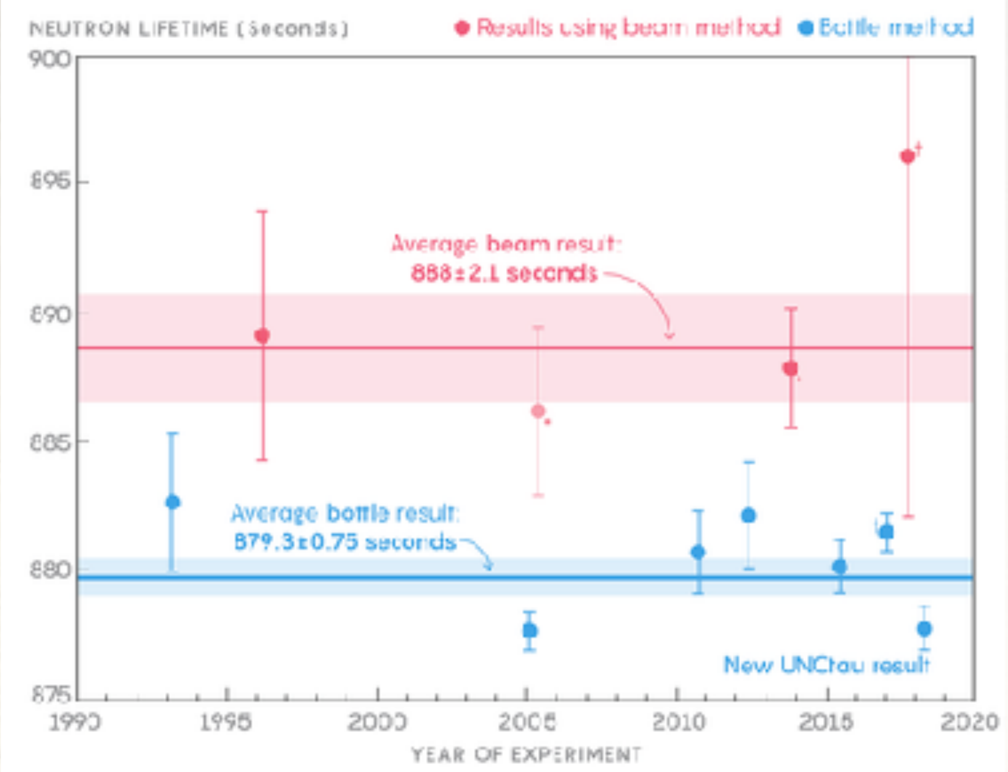
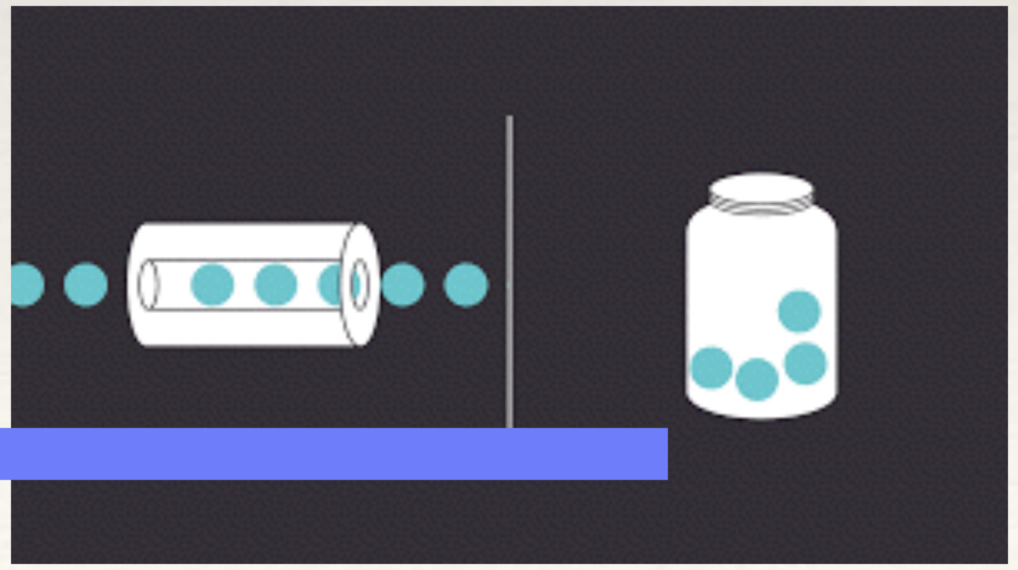
I learned recently in Steven Clayton's (LANL) talk at MENU on their measurement of the neutron lifetime, that the PDG no longer considers this to be a discrepancy - they've dropped the beam measurements 😱

- The neut … del
- We shoul …
- To gain c … orate) our calculatio … k
- In order … ifetime measurements, we must determine g_A with < 0.2% uncertainty - **is this crazy?**





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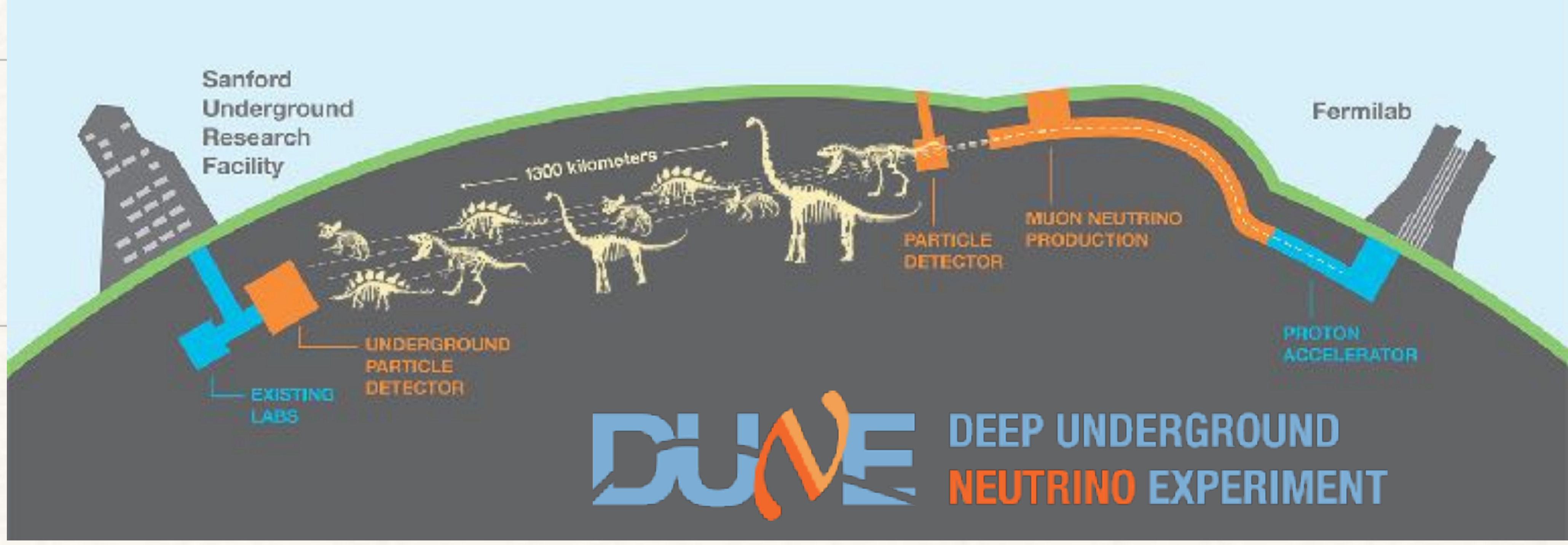
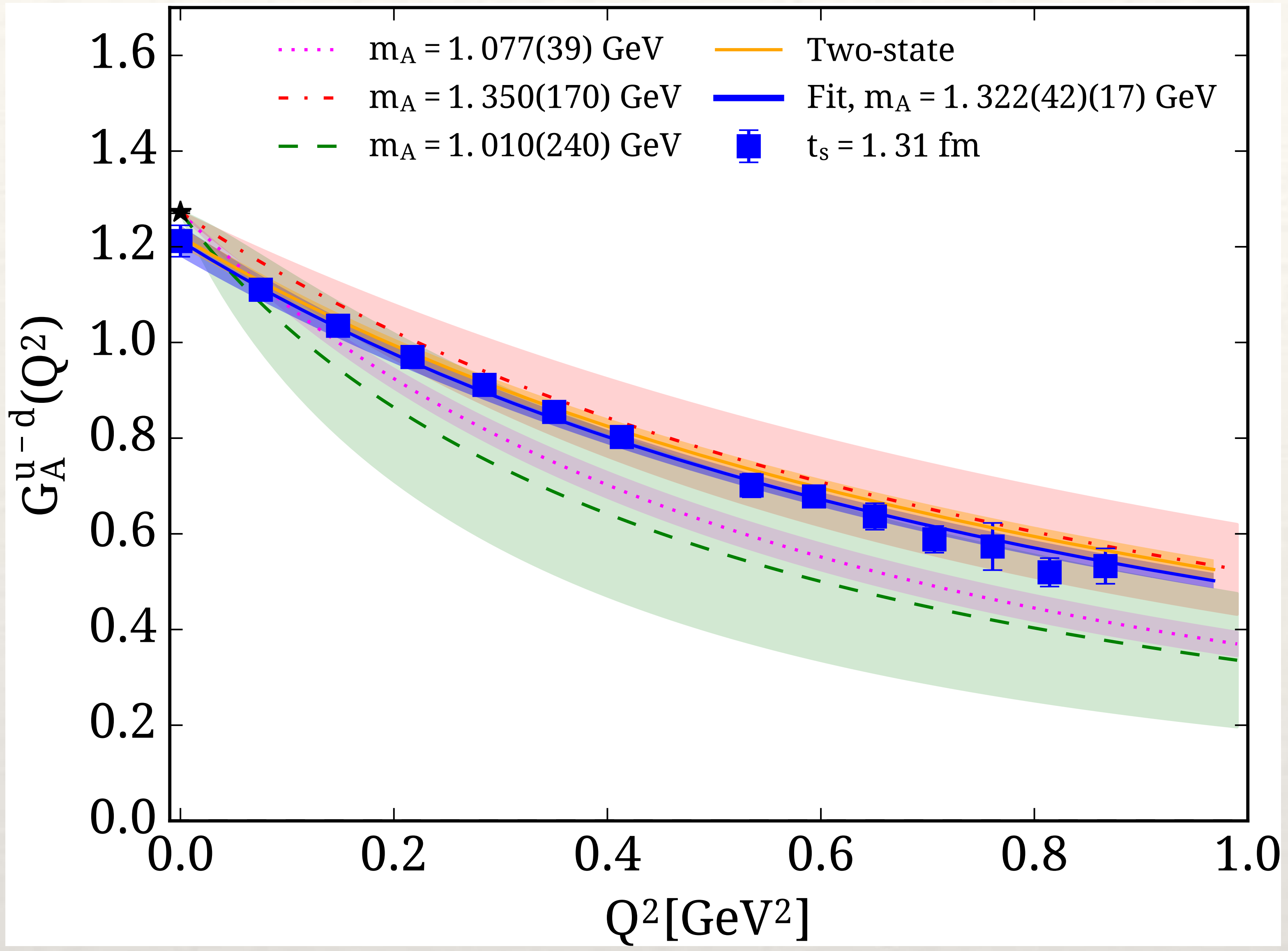
Nucleon Axial FormFactor



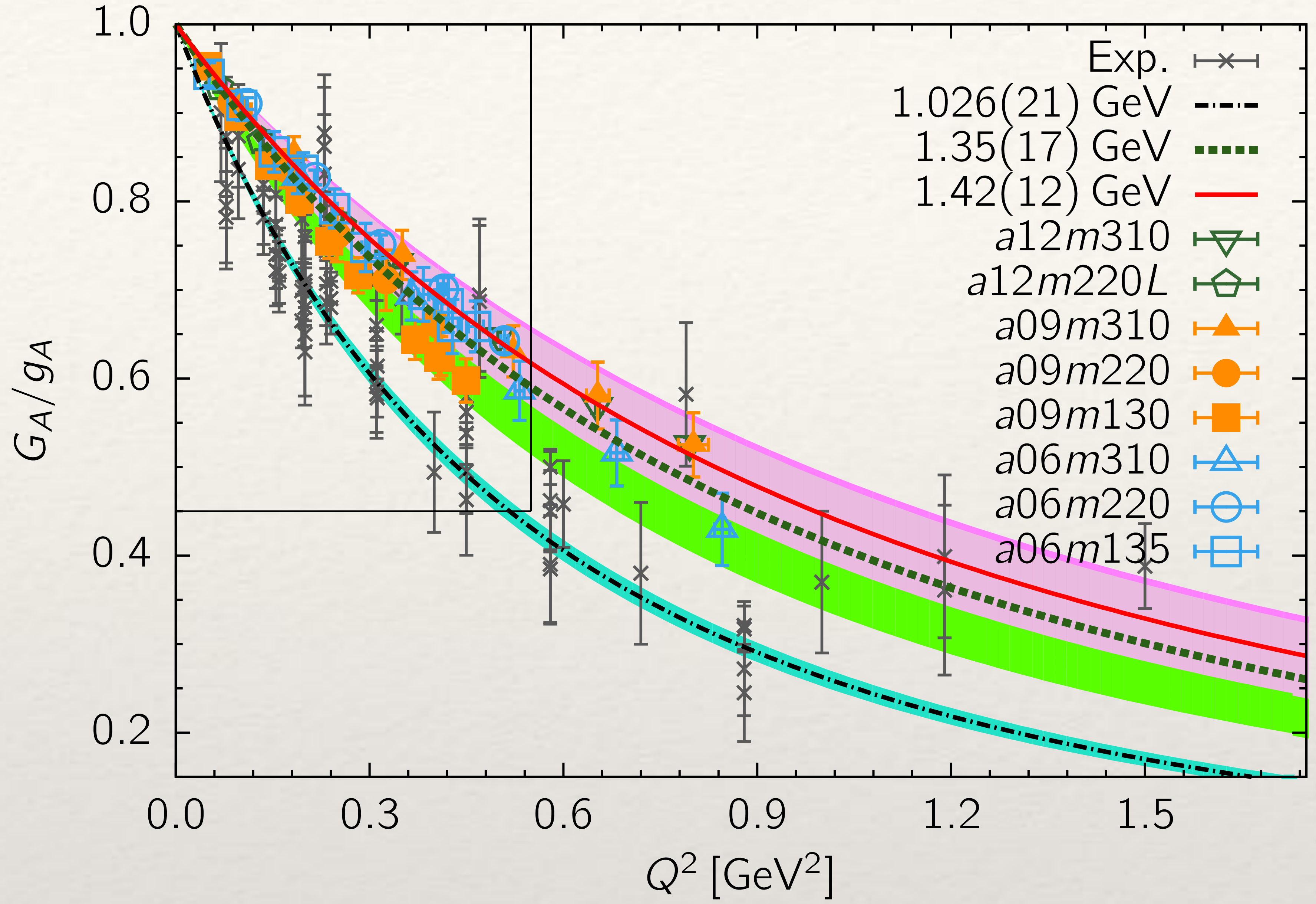
- DUNE is a future neutrino oscillation experiment that will fire a beam of neutrinos from FNAL into an Argonne target in South Dakota.
  - A determination of the CP-violating phase in the neutrino-mixing (PMNS) matrix is one of the goals
  - enough CP violation could explain the matter/anti-matter asymmetry of the universe through Leptogenesis
- The T2K and NOVA experiments are also conducting oscillation experiments
  - "A determination of the nucleon axial form factor at the 5% level would be very helpful, possibly allowing for the isolation of nuclear effects" [private communications with T2K members, Y. Hayato and K. Mcfarland]
- Ultimately, we need to understand neutrino-NUCLEUS cross sections which begins with neutrino-nucleon cross sections
  - The experimental data on $g_A(Q^2)$ is sufficiently limited that a simple dipole-formfactor is assumed
  - The dipole model is too simplistic and overly constraining (the quoted uncertainties do not reflect the true uncertainty of our understanding)

Nucleon Axial FormFactor

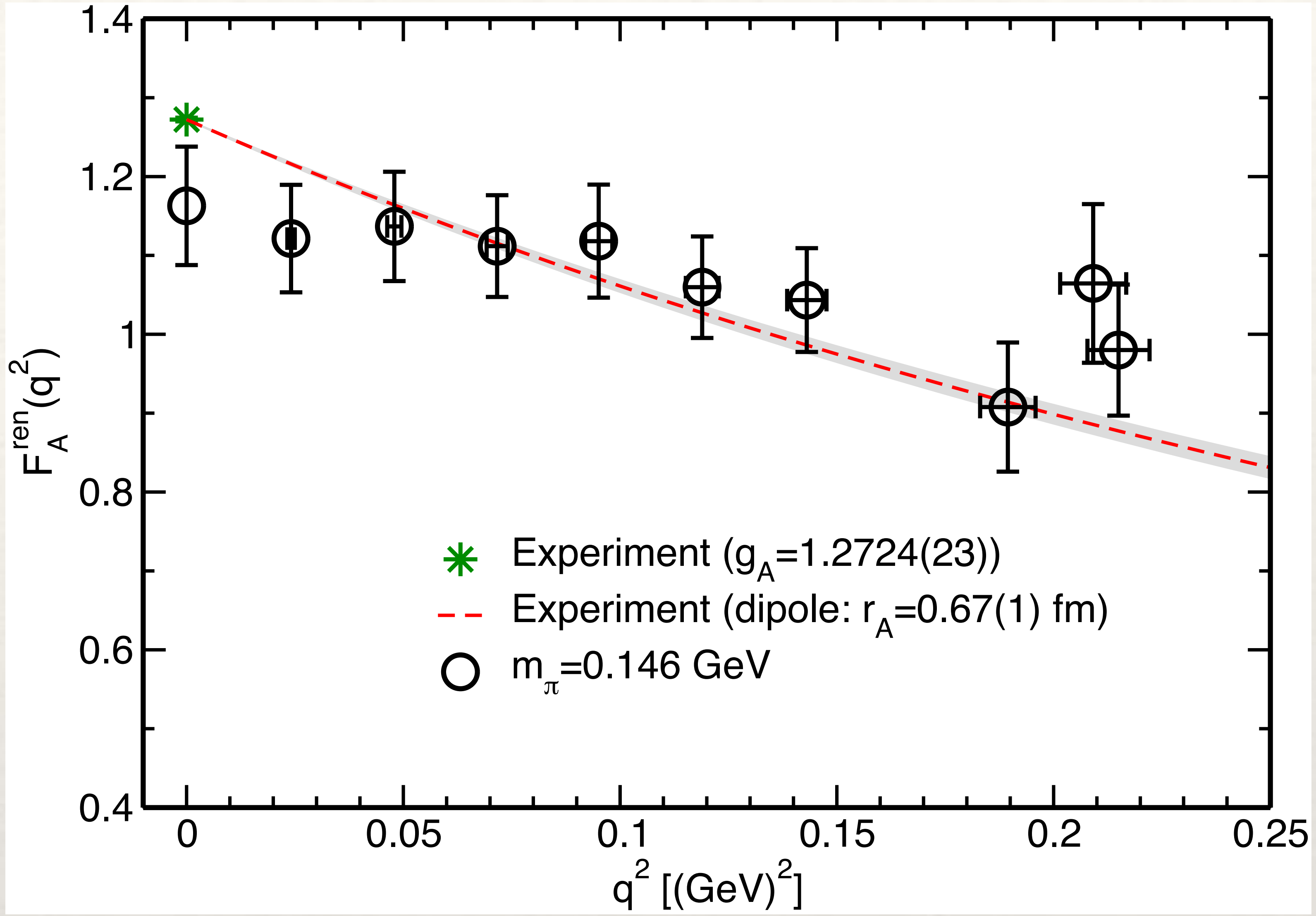
Alexandrou et al. PRD96 (2017) [1705.03399]



Gupta et al. PRD96 (2017) [1705.06834]



Ishikawa et al. PRD98 (2018) [1807.03974]



- All lattice QCD results determine an axial form factor with a significantly different slope (30%) than that determined from the phenomenological determination - two recent examples here
- Examining the LQCD results, it is difficult to understand/guess where this discrepancy is coming from
- A few years ago - this was the same situation with g_A (no one understood why gA results were consistently low compared to the experimental value)
- For g_A, we made progress by pushing to the extreme the LQCD calculations - a similar strategy here seems warranted

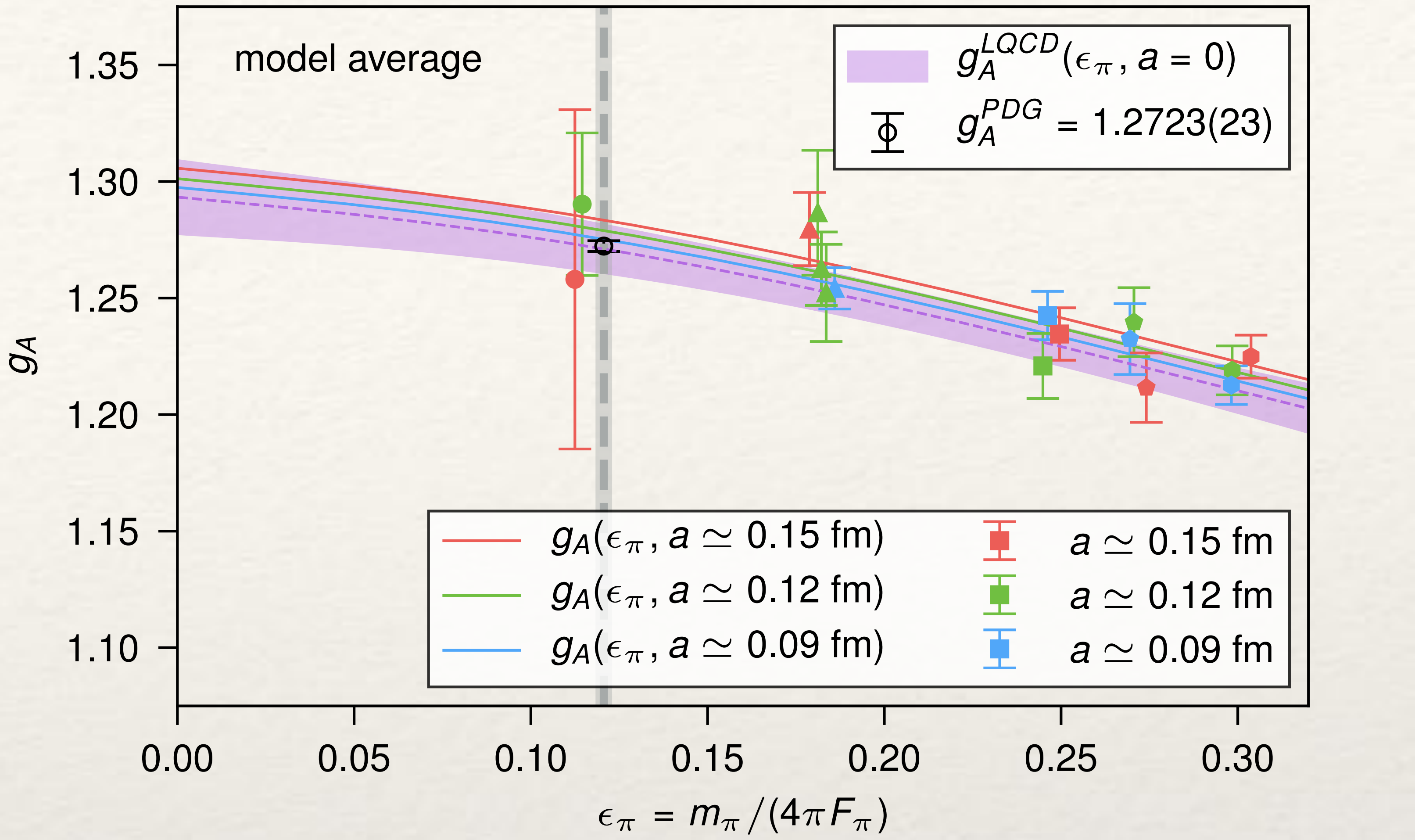
Bart van Lith
E. Berkowitz

improving the determination of g_A

Nature 558 (2018) no.7708, 91-94
Chang et al.
[arXiv:1805.12130]

