Neutron Electric Dipole Moment from QCD and BSM using Clover-on-HISQ

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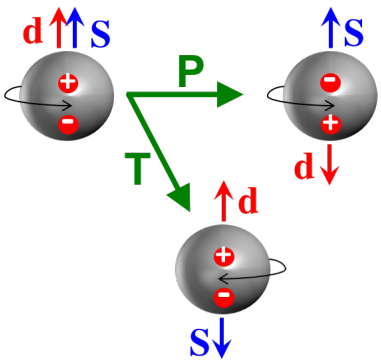
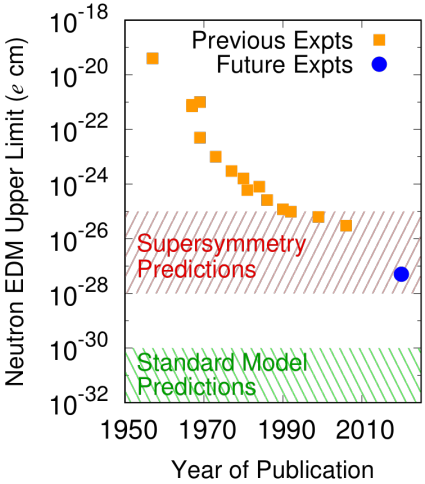
Lattice 2019, June 16 – 22, 2019, Wuhan, China

LA-UR-19-25441

Neutron Electric Dipole Moment (EDM)

- EDM measures separation between centers of (+) and (-) charges
- Nonzero nEDM violates P and T, so CP





- Searches for new source of CPV
  - CPV in SM is not enough to explain observed baryon asymmetry
- Test of Supersymmetry and other BSM models

Effective CPV Lagrangian at 1 GeV

$$
\begin{aligned}
\mathcal{L}_{\text{CPV}}^{d\leq 6} = & -\frac{g_s^2}{32\pi^2}\overline{\theta} G\tilde{G} && \text{dim=4 QCD } \theta\text{-term} \\
& -\frac{i}{2}\sum_{q=u,d,s} d_q \overline{q}(\sigma\cdot F)\gamma_5 q && \text{dim=5 Quark EDM (qEDM)} \\
& -\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q && \text{dim=5 Quark Chromo EDM (CEDM)} \\
& + d_w \frac{g_s}{6} G\tilde{G}G && \text{dim=6 Weinberg's 3g operator} \\
& + \sum_i C_i^{(4q)} O_i^{(4q)} && \text{dim=6 Four-quark operators}
\end{aligned}
$$

- $\overline{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: Strong CP problem
- Dim=5 terms suppressed by $d_q \approx \nu/\Lambda_{BSM}^2$; effectively dim=6

Calculation of Neutron and Proton EDM $d_{n,p}$

$$d_n = \overline{\theta} \cdot C_\theta + d_q \cdot C_{\mathrm{qEDM}} + \tilde{d}_q \cdot C_{\mathrm{CEDM}} + \cdots$$

- SM and BSM theories
  $\longrightarrow$ Coefficients of the effective CPV Lagrangian ($\overline{\theta}, d_q, \tilde{d}_q, \ldots$)
- Lattice QCD
  $\longrightarrow$ Nucleon matrix elements in presence of CPV interactions ($C_\theta, C_{\mathrm{qEDM}}, C_{\mathrm{CEDM}}, \ldots$)

Neutron EDM from QCD θ-term

QCD θ-term

$$S = S_{QCD} + i\theta Q, \qquad Q = \int d^4x \frac{G\tilde{G}}{32\pi^2}$$

- Known approaches:
  - External electric field method: $\langle N\overline{N}\rangle_\theta(\vec{\mathcal{E}}, t) = \langle N(t)\overline{N}(0)e^{i\theta Q}\rangle_{\vec{\mathcal{E}}}$
  - Simulation with imaginary θ: $\theta = i\tilde{\theta}, \quad S^q_\theta = \tilde{\theta}\frac{m_l m_s}{2m_s+m_l}\sum_x \overline{q}\gamma_5 q$
  - Expansion in θ

$$\langle O(x)\rangle_\theta = \frac{1}{Z_\theta}\int d[U, q, \overline{q}]O(x)e^{-S_{QCD}-i\theta Q}$$
$$= \langle O(x)\rangle_{\theta=0} - i\theta\langle O(x)Q\rangle_{\theta=0} + \mathcal{O}(\theta^2)$$

