Exclusive Channel Study of the Muon HVP

Aaron S. Meyer (ameyer@quark.phy.bnl.gov)

in collaboration with:
Mattia Bruno, Taku Izubuchi, Christoph Lehner

for the RBC/UKQCD Collaboration

Brookhaven National Laboratory

June 17, 2019

37th International Symposium on Lattice Field Theory

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu

Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

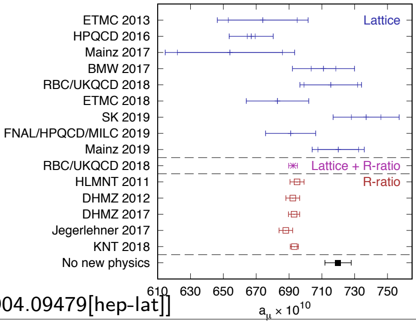
- Introduction
  - Motivation from muon $g-2$
  - Tensions in $\pi\pi$ Scattering
  - Calculation of the HVP using Lattice QCD
- Correlation Function Spectrum & Overlap
  - Lattice Parameters
  - GEVP Spectrum & Overlaps
  - $\pi\pi$ Scattering Phase Shift
  - 4π Correlation Functions
- Bounding Method and the Muon HVP
  - Correlation Function Reconstruction
  - (Improved) Bounding Method
  - Results
- Conclusions/Outlook

Introduction

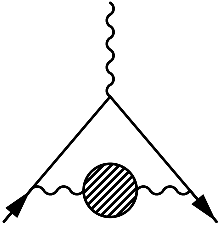
Pieces of Muon $g-2$ Theory Prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.5	2.7
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	2.6
Total SM prediction	11 659 181.7	3.8
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		≈ 1.6

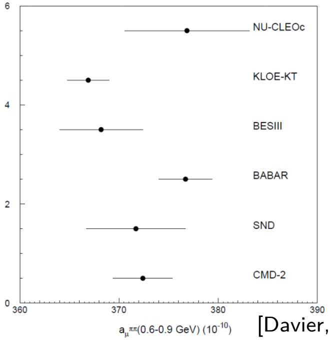
Experiment-Theory difference is 27.4(7.3) $\Longrightarrow$ 3.7σ tension!



[1904.09479[hep-lat]]



Tensions in Experiment

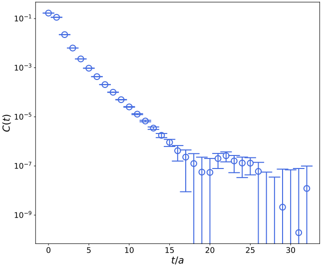


[Davier, KEK (2018)]

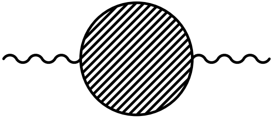
R-ratio data for $ee \to \pi\pi$ exclusive channel, $\sqrt{s} = 0.6 - 0.9\ \mathrm{GeV}$ region
Tension between most precise measurements
Other measurements not precise enough to favor one over the other

Avoid tension by computing precise lattice-only estimate of a_μ^{HVP}
Use lattice QCD to inform experiment, resolve discrepancy

Exclusive Channels in the HVP



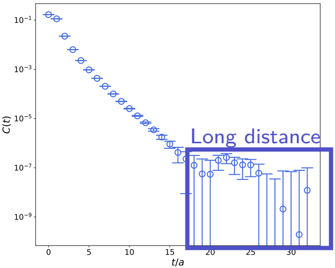
$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle\Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$



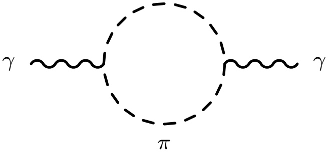
Goal is to compute local vector current precisely,
then integrate with a weighting kernel to get a_μ^{HVP}

Correlator has large statistical error in long-distance region,
but contributions from high energy states are exponentially suppressed

Exclusive Channels in the HVP



$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle \Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$



Goal is to compute local vector current precisely,
then integrate with a weighting kernel to get a_μ^{HVP}

Correlator has large statistical error in long-distance region,
but contributions from high energy states are exponentially suppressed

