Determining the glue component of the nucleon

R. Horsley, T. Howson, W. Kamleh, Y. Nakamura, H. Perlt,
P. E. L. Rakow, G. Schierholz, H. Stüben, R. Young and J. M. Zanotti

– QCDSF-UKQCD–CSSM Collaboration –

Edinburgh – Adelaide – RIKEN (Kobe) – Leipzig – Liverpool – DESY – Hamburg

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Nucleon Momentum

- How is the nucleon's momentum distributed among its constituents ?
- $$\sum_q \langle x \rangle_q + \langle x \rangle_g = 1$$

  where

  $\langle x \rangle_f$ = fraction of nucleon momentum carried by parton $f = q, \overline{q}, g$
- Expt: $\langle x \rangle_g \sim \frac{1}{2}$ – evidence for existence of the gluon
- Previous work includes:

  Göckeler et al, arXiv:hep-lat/9608017; Meyer et al, arXiv:0707.3225; QCDSF-UKQCD, arXiv:1205.6410;

  χQCD, arXiv:1312.4816; Shanahan et al, arXiv:1810.04626
- This talk will describe a determination of

  $$\langle x \rangle_g$$

  (mainly) using Feynman–Hellmann theorem

  Aim: To compare $RI - MOM$ renormalisation procedure with previous QCDSF-UKQCD result

Operators, just consider $n = 2$, but for both quark/gluon

- $\langle x \rangle \equiv v_2$ Euclidean

$$\langle N(\vec{p})|\widehat{\mathcal{O}}_f^{(b)}|N(\vec{p})\rangle = 2(m_N^2 + \tfrac{4}{3}\vec{p}^{\,2})\langle x \rangle_f$$

where

$$\mathcal{O}^{(b)} = O_{44} - \tfrac{1}{3} O_{ii}$$

with

$$O_{\mu\nu}^{(g)} = -tr_c F_{\mu\alpha} F_{\nu\alpha} \qquad O_g^{(b)} = \tfrac{2}{3} tr_c(-\vec{E}^2 + \vec{B}^2)$$

$$O_{\mu\nu}^{(q)} = \bar{q}\gamma_\mu \overleftrightarrow{D}_\nu \, q \qquad O_q^{(b)} = \bar{q}\gamma_4 \overleftrightarrow{D}_4 \, q - \tfrac{1}{3}\bar{q}\gamma_i \overleftrightarrow{D}_i \, q$$

with $\mathcal{O}(t) = \int d^3x \, O(t, \vec{x})$ normalisation $\langle N(\vec{p})|N(\vec{p}\,')\rangle = 2E_N \delta(\vec{p} - \vec{p}\,')$

- Representation allows $\vec{p} = \vec{0}$ (other representations, $O^{(a)} \sim \vec{E} \times \vec{B}$ with $\vec{p} \neq \vec{0}$ are possible)
- Related to decomposition of nucleon mass via energy–momentum tensor [Ji: hep-ph/9410274]

Lattice determination – Feynman–Hellmann

- Rather than forming ratios of 3-point to 2-point correlation functions (very noisy):
    - Add operator of interest to action,

$$S \to S(\lambda) = S + \lambda S_O$$

    - Perform subsidiary runs at different λ
    - Determine $E_N(\lambda)$ and use Feynman–Hellmann theorem to find matrix element of interest

$$\left.\frac{\partial E_N(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_N(\lambda)} \left. \left\langle N \middle| : \widehat{\frac{\partial S(\lambda)}{\partial \lambda}} : \middle| N \right\rangle_\lambda \right|_{\lambda=0}$$

    (gradient at $\lambda = 0$)
    - includes both quark line connected and disconnected terms

The modified action

- Wilson gluonic action

$$S = \tfrac{1}{3}\beta \sum_{x\,\mu<\nu} \mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{\mu\nu}^{\Box}(x)\right]$$

- As

$$\mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{\mu\nu}^{\Box}(x)\right] = \tfrac{1}{4} a^4 g^2 F_{\mu\nu}^{a}(x)^2 + \ldots$$

- This motivates the simplest definition of electric and magnetic field on each time slice as