- We use expansion in θ method in order to reuse existing nucleon correlators

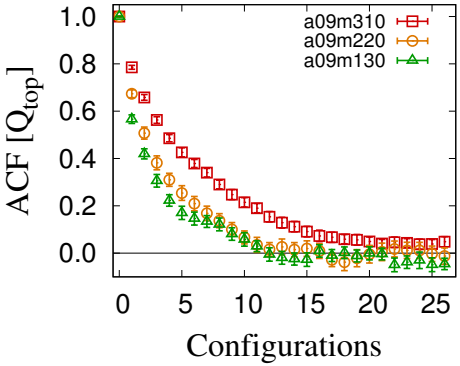
Lattices and Simulation Setup

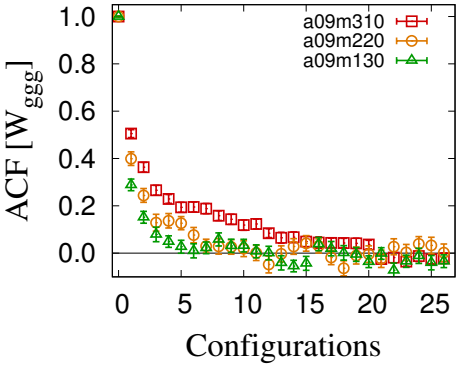
- Clover fermions on MILC HISQ lattices; $N_F = 2 + 1 + 1$

Ensemble	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	N_{conf}	N_{meas}
$a12m220L$	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	1000	128k
$a09m310$	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	2196	140k
$a09m220$	0.0872(07)	225.9(1.8)	$48^3 \times 96$	4.79	961	123k
$a09m130$	0.0871(06)	138.1(1.0)	$64^3 \times 96$	3.90	1289	165k

- Chroma QCD software suite
- Nucleon two- and three-point correlators
  - Truncated solver method (AMA) with 4HP + 128 or 64 LP per configuration
  - Current is renormalized by isovector vector charge g_V
- Topological charge and Weinberg's three-gluon operator
  - $\mathcal{O}(a^4)$-improved gluon field strength tensor [Bilson-Thompson, et al., 2002]
  - Gradient flow for cooling/renormalization

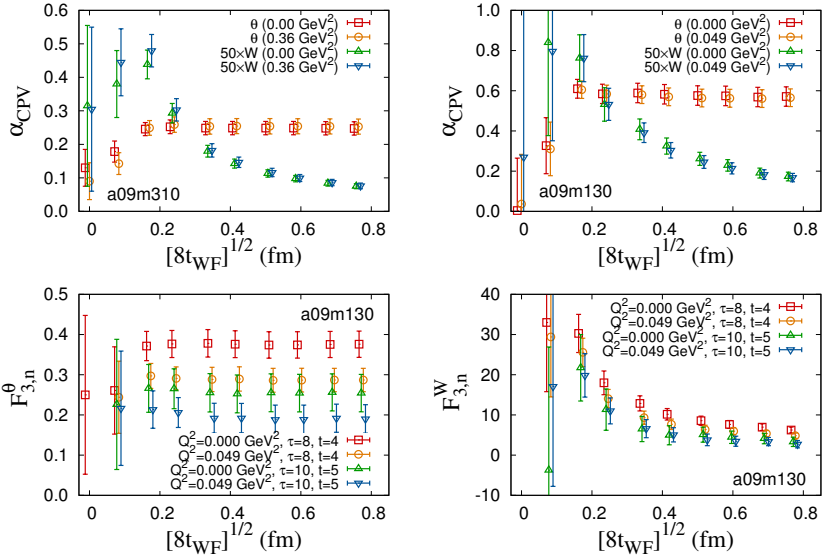
Autocorrelation of Q and W_{ggg} ($\sqrt{8t_{WF}} = 0.34\,\mathrm{fm}$)





Ensemble	a09m130	a09m220	a09m310	a12m220L
$\langle Q_{top} \rangle$	−0.30(31)	−0.30(27)	−0.01(13)	0.24(46)
$\langle W_{ggg} \rangle$	8(21)	−13(16)	−0.07(6.91)	−25(43)
F_3 anly. binsize	11	8	18	10

Gradient Flow Time Dependence



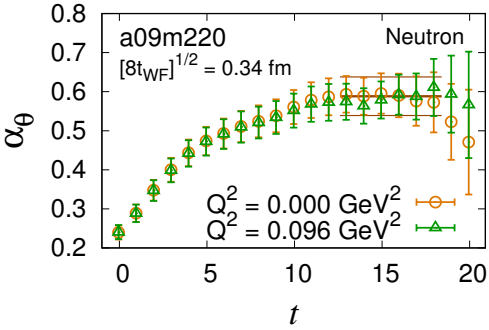
- Topological charge observables are saturating at $\sqrt{8t_{WF}} = 0.34\,\mathrm{fm}$

