Review of results of recent nucleon structure and matrix element calculations

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Thanks to:

- Rajan Gupta, Boram Yoon and Yong-chull Jang for help with this presentation.
- Everybody that sent me unpublished materials or drew my attention to their work.
- OLCF, NERSC, USQCD for computer allocations allowing our calculations.
- MILC Collaboration for HISQ lattices.
- LANL LDRD and DOE office of science for supporting this research.

I will discuss matrix elements in neutrons and protons, but

- Only in isospin symmetric ($m_u = m_d$) theory;
- Only single-point operators;
- Mostly parity conserving matrix elements; and
- Mostly of dimension-3 quark bilinears.

I will not discuss

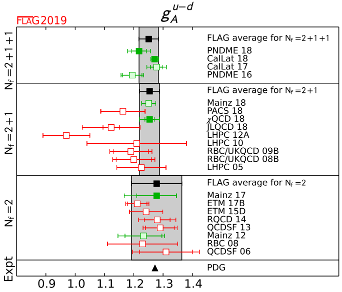
- Products/commutators of currents;
- Electric Dipole Moments from QCD Θ-term (Ohki); or
- PDFs and their moments (Zhao , Karthik)
- Other baryons (Weishäupl)
- Decays (Aoki)

- Calculations of isovector neutron matrix elements have matured.
- Main issue is estimating systematics properly.
- However, more work needed for
  - Scalar matrix elements;
  - Isoscalar matrix elements; and
  - Matrix elements at small nonzero momentum.
- Major systematics that need to be estimated
  - Renormalization,
  - Finite volume,
  - Discretization and scale-setting,
  - Light quark mass, and
  - Excited state contamination.

FLAG review

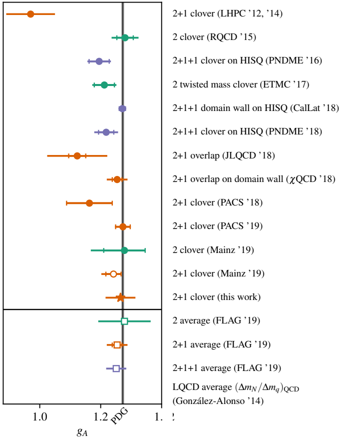
Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	g_A^{u-d}
PNDME 18[a]	2+1+1	A	★‡	★	★	★	★	1.218(25)(30)
CalLat 18	2+1+1	A	○	★	★	★	★	1.271(10)(7)
CalLat 17	2+1+1	P	○	★	★	★	★	1.278(21)(26)
PNDME 16[a]	2+1+1	A	○‡	★	★	★	★	1.195(33)(20)
Mainz 18	2+1	C	★	○	★	★	★	1.251(24)
PACS 18	2+1	A	■	■	★	★	■	1.163(75)(14)
χQCD 18	2+1	A	○	★	★	★	★	1.254(16)(30)[$]
JLQCD 18	2+1	A	■	○	○	★	★	1.123(28)(29)(90)
LHPC 12A[b]	2+1	A	■‡	★	★	★	★	0.97(8)
LHPC 10	2+1	A	■	○	■	★	■	1.21(17)
RBC/UKQCD 09B	2+1	A	■	■	○	★	■	1.19(6)(4)
RBC/UKQCD 08B	2+1	A	■	■	○	★	■	1.20(6)(4)
LHPC 05	2+1	A	■	■	★	★	■	1.226(84)
Mainz 17	2	A	★	★	★	★	○	$1.278(68)(^{+0}_{-0.087})$
ETM 17B	2	A	■	○	○	★	★	1.212(33)(22)
ETM 15D	2	A	■	○	○	★	★	1.242(57)
RQCD 14	2	A	○	★	★	★	■	1.280(44)(46)
QCDSF 13	2	A	○	★	■	★	■	1.29(5)(3)
Mainz 12	2	A	★	○	○	★	○	$1.233(63)(^{+0.035}_{-0.060})$
RBC 08	2	A	■	■	■	★	■	1.23(12)
QCDSF 06	2	A	○	■	■	★	■	1.31(9)(7)

[a] Tree-level tadpole-improved. [b] Tree-level improved. ‡ Not fully O(a) improved. [$] Also has partially quenched analysis.

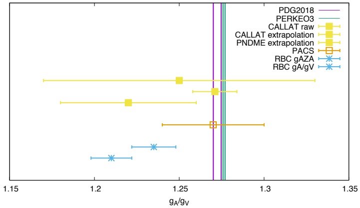


- Clear control at 3%
- Is it controlled at 1%?

LHPC and RBC Update



LHPC comparison

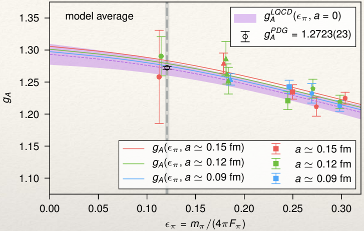


RBC-UKQCD $N_f = 2+1$ DWF, $a^{-1} = 1.73$ GeV.