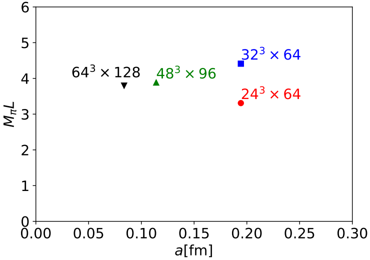
Use exclusive study to replace long-distance region with reconstruction of
exact functional form to trade large statistical uncertainty for smaller systematics

Long distance correlator dominated by two-pion states,
but overlap of vector current with two-pion states is minimal

- Construct & measure operators that overlap strongly with these states
- Correlate these operators with the local vector current

Correlation Function Spectrum & Overlap

Computation Details



Computed on $2+1$ flavor Möbius Domain Wall Fermions for valance and sea, M_π at physical value on all ensembles

Results in this talk will use three ensembles:

- "24ID": $24^3 \times 64$ $(4.8\ \mathrm{fm})$, $a \approx 0.194\ \mathrm{fm} \approx 1.015\ \mathrm{GeV}^{-1}$
- "32ID": $32^3 \times 64$ $(6.2\ \mathrm{fm})$, $a \approx 0.194\ \mathrm{fm} \approx 1.015\ \mathrm{GeV}^{-1}$
- "48I": $48^3 \times 96$ $(5.5\ \mathrm{fm})$, $a \approx 0.114\ \mathrm{fm} \approx 1.730\ \mathrm{GeV}^{-1}$

Additional 64^3 ensemble for continuum extrapolation with 48^3 ensemble
$\Longrightarrow$ to be included in future work

Operators

Distillation used to build large operator basis $\implies$ smearing kernel f
Operators constructed in $I = 1$, P-wave channel to impact upon HVP_μ

Vector current operators:

- Local $\mathcal{O}_{J_\mu} = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x)$, $\mu \in \{1,2,3\}$
- Smeared $\mathcal{O}_{j_\mu} = \sum_{xyz} \bar{\psi}(x)f(x-z)\gamma_\mu f(z-y)\psi(y)$

2π operators with $\mathcal{O}_n$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1),(2,0,0)\}$

$$\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2$$

Also test a 4π operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (1,0,0)$:

$$\mathcal{O}_{4\pi} = \left|\sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2 \left|\sum_{xy} \bar{\psi}(x)f(x-y)\gamma_5\psi(y)\right|^2$$

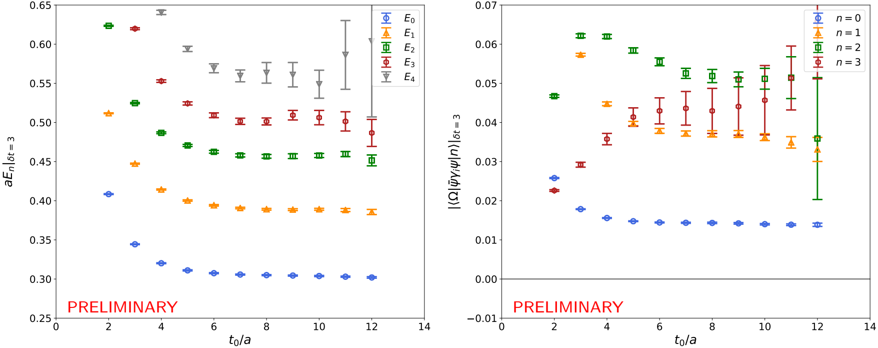
Spectrum & overlap estimates from Generalized EigenValue Problem (GEVP):

$$C(t_0)\,V = C(t_0+\delta t)\,V\,\Lambda(\delta t)\,; \quad \Lambda_{nn}(\delta t) \sim e^{+E_n\delta t}\,,\ V_{im} \propto \langle\Omega|\,\mathcal{O}_i\,|m\rangle$$

Exponential dependence of local vector correlation function reconstructed as

$$C_{ij}^{\text{latt.}}(t) = \sum_n^N \langle\Omega|\,\mathcal{O}_i\,|n\rangle\,\langle n|\,\mathcal{O}_j\,|\Omega\rangle\,e^{-E_n t}$$