CPV Phase α

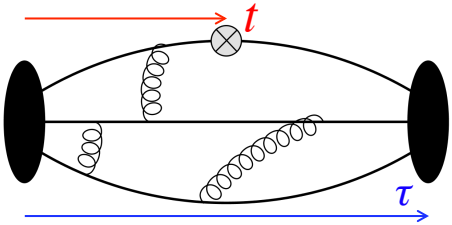
- CPV Phase α is extracted from γ_5-projected $C_{2\mathrm{pt}}$

$$\frac{\mathrm{Im}C^P_{2\mathrm{pt}}(t)}{\mathrm{Re}C_{2\mathrm{pt}}(t)} \equiv \frac{\mathrm{Im\,Tr}\left[\gamma_5 \frac{1}{2}(1+\gamma_4)\langle N(t)\overline{N}(0)\rangle\right]}{\mathrm{Re\,Tr}\left[\frac{1}{2}(1+\gamma_4)\langle N(t)\overline{N}(0)\rangle\right]} = \frac{M_N \sin\left(2\alpha(t)\right)}{E_N + M_N \cos\left(2\alpha(t)\right)}$$

- Final α is obtained from plateau average over $\alpha(t \gg 1)$ where ESC is small



CPV Form Factor F_3



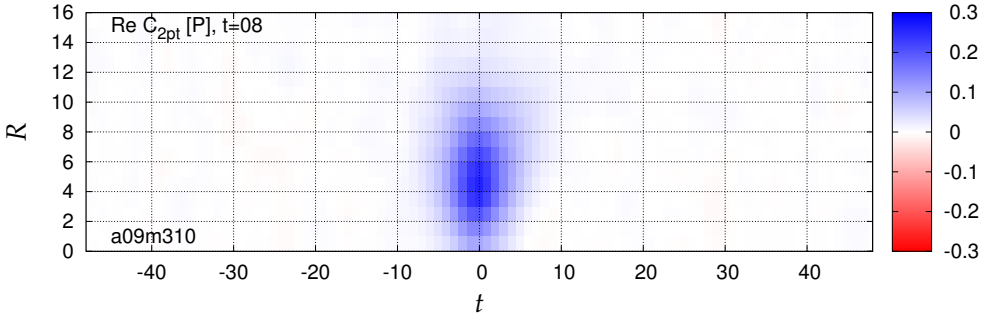
- Neutron EDM $d_n = |e| F_3(Q^2 = 0) \ / \ 2M_N$
- Source and sink separation in Euclidean τ, current insertion timeslice t
- F_3 is extracted from $C_{3\mathrm{pt}}$ with vector current insertion

$$\langle N|V_\mu(q)|N\rangle_{\mathrm{CPV}} = \overline{u}_N \left[F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu - \frac{F_3(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu\gamma_5 \right] u_N(p)$$

Caveat: CPV interactions introduce phase in neutron mass
$\rightarrow \gamma_4$ no longer a parity operator of neutron state [Abramczyk, et al., 2017]

- Neutron F_3 obtained from $V_{\mu=4}$ is insensitive to α_{CPV} because α contribution comes with G_E, which vanishes when $Q^2 = 0$

Correlation between Q and $C_{2\mathrm{pt}}$

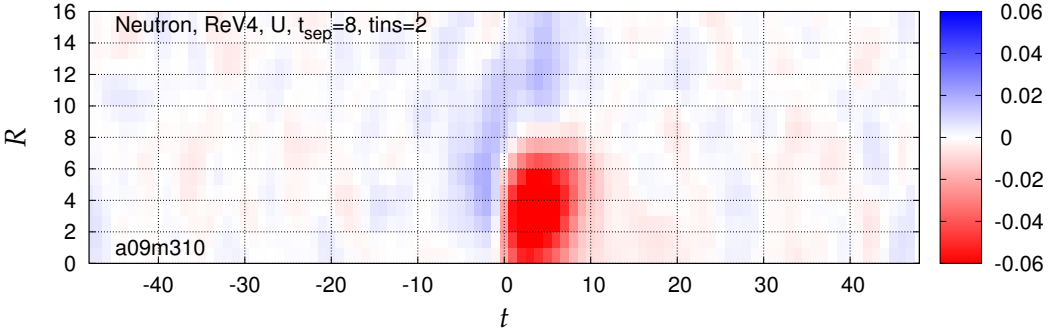


- Correlation map of $Q^{\mathrm{local}}(R,t)$ and $\mathrm{Re}C^{P}_{2\mathrm{pt}}(8) \equiv \mathrm{Re}\,\mathrm{Tr}\left[\gamma_5 \frac{1}{2}(1+\gamma_4)\langle N(4)\overline{N}(0)\rangle\right]$
- Local topological charge density is calculated