CalLat update

Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

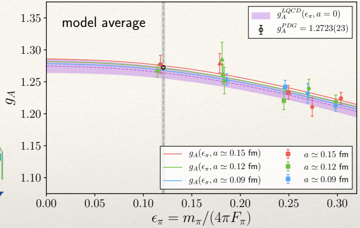
1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



California Lattice Collaboration

Sierra Early Science

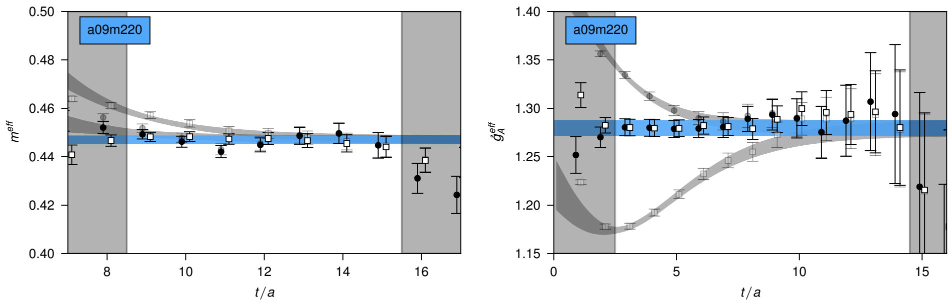
PRELIMINARY



- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$
- We are running $g_A(Q^2)$ on Summit this year (DOE INCITE)
  - We anticipate improving g_A to ~0.5%

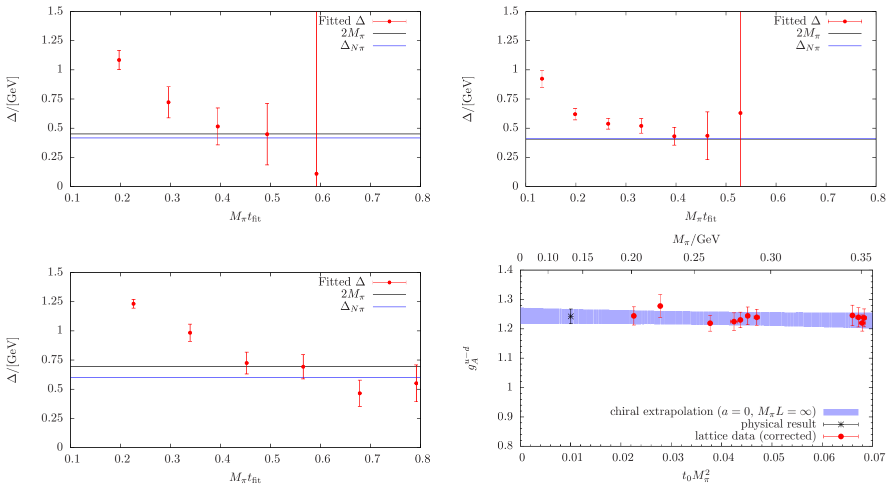
$$g_A = 1.2711(125) \rightarrow 1.2641(93) \quad [0.74\%]$$

CalLat update



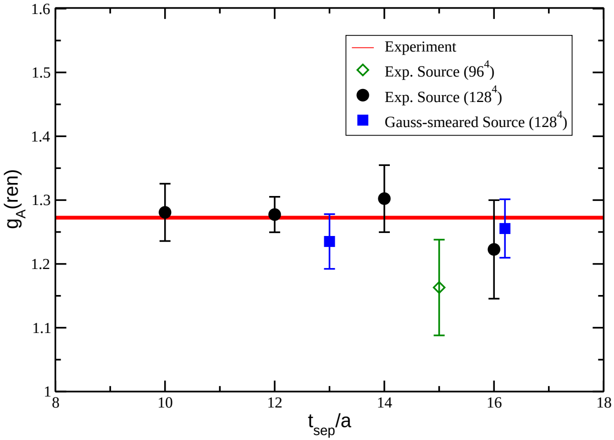
- g_A extracted from $t \sim 3a$.
- Source creates excited states even up to $t \sim 8a$.
- Is the transition matrix element small?
- Or, is statistical precision traded for unaccounted-for systematics?

Mainz Update

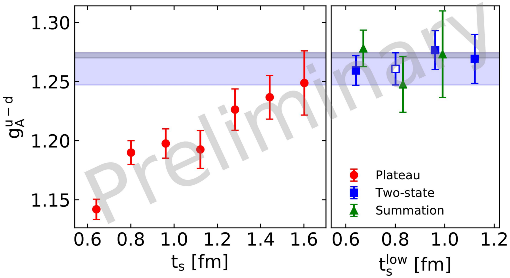


See also Bär.

PACS Update



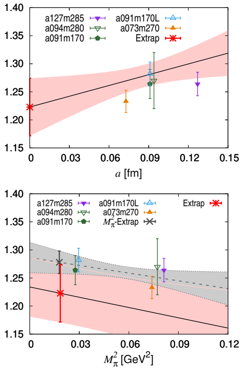
Coulomb-gauge exponentia source works well!



$N_f = 2+1+1$, physical mass, $a = 0.08$ fm, $64^3 \times 128$

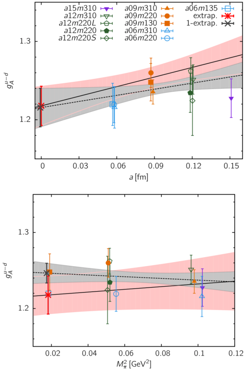
$g_A^{u-d} = 1.261(14)$

Clover (Preliminary)



1.247(64)

HISQ (PNDME '18)

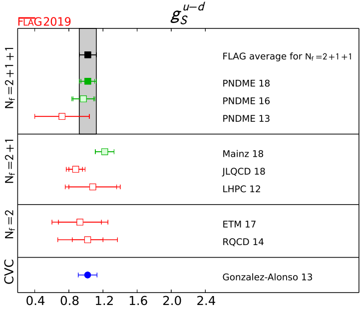


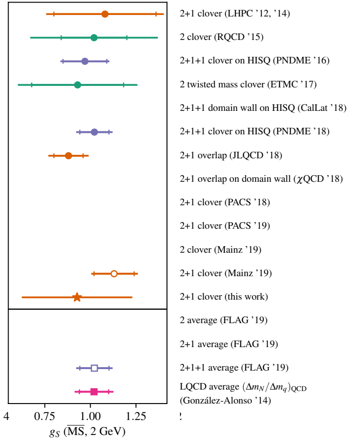
1.218(25)

