Exclusive Channel Study of the Muon HVP

Aaron S. Meyer (ameyer@quark.phy.bnl.gov)

in collaboration with:
Mattia Bruno, Taku Izubuchi, Christoph Lehner

for the RBC/UKQCD Collaboration

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The RBC & UKQCD collaborations

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder
Oliver Witzel

CERN
Mattia Bruno

Columbia University
Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu

Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University
Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

KEK
Julien Frison

University of Liverpool
Nicolas Garron

MIT
David Murphy

Peking University
Xu Feng

University of Regensburg
Christoph Lehner (BNL)

University of Southampton
Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)
Introduction
▶ Motivation from muon $g - 2$
▶ Tensions in $\pi\pi$ Scattering
▶ Calculation of the HVP using Lattice QCD

Correlation Function Spectrum & Overlap
▶ Lattice Parameters
▶ GEVP Spectrum & Overlaps
▶ $\pi\pi$ Scattering Phase Shift
▶ $4\pi$ Correlation Functions

Bounding Method and the Muon HVP
▶ Correlation Function Reconstruction
▶ (Improved) Bounding Method
▶ Results

Conclusions/Outlook
Introduction
### Pieces of Muon $g - 2$ Theory Prediction

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value $\times 10^{10}$</th>
<th>Uncertainty $\times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>11 658 471.895</td>
<td>0.008</td>
</tr>
<tr>
<td>EW</td>
<td>15.4</td>
<td>0.1</td>
</tr>
<tr>
<td>HVP LO</td>
<td>692.5</td>
<td>2.7</td>
</tr>
<tr>
<td>HVP NLO</td>
<td>-9.84</td>
<td>0.06</td>
</tr>
<tr>
<td>HVP NNLO</td>
<td>1.24</td>
<td>0.01</td>
</tr>
<tr>
<td>Hadronic light-by-light</td>
<td>10.5</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Total SM prediction</strong></td>
<td><strong>11 659 181.7</strong></td>
<td><strong>3.8</strong></td>
</tr>
<tr>
<td>BNL E821 result</td>
<td>11 659 209.1</td>
<td>6.3</td>
</tr>
<tr>
<td>Fermilab E989 target</td>
<td>$\approx 1.6$</td>
<td></td>
</tr>
</tbody>
</table>

Experiment-Theory difference is $27.4(7.3) \implies 3.7\sigma$ tension!

[1904.09479[hep-lat]]
Tensions in Experiment

R-ratio data for $ee \rightarrow \pi\pi$ exclusive channel, $\sqrt{s} = 0.6 - 0.9$ GeV region
Tension between most precise measurements
Other measurements not precise enough to favor one over the other

Avoid tension by computing precise lattice-only estimate of $a_{\mu}^{HVP}$
Use lattice QCD to inform experiment, resolve discrepancy
Exclusive Channels in the HVP

\[ C(t) = \frac{1}{3} \sum_i \left\langle \left[ \bar{\psi} \gamma_i \psi \right]_t \left[ \bar{\psi} \gamma_i \psi \right]_0 \right\rangle \approx \sum_n \left| \left\langle \Omega | \bar{\psi} \gamma_i \psi | n \right\rangle \right|^2 e^{-E_n t} \]

Goal is to compute local vector current precisely, then integrate with a weighting kernel to get \( a_{\mu}^{HVP} \).

Correlator has large statistical error in long-distance region, but contributions from high energy states are exponentially suppressed.
Exclusive Channels in the HVP

\[ C(t) = \frac{1}{3} \sum_i \langle [\bar{\psi} \gamma_i \psi]_t [\bar{\psi} \gamma_i \psi]_0 \rangle \approx \sum_n |\langle \Omega |\bar{\psi} \gamma_i \psi |n \rangle|^2 e^{-E_n t} \]

Goal is to compute local vector current precisely, then integrate with a weighting kernel to get \( a^{HVP}_\mu \)

Correlator has large statistical error in long-distance region, but contributions from high energy states are exponentially suppressed

Use exclusive study to replace long-distance region with reconstruction of exact functional form to trade large statistical uncertainty for smaller systematics

Long distance correlator dominated by two-pion states, but overlap of vector current with two-pion states is minimal

- Construct & measure operators that overlap strongly with these states
- Correlate these operators with the local vector current
Correlation Function Spectrum & Overlap
Computed on $2 + 1$ flavor Möbius Domain Wall Fermions for valance and sea, $M_\pi$ at physical value on all ensembles

Results in this talk will use three ensembles:

- **“24ID”**: $24^3 \times 64$ (4.8 fm), $a \approx 0.194$ fm $\approx 1.015$ GeV$^{-1}$
- **“32ID”**: $32^3 \times 64$ (6.2 fm), $a \approx 0.194$ fm $\approx 1.015$ GeV$^{-1}$
- **“48I”**: $48^3 \times 96$ (5.5 fm), $a \approx 0.114$ fm $\approx 1.730$ GeV$^{-1}$