$$Q^{\mathrm{local}}(R,t) = \sum_{R<|\mathbf{x}_{\mathrm{src}}-\mathbf{x}|<R+1} q(\mathbf{x},t)$$

- Nucleon two-point correlator with source at $(\mathbf{x}_{\mathrm{src}}, t=0)$, sink at $t=8$

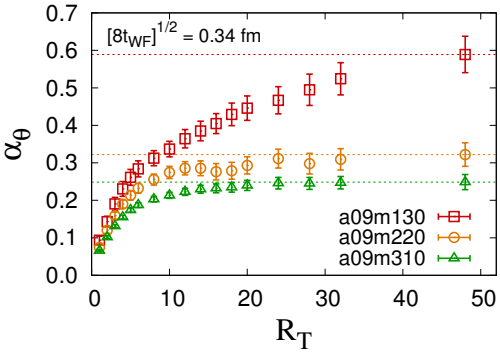
Correlation between Q and $C_{3\mathrm{pt}}$

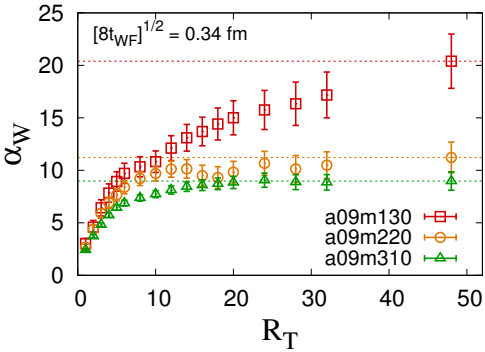


- Correlation map of $Q^{\mathrm{local}}(R,t)$ and $\mathrm{Re}C_{3\mathrm{pt}}^{V_4,U}(\tau=8,t=2)$
- Local topological charge density is calculated

$$Q^{\mathrm{local}}(R,t) = \sum_{R<|\mathbf{x}_{\mathrm{src}}-\mathbf{x}|<R+1} q(\mathbf{x},t)$$

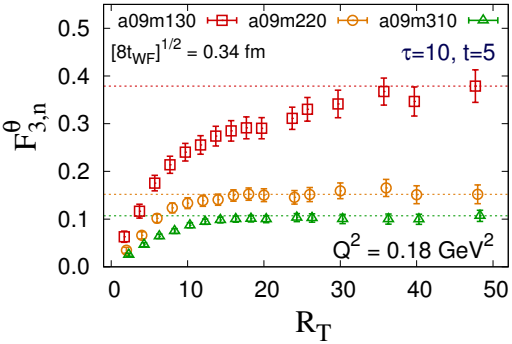
Error Reduction using Localized Correlation - $C_{2\mathrm{pt}}$

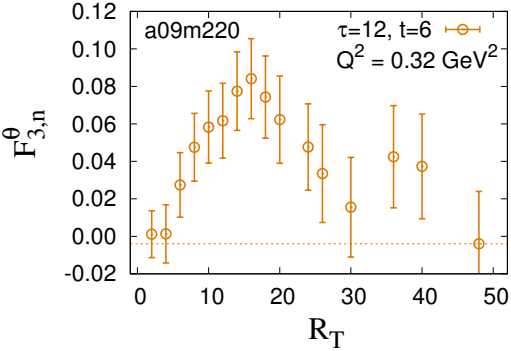




- Cylinder method needs large spatial R [Syritsyn, Lattice 2018], so we take $R = \infty$
- Topological charge summed only within $|t_Q - t_{\mathrm{src}}| \leq R_T$
- CPV phase α forms plateau at $R_T \approx 25a$ for a09m310 and a09m220, but no saturation in a09m130; we use $R_T = \infty$ for α in this study
- Neutron F_3 at small Q^2 is insensitive to α because of the $G_E(Q^2)$ suppression