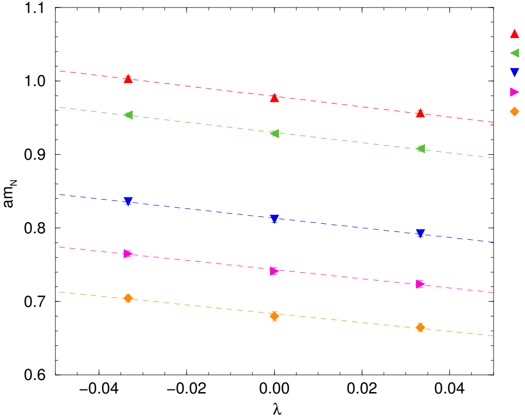
$$\begin{aligned} \tfrac{1}{2}\mathcal{E}^{a\,2}(\tau) &= \tfrac{1}{3}\beta \textstyle\sum_{\vec{x}\,i} \mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{i4}^{\Box}(\vec{x},\tau)\right] \\ \tfrac{1}{2}\mathcal{B}^{a\,2}(\tau) &= \tfrac{1}{3}\beta \textstyle\sum_{\vec{x}\,i<j} \mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{ij}^{\Box}(\vec{x},\tau)\right] \end{aligned}$$

- For the action we thus take

$$S(\lambda) = \sum_{\tau} \tfrac{1}{2}[\mathcal{E}^{a\,2}(\tau) + \mathcal{B}^{a\,2}(\tau)] - \lambda \textstyle\sum_{\tau} \underbrace{\tfrac{1}{2}[-\mathcal{E}^{a\,2}(\tau) + \mathcal{B}^{a\,2}(\tau)]}_{\frac{3}{2}O^{(b)}}$$

Lattice results

$$\langle x \rangle_g^{LAT} = -\frac{2E_N}{3(m_N^2 + \frac{4}{3}\vec{p}^2)} \left. \frac{\partial E_N(\lambda)}{\partial \lambda} \right|_{\lambda=0}$$



- Quenched, $\beta \equiv 6/g^2 = 6.0$, $24^3 \times 48$ lattice, $O^{(b)}$, $\vec{p} = \vec{0}$, $O(500)$ configs

Quenched Renormalisation

$$\begin{pmatrix} \langle x\rangle_g \\ \langle x\rangle_{v_u} \\ \langle x\rangle_{v_d} \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} & Z_{gq} \\ 0 & 0 & Z_{qq} \\ 0 & 0 & Z_{qq} \end{pmatrix} \begin{pmatrix} \langle x\rangle_g \\ \langle x\rangle_{v_u} \\ \langle x\rangle_{v_d} \end{pmatrix}^{LAT}$$

- Bottom rows of zeros:
  if don't put in a valence $\langle x\rangle_v$ 'by hand' then it remains zero
- Last column:
  valence quark surrounded by cloud of g, $u-\overline{u}, \ldots$ quark bubbles
- Valence means only for 'quark line connected'
- $$\left(\langle x\rangle_g + \langle x\rangle_{v_u} + \langle x\rangle_{v_d}\right)^R = Z_g \langle x\rangle_g^{LAT} + Z_q \left(\langle x\rangle_{v_u} + \langle x\rangle_{v_d}\right)^{LAT} = 1$$
- so Z_g, Z_q just depend on coupling, g, with [hence in quenched limit so does Z_{gg}]

$$Z_g = Z_{gg}\,, \qquad Z_q = Z_{gq}^{\overline{\mathrm{MS}}} + Z_{qq}^{\overline{\mathrm{MS}}}$$

Quenched Renormalisation I – estimating Z_g from $RI - MOM$ and FH

$$A_\mu(x+\hat\mu/2) = \frac{1}{2ig}\left(U_\mu(x) - U_\mu(x)^\dagger\right) - \mathrm{tr} \qquad A^a_\mu(p) = \sum_x e^{ip\cdot(x+\hat\mu/2)} A^a_\mu(x+\hat\mu/2)$$

$$\begin{aligned} \langle A(p)A(-p)\rangle &= D(p) \\ \langle A(p)O^{(b)}A(-p)\rangle &= D(p)\Gamma^{(b)}(p)D(p) \end{aligned}$$