Additional $64^3$ ensemble for continuum extrapolation with $48^3$ ensemble $\implies$ to be included in future work
Operators

Distillation used to build large operator basis \implies smearing kernel $f$

Operators constructed in $I = 1$, $P$-wave channel to impact upon $HVP_\mu$

Vector current operators:

\begin{itemize}
  \item Local $O_{J\mu} = \sum_x \bar{\psi}(x) \gamma_\mu \psi(x)$, $\mu \in \{1, 2, 3\}$
  \item Smeread $O_{j\mu} = \sum_{xyz} \bar{\psi}(x) f(x - z) \gamma_\mu f(z - y) \psi(y)$
\end{itemize}

2\pi operators with $O_n$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 0, 0)\}$

$$O_n = \left| \sum_{xyz} \bar{\psi}(x) f(x - z) e^{-i \vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z - y) \psi(y) \right|^2$$

Also test a 4\pi operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (1, 0, 0)$:

$$O_{4\pi} = \left| \sum_{xyz} \bar{\psi}(x) f(x - z) e^{-i \vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z - y) \psi(y) \right|^2 \left| \sum_{xy} \bar{\psi}(x) f(x - y) \gamma_5 \psi(y) \right|^2$$

Spectrum & overlap estimates from Generalized EigenValue Problem (GEVP):

$$C(t_0) V = C(t_0 + \delta t) V \Lambda(\delta t); \quad \Lambda_{nn}(\delta t) \sim e^{+E_n\delta t}, \quad V_{im} \propto \langle \Omega | O_i | m \rangle$$

Exponential dependence of local vector correlation function reconstructed as

$$C_{ij}^{\text{latt.}}(t) = \sum_{n}^{N} \langle \Omega | O_i | n \rangle \langle n | O_j | \Omega \rangle e^{-E_nt}$$
GEVP Results - $J_\mu + 2\pi$ Operators only

PRELIMINARY

6-operator basis on 48I ensemble: local+smeared vector, $4 \times (2\pi)$

Data points from solving GEVP at fixed $\delta t$

$$C(t_0) \, V = C(t_0 + \delta t) \, V \, \Lambda(\delta t) \, , \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Excited state contaminations decay as $t_0, \delta t \to \infty$

moving right on plot $\implies$ asymptote to lowest states’ spectrum & overlaps

Left: Spectrum; Right: Overlap with local vector current
From spectrum, can compute pion scattering phase shifts in $l = 1$ channel
Statistics + systematic uncertainties included

Used to explicitly calculate FV corrections at physical $M_\pi$ (C.Lehner, Lattice 2018)

Compare to simple Breit-Wigner parametrization and pheno (courtesy of M.Bruno)
Good agreement with pheno for 32ID, 48I
24ID data not at plateau, but improved with fit to data

Scattering phase shift results to appear as part of series of papers by RBC+UKQCD
Phase Shift

PRELIMINARY

BW: \( \cot \delta_{11} = -\frac{2}{\Gamma_{\rho}} (E - E_{\rho}) \)

From spectrum, can compute pion scattering phase shifts in \( l = 1 \) channel
Statistics + systematic uncertainties included

Used to explicitly calculate FV corrections at physical \( M_\pi \) (C. Lehner, Lattice 2018)

Compare to simple Breit-Wigner parametrization and pheno (courtesy of M. Bruno)
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Aaron S. Meyer  Section: Correlation Function Spectrum & Overlap  12/25
Breakdown of formalism for phase shifts +FVC could occur at $4\pi$ threshold
Compute $2\pi \rightarrow 4\pi$ and $4\pi \rightarrow 4\pi$ correlation functions and check explicitly
$4\pi \rightarrow 4\pi$ has $\sim 1000$ independent Wick contractions
Spectrum unaffected by inclusion of $4\pi$ operator, but state is resolvable
Breakdown of formalism for phase shifts +FVC could occur at $4\pi$ threshold
Compute $2\pi \rightarrow 4\pi$ and $4\pi \rightarrow 4\pi$ correlation functions and check explicitly
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Spectrum unaffected by inclusion of $4\pi$ operator, but state is resolvable
Overlap of $4\pi$ state with local vector current unresolvable
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Compute $2\pi \rightarrow 4\pi$ and $4\pi \rightarrow 4\pi$ correlation functions and check explicitly
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Spectrum unaffected by inclusion of $4\pi$ operator, but state is resolvable

Overlap of $4\pi$ state with local vector current unresolvable

Overlap of state with $4\pi$ operator significant
$\Rightarrow$ $4\pi$ state safely negligible in local vector current
Bounding Method and the Muon HVP
GEVP results to reconstruct long-distance behavior of local vector correlation function needed to compute connected HVP

Explicit reconstruction good estimate of correlation function at long-distance, missing excited states at short-distance