GEVP Results - $J_\mu + 2\pi$ Operators only



6-operator basis on 48I ensemble: local+smeared vector, $4\times(2\pi)$

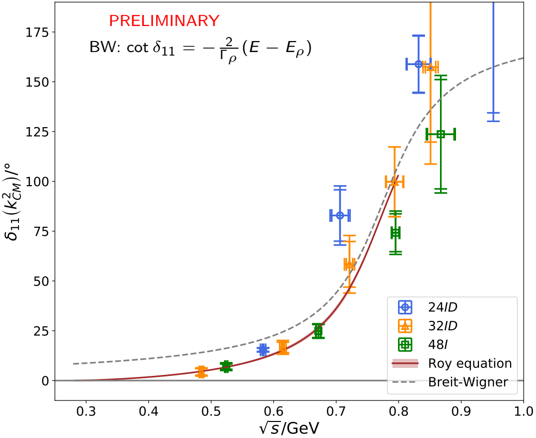
Data points from solving GEVP at fixed δt

$$C(t_0)\,V = C(t_0+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Excited state contaminations decay as $t_0, \delta t \to \infty$
moving right on plot $\implies$ asymptote to lowest states' spectrum & overlaps

Left: Spectrum; Right: Overlap with local vector current

Phase Shift



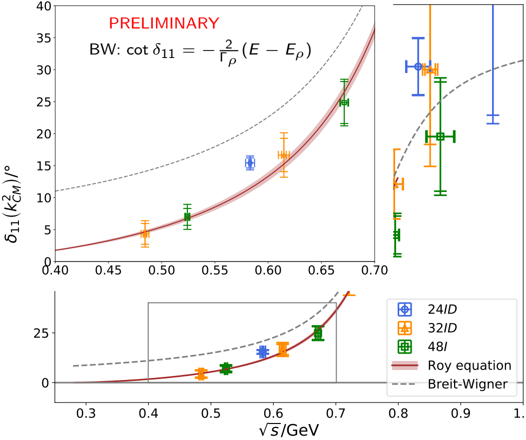
From spectrum, can compute pion scattering phase shifts in $I = 1$ channel
Statistics + systematic uncertainties included

Used to explicitly calculate FV corrections at physical M_π (C.Lehner, Lattice 2018)

Compare to simple Breit-Wigner parametrization and pheno (courtesy of M.Bruno)
Good agreement with pheno for 32ID, 48I
24ID data not at plateau, but improved with fit to data

Scattering phase shift results to appear as part of series of papers by RBC+UKQCD

Phase Shift



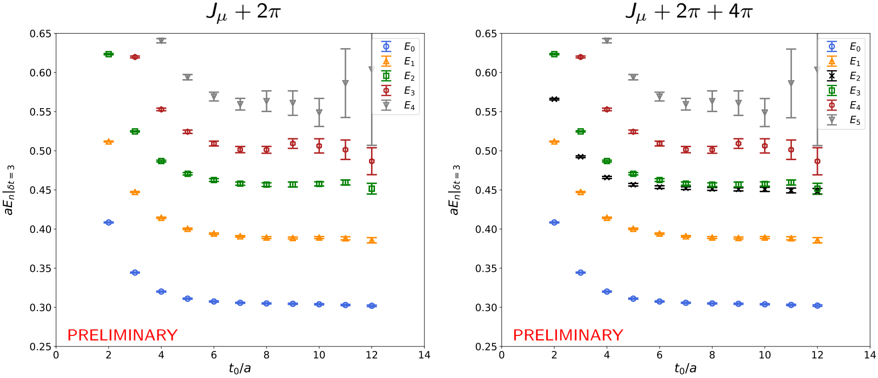
From spectrum, can compute pion scattering phase shifts in $I = 1$ channel
Statistics + systematic uncertainties included

Used to explicitly calculate FV corrections at physical M_π (C.Lehner, Lattice 2018)

Compare to simple Breit-Wigner parametrization and pheno (courtesy of M.Bruno)
Good agreement with pheno for 32ID, 48I
24ID data not at plateau, but improved with fit to data

Scattering phase shift results to appear as part of series of papers by RBC+UKQCD