Final result

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- More precise results at the physical pion mass will improve the three largest uncertainties:
  - statistical (s), extrapolation (χ) and model selection (M)
- Following our existing strategy, we anticipate getting to 0.5% by the end of this year
- Getting below (or maybe to 0.5%) will require a 4th lattice spacing as well (~0.06fm)
- Adding a FV study on additional pion mass points will improve the FV uncertainty
- The isospin uncertainty seems unnecessary…

improving the determination of g_A

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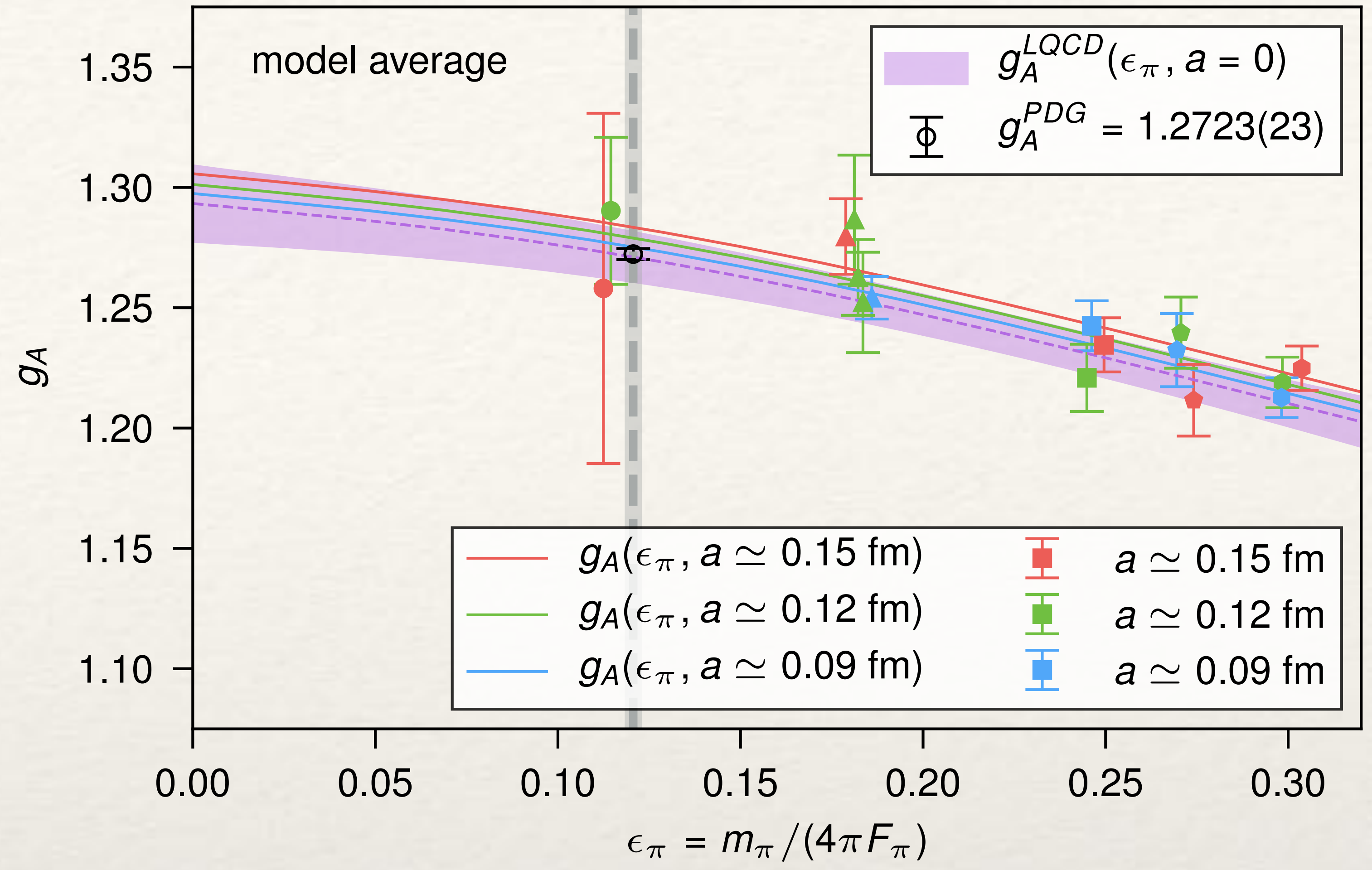

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

Isospin corrections **(my understanding prior to May 18 this year - see ACFI workshop)**

- ❑ The leading radiative corrections are subtracted from the experimental measurement leaving corrections of $\mathcal{O}\left(\frac{\alpha_{EM}^2}{\pi^2}\right) \sim 0.0005\%$
- ❑ There are $(m_d\text{-}m_u)^2$ corrections $\mathcal{O}\left(\frac{(m_d-m_u)^2}{(m_d+m_u)^2}\epsilon_\pi^4\right) \sim 0.002\%$
- ❑ There are mixed corrections of $\mathcal{O}\left(\alpha_{EM}\frac{m_d-m_u}{m_d+m_u}\epsilon_\pi^2\right) \sim 0.004\%$
- ❑ The largest isospin correction comes from the extrapolation to $\epsilon_{\pi^-} = \frac{m_{\pi^-}}{4\pi F_{\pi^-}}$ $\epsilon_{\pi^0} = \frac{m_{\pi^0}}{4\pi F_{\pi^0}}$

improving the determination of g_A

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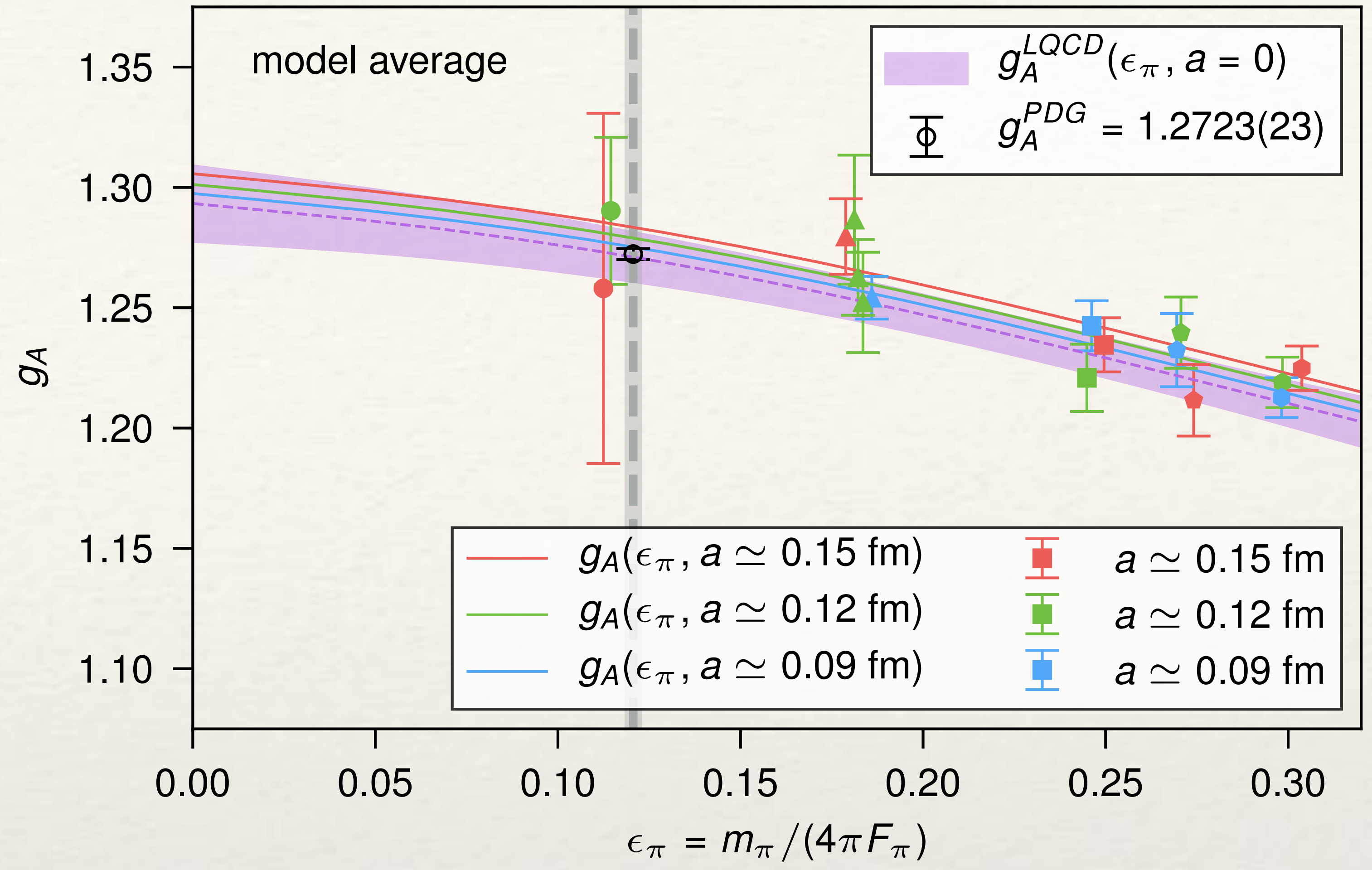
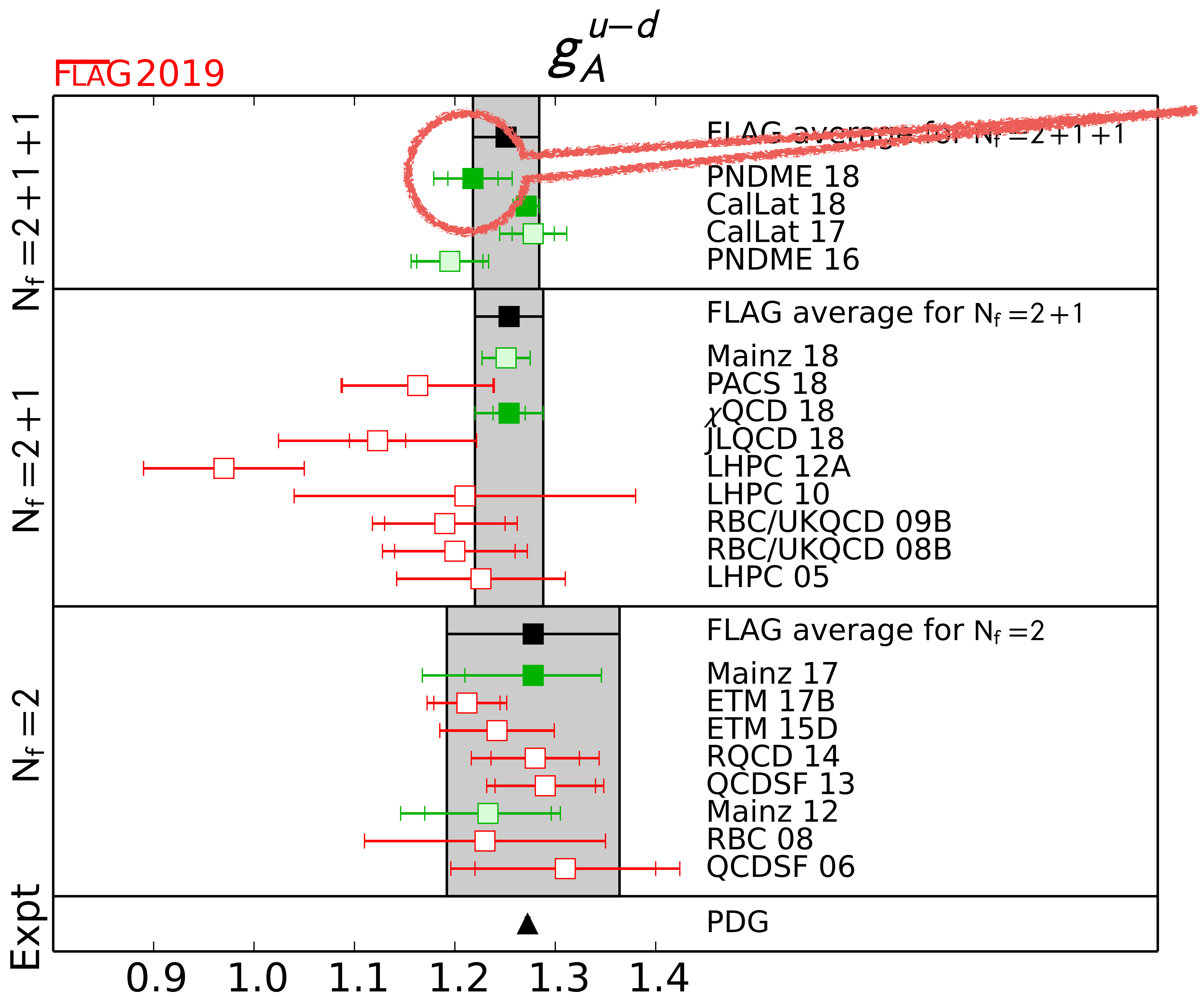


$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

Isospin corrections

- There is a radiative correction - not previously computed - the gamma-W box diagrams for an axial matrix element (instead of vector) - that leads to a 0.4% correction - **See talk of Leendert Hayen**
- To push the LQCD calculation below this precision, we should incorporate isospin breaking corrections to validate our results and provide a nice cross check of our understanding of these effects

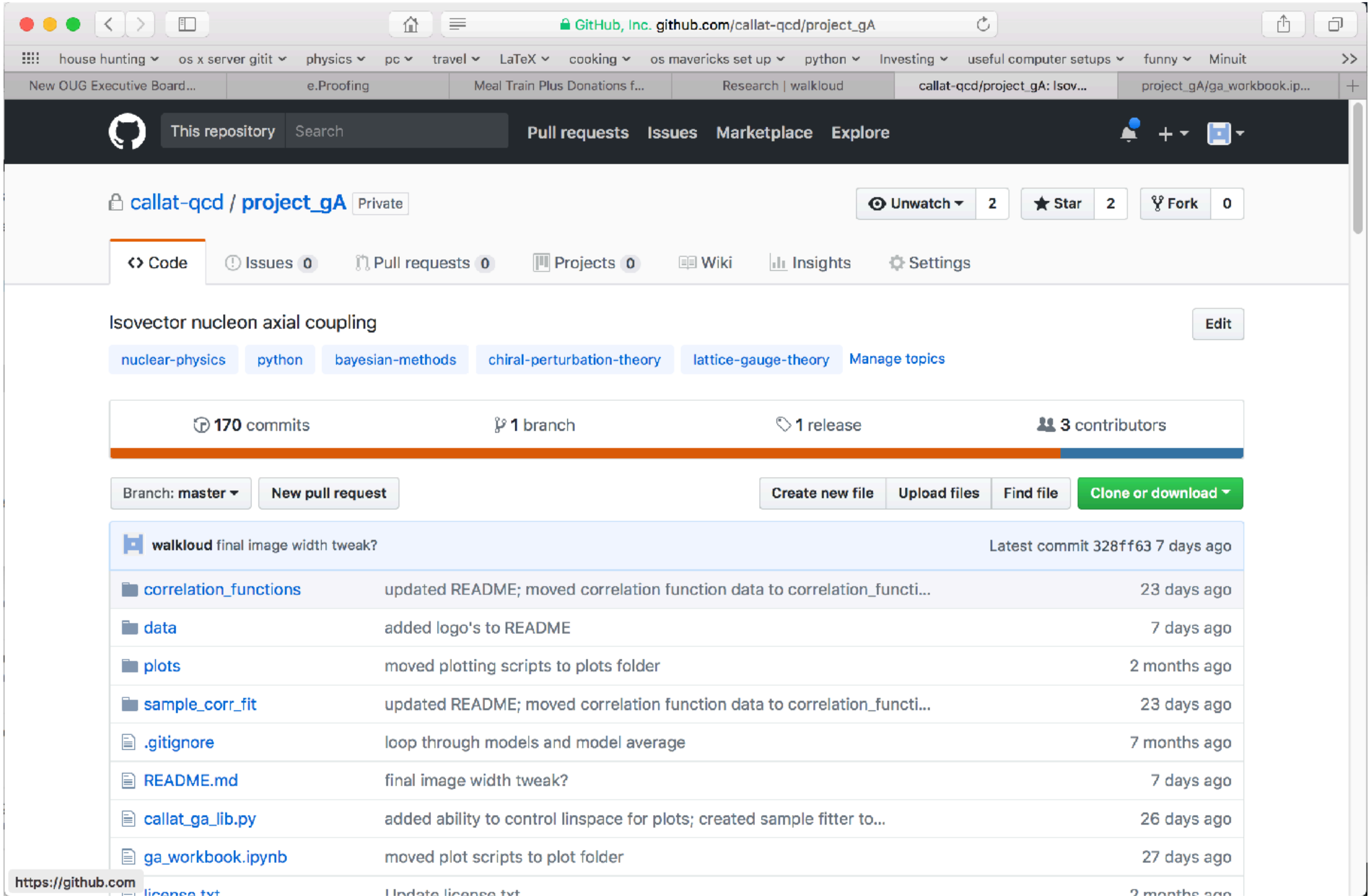
Comparison with other LQCD results



- Only one other group with three lattice spacings and physical pion mass results: Gupta et al. Phys.Rev. D98 (2018) [arXiv:1806.09006]
- *The difference in the chiral fit is a consequence of the "jump" in the CalLat data between* $M\pi = \{400, 350, 310\}$ *and the 220 MeV data*
- *The CalLat data at* $M\pi \approx 130$ *MeV do not contribute much to the fit because of the larger errors*
- *The difference in the continuum extrapolation is driven by the smaller estimates on all three fine* $a \approx 0.06$*fm ensembles*

https://github.com/callat-qcd/project_gA

raw correlation functions, correlation function analysis results, extrapolation analysis



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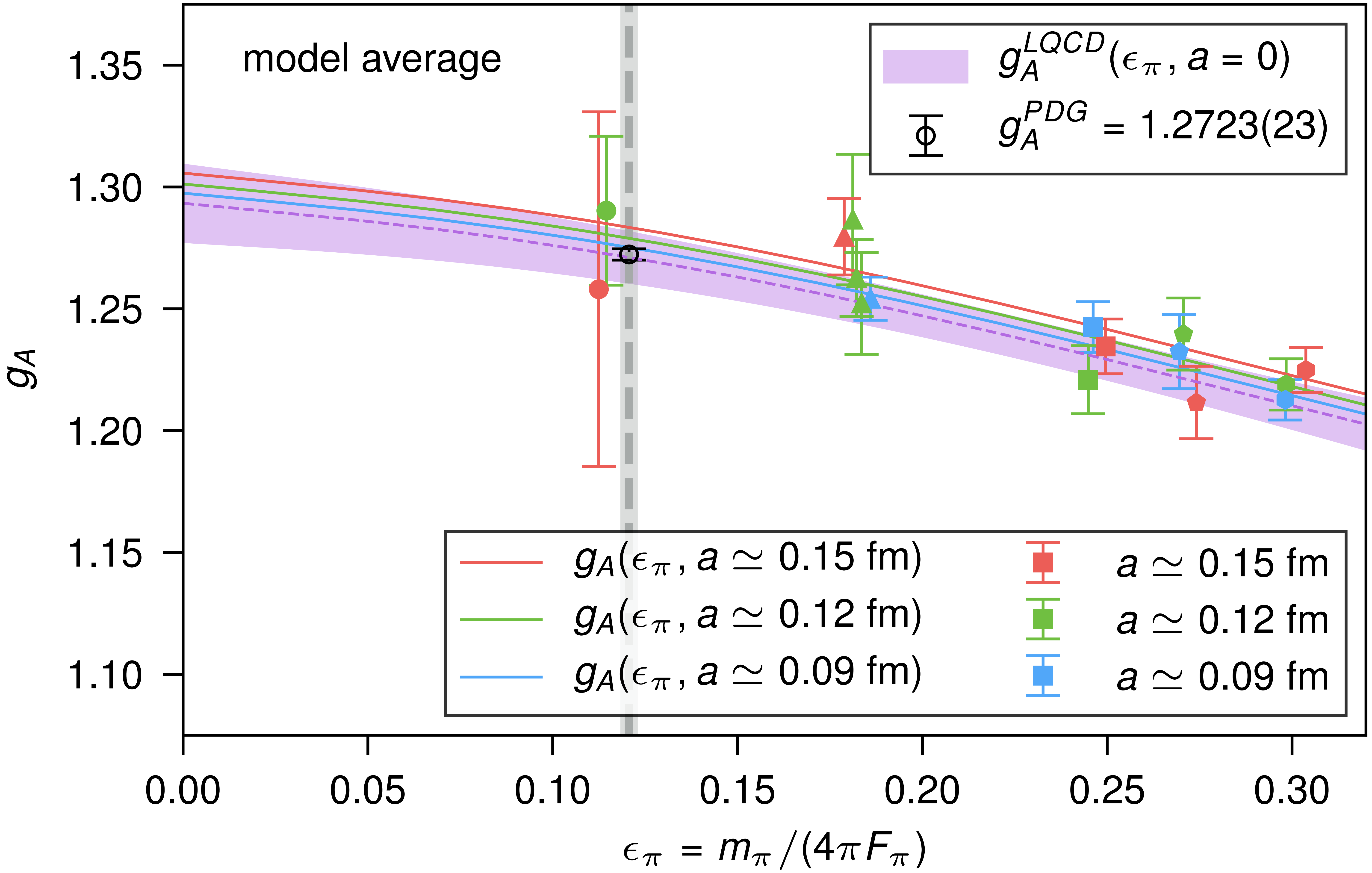
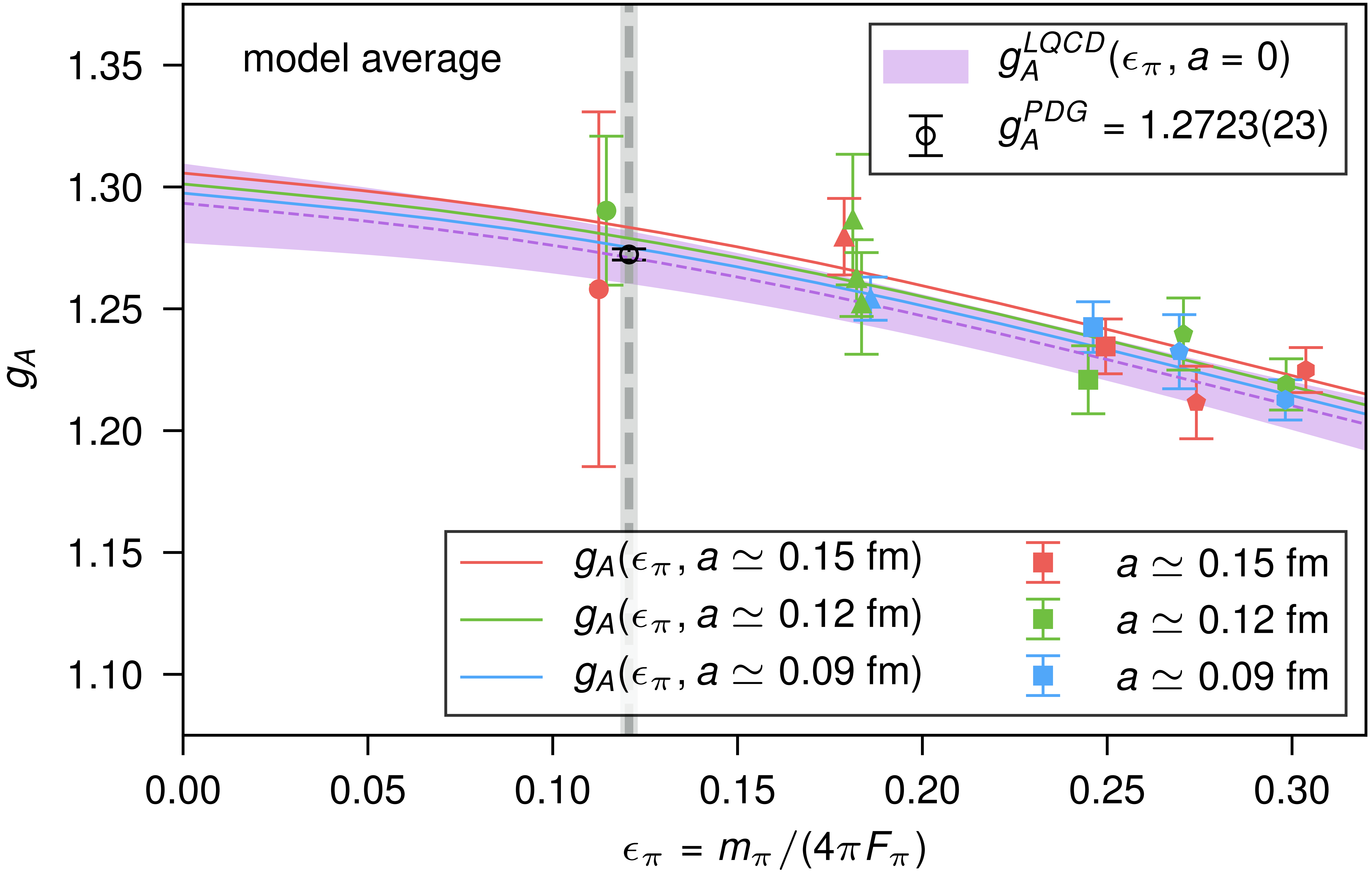