More states \(\rightarrow\) better reconstruction, can replace \(C(t)\) at shorter distances
Improved Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on $a^{HVP}_\mu$

$$\tilde{C}(t; t_{\text{max}}, E) = \begin{cases} 
C(t) & t < t_{\text{max}} \\
C(t_{\text{max}})e^{-E(t - t_{\text{max}})} & t \geq t_{\text{max}}
\end{cases}$$

Upper bound: $E \leq E_0$, lowest state in spectrum

Lower bound: $E \geq \log\left(\frac{C(t_{\text{max}})}{C(t_{\text{max}} + 1)}\right)$

BMW (K.Miura, Lattice2018) take $E \to \infty$

With good control over lower states in spectrum from exclusive reconstruction:

Replace $C(t) \to C(t) - \sum_{n}^{N} |c_n|^2 e^{-E_n t}$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\implies$ Smaller overall contribution from neglected states

Add back contribution from reconstruction after bounding correlator
No bounding method:
Bounding method \( t_{\text{max}} = 3.3 \) fm, no reconstruction:
\[ a_\mu^{\text{HVP}} = 646(38), \]
\[ a_\mu^{\text{HVP}} = 631(16) \]

Bounding method gives factor of 2 improvement over no bounding method
Improving the bounding method increases gain to factor of 7, including systematics
Bounding Method Results - 481

No bounding method:  \( a_{\mu}^{HVP} = 646(38) \)
Bounding method  \( t_{\text{max}} = 3.3 \text{ fm} \), no reconstruction:  \( a_{\mu}^{HVP} = 631(16) \)
Bounding method  \( t_{\text{max}} = 3.0 \text{ fm} \), 1 state reconstruction:  \( a_{\mu}^{HVP} = 631(12) \)

Bounding method gives factor of 2 improvement over no bounding method
Improving the bounding method increases gain to factor of 7, including systematics
Bounding Method Results - 48I

No bounding method:

Bounding method $t_{\text{max}} = 3.3$ fm, no reconstruction:

Bounding method $t_{\text{max}} = 3.0$ fm, 1 state reconstruction:

Bounding method $t_{\text{max}} = 2.9$ fm, 2 state reconstruction:

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

$\sum_{{t}} \frac{w(t)C(t)}{10^{-10}}$

PRELIMINARY
Bounding Method Results - 48I

No bounding method:
Bounding method $t_{\text{max}} = 3.3$ fm, no reconstruction: $a_{\mu}^{\text{HVP}} = 646(38)$
Bounding method $t_{\text{max}} = 3.0$ fm, 1 state reconstruction: $a_{\mu}^{\text{HVP}} = 631(16)$
Bounding method $t_{\text{max}} = 2.9$ fm, 2 state reconstruction: $a_{\mu}^{\text{HVP}} = 633(12)$
Bounding method $t_{\text{max}} = 2.2$ fm, 3 state reconstruction: $a_{\mu}^{\text{HVP}} = 624.3(7.5)$

Bounding method gives factor of 2 improvement over no bounding method
Improving the bounding method increases gain to factor of 7, including systematics
Bounding Method Results - 48l

No bounding method: \( a^{HVP}_\mu = 646(38) \)

Bounding method \( t_{\text{max}} = 3.3 \) fm, no reconstruction: \( a^{HVP}_\mu = 631(16) \)

Bounding method \( t_{\text{max}} = 3.0 \) fm, 1 state reconstruction: \( a^{HVP}_\mu = 631(12) \)

Bounding method \( t_{\text{max}} = 2.9 \) fm, 2 state reconstruction: \( a^{HVP}_\mu = 633(10) \)

Bounding method \( t_{\text{max}} = 2.2 \) fm, 3 state reconstruction: \( a^{HVP}_\mu = 624.3(7.5) \)

Bounding method \( t_{\text{max}} = 1.8 \) fm, 4 state reconstruction: \( a^{HVP}_\mu = 625.0(5.4) \)

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics
Outlook and Conclusions
Summary

▶ $g - 2$ is an interesting and exciting topic to work on!
▶ Tensions in experimental $ee \rightarrow \pi\pi$ data make independent study of exclusive channels valuable
▶ Progress this year in extending our analysis to include three lattice ensembles
▶ Computed $2\pi \rightarrow 4\pi, 4\pi \rightarrow 4\pi$ correlation functions to show explicitly that $4\pi$ state has negligible effect on HVP
▶ Study of exclusive channels able to significantly reduce statistical uncertainty on an all-lattice computation of muon HVP
  $\implies$ factor of 4 more statistics on 48l now
  $\implies$ expect to reach precision of $O(5 \times 10^{-10})$ by the end of year
▶ This calculation enables direct calculation of FV correction at physical $M_\pi$
  (see C.Lehner, Lattice2018)
▶ Part of ongoing lattice study to address all lattice systematics in RBC+UKQCD HVP computation (see talk by C.Lehner, Lattice2019)
▶ Several configurations of $64^3$ data computed, to be included in future studies

Thank you!