GEVP Results - 4π Operators



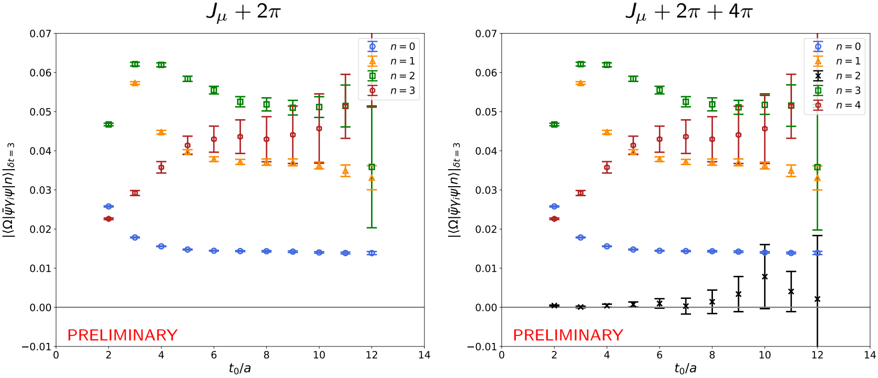
Breakdown of formalism for phase shifts +FVC could occur at 4π threshold

Compute $2\pi \to 4\pi$ and $4\pi \to 4\pi$ correlation functions and check explicitly

- $4\pi \to 4\pi$ has ~ 1000 independent Wick contractions

Spectrum unaffected by inclusion of 4π operator, but state is resolvable

GEVP Results - 4π Operators



Breakdown of formalism for phase shifts +FVC could occur at 4π threshold

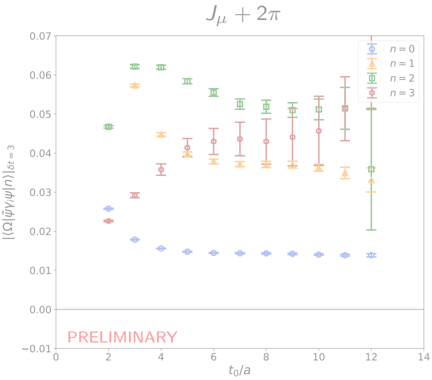
Compute $2\pi \to 4\pi$ and $4\pi \to 4\pi$ correlation functions and check explicitly

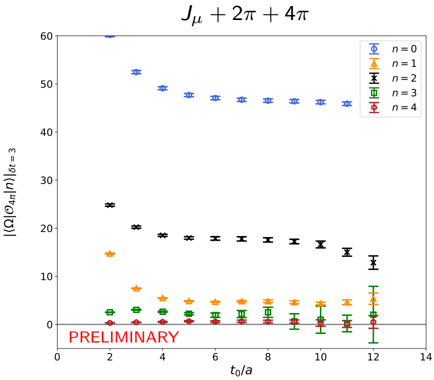
- $4\pi \to 4\pi$ has ~ 1000 independent Wick contractions

Spectrum unaffected by inclusion of 4π operator, but state is resolvable

Overlap of 4π state with local vector current unresolvable

GEVP Results - 4π Operators





Breakdown of formalism for phase shifts +FVC could occur at 4π threshold
Compute $2\pi \to 4\pi$ and $4\pi \to 4\pi$ correlation functions and check explicitly
$4\pi \to 4\pi$ has $\sim$ 1000 independent Wick contractions

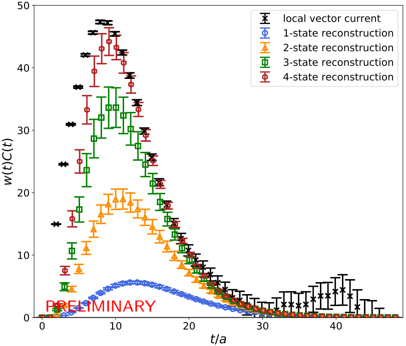
Spectrum unaffected by inclusion of 4π operator, but state is resolvable

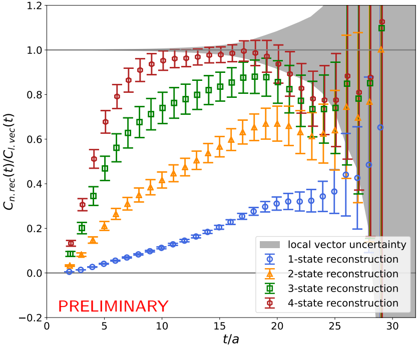
Overlap of 4π state with local vector current unresolvable

Overlap of state with 4π operator significant
$\Longrightarrow$ 4π state safely negligible in local vector current

Bounding Method and the Muon HVP

Correlation Function Reconstruction - 48I





GEVP results to reconstruct long-distance behavior of
local vector correlation function needed to compute connected HVP