- *The difference in the chiral fit is a consequence of the "jump" in the CalLat data between $M\pi = \{400, 350, 310\}$ and the 220 MeV data*

"jump" → smooth transition

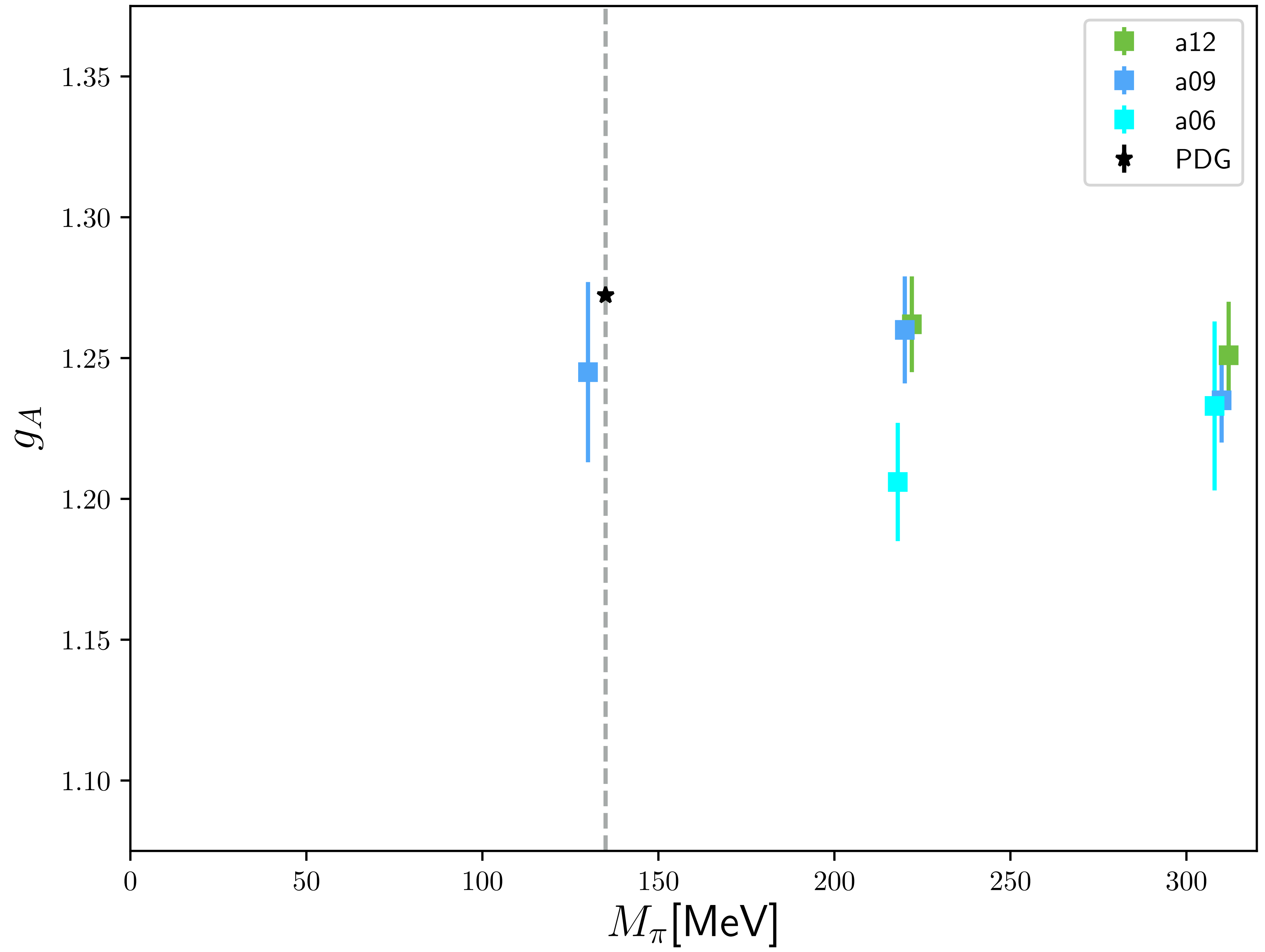
Comparison with other LQCD results



- *The CalLat data at $M\pi \approx 130$ MeV do not contribute much to the fit because of the larger errors*
  - True - but - we'll come back to this

Comparison with other LQCD results

- *The difference in the continuum extrapolation is driven by the smaller estimates on all three fine a ≈ 0.06fm ensembles*



arXiv:1606.07049

$g_A = 1.195(33)(22)$

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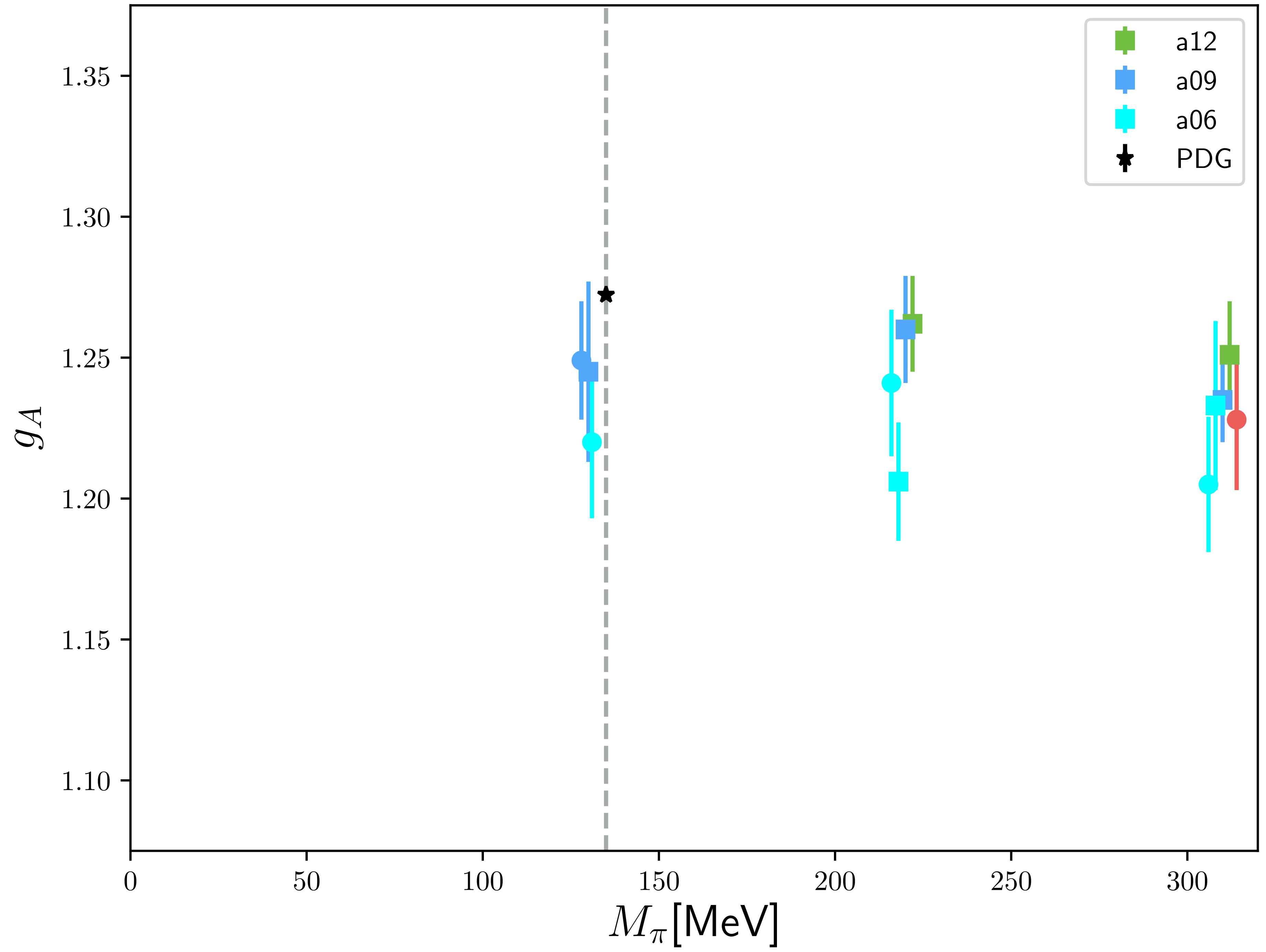

arXiv:1606.07049 $g_A = 1.195(33)(22)$

arXiv:1806.09006 $g_A = 1.218(25)(30)$

g_A[no a06] $= 1.245(42)(xx)$

- A change in quark smearing caused a ~2 sigma shift in the a06m220 point
  - The correlated difference will be ~5-sigma
  - suggests an underestimate of systematic uncertainties on this ensemble
- In conferences - it has been stated it is only the physical pion mass a06m135 point that causes the discrepancy - **note - this is the very endpoint result in an extrapolation**

Comparison with other LQCD results

- *The difference in the continuum extrapolation is driven by the smaller estimates on all three fine $a \approx 0.06$fm ensembles*
- "Surely, You're Joking, Mr. Feynman!"

I went out and found the original article on the experiment that said the neutron-proton coupling is T, and I was shocked by something. I remembered reading that article once before (back in the days when I read every article in the Physical Review—it was small enough). And I remembered, when I saw this article again, looking at that curve and thinking, "That doesn't prove anything!"

You see, it depended on one or two points at the very edge of the range of the data, and there's a principle that a point on the edge of the range of the data—the last point—isn't very good, because if it was, they'd have another point further along. And I had realized that the whole idea that neutron-proton coupling is T was based on the last point, which wasn't very good, and therefore it's not proved. I remember noticing that!

And when I became interested in beta decay, directly, I read all these reports by the "beta-decay experts," which said it's T. I never looked at the original data; I only read those reports, like a dope. Had I been a good physicist, when I thought of the original idea back at the Rochester Conference I would have immediately looked up "how strong do we know it's T?"—that would have been the sensible thing to do. I would have recognized right away that I had already noticed it wasn't satisfactorily proved.

Comparison with other LQCD results

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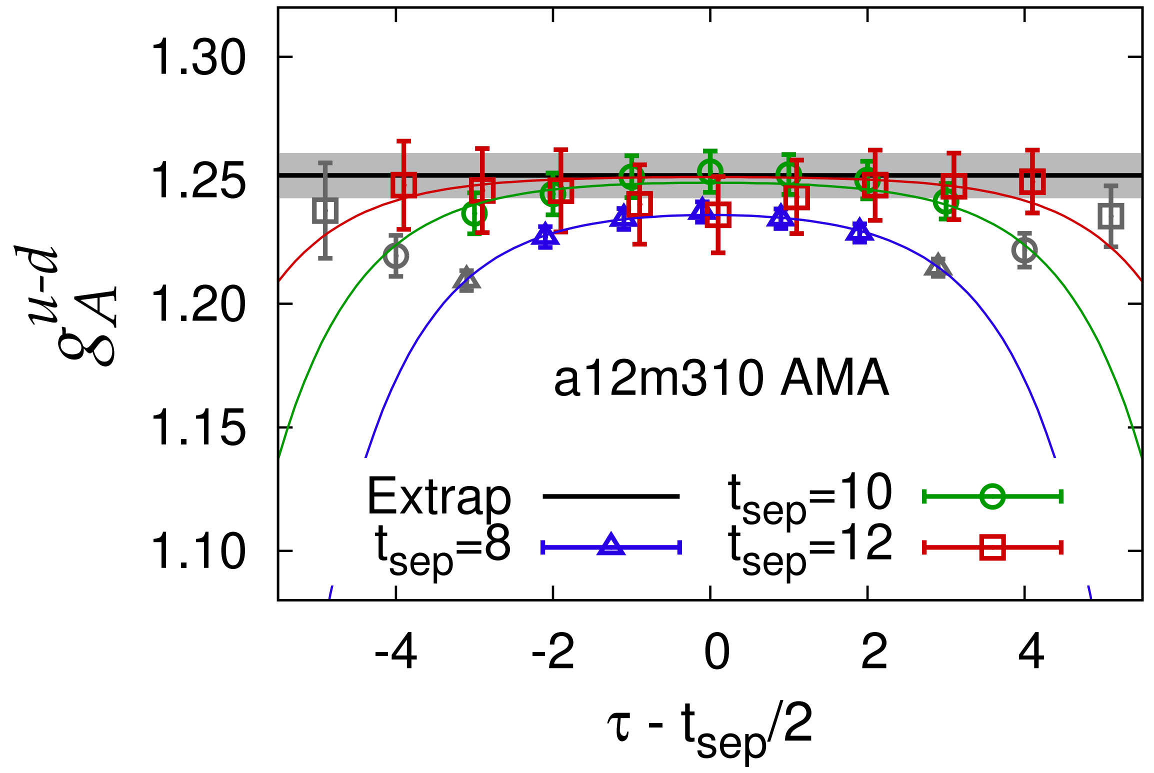
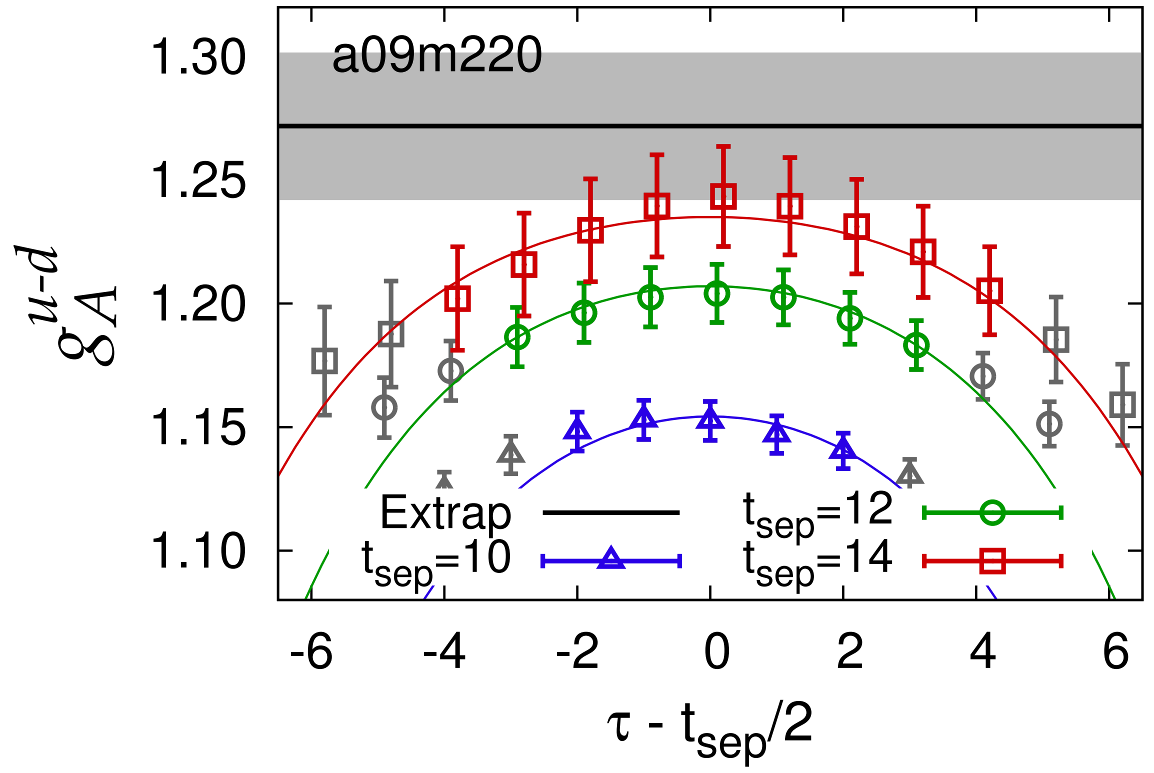
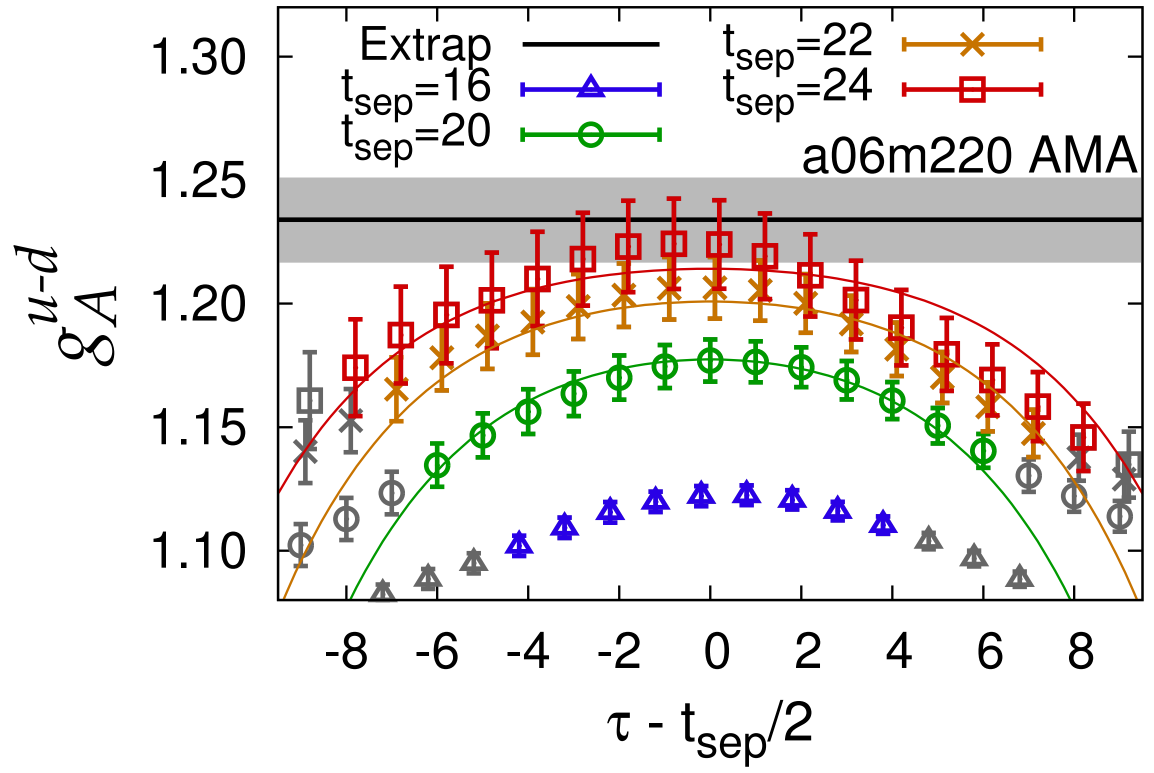


0.49 fm



0.35 fm

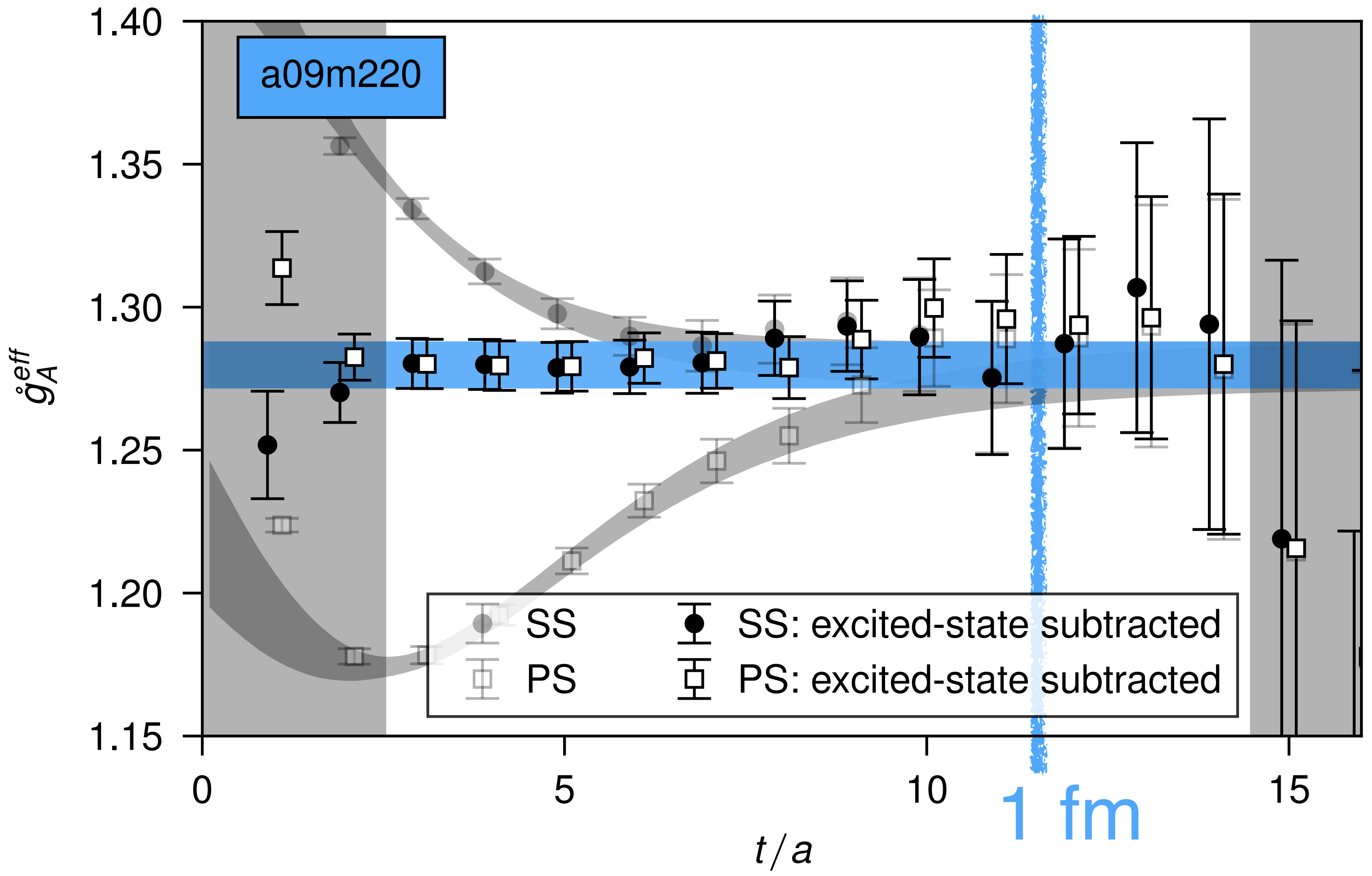


0.23 fm

lever arm to extrapolate to infinite $\mathbf{t_{sep}}$

$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \dots$$

from curvature at fixed $\mathbf{t_{sep}}$



Also, the a06 results, from time-slice to time-slice, are more correlated than the other ensembles, and therefore more susceptible to correlated fluctuations that will fool the eye (analysis)