Free/Born/Tree-level case: numerically: Landau gauge $\xi = 0$, D non-invertible

$$D^{\mathrm{Born}\,ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2}$$

$$\Gamma^{(b)\,\mathrm{Born}\,ab}_{\mu\nu}(p) = \frac{2}{3}\left(\begin{array}{c|c} (\vec{p}^{\,2} - p_4^2)\delta_{ij} - p_i p_j & p_i p_4 \\ \hline p_4 p_j & -\vec{p}^{\,2} \end{array}\right)\delta^{ab}$$

Interacting case: expect similar structures

$$\begin{aligned} D^{ab}_{\mu\nu}(p) &= \delta^{ab}\left(\delta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right) D_0(p^2) \\ \Gamma^{(b)\,ab}_{\mu\nu}(p) &= \Gamma^{(b)\,\mathrm{Born}\,ab}_{\mu\nu}(p)\,\Lambda(p^2) \end{aligned}$$

In particular $\langle A(p)O^{(b)}A(-p)\rangle = \Gamma^{(b)}(p)D_0(p)^2$

Renormalised fields

$$A^{\mathrm{R}} = Z_3^{1/2} A \qquad O^{(b)\,\mathrm{R}} = Z_g O^{(b)} \quad \Rightarrow \quad \Gamma^{(b)\,\mathrm{R}} = Z_g Z_3^{-1} \Gamma^{(b)}$$

Renormalisation condition [$\Gamma^{(b)\,\mathrm{Born}}$ non-invertible]

$$\mathrm{tr}_{\mathrm{CL}} \Gamma^{(b)\,\mathrm{R}}(p)\Big|_{p^2=\mu^2} = \mathrm{tr}_{\mathrm{CL}} \Gamma^{(b)\,\mathrm{Born}}(p)\Big|_{p^2=\mu^2} \quad \Rightarrow \quad Z_g = \frac{Z_3}{\Lambda}$$

Defining

$$\langle A^a_\mu(p) O^{(b)} A^a_\mu(-p) \rangle = \frac{2}{3} \frac{\partial}{\partial \lambda} \underbrace{\langle A^a_\mu(p) A^a_\mu(-p) \rangle_\lambda}_{\text{define: } 3\times 3\times \mathrm{D}_\lambda(\mathrm{p})} \Big|_{\lambda=0}$$

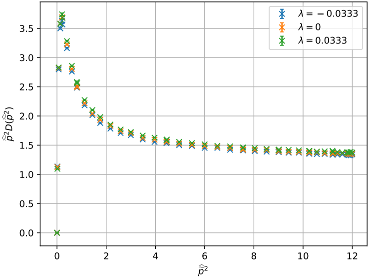
gives

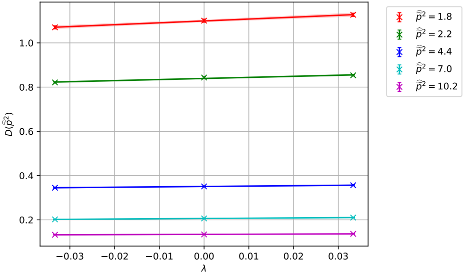
$$Z_g = \frac{\vec{p}^{\,2} - 3p_4^2}{3(\vec{p}^{\,2} + p_4^2)} \frac{p^2 D_0(p)}{\frac{\partial}{\partial \lambda} p^2 D_\lambda(p)\big|_{\lambda=0}}\Bigg|_{p^2=\mu^2}$$

Z_3 found by comparing gluon propagator to the Born value: $Z_3 = 1/(p^2 D_0)|_{p^2=\mu^2}$

PRELIMINARY RESULTS

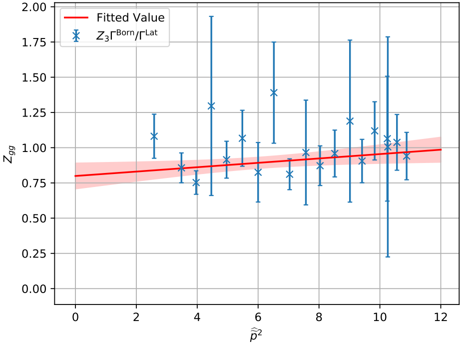
$p^2 D_\lambda$:





