Proton momentum and angular momentum fractions at physical pion mass

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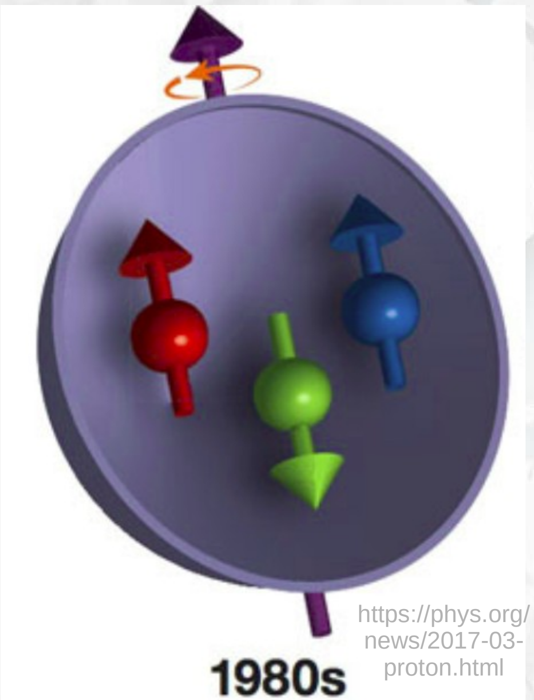
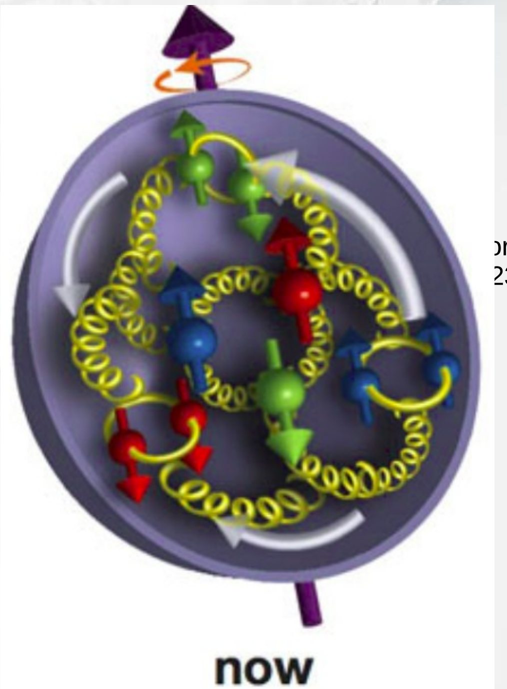
Motivation



Only 30% of the proton's spin is carried by the spin of quark constituents. [1]



[1] C. A. Aidala, S. D. Bass, D. Hasch, and G. K. Mallot, "The Spin Structure of the Nucleon," Rev. Mod. Phys. 85, 655 (2013)

Energy-momentum tensor and angular momentum fractions

Angular momentum operators expressed with energy-momentum tensor

$$J^{\mu\nu} \equiv \int d^3x \left[\mathcal{T}^{\{0\nu\}} x^\mu - \mathcal{T}^{\{0\mu\}} x^\nu\right]$$

The i-th component of the angular momentum operators for quarks and gluons

$$\vec{J}_i^{q,g} = \frac{1}{2}\epsilon^{ijk} \int d^3x \left(\mathcal{T}^{\{0k\}q,g} x^j - \mathcal{T}^{\{0j\}q,g} x^k\right)$$

The matrix element (energy-momentum tensor) between two nucleon states can be written in terms of three form factors (T1, T1, T3)

$$\langle p', s'|\, \mathcal{T}^{\{0i\}q,g}\, |p, s\rangle = \frac{1}{2}\bar{u}(p', s')\left[T_1(q^2)(\gamma^0 \bar{p}^i + \gamma^i \bar{p}^0)\right.$$
$$\left. + \frac{1}{2m}T_2(q^2)\left(\bar{p}^0(i\sigma^{i\alpha}) + \bar{p}^i(i\sigma^{0\alpha})\right) q_\alpha + \frac{1}{m}T_3(q^2)q^0 q^i\right]^{q,g} u(p, s)$$

M. Deka, T. Doi, YBY, et. al., χQCD collaboration, PRD91, 014505 (2015), 1312.4816

Matrix element and form factors

Momentum and angular momentum fractions can be obtained using T1 and T1+T2 form factors at zero momentum transfer

$$\langle x \rangle^{q,g} = T_1(0)^{q,g} \qquad J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}$$

Normalization conditions from momentum and angular momentum conservation

$$\langle x \rangle^q + \langle x \rangle^g = T_1(0)^q + T_1(0)^g = 1$$

$$J^q + J^g = \frac{1}{2}\left\{\left[T_1(0) + T_2(0)\right]^q + \left[T_1(0) + T_2(0)\right]^g\right\} = \frac{1}{2}$$

Three point functions on the Lattice

$$G_{\alpha\beta}^{N\mathcal{T}_{4i}N}(\vec{p}\,', t_2; \vec{q}, t_1; \vec{p}, t_0) = \sum_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}\,'\cdot(\vec{x}_2 - \vec{x}_1)} e^{-\vec{p}\cdot(\vec{x}_1) - \vec{x}_0} \times \langle 0 | \, \mathrm{T}[\chi_\alpha(\vec{x}_2, t_2) \mathcal{T}_{\{4i\}} \bar{\chi}_\beta(\vec{x}_0, t_0)] \, | 0 \rangle$$

$$\mathrm{Tr}\left[\Gamma^{unpol,pol} G^{(f)N\mathcal{T}_{4i}N}(\vec{p}\,', t_2; \vec{q}, t_1; \vec{p}, t_0)\right] \propto a_1 T_1(Q^2) + a_2 T_2(Q^2) + a_3 T_3(Q^2)$$