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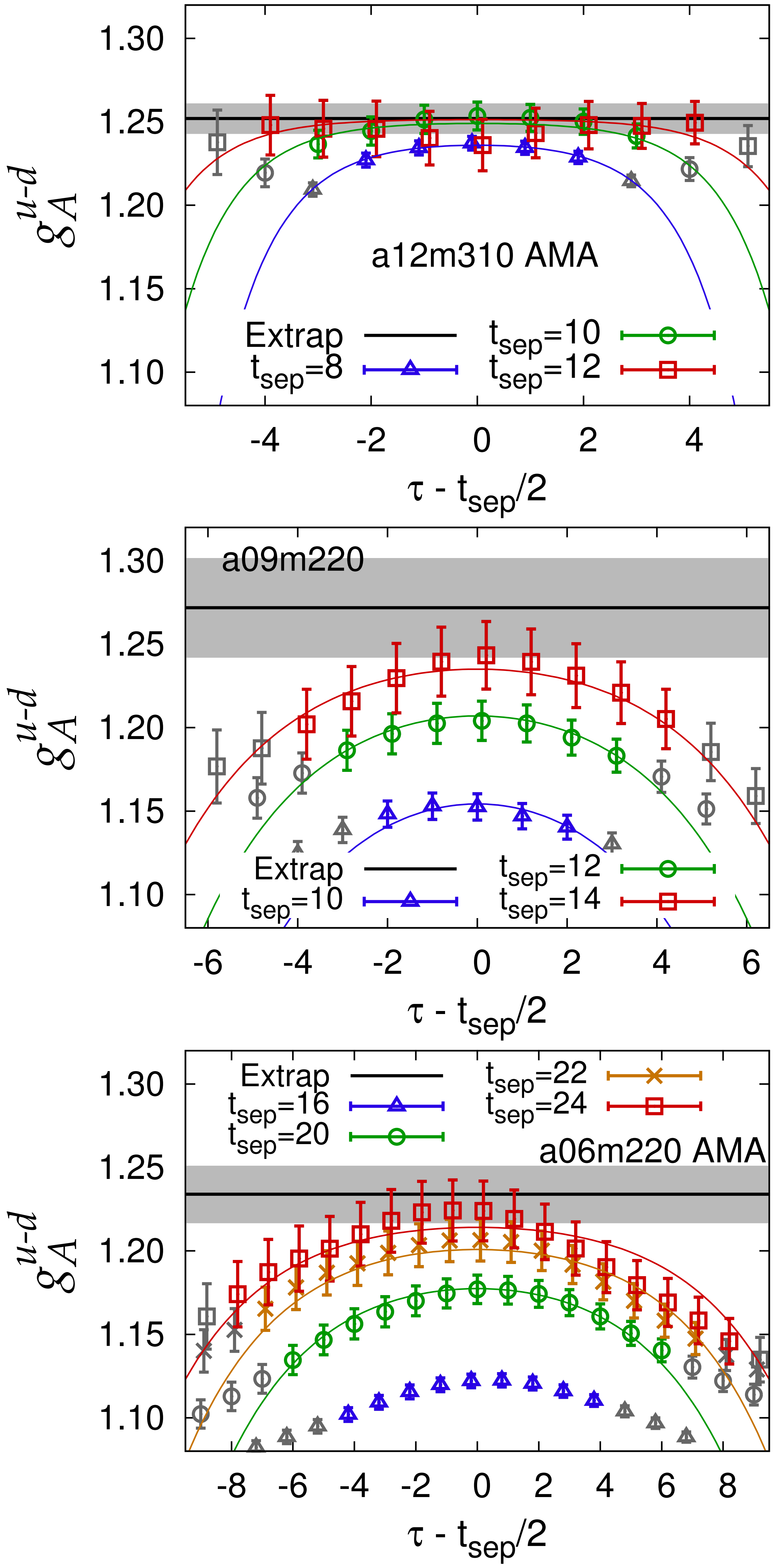

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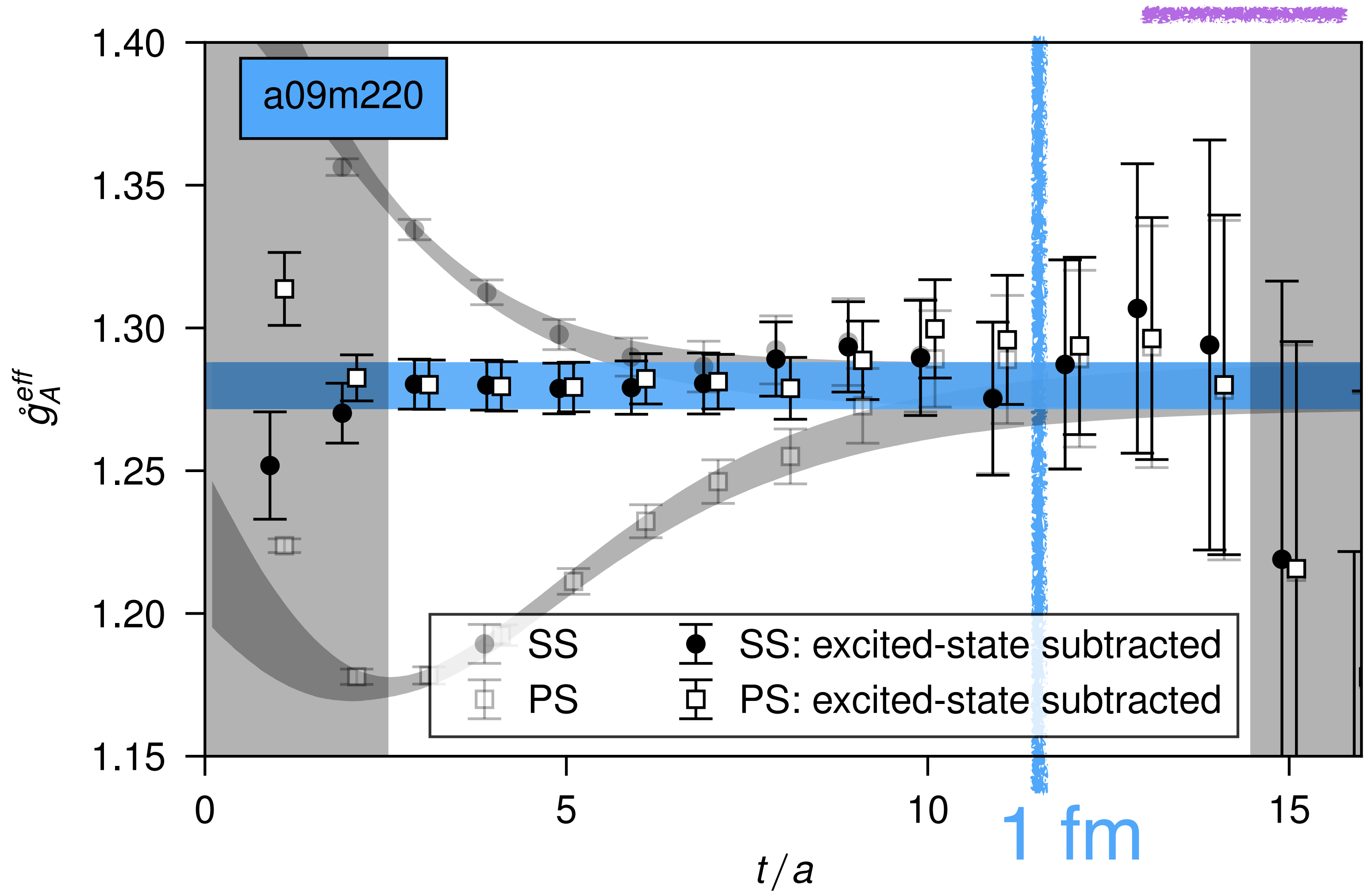

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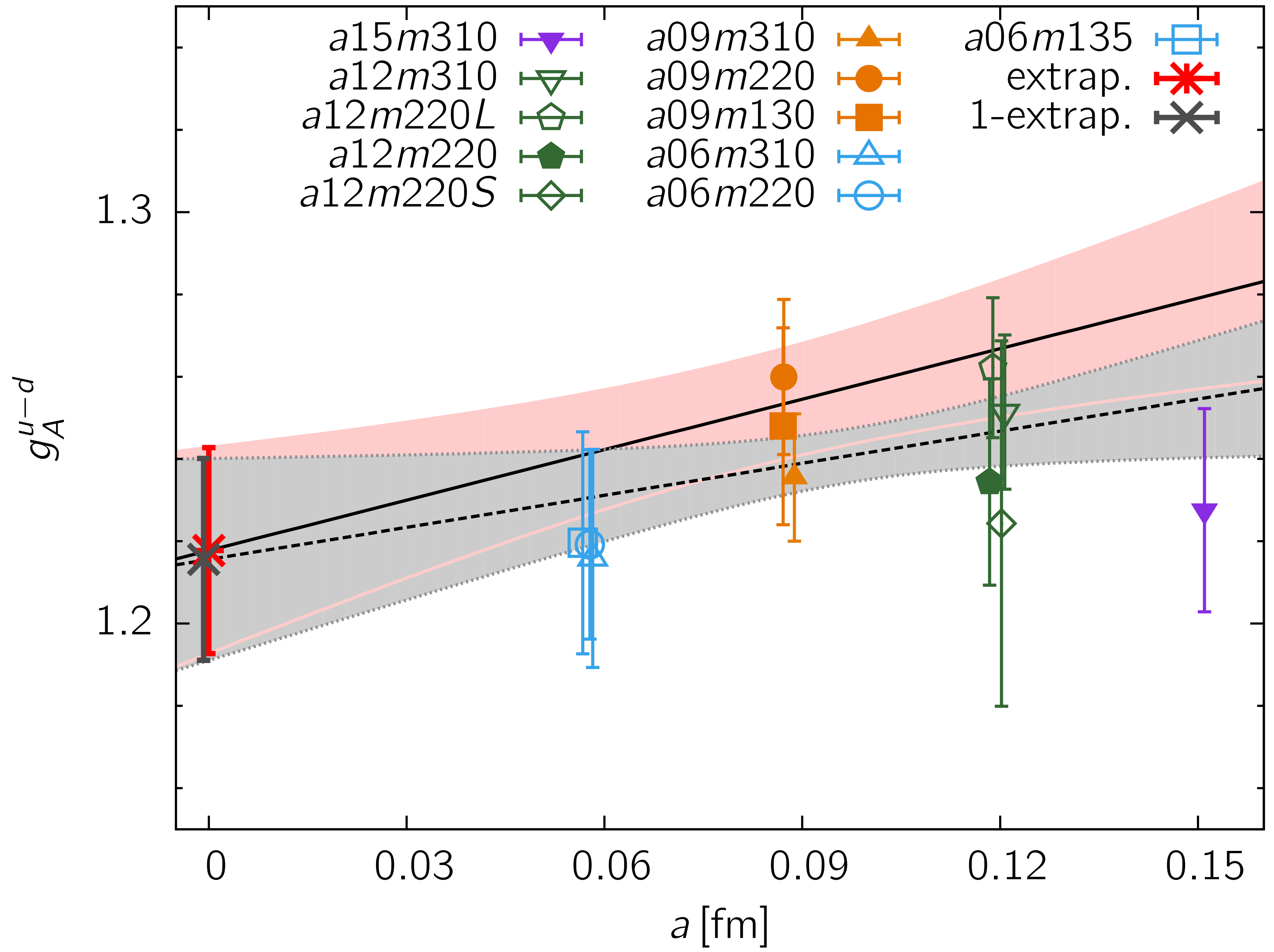
- Suppose the results in arXiv:1806.09006 are correct (not biased by a systematic uncertainty) What are the implications?



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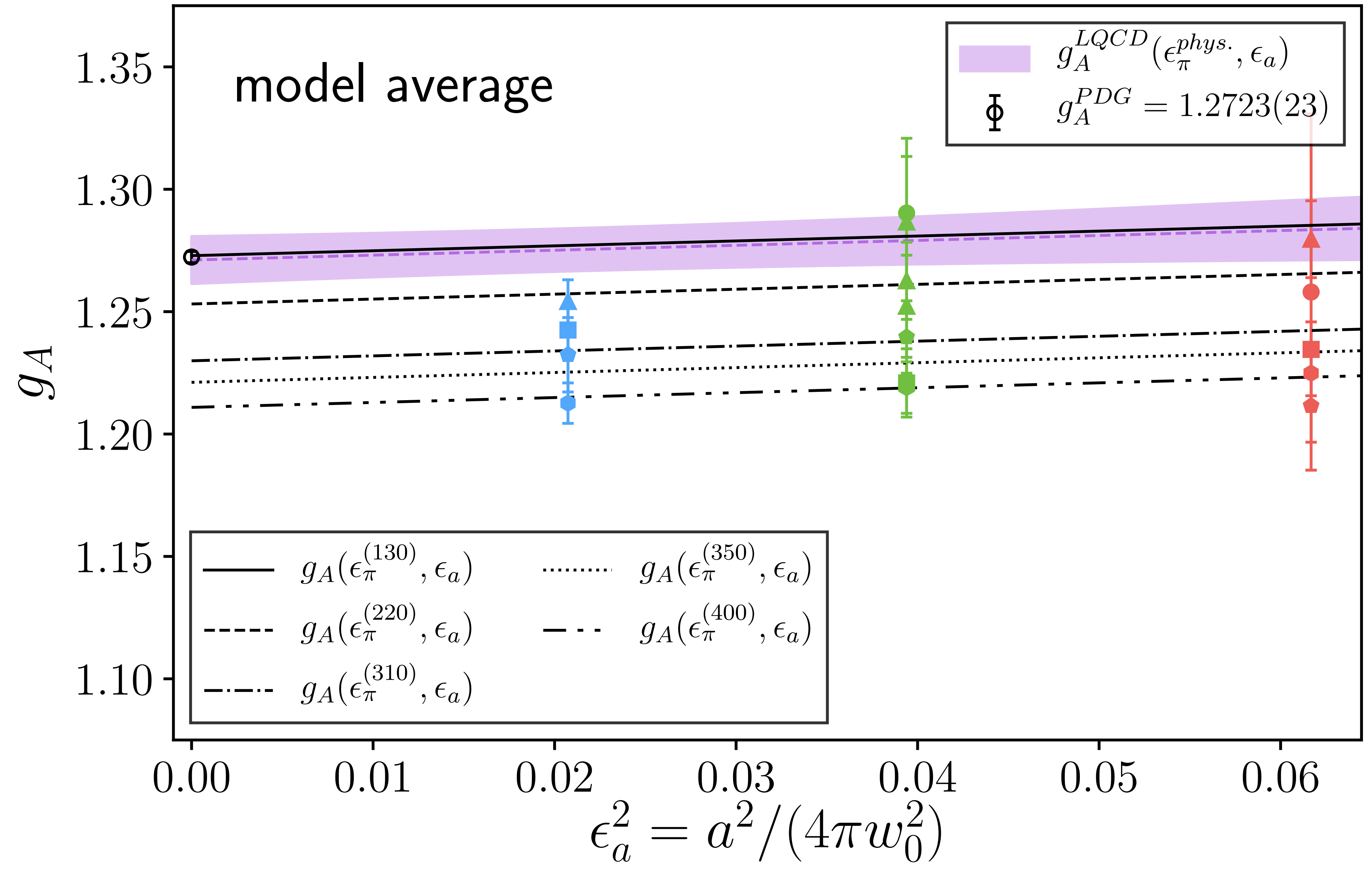

arXiv:1805.12130

- **Either**
  - There is significant new physics in g_A
  - The continuum extrapolation would follow a dramatic curve

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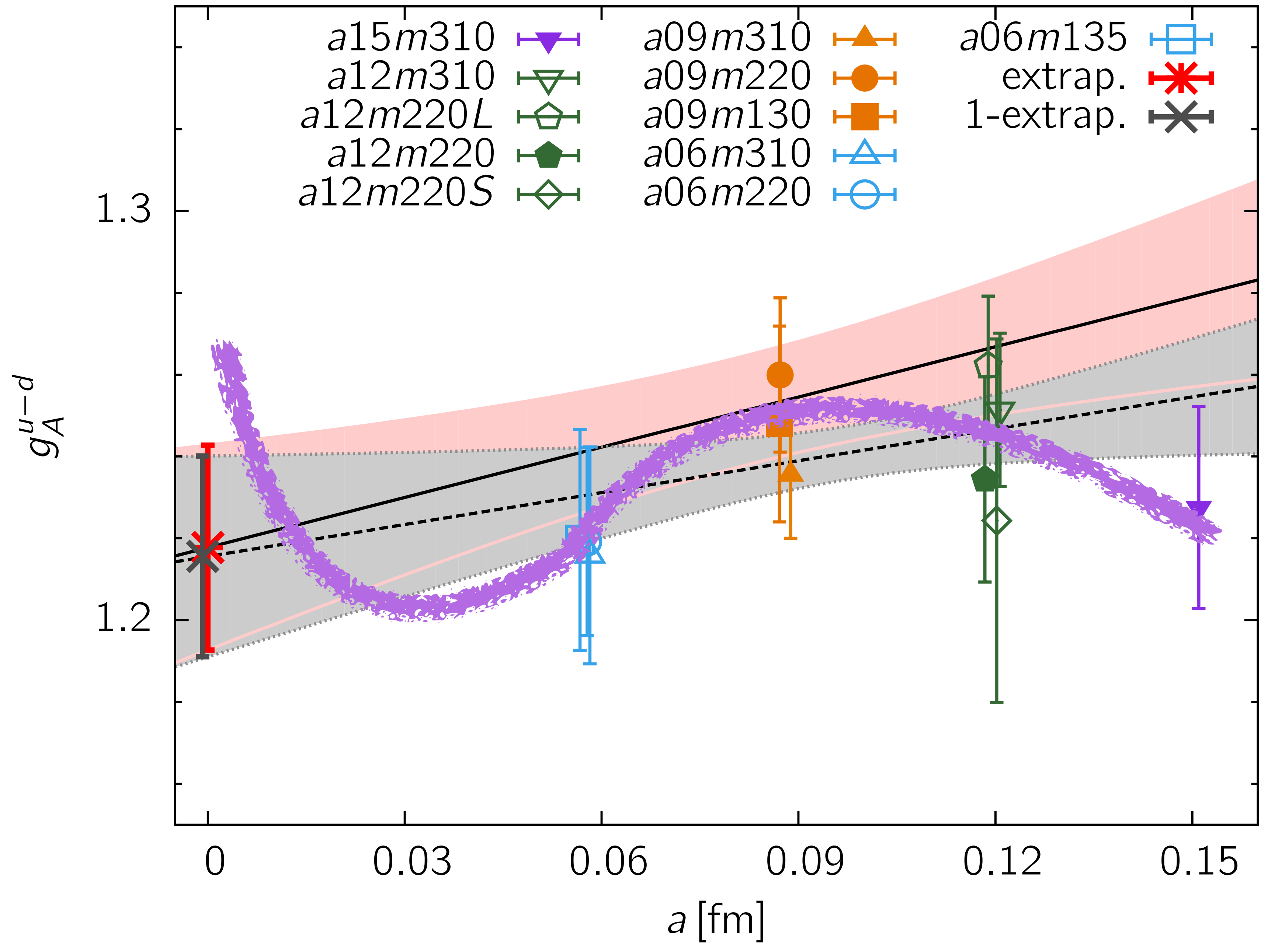

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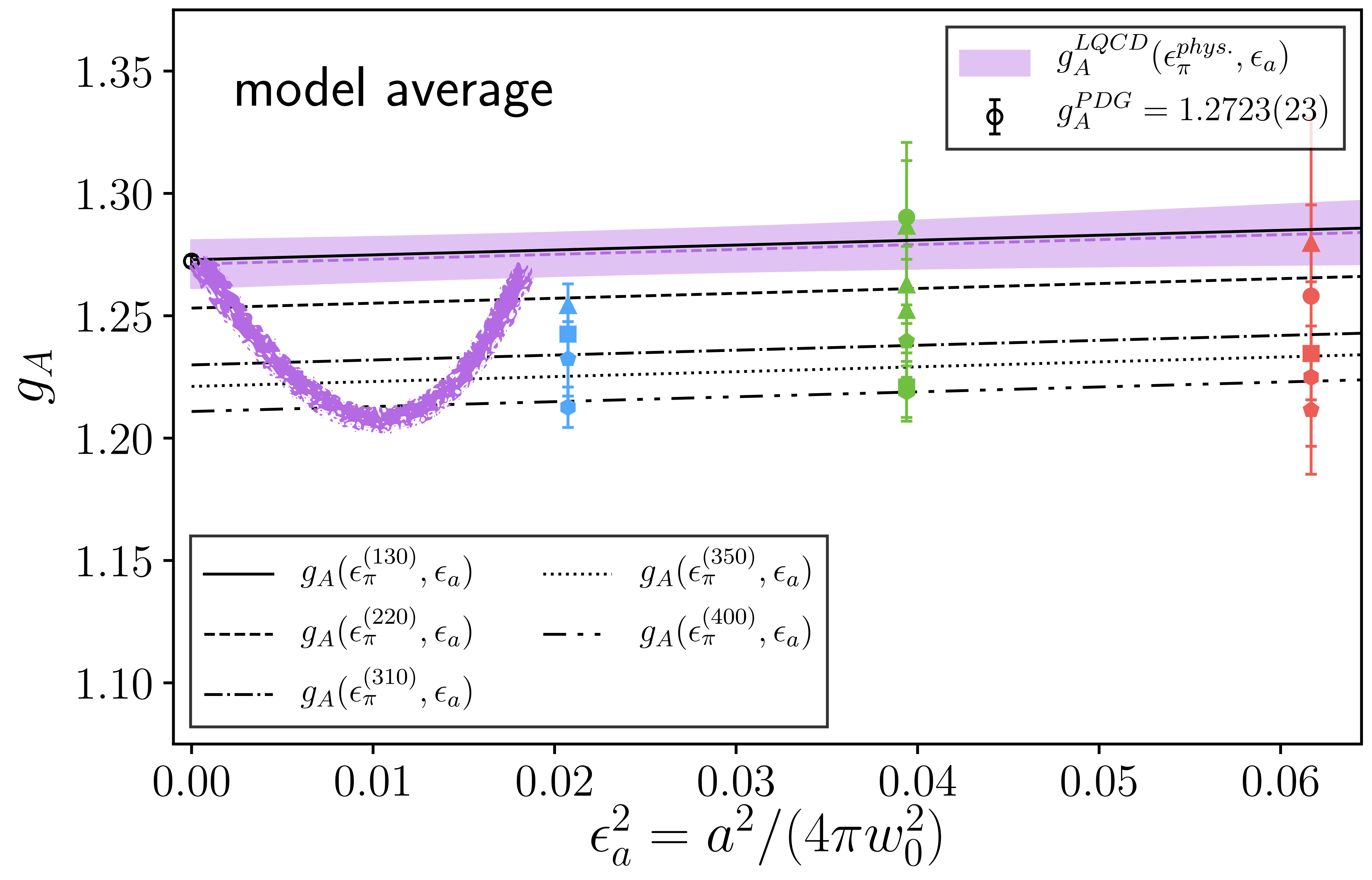