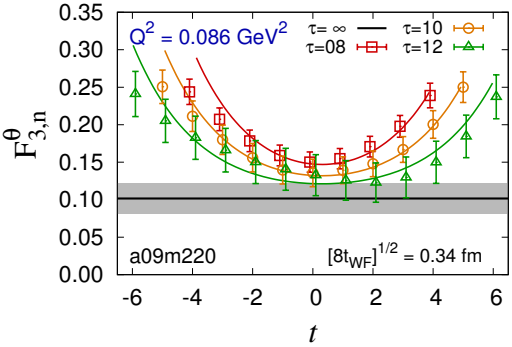
Error Reduction using Localized Correlation - $C_{3\mathrm{pt}}$

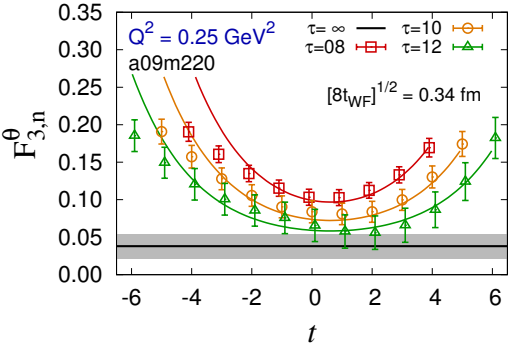




- Saturation depends on the distance from the current insertion t rather than t_{src}
  $\longrightarrow$ Topological charge locally summed in $|t_Q - t| \leq R_T$
- For most of the momenta, τ and t,
  R_T/a = **14** (a09m310), **20** (a09m220), **36** (a09m130) and **12** (a12m220L)
  produce reasonable results
- Warning: sometimes, the localized correlation brings you to 2.5σ away