FLAG review: g_S

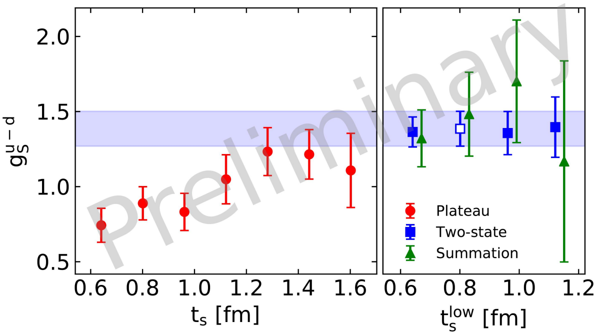
Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	g_S^{u-d}
PNDME 18	2+1+1	A	★ ‡	★	★	★	★	1.022(80)(60)
PNDME 16	2+1+1	A	○ ‡	★	★	★	★	0.97(12)(6)
PNDME 13	2+1+1	A	■ ‡	■	★	★	★	0.72(32)
Mainz 18	2+1	C	★	○	★	★	★	1.22(11)
JLQCD 18	2+1	A	■	○	○	★	★	0.88(8)(3)(7)
LHPC 12	2+1	A	■ ‡	★	★	★	★	1.08(28)(16)
ETM 17	2	A	■	○	○	★	★	0.930(252)(48)(204)
RQCD 14	2	A	○	★	★	★	■	1.02(18)(30)

‡ Not fully O(a) improved.





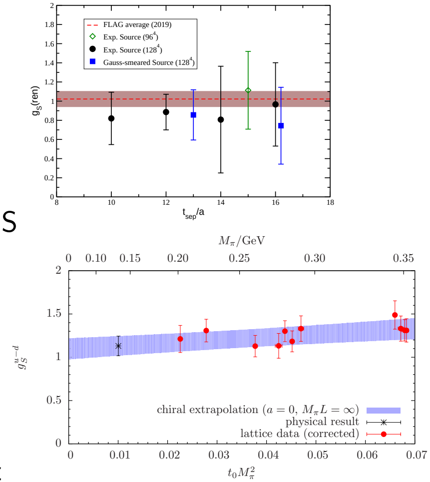
LHPC comparison



ETMC: $N_f = 2+1+1$, physical mass, $a = 0.08$ fm, $64^3 \times 128$

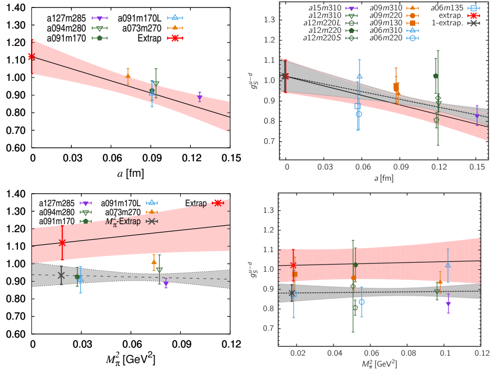
$$g_S^{u-d} = 1.39(12)$$

PACS, Mainz, and PNDME Update: scalar charge



PACS

Mainz



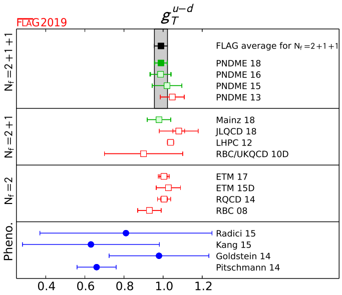
1.15(12) 1.022(80)

See also RBC/UKQCD Update (Liu)

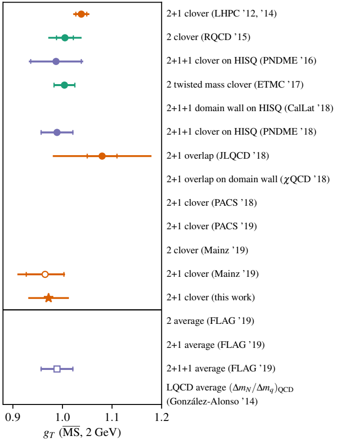
FLAG review: g_T

Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	g_T^{u-d}
PNDME 18	2+1+1	A	★‡	★	★	★	★	0.989(32)(10)
PNDME 16	2+1+1	A	○‡	★	★	★	★	0.987(51)(20)
PNDME 15	2+1+1	A	○‡	★	★	★	★	1.020(76)
PNDME 13	2+1+1	A	■‡	■	★	★	★	1.047(61)
Mainz 18	2+1	C	★	○	★	★	★	0.979(60)
JLQCD 18	2+1	A	■	○	○	★	★	1.08(3)(3)(9)
LHPC 12	2+1	A	■‡	★	★	★	★	1.038(11)(12)
RBC/UKQCD 10D	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	2	A	■	○	○	★	★	1.004(21)(2)(19)
ETM 15D	2	A	■	○	○	★	★	1.027(62)
RQCD 14	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	2	A	■	■	■	★	■	0.93(6)

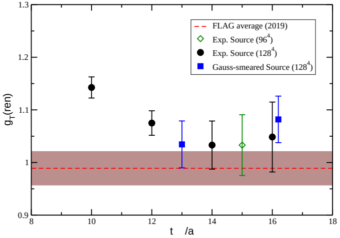
‡Not fully O(a) improved.