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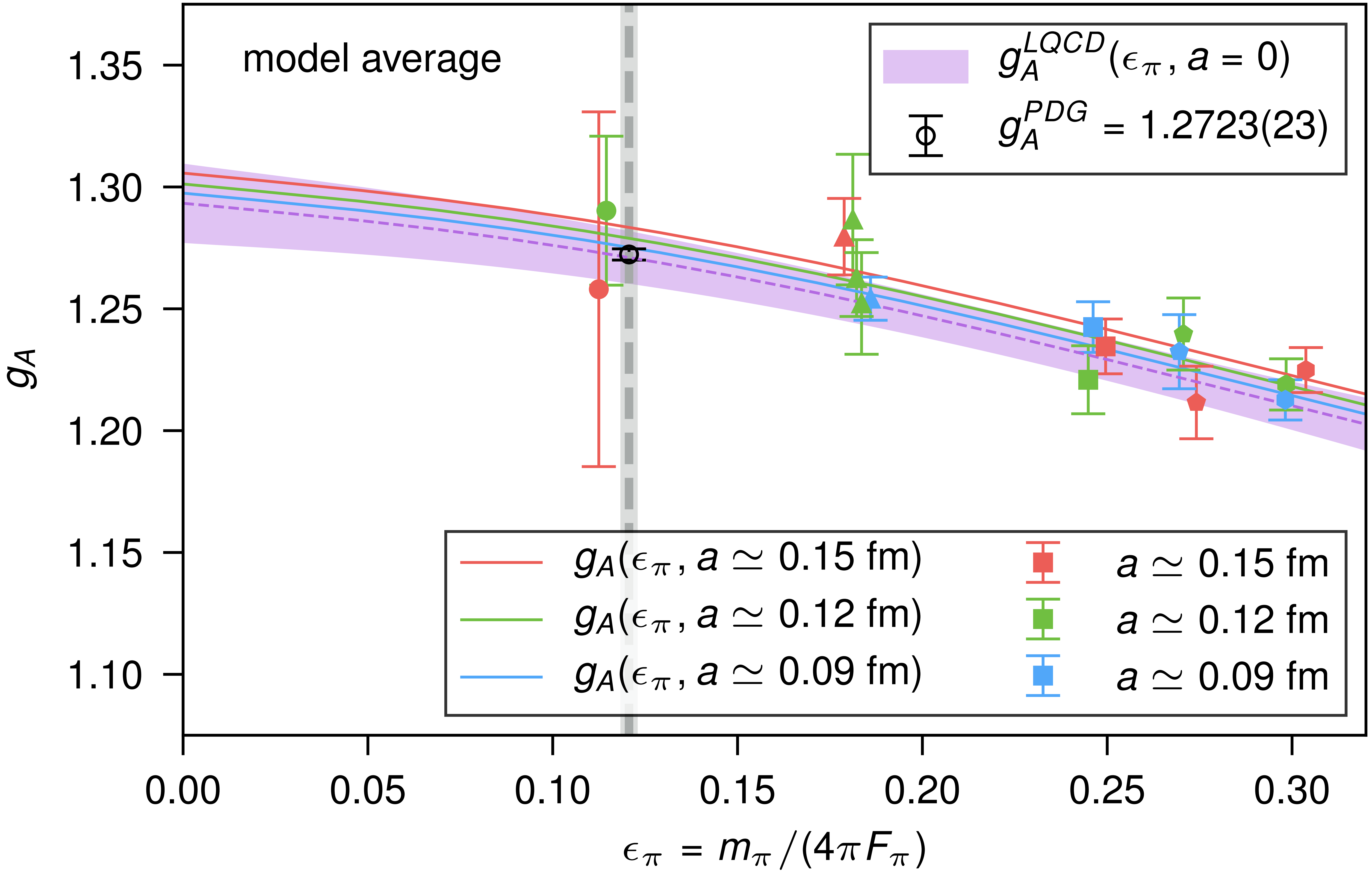

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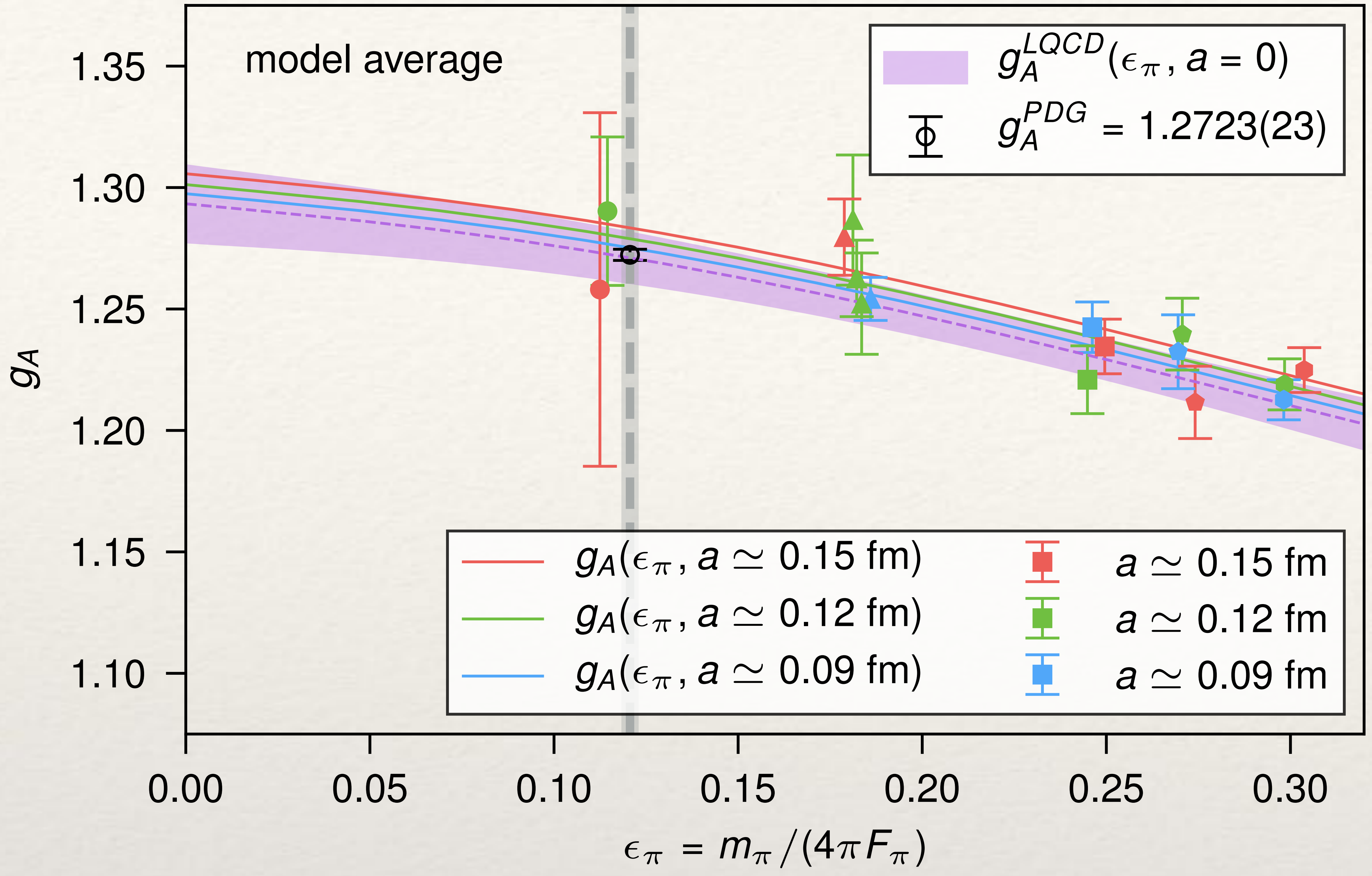
Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

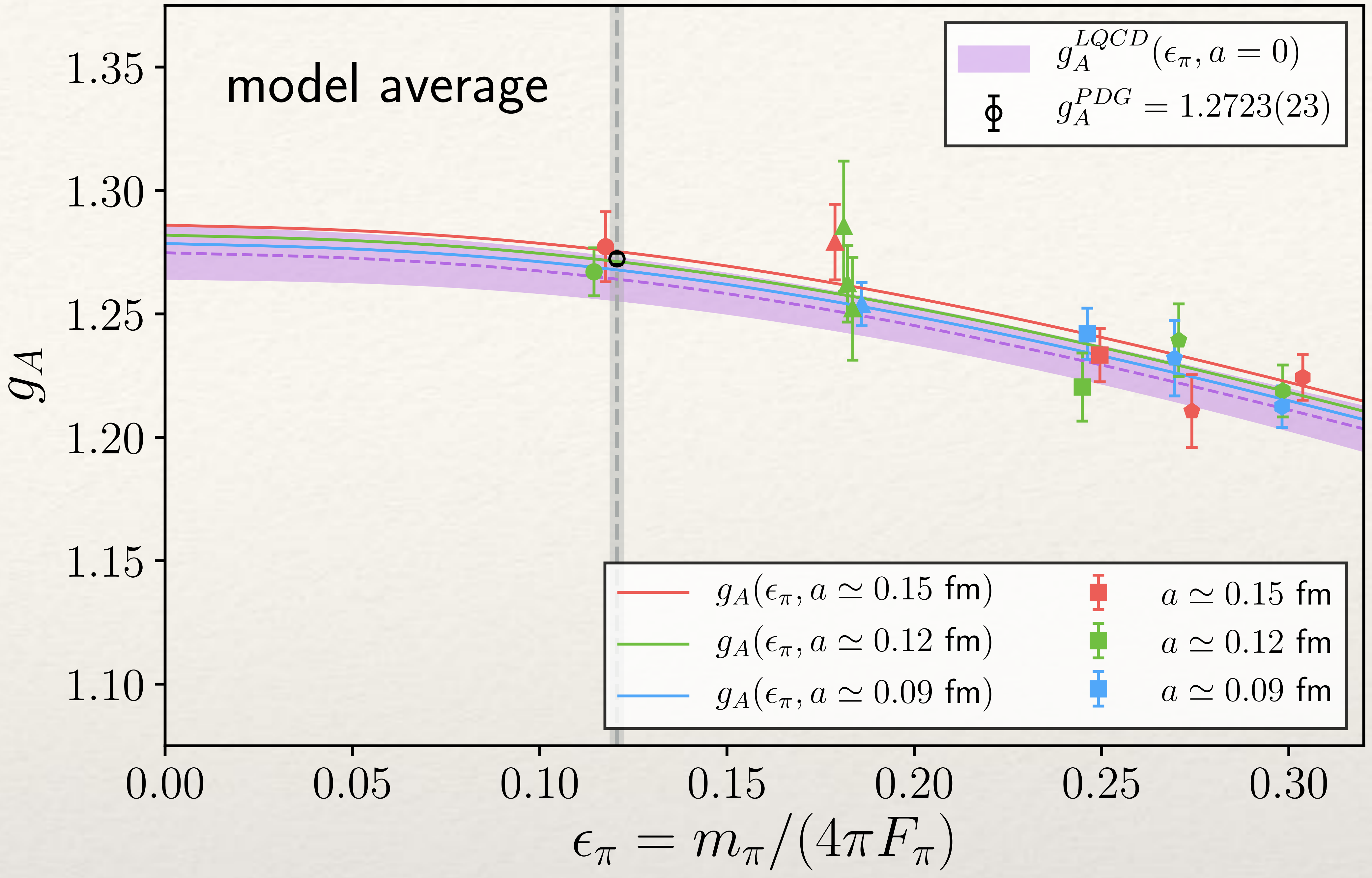
1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



California Lattice Collaboration

Sierra Early Science

PRELIMINARY

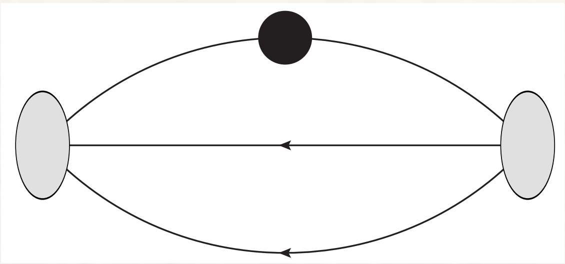


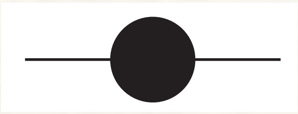
- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$
- We are running $g_A(Q^2)$ on Summit this year (DOE INCITE)
  - We anticipate improving g_A to ~0.5%

$$g_A = 1.2711(125) \to 1.2641(93) \quad [0.74\%]$$

Nucleon Axial FormFactor

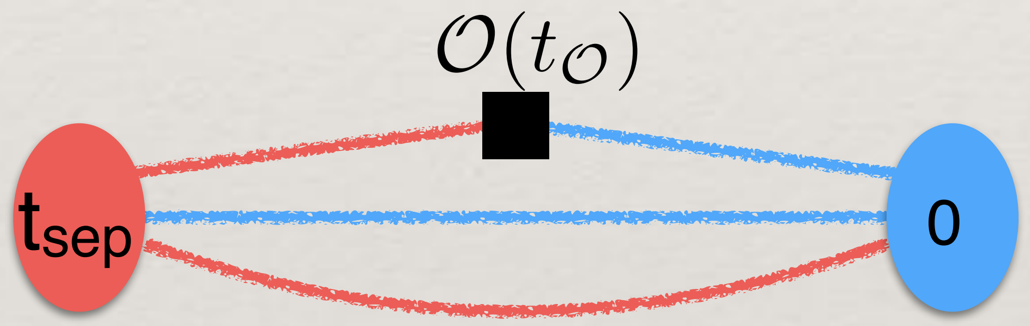
- Inherent to our gA calculation was the "Feynman-Hellmann" Propagator





$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

- For each choice of current and momentum, a new FH propagator is required
- We have tried several variants of stochastic methods to relax this constraint, but the noise is too large
- We have resorted to the standard fixed source-sink separation method (with our tail between our legs a little)



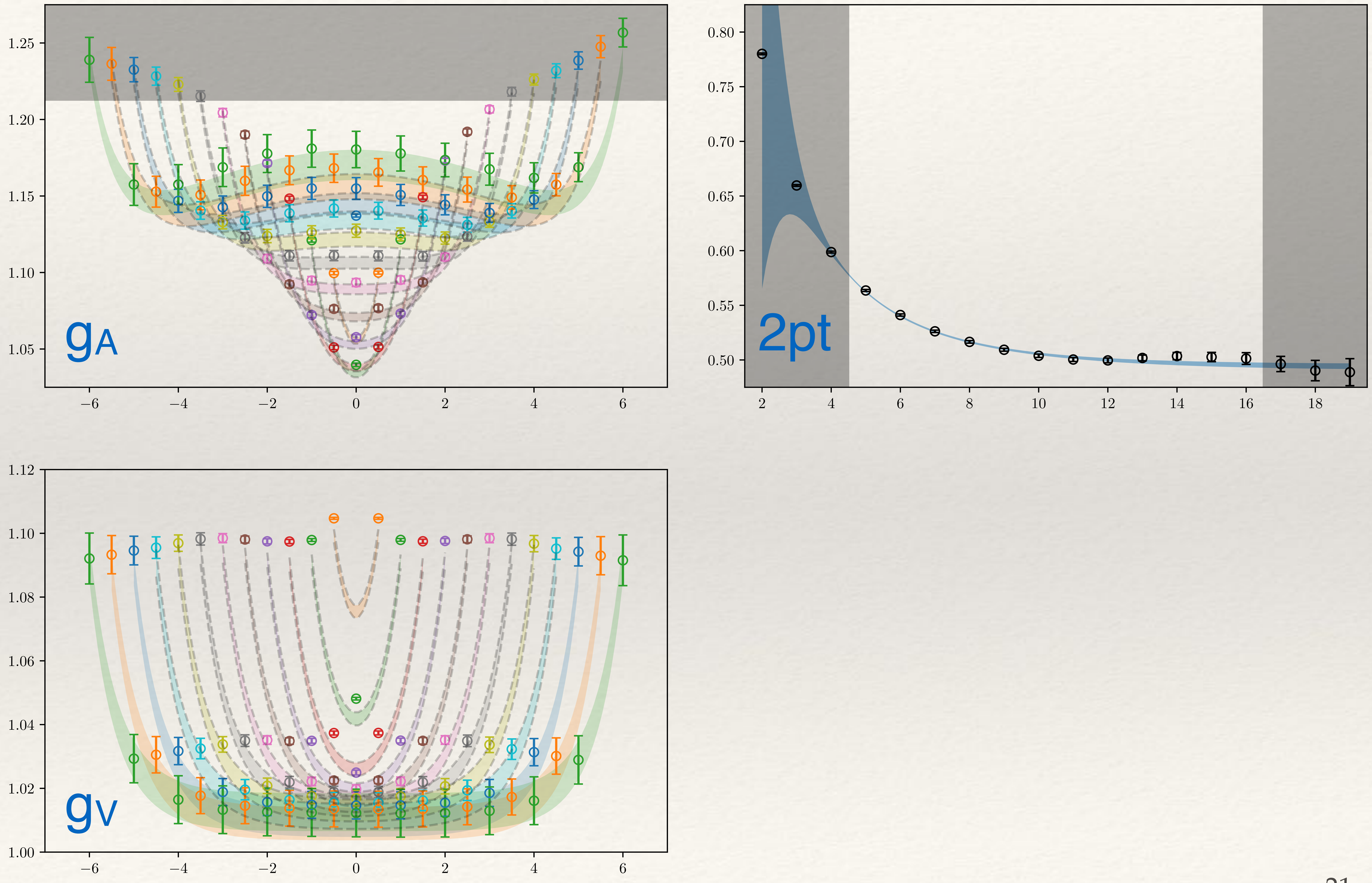
repeat for multiple values of $\mathbf{t_{sep}}$

- However, if there was a lesson to be learned from our g_A calculation when applying the fixed source-sink separation method - it is imperative to use many values of $\mathbf{t_{sep}}$ and also small values
  - See also S. Meinel, Chiral Dynamics 2012 and Hasan et al. (LHPC) 1903.06487

Nucleon Axial FormFactor PRELIMINARY

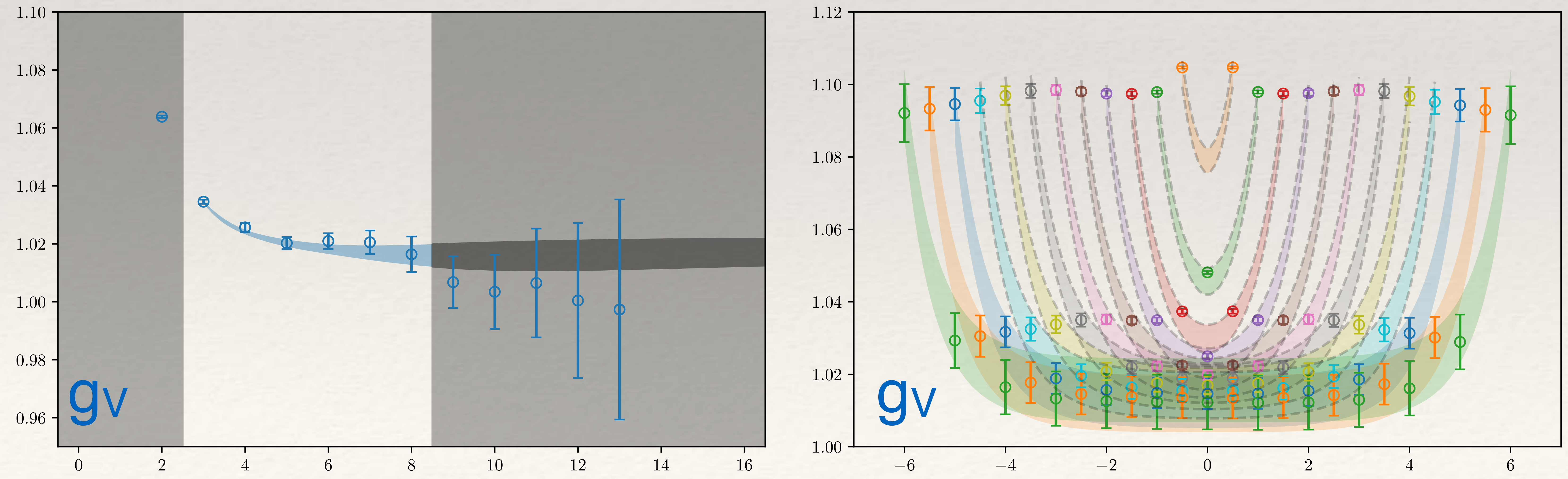
a09m310

t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]



Nucleon Axial FormFactor PRELIMINARY

a09m310 - 8 sources - 1 coherent sink t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]



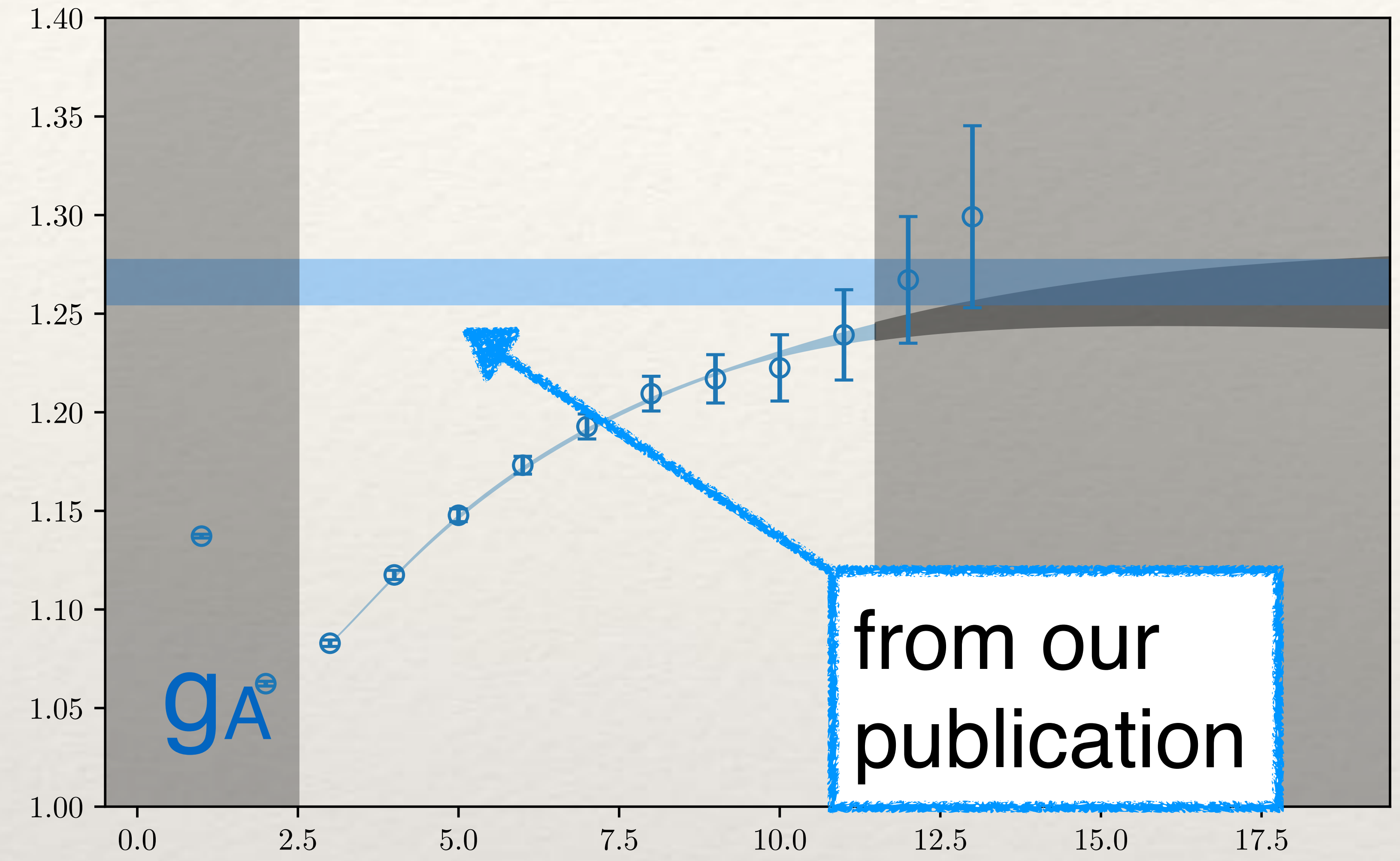
Add summed and then subtracted 3pt data to analysis