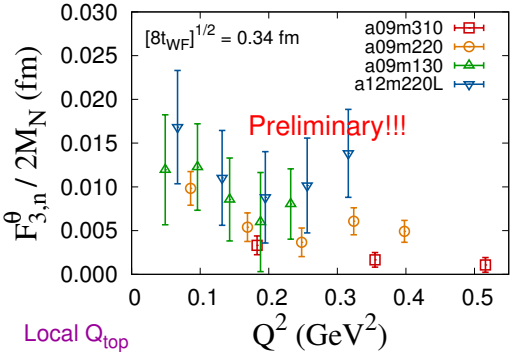
Removing Excited State Contamination



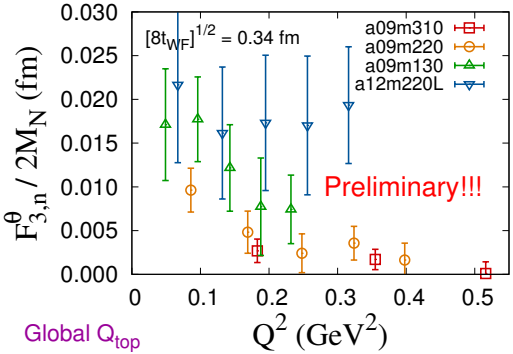


- **Two-state fit** to the F_3/g_V obtained at each τ and t

Results: Neutron EDM from QCD θ-term

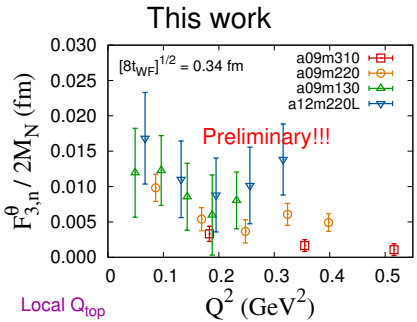


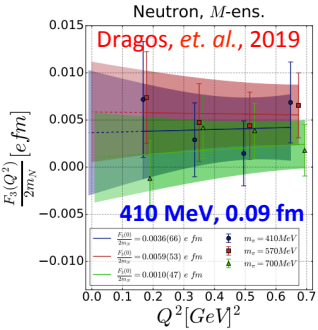
Local Q_{top}

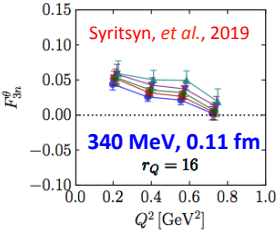


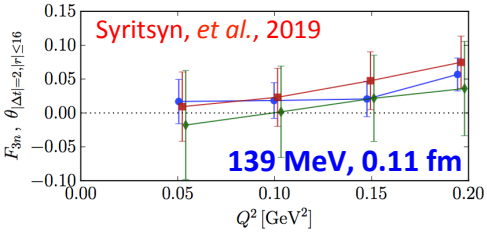
Global Q_{top}

Comparison: Neutron EDM from QCD θ-term

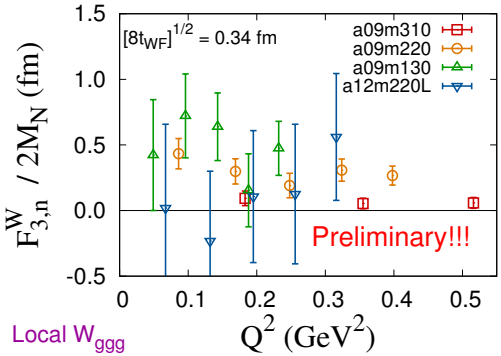




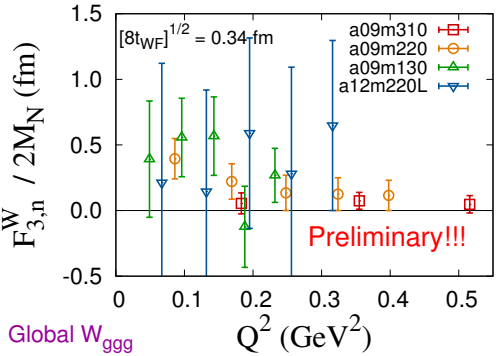




Results: Neutron EDM from QCD Weinberg's Three-gluon term



Local W_{ggg}



Global W_{ggg}

Neutron EDM from quark Chromo-EDM (CEDM)

Schwinger Source Method for CEDM

- Quark chromo EDM operator is a quark bilinear

$$\frac{i}{2}\overline{q}(\sigma \cdot G)\gamma_5 q$$

- Include CEDM term in valence quark propagators by changing Dirac operator

$$D_{clov} \to D_{clov} + \frac{i}{2}\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

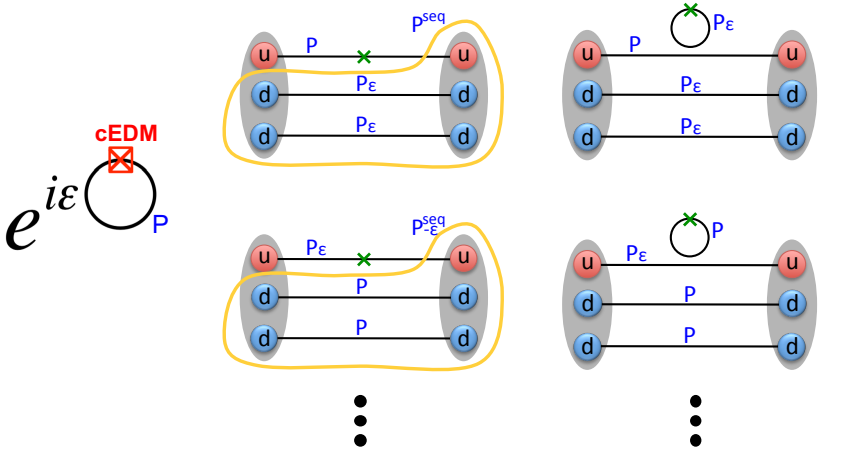
  which is equivalent to shifting the Sheikholeslami-Wohlert coefficient

$$c_{sw}\sigma^{\mu\nu}G_{\mu\nu} \to \sigma^{\mu\nu}(c_{sw} + 2i\varepsilon\gamma_5)G_{\mu\nu}$$

- Avoid four-point correlation function calculation
- Fermion determinant gives reweighting factor (disconnected diagrams)

$$\frac{\det\left(D_{clov} + \frac{i}{2}\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\right)}{\det\left(D_{clov}\right)} \approx \exp\left[\frac{i}{2}\varepsilon\,\mathrm{Tr}\left(\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} D_{clov}^{-1}\right)\right]$$

Schwinger Source Method for CEDM



Reweighting diagrams **Connected Diagrams** **Disconnected**

Lattices and Simulation Setup

- Clover fermions on MILC HISQ lattices; $N_F = 2 + 1 + 1$

Ensemble	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	N_{conf}	N_{meas}
$a12m310$	0.1207(11)	310.2(2.8)	$32^3 \times 64$	4.55	1012	130k
$a12m220L$	0.1189(09)	227.6(1.7)	$40^3 \times 64$	5.49	475	61k
$a09m310$	0.0888(08)	313.0(2.8)	$32^3 \times 96$	4.51	447	57k