- Quenched, $\beta = 6.0$, 24^4 lattice, $\lambda = 0, \pm 0.0333$
- $p_\mu \rightarrow \hat{p}_\mu = 2\sin(p_\mu/2)$, cylinder cut: $(n, n, n, 0)$
- 1000 configs/λ value

Z_g



- Linear fit [$O(a\hat{p})^2$ discretisation errors assumed]

$$Z_g = 0.799(91)$$

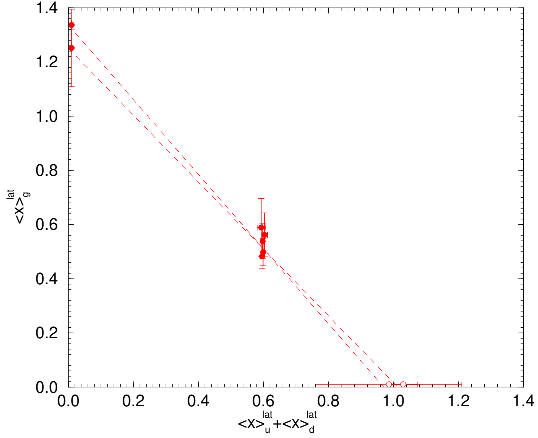
- Benchmark/comparison: Engels et al [hep-lat/9905002]

$$Z_g|_{g^2=1} = \left.\frac{1 - 1.0225g^2 + 0.1305g^4}{1 - 0.8557g^2}\right|_{g^2=1} = 0.748(20)$$

$Z_g = 1 - g^2/2(c_\sigma - c_\tau)$ and NP determination of anisotropic coefficients c_σ, c_τ, see also Meyer: arXiv:0704.1801

Quenched Renormalisation II – estimating Z_q by enforcing sum-rule

$$Z_g \langle x \rangle_g^{LAT} + Z_q \left(\langle x \rangle_{v_u} + \langle x \rangle_{v_d} \right)^{LAT} = 1$$



- intercepts give: $1/Z_g$, $1/Z_q$
- $\langle x \rangle_v$ from QCDSF: hep-ph/0410187
- need: $\langle x \rangle_v$ for other hadrons (to give a range of points)

Quenched Renormalisation II

- $Z_g = 0.799(91)$, $Z_q = 0.99(22)$
- Z_{qq}: [QCDSF: hep-ph/0410187]

$$\langle x \rangle_v^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = Z_{qq} \langle x \rangle^{LAT}$$

gives

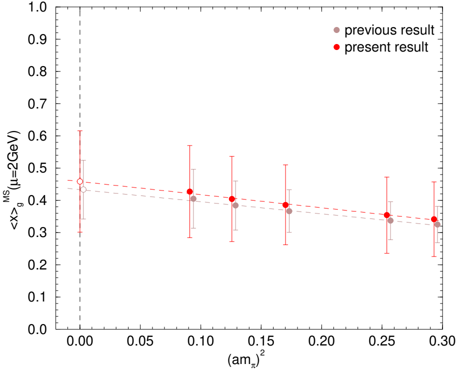
$$Z_{qq}^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = Z_{v_{2b}}^{RGI} \times [\Delta Z_{v_2}^{\overline{MS}}(\mu = 2\,\mathrm{GeV})]^{-1} = 1.45 \times 0.732(9) = 1.06(1)$$

- finally

$$\langle x \rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = Z_g \langle x \rangle_g^{LAT} + Z_{gq}^{\overline{MS}} \left(\langle x \rangle_{v_u} + \langle x \rangle_{v_d} \right) , \quad Z_{gq}^{\overline{MS}} = Z_q - Z_{qq}^{\overline{MS}}$$

where we now know everything

$\langle x \rangle_g$



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$$\langle x \rangle_g^{\overline{MS}}(\mu = 2\,\text{GeV}) = 0.46(16)$$

- Consistent with previous determination
- Need to reduce errors – work in progress