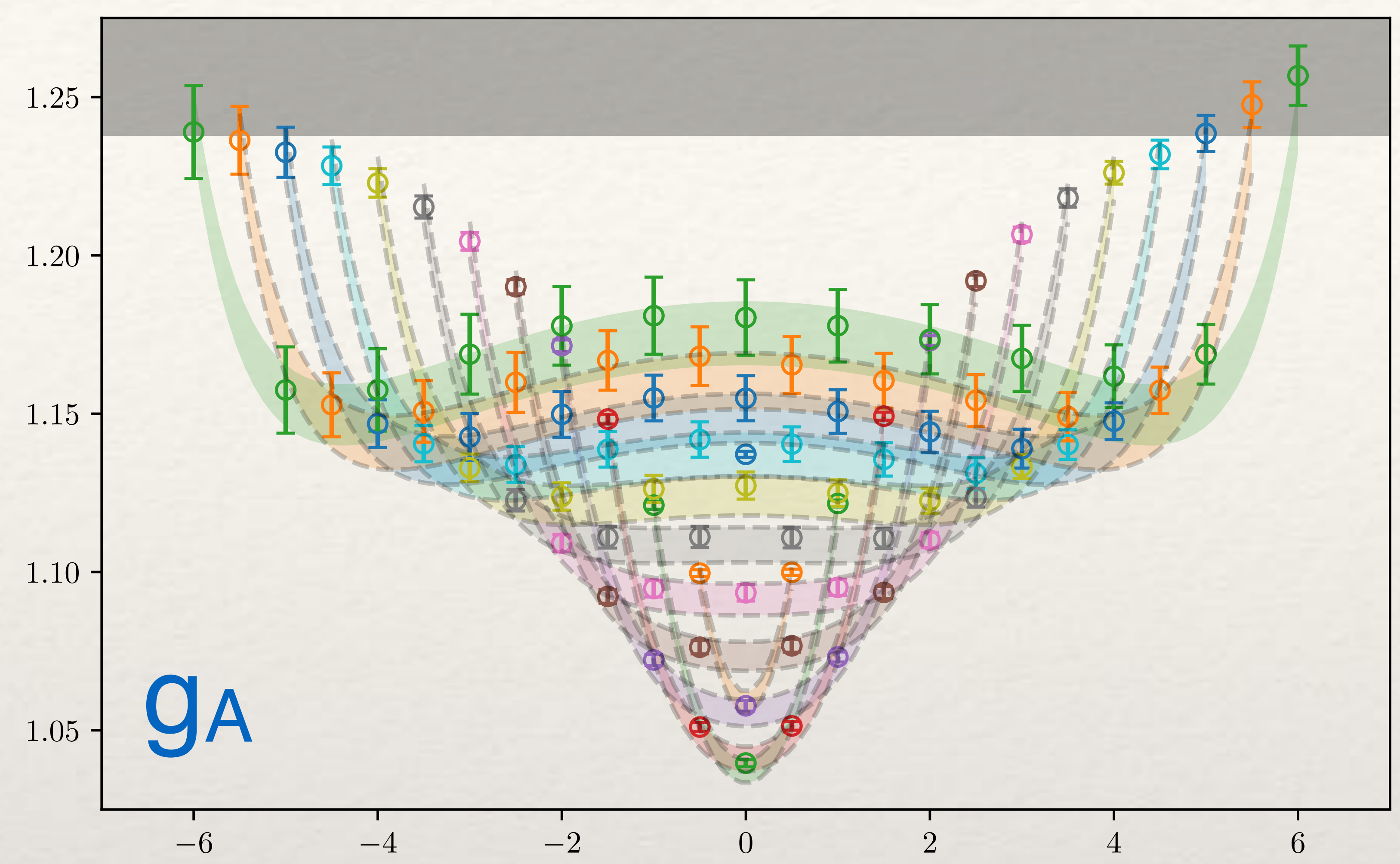
$$C_\Gamma^{sum}(t_{sep}) = \sum_{\tau=t_0+1}^{t_{sep}-1} C_3(t_{sep}, \tau_\Gamma)$$

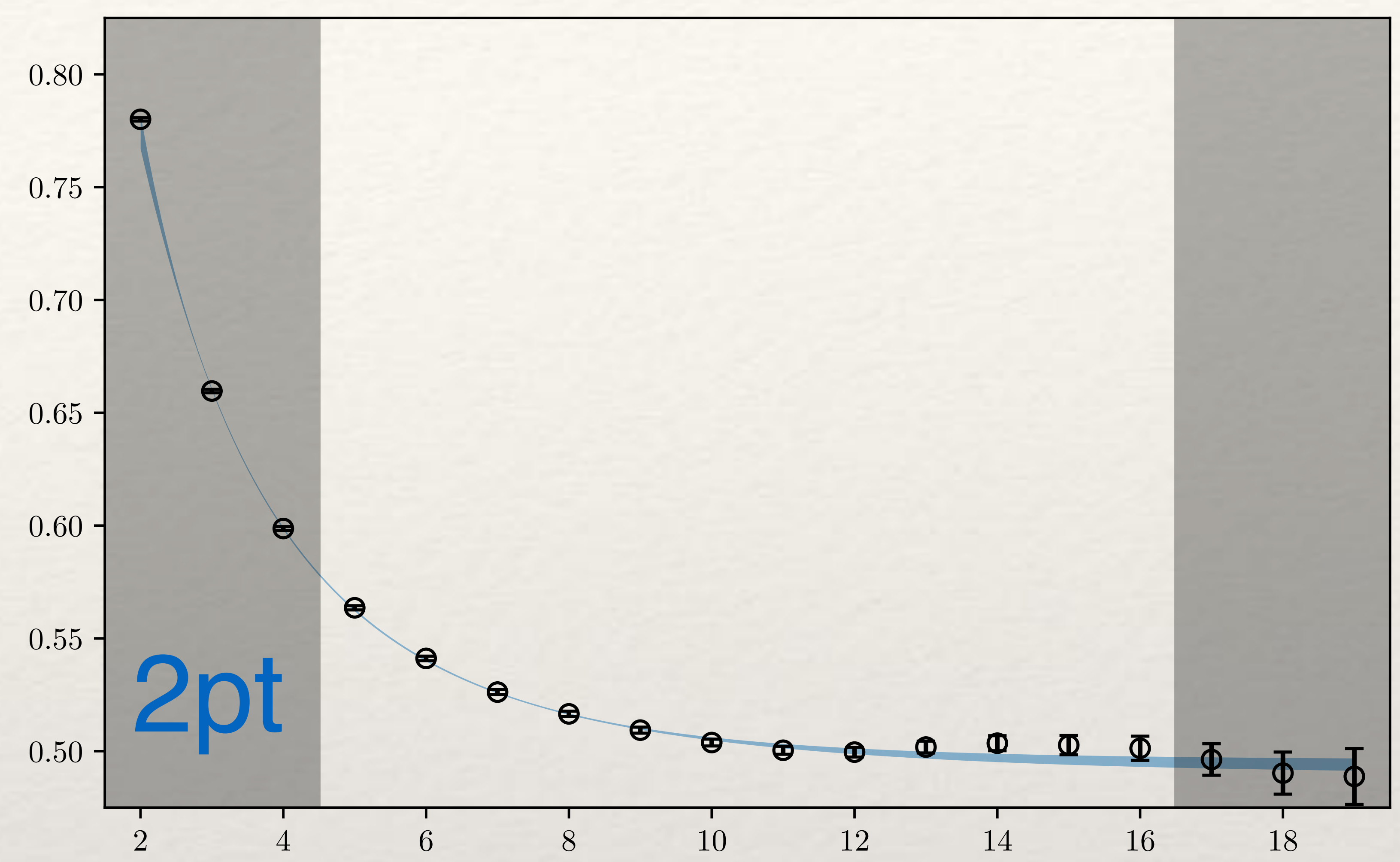
$$C_\Gamma^{\text{“}FH\text{”}}(t_{sep}) = \frac{C_\Gamma^{sum}(t_{sep}+1)}{C_2(t_{sep}+1)} - \frac{C_\Gamma^{sum}(t_{sep})}{C_2(t_{sep})}$$

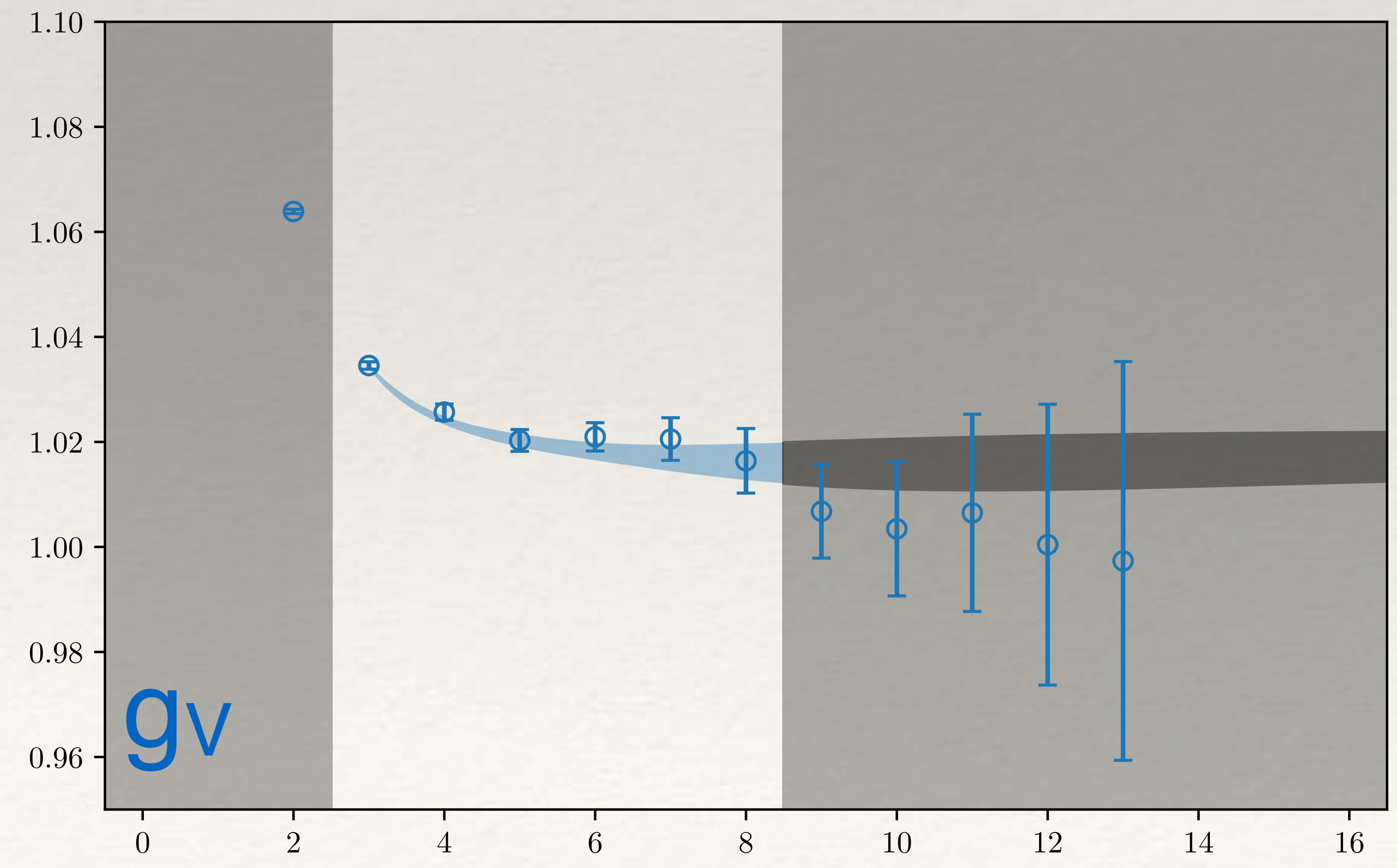
Nucleon Axial FormFactor PRELIMINARY

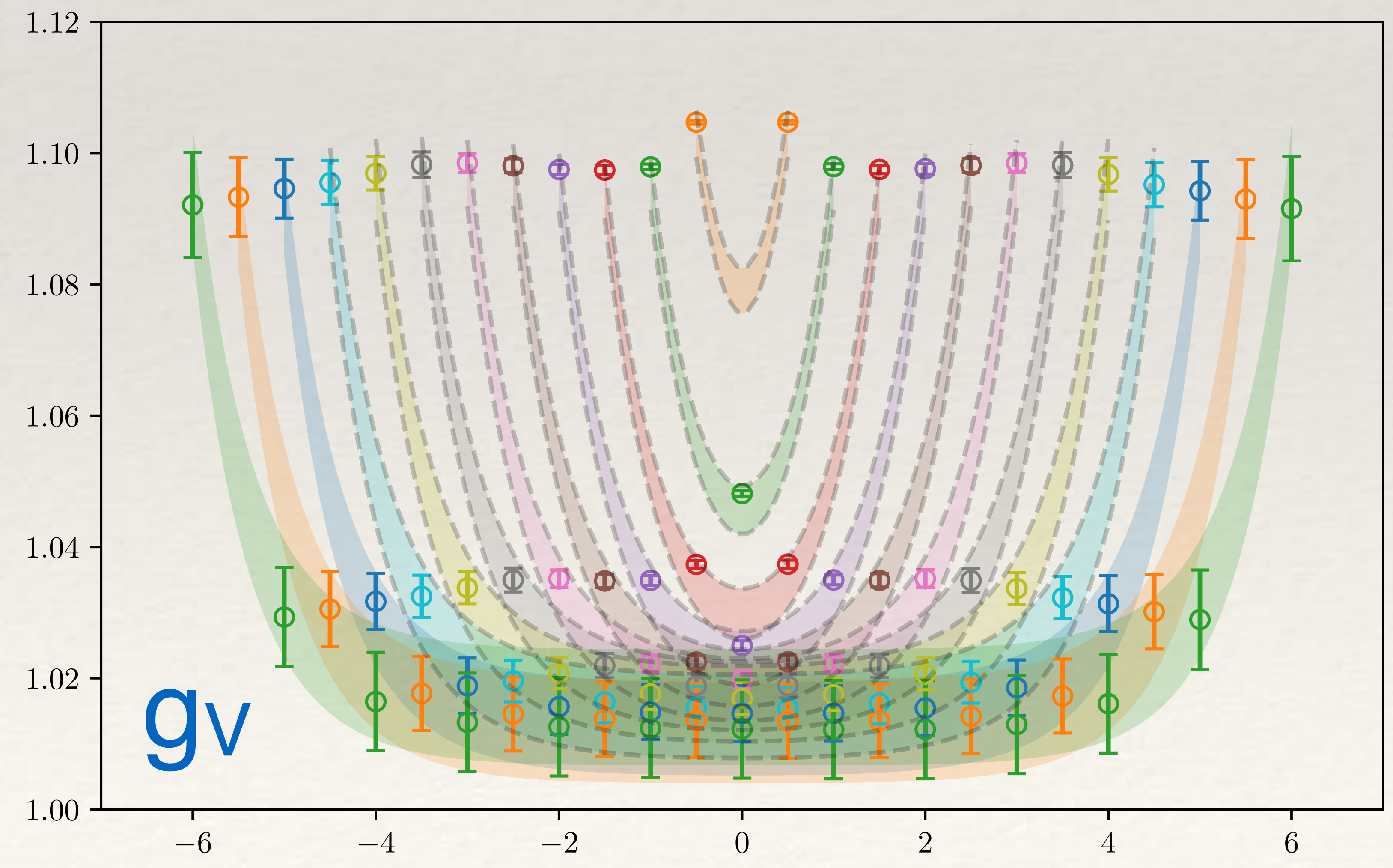
a09m310 - 8 sources - 1 coherent sink t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]











Add summed and then subtracted 3pt data to analysis

$$C_\Gamma^{sum}(t_{sep}) = \sum_{\tau=t_0+1}^{t_{sep}-1} C_3(t_{sep}, \tau_\Gamma)$$

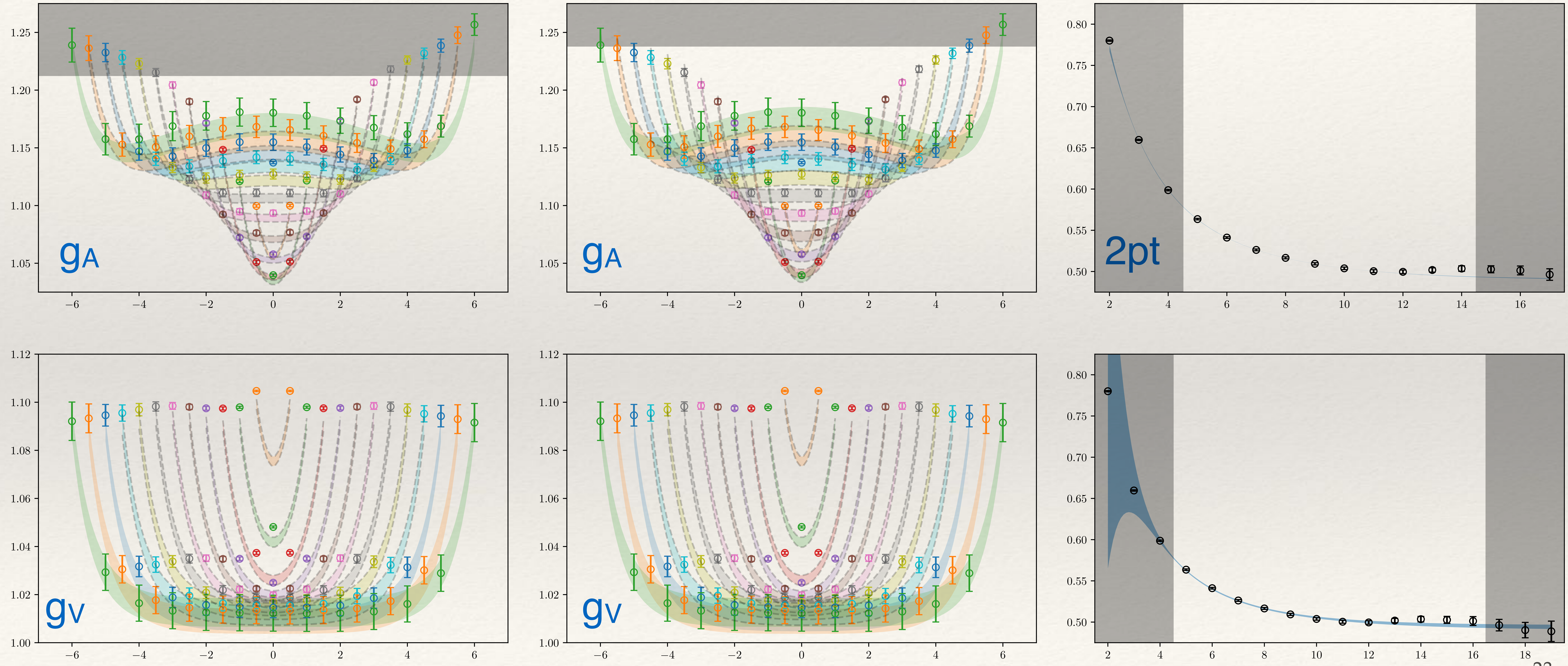
$$C_\Gamma^{\text{“}FH\text{”}}(t_{sep}) = \frac{C_\Gamma^{sum}(t_{sep}+1)}{C_2(t_{sep}+1)} - \frac{C_\Gamma^{sum}(t_{sep})}{C_2(t_{sep})}$$

Nucleon Axial FormFactor

PRELIMINARY

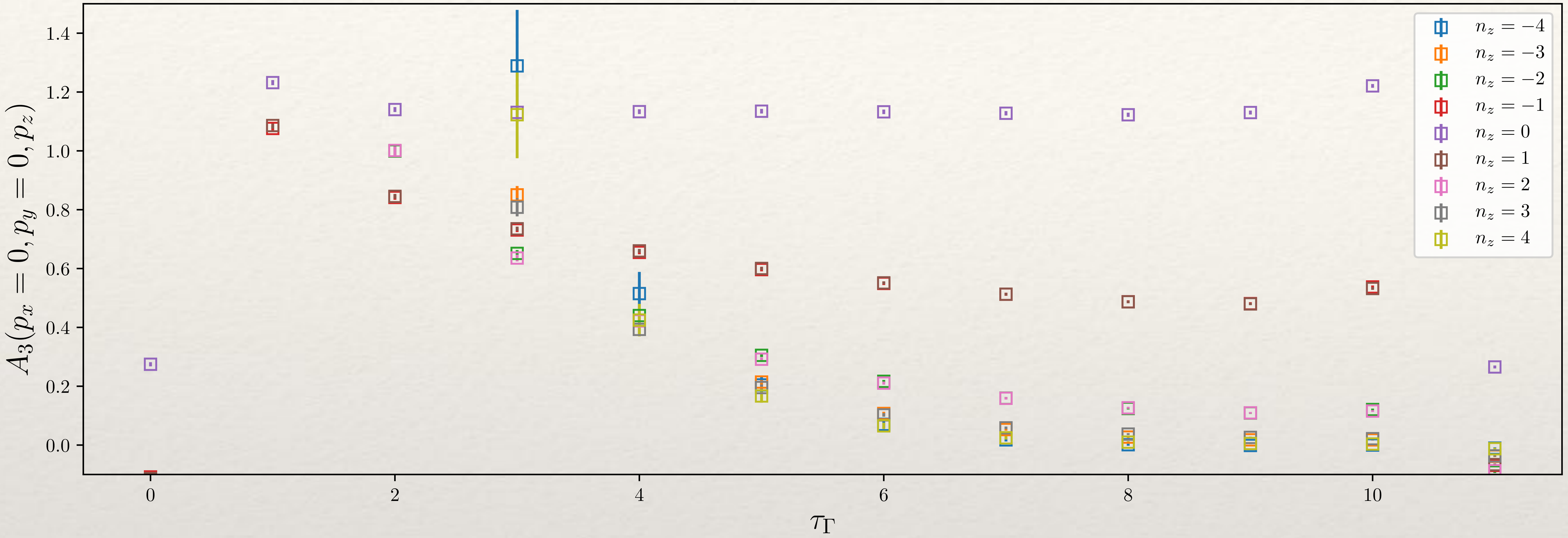
a09m310

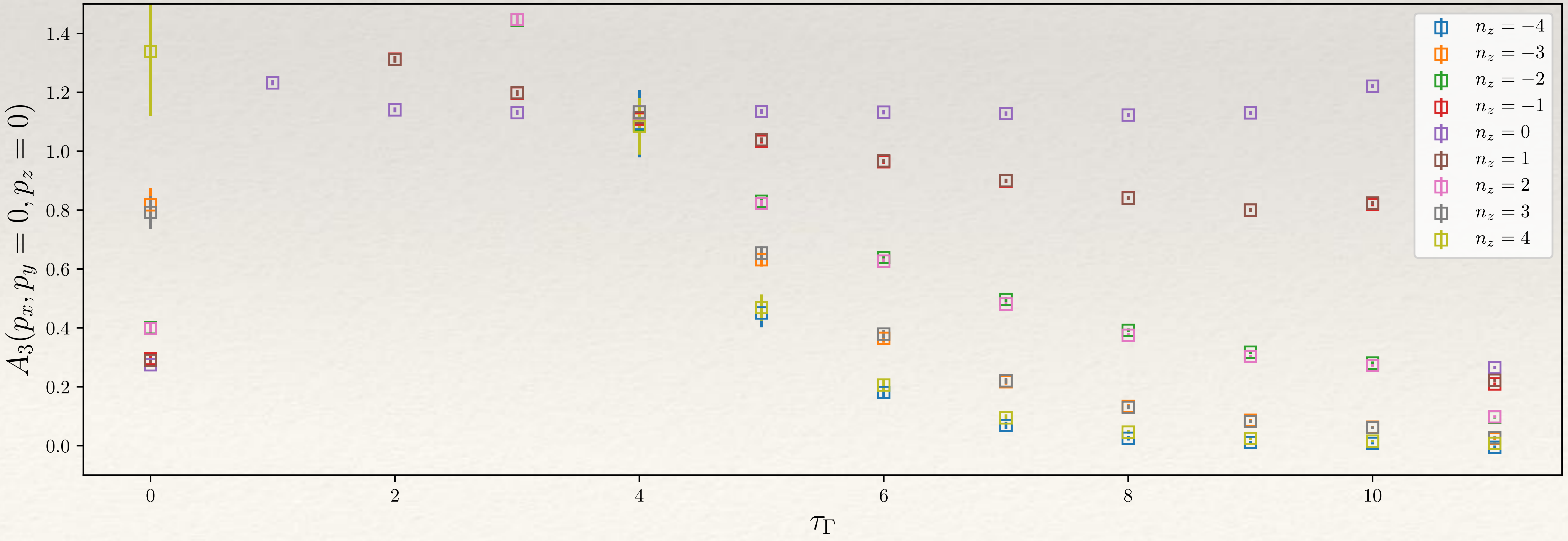
t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]