Explicit reconstruction good estimate of correlation function at long-distance,
missing excited states at short-distance

More states $\implies$ better reconstruction, can replace $C(t)$ at shorter distances

Improved Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on a_μ^{HVP}

$$\widetilde{C}(t; t_{\text{max}}, E) = \begin{cases} C(t) & t < t_{\text{max}} \\ C(t_{\text{max}}) e^{-E(t - t_{\text{max}})} & t \geq t_{\text{max}} \end{cases}$$

Upper bound: $E \leq E_0$, lowest state in spectrum

Lower bound: $E \geq \log[\frac{C(t_{\text{max}})}{C(t_{\text{max}}+1)}]$

BMW (K.Miura, Lattice2018) take $E \to \infty$

With good control over lower states in spectrum from exclusive reconstruction:

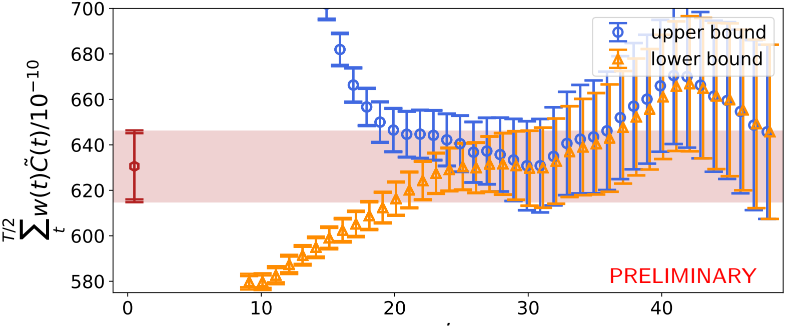
Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\implies$ Smaller overall contribution from neglected states

Add back contribution from reconstruction after bounding correlator

Bounding Method Results - 48I



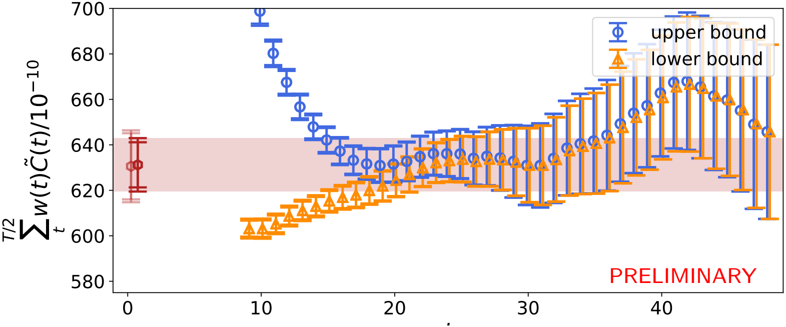
No bounding method: $a_\mu^{HVP} = 646(38)$

Bounding method $t_{\text{max}} = 3.3\ \text{fm}$, no reconstruction: $a_\mu^{HVP} = 631(16)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 646(38)$

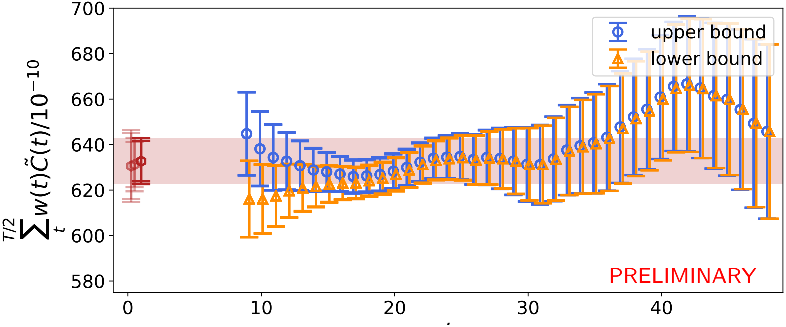
Bounding method $t_{\mathrm{max}} = 3.3\ \mathrm{fm}$, no reconstruction: $a_\mu^{HVP} = 631(16)$

Bounding method $t_{\mathrm{max}} = 3.0\ \mathrm{fm}$, 1 state reconstruction: $a_\mu^{HVP} = 631(12)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