LHPC, PACS, and ETMC Update



LHPC comparison

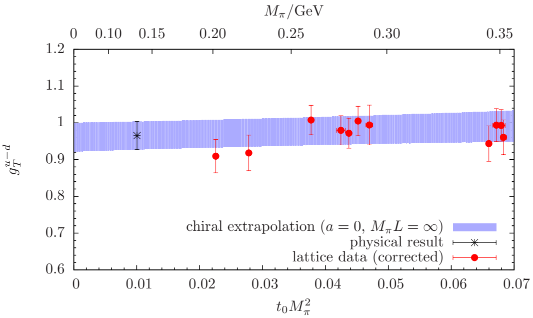


PACS

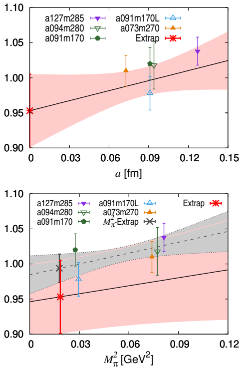
ETMC Update: $N_f = 2+1+1$, physical mass, $a = 0.08$ fm, $64^3 \times 128$

$$g_T^{u-d} = 0.929(14)$$

Mainz and PNDME Update: tensor charge

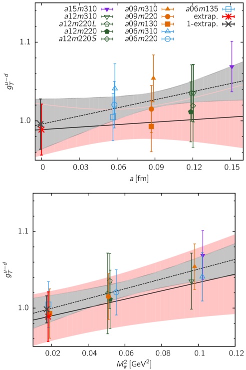


Clover (Preliminary)



0.967(31)

HISQ (PNDME '18)



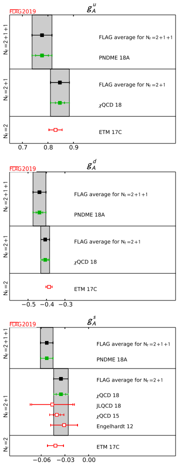
0.989(32)

FLAG review: Flavor Diagonal g_A

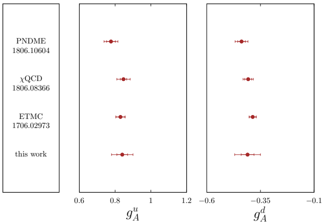
Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	Δu	Δd
PNDME 18A	2+1+1	A	★ ‡	★	★	★	★	0.777(25)(30) #	−0.438(18)(30) #
χQCD 18	2+1	A	○	★	★	★	★	0.847(18)(32) $	−0.407(16)(18) $
ETM 17C	2	A	■	○	○	★	★	0.830(26)(4)	−0.386(16)(6)

Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	Δs
PNDME 18A	2+1+1	A	★ ‡	★	★	★	★	−0.053(8) #
χQCD 18	2+1	A	○	★	★	★	★	−0.035(6)(7) $
JLQCD 18	2+1	A	■	○	○	★	★	−0.046(26)(9) #
χQCD 15	2+1	A	■	○	■	★	★	−0.0403(44)(78) #
Engelhardt 12	2+1	A	■	○	■	★	★	−0.031(17) #
ETM 17C	2	A	■	○	○	★	★	−0.042(10)(2)

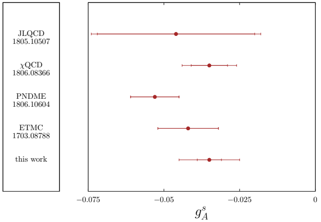
$Z_A^{n.s.} = Z_A^s$ assumed. $ Also partially quenched analysis. ‡ Not fully O(a) improved.



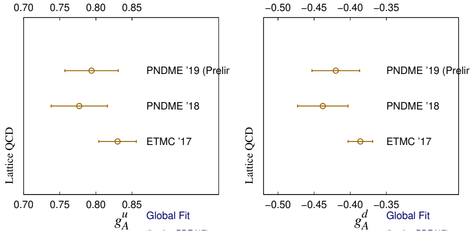
Mainz and PNDME update: Flavor Diagonal g_A

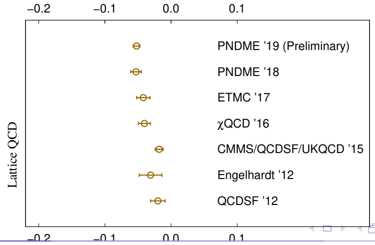


$$g_A^u = 0.84(3)(6) \qquad g_A^d = -0.41(3)(6)$$



$$g_A^s = -0.035(4)(5)$$

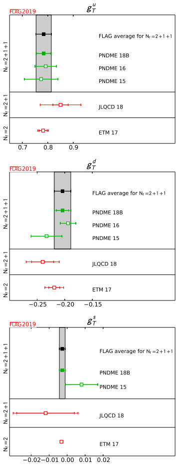




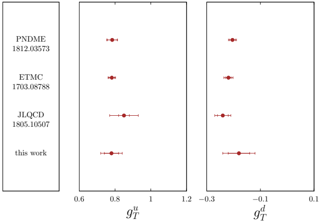
FLAG review: Flavor Diagonal g_T

Collaboration	N_f	pub.	cont.	chiral	vol.	ren.	states	g_T^u	g_T^d
PNDME 18B	2+1+1	P	★ ‡	★	★	★	★	0.784(28)(10) #	−0.204(11)(10) #
PNDME 16	2+1+1	A	○ ‡	★	★	★	★	0.792(42) #&	−0.194(14) #&
PNDME 15	2+1+1	A	○ ‡	★	★	★	★	0.774(66) #	−0.233(28) #
JLQCD 18	2+1	A	■	○	○	★	★	0.85(3)(2)(7)	−0.24(2)(0)(2)
ETM 17	2	A	■	○	○	★	★	0.782(16)(2)(13)	−0.219(10)(2)(13)
								g_T^s	
PNDME 18B	2+1+1	P	★ ‡	★	★	★	★	−0.0027(16) #	
PNDME 15	2+1+1	A	○ ‡	★	★	★	★	0.008(9) #	
JLQCD 18	2+1	A	■	○	○	★	★	−0.012(16)(8)	
ETM 17	2	A	■	○	○	★	★	−0.00319(69)(2)(22)	

$Z_T^{n.s.} = Z_T^s$ assumed. & Only 'connected'. ‡ Not fully O(a) improved.