Nucleon Axial FormFactor PRELIMINARY

a09m310 non-zero momentum, t_{sep} = 11





|n|=1, Q=0.196 GeV
|n|=2, Q=0.393 GeV
|n|=3, Q=0.589 GeV
|n|=4, Q=0.785 GeV

Nucleon Axial FormFactor

Spin averaging

PRELIMINARY

In the literature - we see both

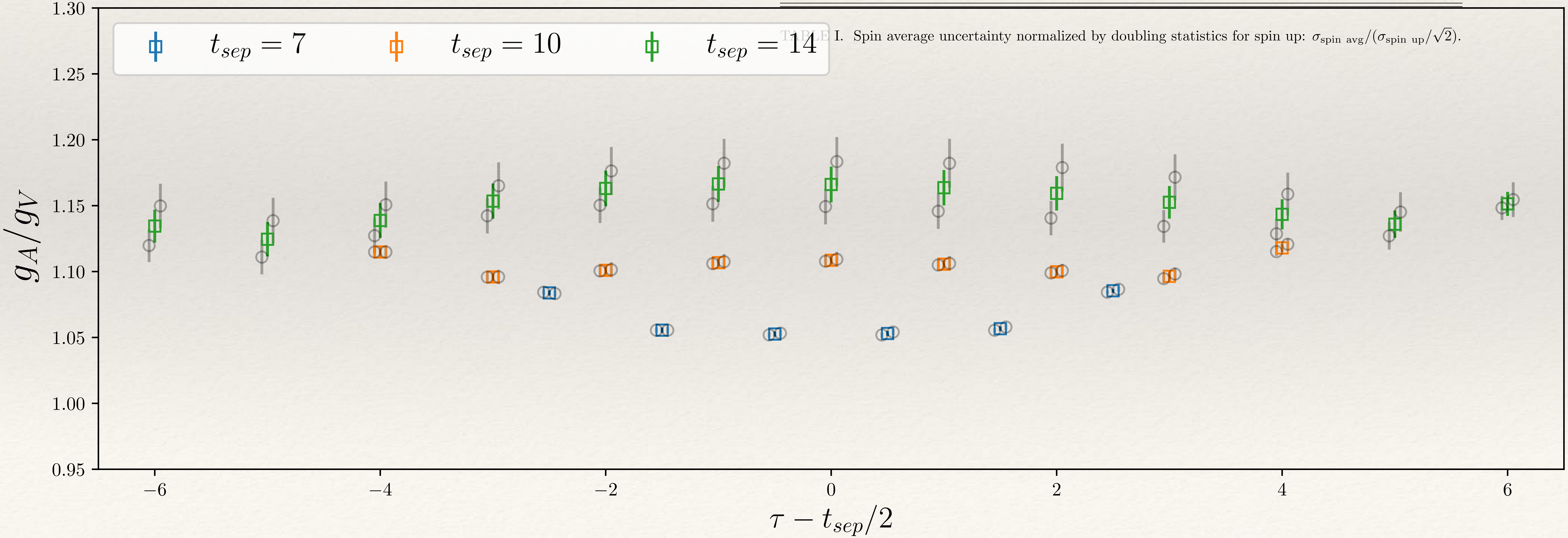
$\mathcal{P}_{3pt} \propto 1 + i\gamma_5\gamma_3$ spin up only

$\mathcal{P}_{3pt} \propto i\gamma_5\gamma_3$ spin up - spin dn

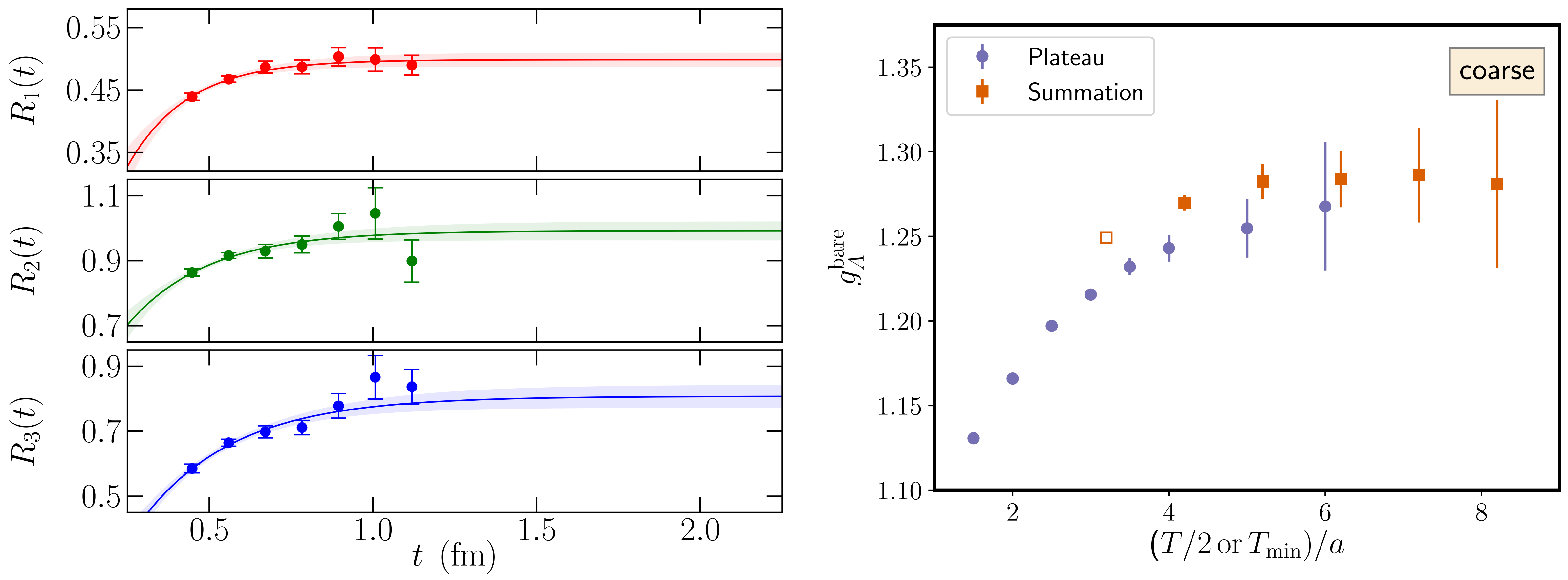
t_{sep}	$t_0 + \tau_T$												
3	1.23	1.23											
4	1.19	1.20	1.18										
5	1.14	1.15	1.15	1.12									
6	1.09	1.11	1.12	1.10	1.06								
7	1.07	1.08	1.10	1.09	1.06	1.03							
8	1.06	1.07	1.09	1.09	1.06	1.03	1.01						
9	1.05	1.06	1.08	1.08	1.07	1.05	1.03	1.02					
10	1.04	1.05	1.06	1.07	1.07	1.06	1.05	1.05	1.04				
11	1.03	1.03	1.04	1.05	1.05	1.06	1.07	1.07	1.07	1.07			
12	1.02	1.02	1.02	1.03	1.04	1.06	1.08	1.10	1.10	1.10	1.09		
13	1.02	1.02	1.01	1.01	1.03	1.06	1.08	1.10	1.12	1.12	1.13	1.12	
14	1.01	1.01	1.01	1.01	1.03	1.05	1.08	1.10	1.12	1.13	1.14	1.14	1.13

TABLE I. Spin average uncertainty normalized by doubling statistics for spin up: $\sigma_{\text{spin avg}}/(\sigma_{\text{spin up}}/\sqrt{2})$.

$$\frac{\sigma_{\text{spin avg}}}{\sigma_{\text{spin up}}/\sqrt{2}}$$



- The success of this (and future) result(s) was enabled through several key features:
  - an unconventional strategy that can exploit **exponentially more precise data** at early time and has **demonstrable control of excited state** contributions
  - access to a set of ensembles (HISQ 2+1+1 from **MILC**) that allowed for control over all standard lattice systematics,
  - ***ludicrously fast*** GPU code - QUDA
  - an action with **improved stochastic behavior** and a **mild continuum** extrapolation $m_\pi \to m_\pi^{phys}, a \to 0, L \to \infty$
  - access to Leadership Computing
- Making progress in understanding $g_A(Q^2)$ - it seems essential to have enough $\mathbf{t_{sep}}$ values to control the infinite separation extrapolation - more than is common
  - See also
    - Detmold, Lin, Meinel PRL 108 (2012) [arXiv:1109.2480]
    - N. Hasan et al (LHPC) arXiv:1903.06487



LETTER
https://doi.org/10.1038/s41586-018-0161-8
Nature 558 (2018) no.7708, 91-94
arXiv:1805.12130
https://github.com/callat-qcd/project_gA

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

Lattice QCD Team

(postdoc, grad student)

California Lattice Collaboration

plus a few friends

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Amy Nicholson	Berkeley —> UNC, Chapel Hill
Enrico Rinaldi	RIKEN-BNL
Evan Berkowitz	Forschungszentrum Jülich
Nicolas Garron	Liverpool
David Brantley	W&M, LBNL —> LLNL
Henry Monge-Camacho	W&M, LBNL —> UNC
Chris Monahan	INT —> W&M
Chris Bouchard	Glasgow
Kate Clark	NVIDIA
Bálint Joó	JLab
Thorsten Kurth	NERSC, LBNL
Kostas Orginos	W&M, JLab
Pavlos Vranas	LLNL
André Walker-Loud	LBNL

New characters
Chris Koerber
Ben Hörz
Dean Howarth
Arjun Gambhir
Ken McElvain

Bart van Lith
E. Berkowitz

INCITE Leadership Computing, U.S. Department of Energy
Oak Ridge National Laboratory
Leadership Computing Facility
Lawrence Livermore National Laboratory
SciDAC Scientific Discovery through Advanced Computing
U.S. Department of Energy Office of Science
NNSA National Nuclear Security Administration

DOE Topical Collaboration
Double Beta Decay

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Chris Koerber
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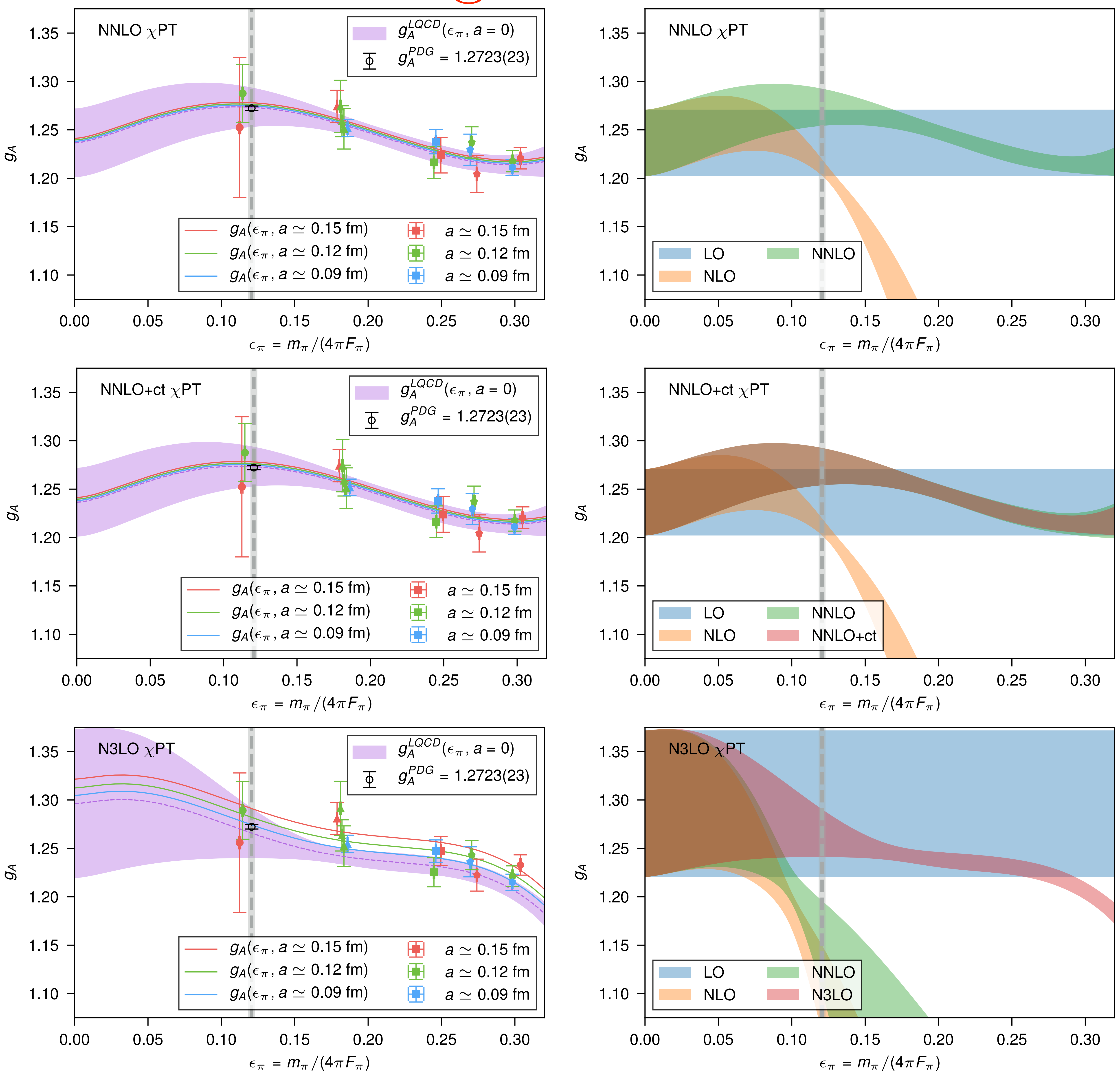
Dean Howarth
Arjun Gambhir
Ken McElvain

Thank You

Bart van Lith
E. Berkowitz

INCITE LEADERSHIP COMPUTING — U.S. DEPARTMENT OF ENERGY
OAK RIDGE National Laboratory — LEADERSHIP COMPUTING FACILITY
Lawrence Livermore National Laboratory
SciDAC Scientific Discovery through Advanced Computing
DOE Topical Collaboration Double Beta Decay
U.S. DEPARTMENT OF ENERGY Office of Science
NNSA National Nuclear Security Administration

convergence of the chiral expansion…



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

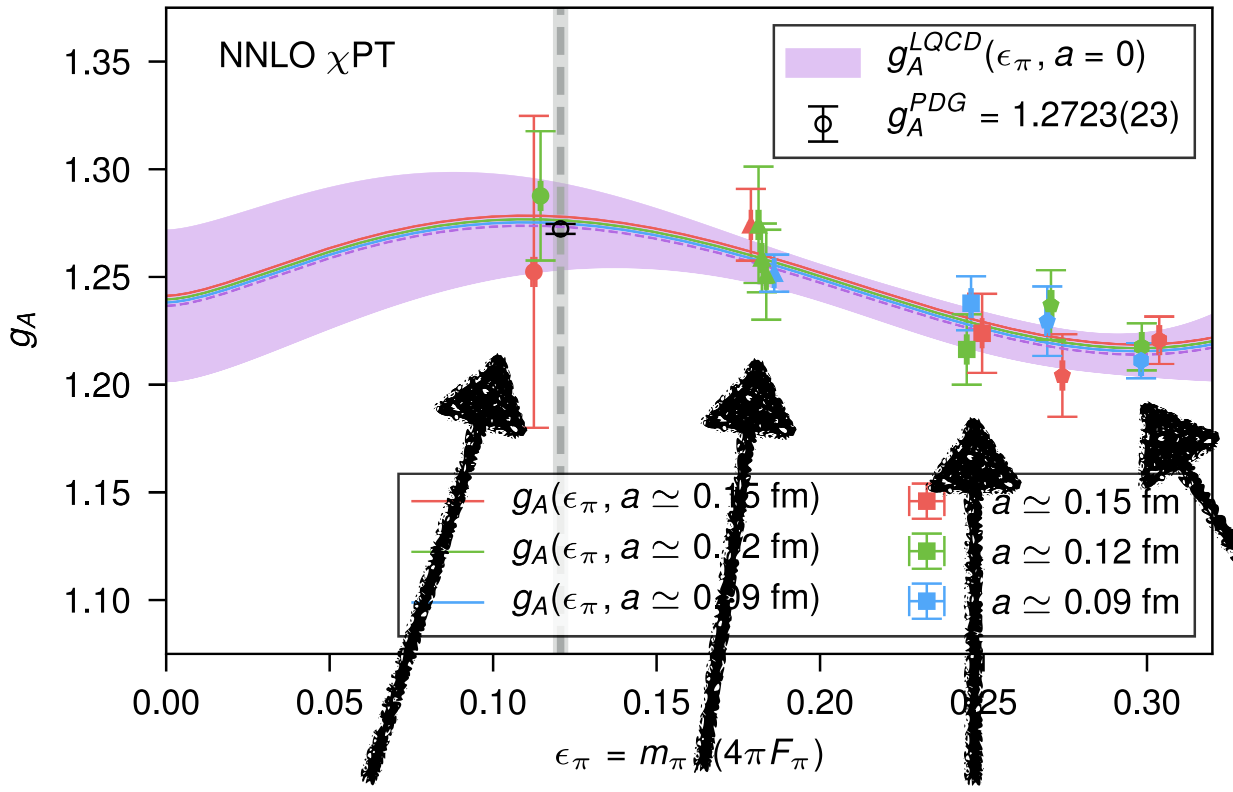
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

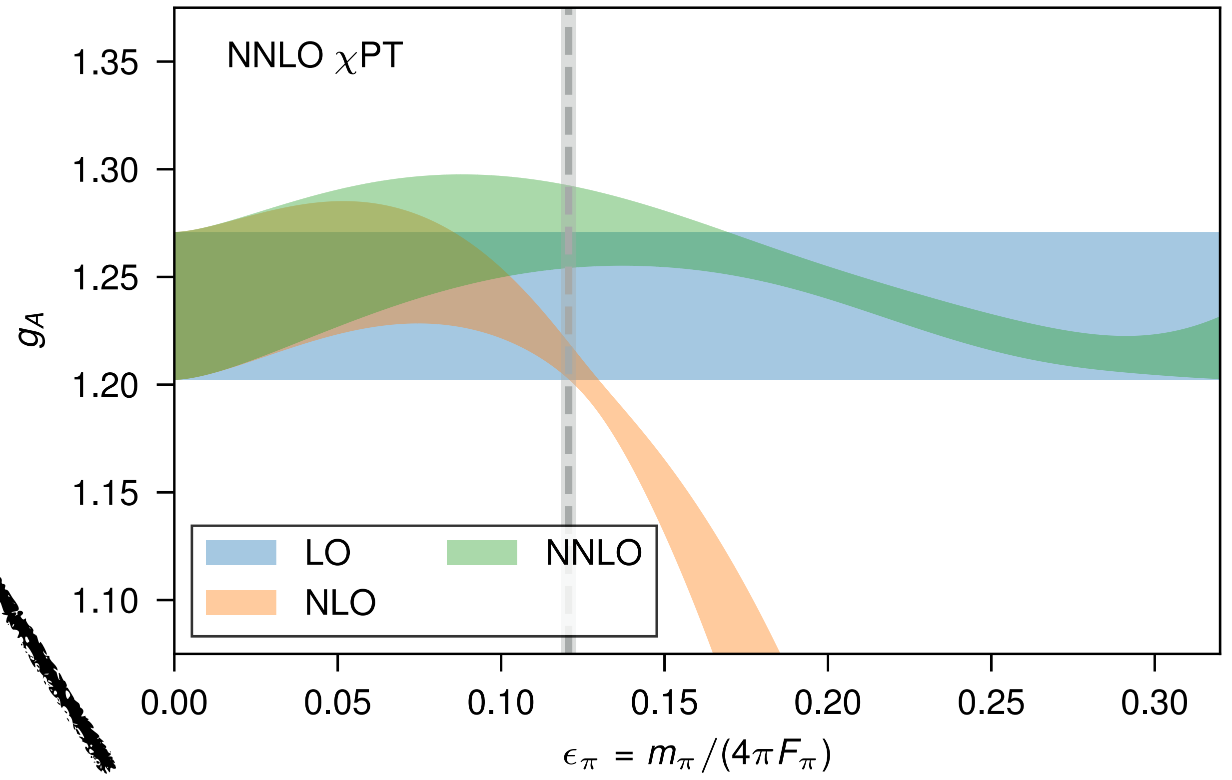
$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4\ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

convergence of the chiral expansion…





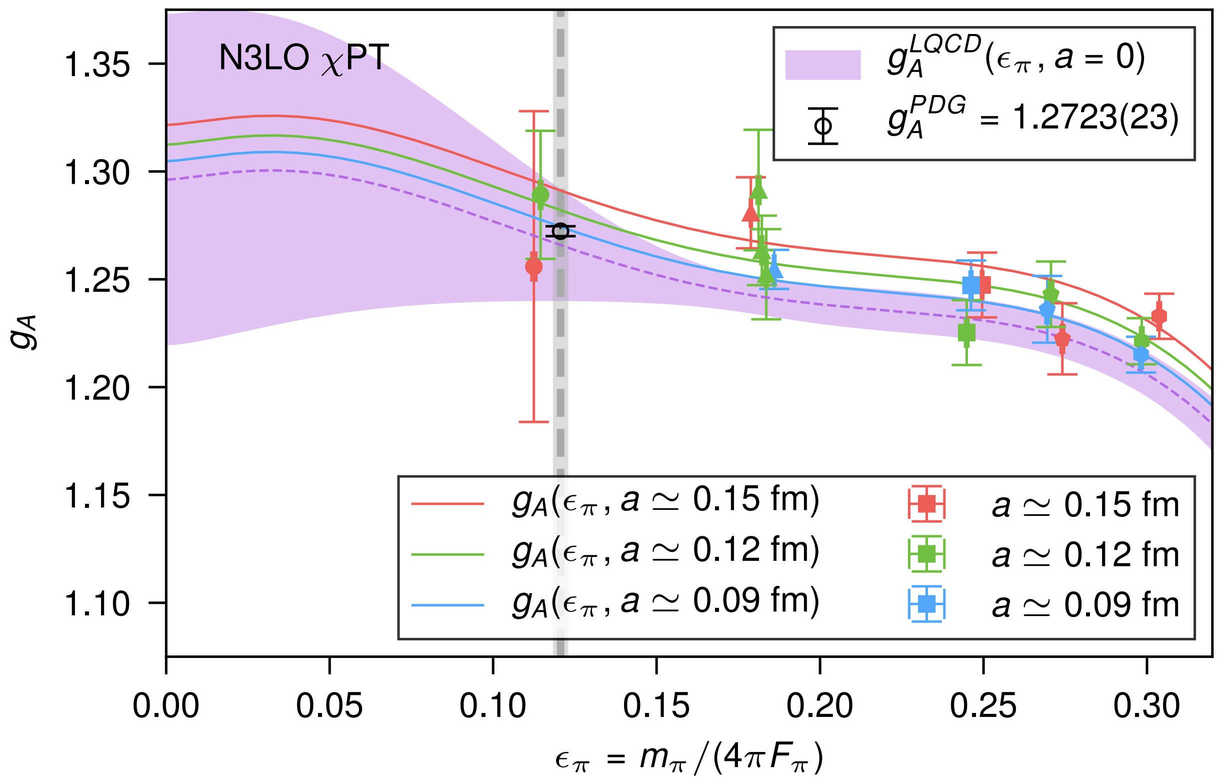
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

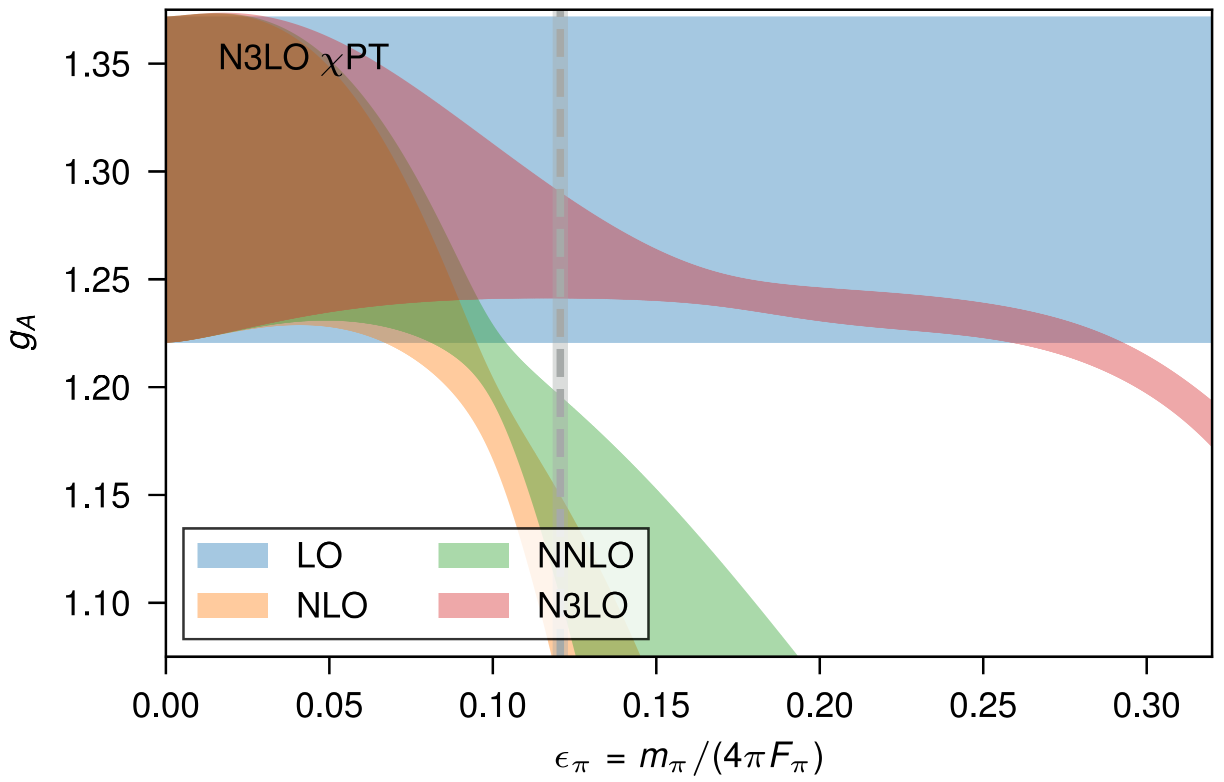
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130$ MeV 220 310 400

can we trust extrapolation of quantities with chiraly-enhanced behavior?

if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?