Bounding Method Results - 48I

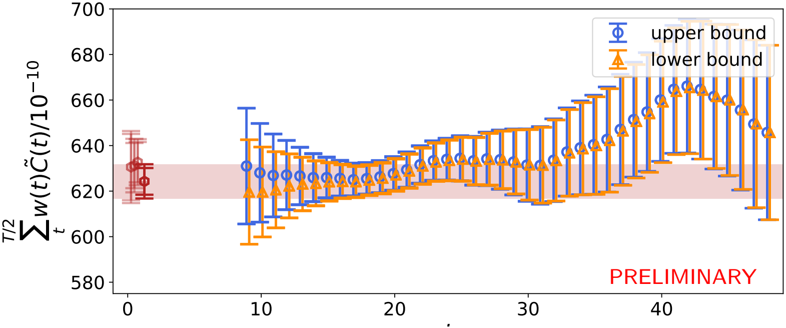


No bounding method:	$a_\mu^{HVP} = 646(38)$
Bounding method $t_{\max} = 3.3$ fm, no reconstruction:	$a_\mu^{HVP} = 631(16)$
Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction:	$a_\mu^{HVP} = 631(12)$
Bounding method $t_{\max} = 2.9$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 633(10)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

Bounding Method Results - 48I

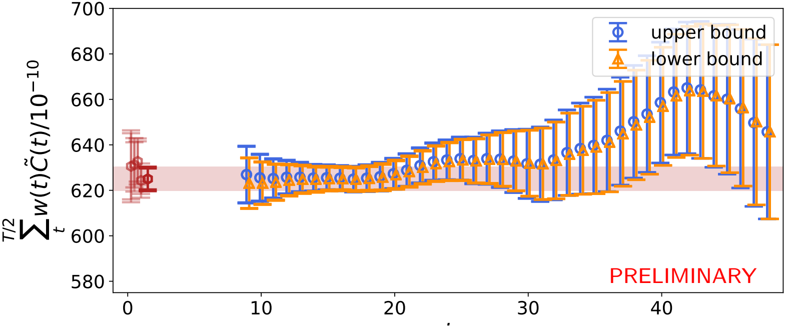


No bounding method:	$a_\mu^{HVP} = 646(38)$
Bounding method $t_{\max} = 3.3$ fm, no reconstruction:	$a_\mu^{HVP} = 631(16)$
Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction:	$a_\mu^{HVP} = 631(12)$
Bounding method $t_{\max} = 2.9$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 633(10)$
Bounding method $t_{\max} = 2.2$ fm, 3 state reconstruction:	$a_\mu^{HVP} = 624.3(7.5)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

Bounding Method Results - 48I



No bounding method: $a_\mu^{HVP} = 646(38)$
Bounding method $t_{\max} = 3.3$ fm, no reconstruction: $a_\mu^{HVP} = 631(16)$
Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 631(12)$
Bounding method $t_{\max} = 2.9$ fm, 2 state reconstruction: $a_\mu^{HVP} = 633(10)$
Bounding method $t_{\max} = 2.2$ fm, 3 state reconstruction: $a_\mu^{HVP} = 624.3(7.5)$
Bounding method $t_{\max} = 1.8$ fm, 4 state reconstruction: $a_\mu^{HVP} = 625.0(5.4)$

Bounding method gives factor of 2 improvement over no bounding method

Improving the bounding method increases gain to factor of 7, including systematics

Outlook and Conclusions

Summary

- $g - 2$ is an interesting and exciting topic to work on!
- Tensions in experimental $ee \to \pi\pi$ data make independent study of exclusive channels valuable

- Progress this year in extending our analysis to include three lattice ensembles
- Computed $2\pi \to 4\pi$, $4\pi \to 4\pi$ correlation functions to show explicitly that 4π state has negligible effect on HVP
- Study of exclusive channels able to significantly reduce statistical uncertainty on an all-lattice computation of muon HVP
  $\implies$ factor of 4 more statistics on 48I now
  $\implies$ expect to reach precision of $O(5 \times 10^{-10})$ by the end of year
- This calculation enables direct calculation of FV correction at physical M_π (see C.Lehner, Lattice2018)
- Part of ongoing lattice study to address all lattice systematics in RBC+UKQCD HVP computation (see talk by C.Lehner, Lattice2019)
- Several configurations of 64^3 data computed, to be included in future studies

Thank you!