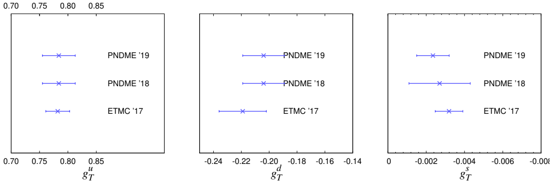


Mainz and PNDME Update: Flavor Diagonal g_T



$$g_T^u = 0.78(4)(3) \qquad g_T^d = -0.18(4)(3)$$

Preliminary

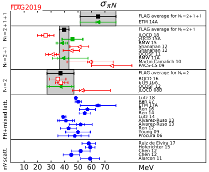


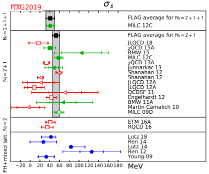
[PNDME, PRD98, 091501 (2018)]

FLAG review: $\sigma_{\pi N}$ and σ_s

Collaboration	N_f	pub.	cont.	chiral	vol.	states	$\sigma_{\pi N}$ [MeV]	σ_s [MeV]
MILC 12C	2+1+1	A	★	★	★	★	—	$0.44(8)(5) \times m_s$ ¶§
JLQCD 18	2+1	A	■	○	○	★	26(3)(5)(2)	17(18)(9)
χQCD 15A	2+1	A	○	★	★	★	45.9(7.4)(2.8)$	40.2(11.7)(3.5)$
χQCD 13A	2+1	A	■	■	○	★	—	33.3(6.2)$
JLQCD 12A	2+1	A	■	○	○	★	—	$0.009(15)(16) \times m_N$ †
Engelhardt 12	2+1	A	■	○	■	★	—	$0.046(11) \times m_N$ †
MILC 12C	2+1	A	★	○	★	★	—	$0.637(55)(74) \times m_s$ ¶§
MILC 09D	2+1	A	★	○	★	★	—	59(6)(8)§
ETM 16A	2	A	■	○	○	★	$37.2(2.6)(^{4.7}_{2.9})$	$41.1(8.2)(^{7.8}_{5.8})$
RQCD 16	2	A	○	★	★	■	35(6)	35(12)
ETM 14A	2+1+1	A	★	○	○		64.9(1.5)(13.2)△	—
BMW 15	2+1	A	★‡	★	★		38(3)(3)	105(41)(37)
Junnarkar 13	2+1	A	○	○	○		—	48(10)(15)
Shanahan 12	2+1	A	■	○	○		45(6)/51(7)*	21(6)/59(6)*
JLQCD 12A	2+1	A	■	○	○		—	$0.023(29)(28) \times m_N$ †
QCDSF 11	2+1	A	■	■	○		31(3)(4)	71(34)(59)
BMW 11A	2+1	A	○‡	★	○		$39(4)(^{18}_{7})$	$67(27)(^{55}_{47})$
Martin Camalich 10	2+1	A	■	★	■		59(2)(17)	−4(23)(25)
PACS-CS 09	2+1	A	■	★	■		75(15)	—
QCDSF 12	2	A	○	★	○		37(8)(6)	—
JLQCD 08B	2	A	■	○	■		$53(2)(^{+21}_{-7})$	—

△Multiple results. ‡Not fully O(a) improved. *Two results are quoted. †From $f_{T_s} = \sigma_s / m_N$ $Also partially quenched §Uses a hybrid method ¶At $\mu = 2$ GeV in $\overline{\text{MS}}$ scheme.

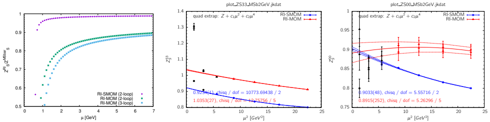




PNDME Update: $\sigma_{\pi N}$ and σ_s

Comparing $Z_S^{isovector}$ and $Z_S^{isoscalar}$

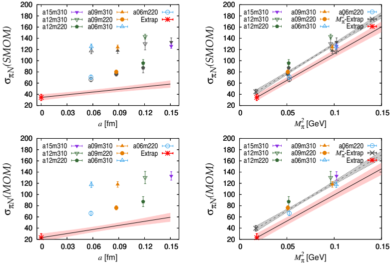
$\mathcal{O}_S^{isovector} = \mathcal{O}_S^{u-d}$ $\qquad$ $\mathcal{O}_S^{isoscalar} = \mathcal{O}_S^{u+d+s}$



a	$Z_S^{isovector} / Z_S^{isoscalar}$ ($\overline{MS}$ 2GeV) RI-SMOM	RI-MOM
0.15	1.063(13)	1.362(83)
0.12	1.0229(38)	1.35(14)
0.09	1.0223(54)	1.161(33)
0.06	1.0103(44)	1.159(34)

- Large difference between RI-MOM and RI-SMOM schemes
  - $Z_S^{isovector}$ (5% $\sim$ 30%)
  - $Z_S^{isoscalar}$ (5% $\sim$ 10%)
- $Z_S^{isovector} / Z_S^{isoscalar} \to 1$ as $a \to 0$

Nucleon $\sigma_{\pi N}$ term (Preliminary)