- Chroma QCD software suite
- Truncated solver method (AMA) with 4HP + 128LP per configuration
- Variance reduction using correlated zeros
- Results without renormalization
- Two CPV operators that mix under renormalization: $\frac{i}{2}\overline{q}(\sigma \cdot G)\gamma_5 q$ and $-i\overline{q}\gamma_5 q$

Variance Reduction using Correlated Zeros

- Expectation value of a target observable $\langle O \rangle$ does not change by adding observables that average to zero $\langle z_i \rangle = 0$ with any coefficient c_i

$$\langle O \rangle = \langle O + \textstyle\sum_i c_i z_i \rangle$$

By taking $c_i = \sigma_O^2 [\sigma_z^{-2}]_{ij} z_j$, the new combination minimizes variance

[Bhattacharya, Lattice 2018]

- For α and F_3, subtracting those calculated with $\varepsilon = 0$ **cancel noise**

$$\alpha^{\mathrm{VR}}(\varepsilon) = \alpha(\varepsilon) - c_\alpha \alpha(\varepsilon = 0), \qquad F_3^{\mathrm{VR}}(\varepsilon) = F_3(\varepsilon) - c_F F_3(\varepsilon = 0),$$

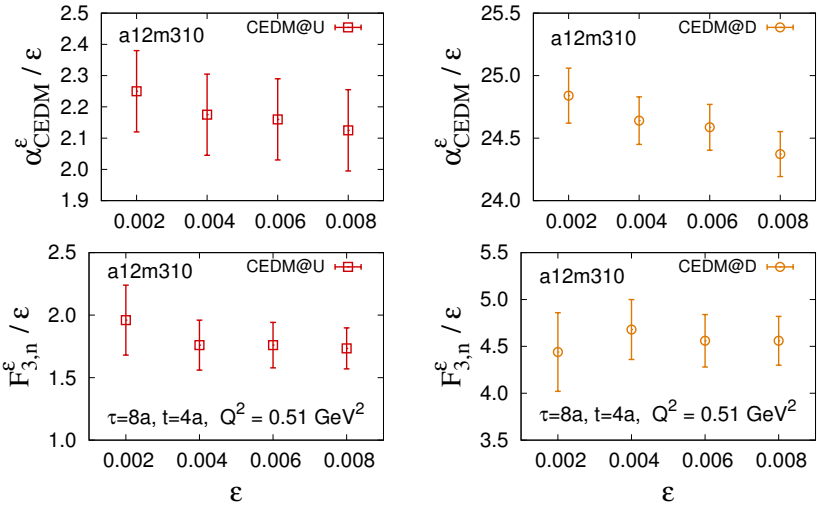
where $c_\alpha \approx 1$, $c_F \approx 1$.

- For tiny ε, CPV signal is buried in gauge noise, so variance reduction is necessary

- Reweighting θ-term with small ε and using VR gives small α (so that linear approximation works) and minimum modification of energies and F_1, F_2

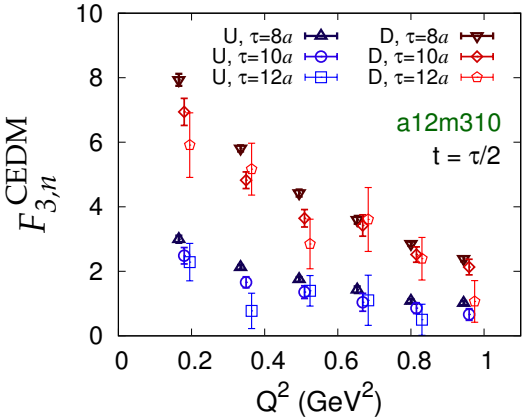
ε	F_3	F_3^{VR}
0.002	0.9(2.2)	0.98(14)
0.004	0.8(1.1)	0.88(10)
0.006	0.84(74)	0.880(91)
0.008	0.84(55)	0.867(82)

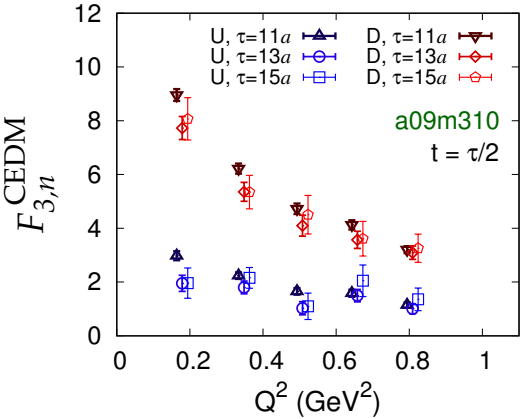
Linearity of α_{CPV} and F_3 in small ε



- CEDM operator insertions in Dirac operator
  $\longrightarrow \alpha^{\varepsilon}_{\mathrm{CPV}} \approx \varepsilon \cdot \alpha_{\mathrm{CPV}}$ and $F_3^{\varepsilon} \approx \varepsilon \cdot F_3$ for small ε