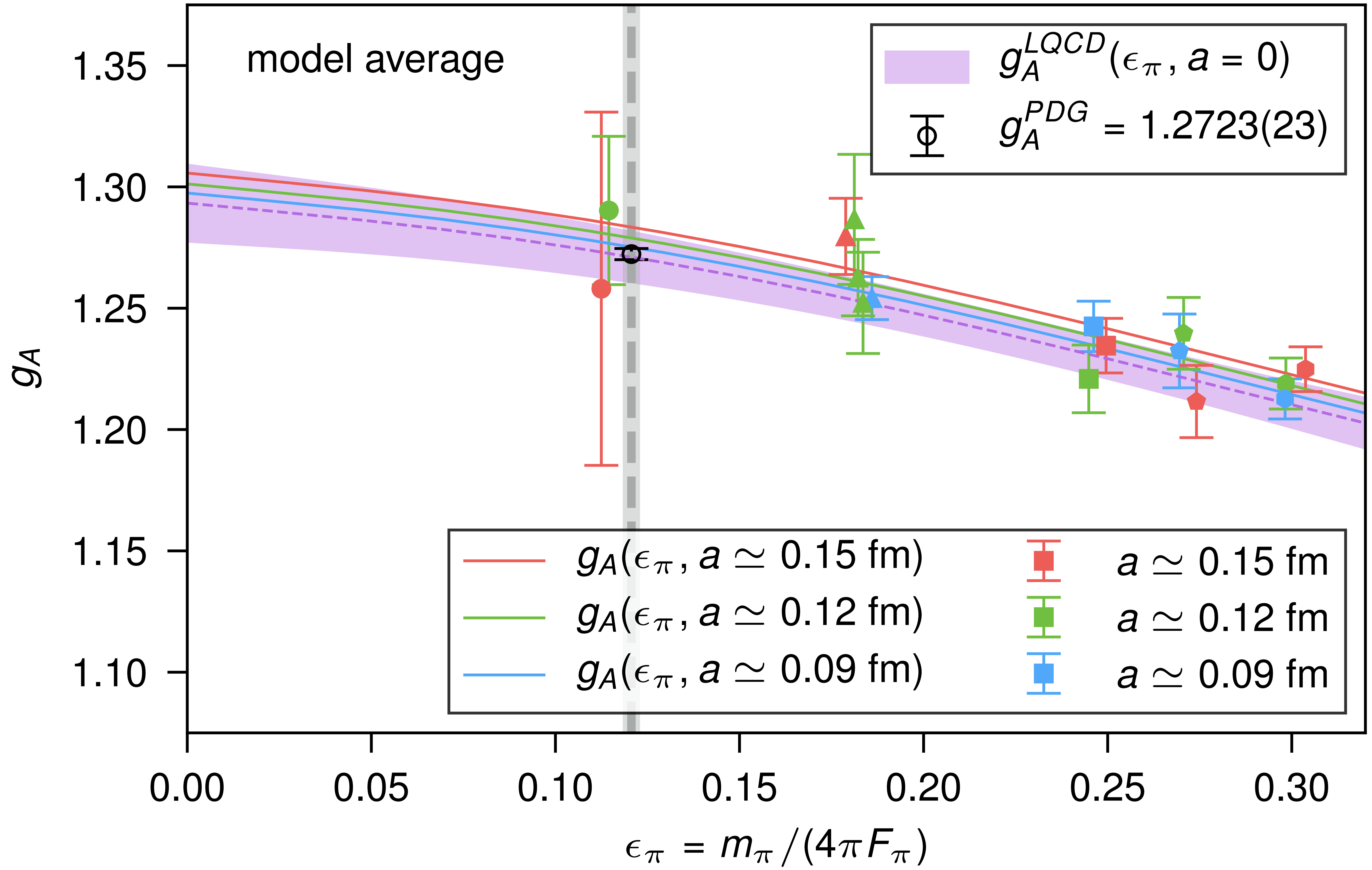




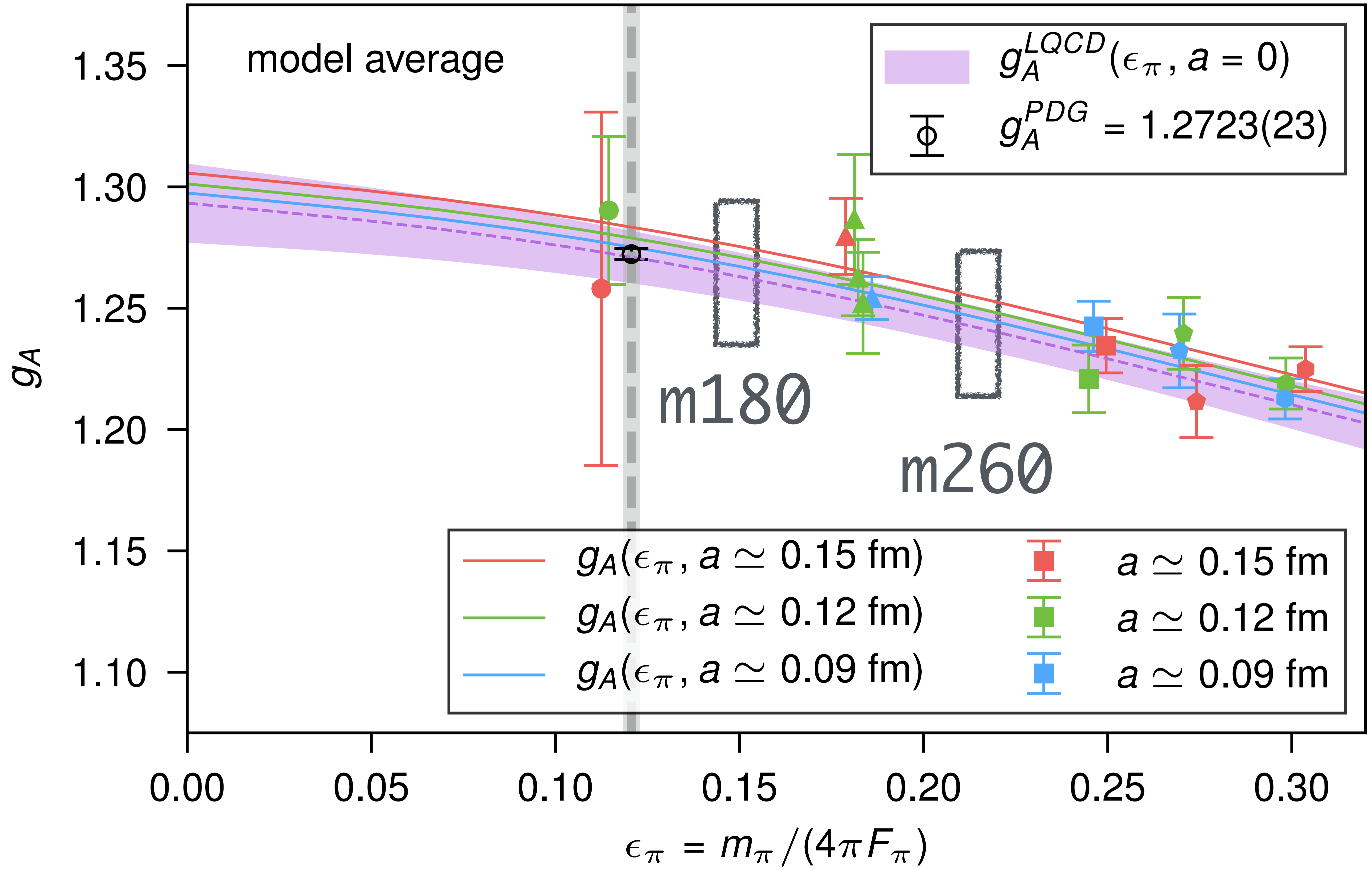
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

Bernard and Meissner (CD06)
Phys.Lett.B639 [hep-lat/0605010]
F —> F_π

Understanding the quark mass dependence



Understanding the quark mass dependence

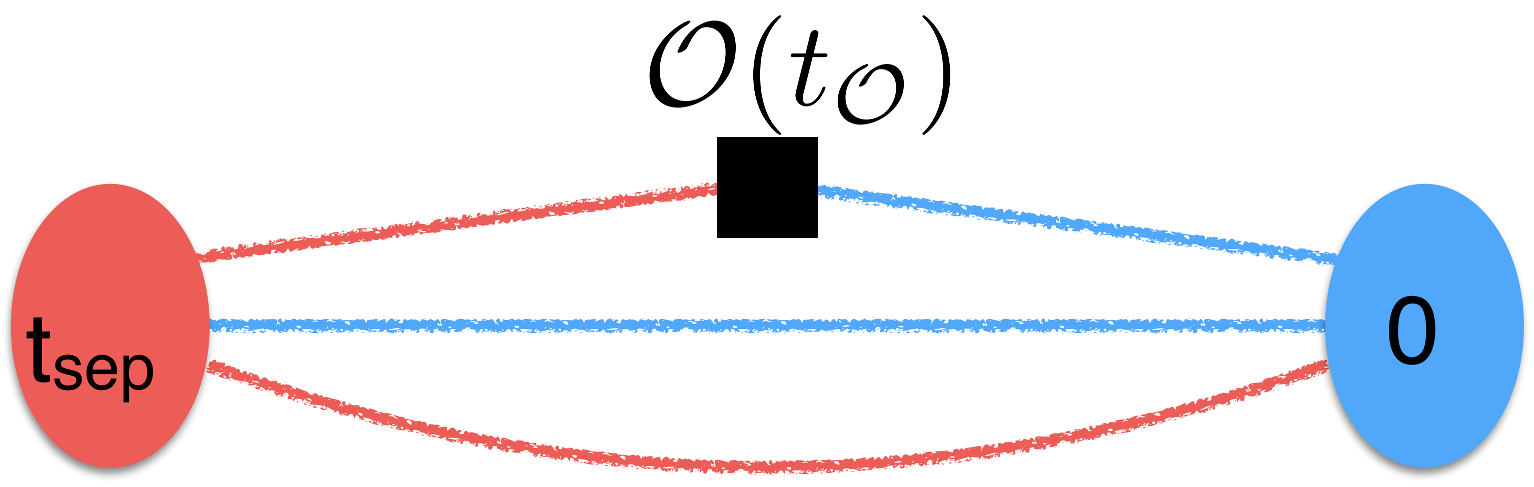


- We are generating new ensembles at $M_\pi \sim 180$, 260 MeV

On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

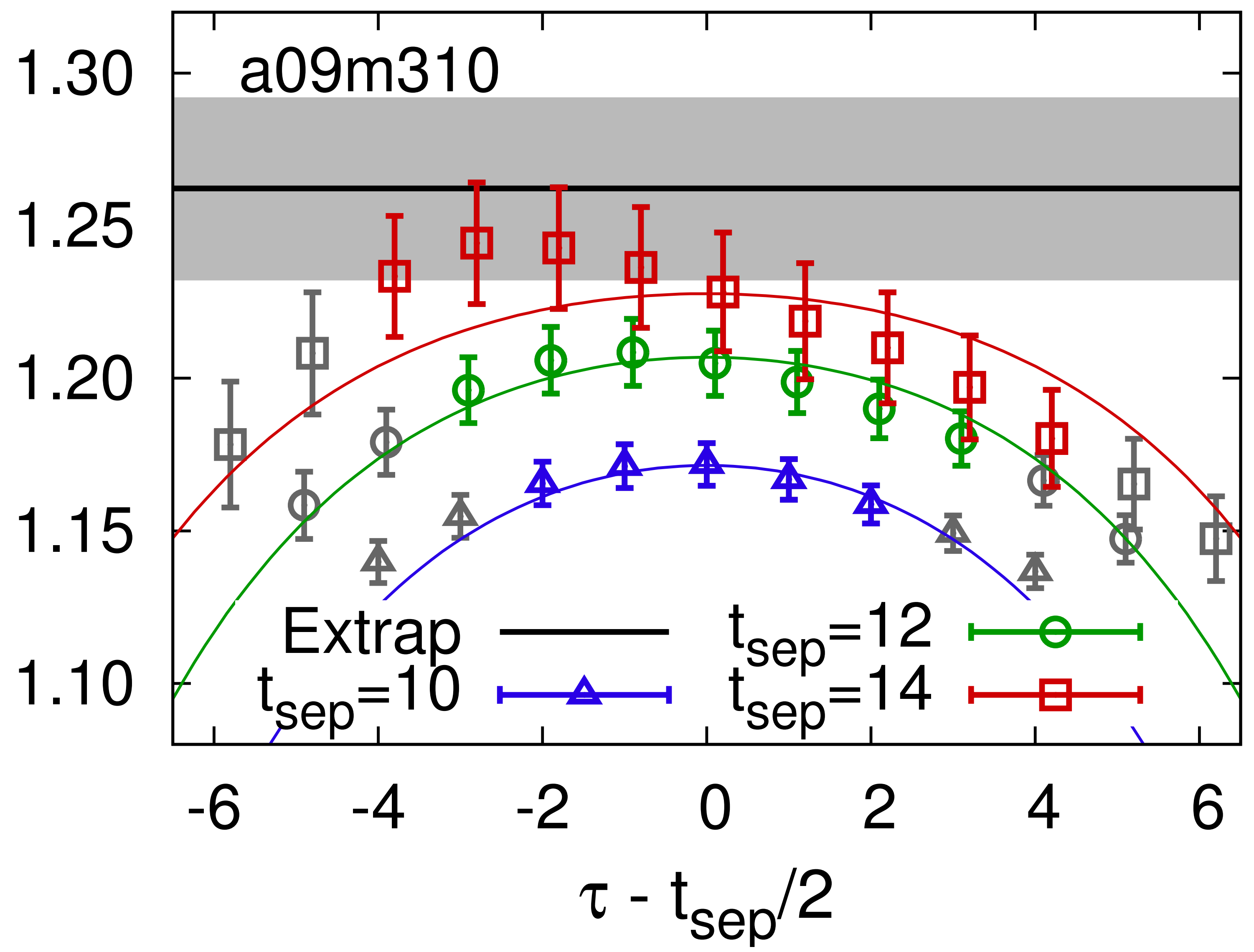
Phys. Rev. D96 (2017)
arXiv:1612.06963

standard method



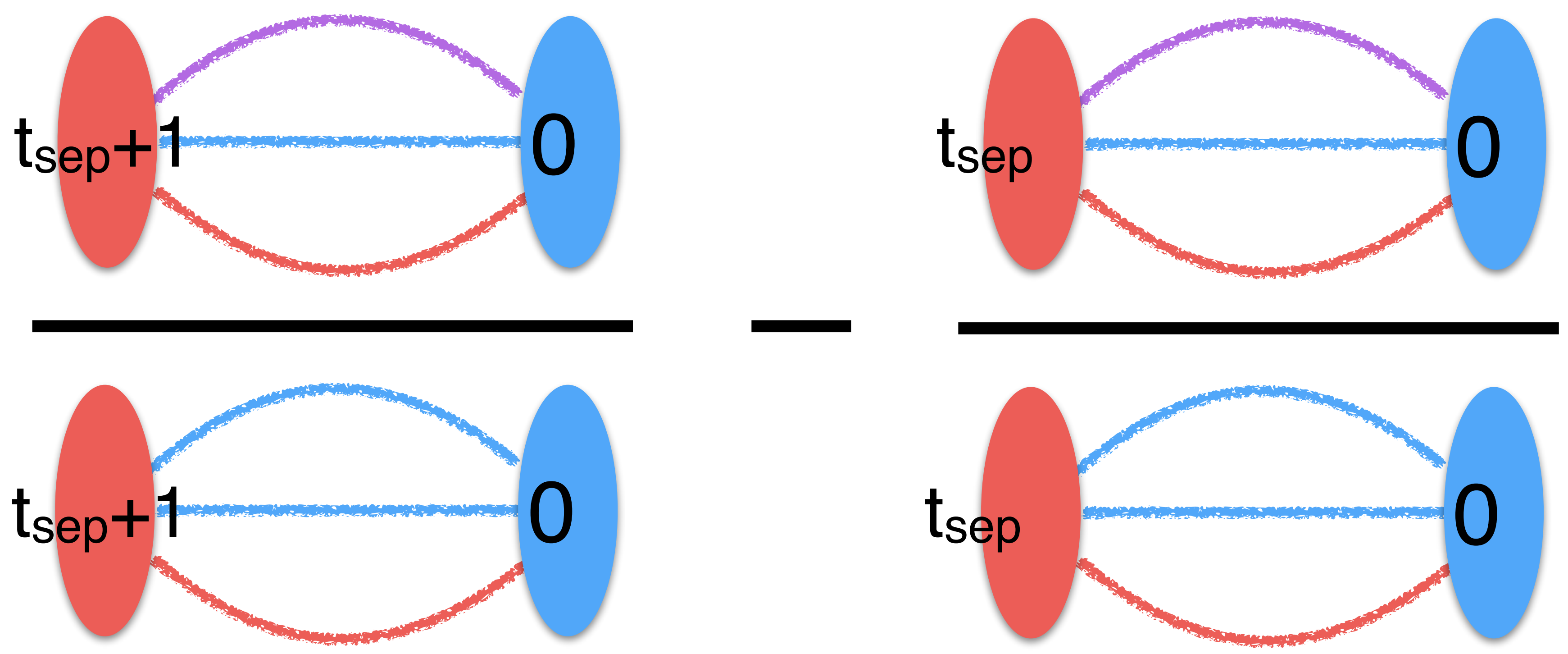
fixed source-sink separation time, $\mathbf{t_{sep}}$
repeat for a few different t_{sep}

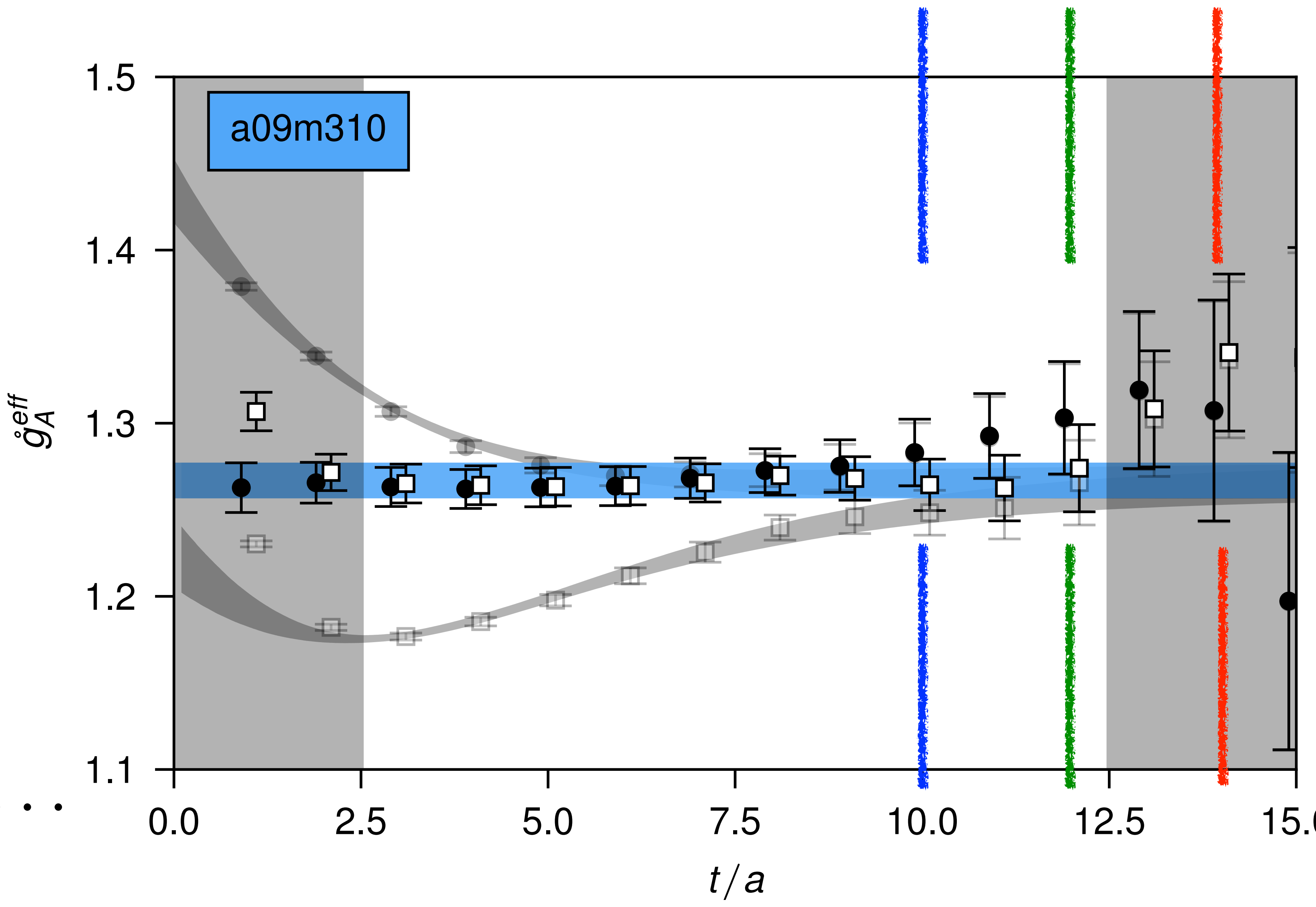
PNDME arXiv:1606.07049



$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

our unconventional method





$$\partial_\lambda m_\lambda \Big|_{\lambda=0} = g_\lambda + z\left(e^{-(t_{sep}+1)\Delta_{10}} - e^{-t_{sep}\Delta_{10}}\right) + \cdots$$