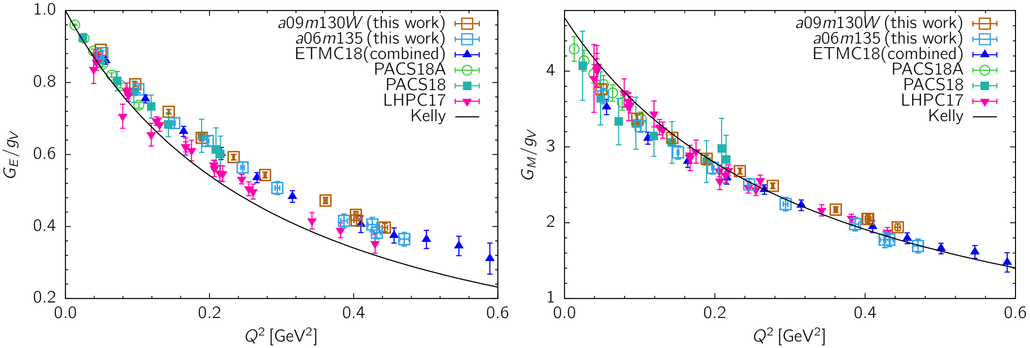
- SMOM, $\sigma_{\pi N} = 34.5(5.5)$ [6.373]
- MOM, $\sigma_{\pi N} = 22.9(7.2)$ [0.426]
- FLAG 19', $N_f = 2+1+1$, $\sigma_{\pi N} = 64.9(1.5)(13.2)$
- FLAG 19', $N_f = 2+1$, $\sigma_{\pi N} = 39.7(3.6)$

See also BMW Update (Varnhorst)

Isovector VFF: Compilation of lattice data

$$\langle N|V_\mu|N\rangle \equiv \bar{u}\left[F_1\gamma_\mu + F_2\sigma_{\mu\nu}\frac{q_\nu}{2M} + F_A\frac{2iMq_\mu - q^2\gamma_\mu}{m_N^2}\gamma_5 - iF_3\sigma_{\mu\nu}\frac{q_\nu}{2M}\gamma_5\right]u$$

$$G_E = F_1 - \frac{Q^2}{4M^2}F_2 \qquad G_M = F_1 + F_2$$



Isovector VFF: Compilation of lattice data

$$\mu \equiv \frac{G_M(0)}{G_E(0)} \qquad \langle r^2_{E,M} \equiv -6 \frac{d}{dQ^2}\left(\frac{G_{E,M}(Q^2)}{G_{E,M}(0)}\right)\bigg|_{Q^2=0}$$

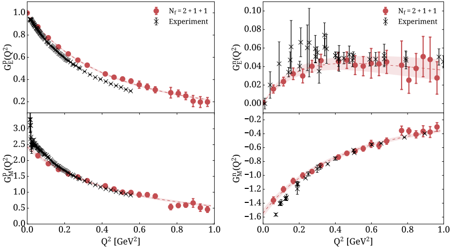
	M_N MeV	a from	Q^2 Fit	r_E (fm)	r_M (fm)	μ
PNDME	953(4)	r_1	z^4	0.769(27)(30)	0.671(48)(76)	3.94(9)(14)
PNDME a06	951(10)	r_1	z^4	0.765(11)(8)	0.704(21)(29)	3.98(8)(13)
LHPC'17	912(8)	M_Ω	z^5	0.887(49)		4.75(15)
ETMC'18	929(6)	$r_0^2 F(r_0) = 1.65$	dipole	0.802(19)(12)(1)	0.714(26)(88)(16)($\binom{1}{0}$)	3.96(14)(3)(7)($\binom{1}{0}$)
ETMC'17	941(2)	$r_0^2 F(r_0) = 1.65$	dipole	0.808(30)(19)	0.732(36)(45)	4.02(21)(28)
PACS'18	958(10)	M_Ω	$z^8 \vert z^7$	0.915(99)	1.437(409)	4.81(79)
PACS'18A	942(11)	M_Ω	dipole	0.875(15)(28)	0.805(32)(274)	4.417(138)(317)

Experimentally,
$\mu^p = 1.79$, $\mu^n = -1.91$, $r_E = 0.875(6)$ from electrons and $0.8409(4)$ from muons.

Disconnected contribution: ETMC and Mainz

Proton and neutron electric and magnetic form factors

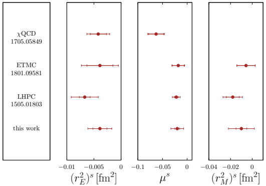
Extended Twisted Mass collaboration using $N_f = 2+1+1$ simulations at the physical point, with $a = 0.08$ fm and $64^3 \times 128$ lattice



Deviations from experimental results under investigation. // May arise from e.g. finite volume and/or excited states