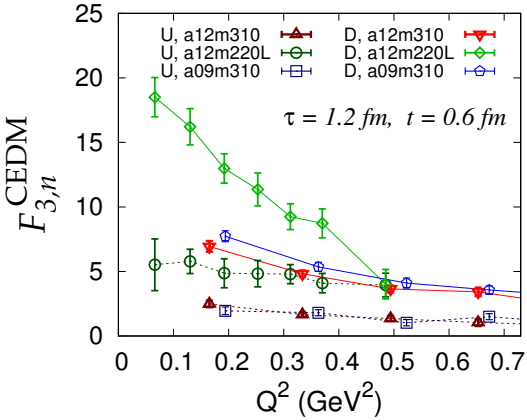
Excited State Effect

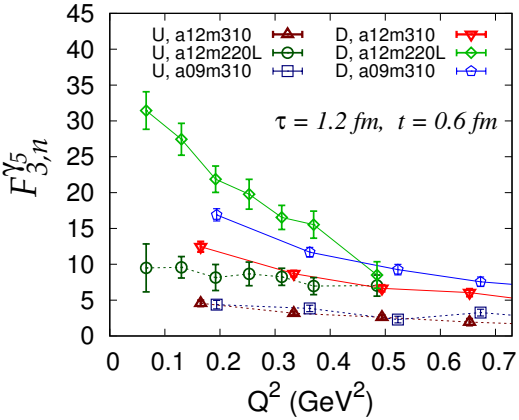




- Excited state effect seems to be small when $\tau \approx 1.2\,\mathrm{fm}$

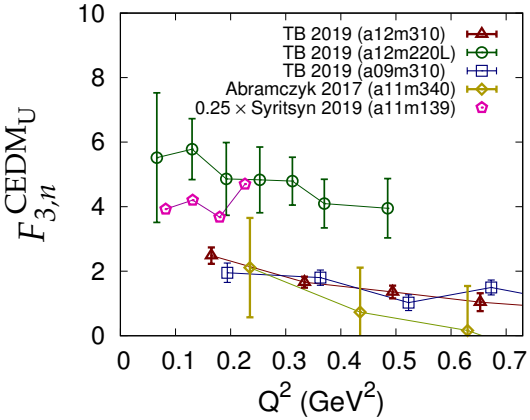
Results: F_3 - Connected Diagrams Only, No Renormalization

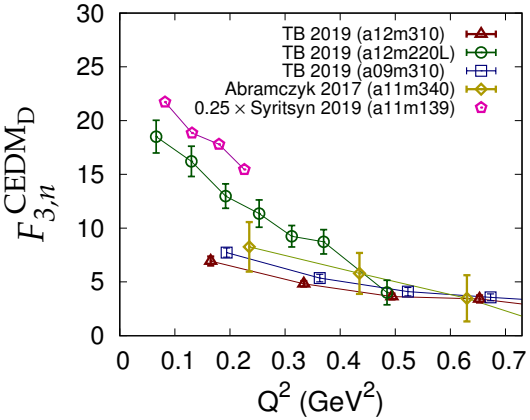




- Bare F_3 without renormalization

Comparison: F_3 - Connected Diagrams Only, No Renormalization





- Syritsyn 2019 (a11m139) results are scaled by $\frac{1}{4}$ to make it fit within range
- Qualitative agreement between different calculations
- Need further study for pion mass dependence

Conclusion

- Neutron EDMs from QCD θ-term, Weinberg's three-gluon term and quark chromo EDM term are calculated using Clover fermions on HISQ lattices
- Correlation between topological charge and $C_{\mathrm{2pt,3pt}}$ is explored
- Preliminary results at multiple pion mass ≤ 310 MeV are presented
- Need further investigation

Acknowledgement

- Simulations were carried out on computer facilities of (i) the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231, (ii) the Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725, (iii) the USQCD Collaboration, which are funded by the Office of Science of the U.S. Department of Energy, and (iv) Institutional Computing at Los Alamos National Laboratory.

- This work is supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under Contract No. DE-AC52-06NA25396 and by the LANL LDRD program.