Next steps

- Z_g
  Modify renormalisation condition

$$\mathrm{tr}_{\mathrm{CL}} \Gamma^{(b)\,\mathrm{R}}(p) \Gamma^{(b)\,\mathrm{Born}}(p) \Big|_{p^2=\mu^2} = \mathrm{tr}_{\mathrm{CL}} \Gamma^{(b)\,\mathrm{Born}}(p) \Gamma^{(b)\,\mathrm{Born}}(p) \Big|_{p^2=\mu^2}$$

  Better geometry possible:
  numerator sum of squares, so never vanishes

- Z_{gq}
  Quark propagator modified, so expect

$$Z_{gq} \propto \mathrm{tr}_{\mathrm{CL}} \Gamma^{(b)\,\mathrm{Born}} (S_\lambda^{-1} - S_0^{-1})$$

Conclusions

- $\langle x \rangle_g$ is a notoriously difficult quantity to compute
  - short distance quantity
  - large fluctuations
  - a ‘disconnected quantity’
- Method I for matrix element and renormalisation Z:
  - straightforward computation of 3-point functions
  - one run, requires $500,000$ configs due to fluctuations in signal
  - extremely expensive
- Method II for matrix element and renormalisation Z:
  - use Feynman–Hellmann theorem
  - several runs required, but each run moderate
  - expensive [Ferrenberg–Swendsen?]
- Result – reasonable agreement with previous result

[Need to reduce errors – work in progress]

Renormalisation – general

$$\begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u \\ \langle x \rangle_d \\ \langle x \rangle_s \\ \langle x \rangle_v \end{pmatrix}^R = \begin{pmatrix} Z_{gg} & Z_{gq} & Z_{gq} & Z_{gq} & Z_{gq} \\ Z_{qg} & Z_a & Z_b & Z_b & Z_b \\ Z_{qg} & Z_b & Z_a & Z_b & Z_b \\ Z_{qg} & Z_b & Z_b & Z_a & Z_b \\ 0 & 0 & 0 & 0 & Z_a - Z_b \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \langle x \rangle_u \\ \langle x \rangle_d \\ \langle x \rangle_s \\ \langle x \rangle_v \end{pmatrix}^{\mathrm{LAT}}$$

- Case: $n_f = 3$ and $n_{f_v} = 1$ partially quenched or valence quarks
- All Zs depend on scheme, renormalisation scale μ
- Non-singlet (eg $\langle x \rangle_u - \langle x \rangle_d$) and singlet (ie $\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s$) so
$$Z_{qq}^{\mathrm{NS}} = Z_a - Z_b \,, \qquad Z_{qq}^{\mathrm{S}} = Z_{qq}^{\mathrm{NS}} + n_f Z_b$$
- Bottom row of zeros:
  if don't put in a valence $\langle x \rangle_v$ 'by hand' then it remains zero
- Last column:
  valence quark surrounded by cloud of g, $u - \overline{u}$, ... quark bubbles
- Valence means only for 'quark line connected', so renormalise as $\langle x \rangle_v^{\mathrm{R}} = Z_{qq}^{\mathrm{NS}} \langle x \rangle_v^{\mathrm{LAT}}$
- Can also split: $\langle x \rangle_q = \langle x \rangle_q^{\mathrm{con}} + \langle x \rangle_q^{\mathrm{dis}}$ with $\langle x \rangle_q^{\mathrm{con\,R}} = Z_{qq}^{\mathrm{NS}} \langle x \rangle_q^{\mathrm{con\,LAT}}$, $\langle x \rangle_q^{\mathrm{dis\,R}} = \ldots$

Renormalisation – general II

$$\left(\langle x \rangle_g + \sum_f \langle x \rangle_f + \sum_{v_f} \langle x \rangle_{v_f}\right)^R = Z_g \langle x \rangle_g^{LAT} + Z_q \left(\sum_f \langle x \rangle_f + \sum_{v_f} \langle x \rangle_{v_f}\right)^{LAT} = 1$$

- with

$$Z_g = Z_{gg}^{\overline{\mathrm{MS}}} + n_f Z_{qg}^{\overline{\mathrm{MS}}}$$
$$Z_q = Z_{gq}^{\overline{\mathrm{MS}}} + Z_{qq}^{NS\,\overline{\mathrm{MS}}}$$

- Z_g, Z_q just depend on coupling, g
  (but individual terms depend on chosen scheme, eg $\overline{MS}$)