C. A., S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou and A. Vaquero Aviles-Casco, arXiv:1812.10311.

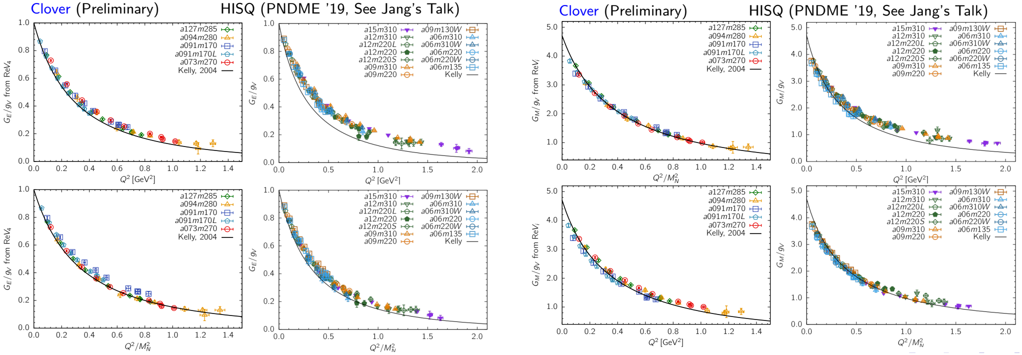
comparison



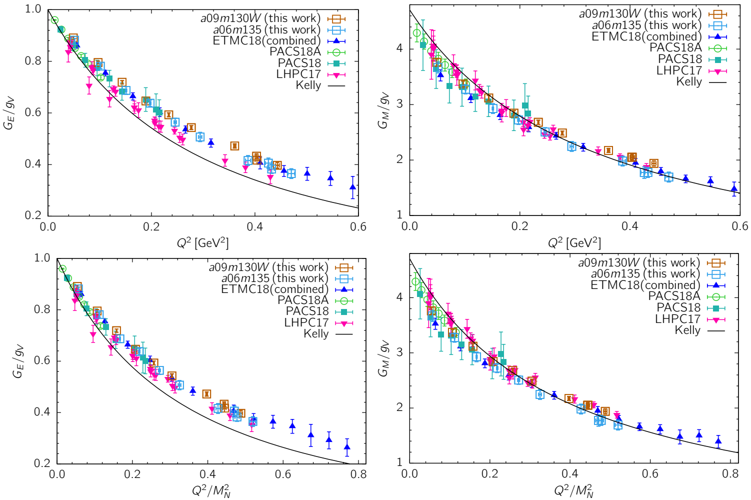
$$(r^2)_E^s = -0.0039(13)(18)\ \mathrm{fm}^2$$
$$\mu^s = -0.020(5)(12)$$
$$(r^2)_M^s = -0.010(5)(11)\ \mathrm{fm}^2$$

Wilhelm

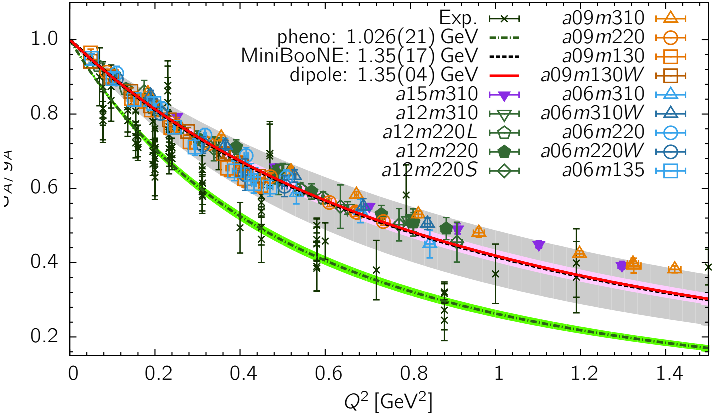
PNDME: Lattice Scale?



PNDME: world data



$$\langle N|A_\mu|N\rangle \equiv \bar{u}\left[G_A\gamma_\mu + \tilde{G}_P\frac{q_\mu}{2M}\right]\gamma_5 u \qquad \langle N|P|N\rangle \equiv \bar{u}G_P\gamma_5 u$$



Also new results (Wilhelm) for G_A^s.

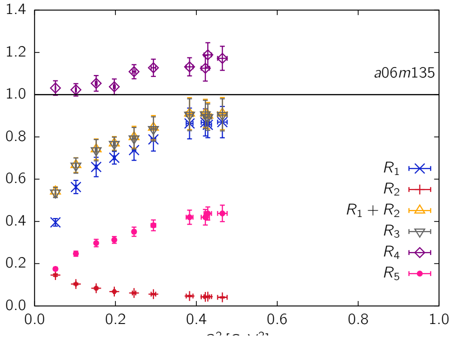
$$\langle r_A^2 \rangle \equiv -6 \, \frac{d}{dQ^2} \left(\frac{G_A(Q^2)}{G_A(0)} \right) \Bigg|_{Q^2=0}$$

- Experimental:
  - neutrino scatterring: $r_A = 0.666(17)$ fm and $M_A = 1.026(21)$ GeV
  - Deuterium: $r_A = 0.68(16)$ fm and $M_A = 1.00(24)$ GeV
  - MiniBooNE prefers $M_A \sim 1.35(17)$ GeV
- Previous: $r_A = 0.48(4)$
- Update:
  - z-expansion: $r_A = 0.481(58)(62)$ fm and $M_A = 1.42(17)(18)$ GeV
  - Dipole fit: $r_A = 0.505(13)(6)$ fm and $M_A = 1.35(3)(2)$ GeV

- Axial Ward Identity: $\partial_\mu A^\mu = 2\hat{m}P$ satisfied
- This impies

$$\left(R_1 \equiv \frac{2\hat{m}G_P}{2MG_A} 2MG_A\right) + \left(R_2 \equiv \frac{Q^2\tilde{G}_P}{4M^2G_A}\right) = 1$$

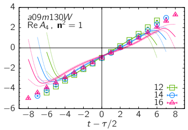
- Pion pole dominance: $R_4 \equiv \frac{4\hat{m}M_N G_P}{M_\pi^2 \tilde{G}_P} = 1$
- Combined, this means: $R_3 \equiv \frac{(Q^2+M_\pi^2)\tilde{G}_P}{4M_N^2 G_A} = 1$
- $O(a)$ correction to R_1, $= c_A R_5$ is small.
- Bali *et al.*:
  - Project A_μ with $g_{\mu\nu} - \bar{p}_\mu\bar{p}_\nu/\bar{p}^2$
  - Enforce $p^\mu A_\mu = 2\hat{m}P$
  - P corrected by $O(a^2)/\hat{m}$, which is large!



PCAC Relation

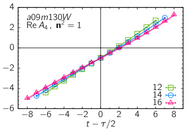
Axial Current A_4 3-pt Correlator

[arXiv:1905.06470]



[3*-state]

- $E_i, \mathcal{A}'_i$ and $M_j, \mathcal{A}_j$ are taken from 4-state fits to nucleon two-point correlator. $(i, j = 0, 1, 2)$

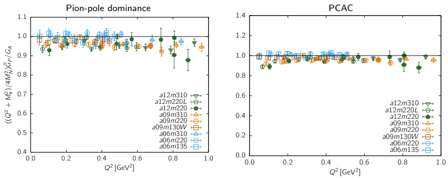


[relaxed 2-state]

- $E_0, \mathcal{A}'_0$ and $M_0, \mathcal{A}_0$ are taken from nucleon two-point correlator fits. Excited state parameters are free.

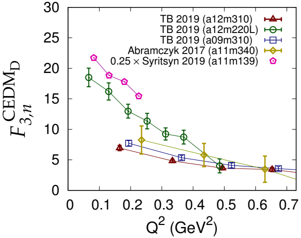
	3*-state		relaxed 2-state	
n^2	χ^2/d.o.f	p-value	χ^2/d.o.f	p-value
1	21.78	$< 5 \times 10^{-5}$	0.698	0.76
2	19.36	$< 5 \times 10^{-5}$	1.654	0.06
3	11.79	$< 5 \times 10^{-5}$	2.018	0.02

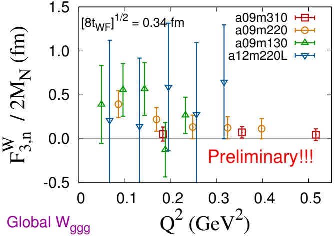
PCAC with Excited States $(N\pi)$ from A_4

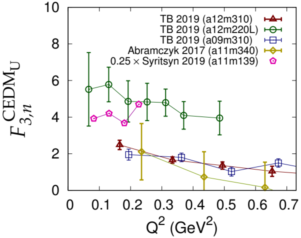


- $C^{3pt}[A_i]$ $(i = 1, 2, 3)$ and $C^{3pt}[P]$ are reanalyzed with 2-state fit using the excited states extracted from $C^{3pt}[A_4]$.
- PCAC is satisfied better for all Q^2.
- Pion-pole dominance becomes prominent.
- The deviation from the exact limit $(y = 1)$ diminises as $a \to 0$, $M_\pi \to 135\text{MeV}$.

CP violation from BSM

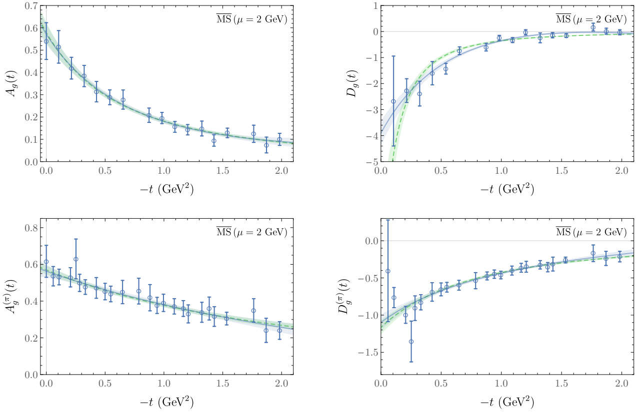






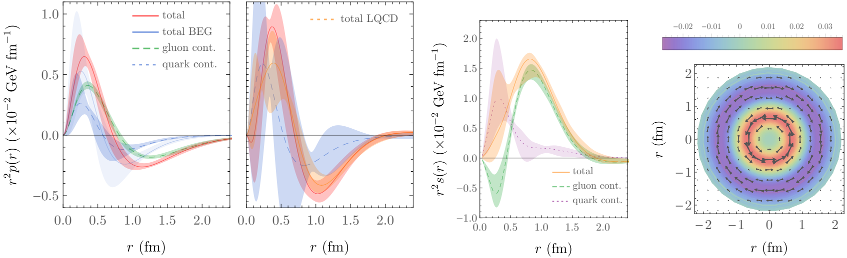
- Qualitative consistency between quark chromo-EDM calculations.
- The Weinberg operator does not yet give a signal.
- Does not yet account for operator mixing and renormalization.

Shanahan and Detmold: Gravitational Moments



- First calculations!
- $m_\pi \sim 450$ MeV

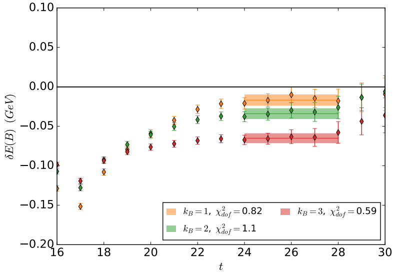
Shanahan and Detmold: Gravitational Moments

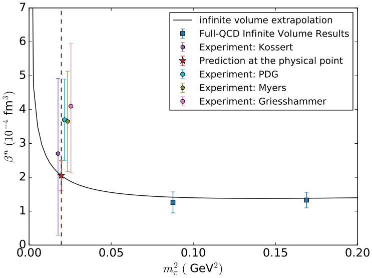


Pressure and shear distribution inside the nucleon

- Model-independent z-fits have larger statistical errors
- Systematics in phenomenological tripole parameterization not clear
- $r_{\text{Mechanical}} \sim 0.71(1)$ fm

Adelaide: Magnetic Polarizability





- Systematic errors need better estimation
  - Easy to convert statistical to systematic errors.
  - "How large can model violation reasonably be?"
  - Model averaging helps, but does not solve the problem.
  - Blind analysis required to inspire confidence.
- Still need:
  - Better control of disconnected diagrams
  - Scalar matrix elements
  - Higher dimensional (BSM) operators