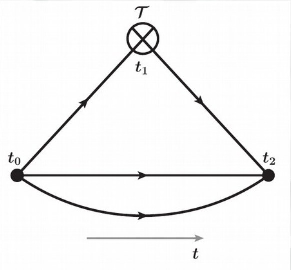
Simulation Setup

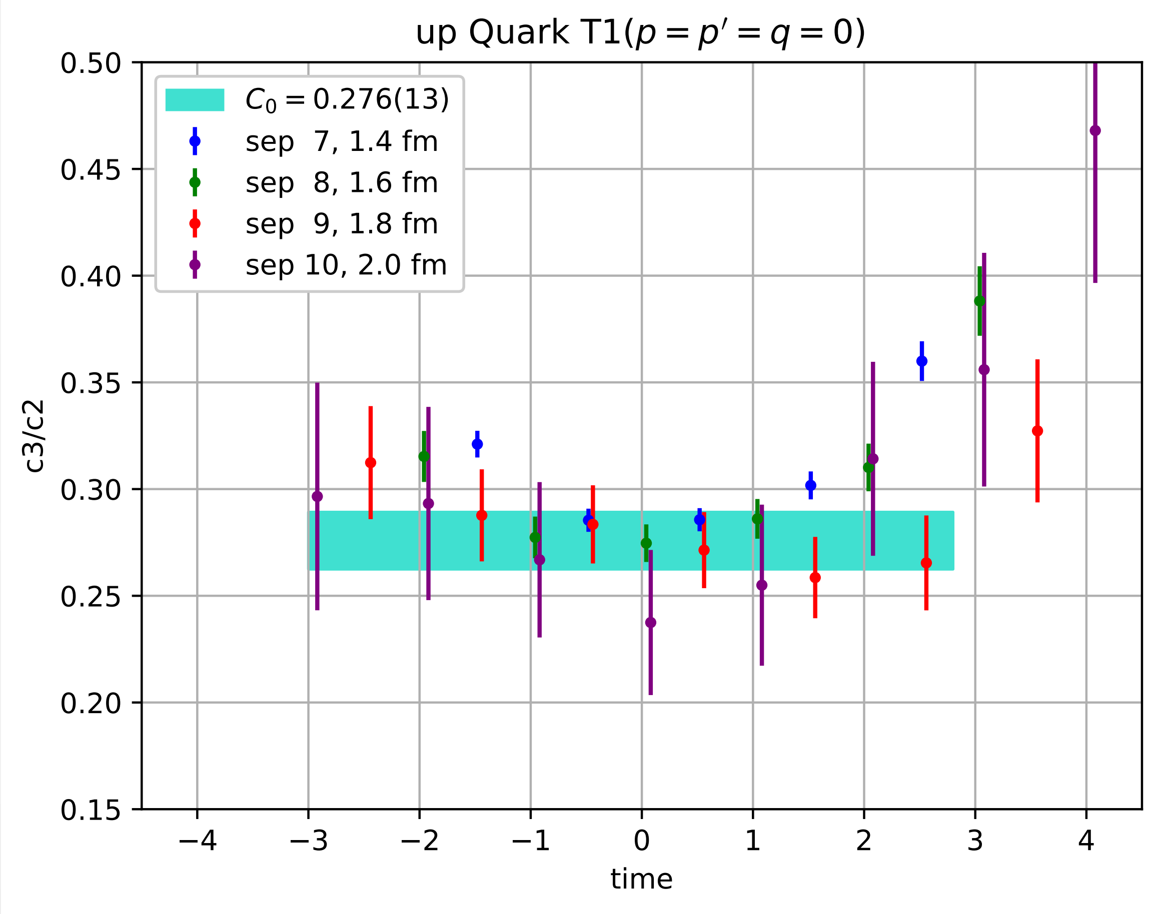
- Lattices
  - 24IDC--Domain Wall 2+1 Lattice, 24^3 *64, a=0.193 fm, Pion 139 MeV
  - Overlap fermions with pion mass 135 MeV as valence quark
- Sources and Sinks
  - Grid source at time 0 with momentum p_i=0
  - Stochastic point sinks with t_f with momentum p_f=q
- Analysis Strategy
  - Two state fit of C_{3pt}/C_{2pt}
  - Z-expansion fitting for the final T1+T2 form factors as a function of Q^2

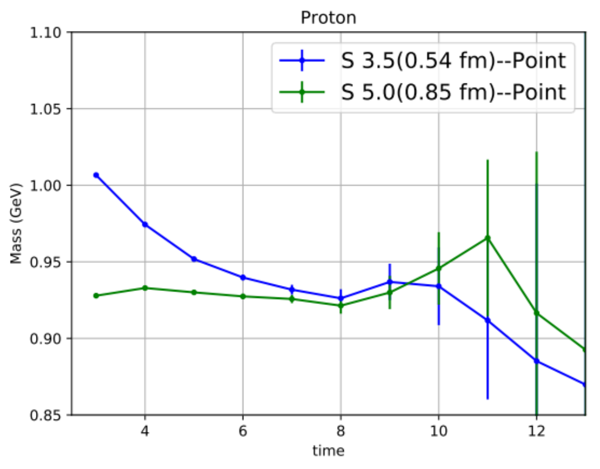
Quark T1(0) from T44 connected contribution

Setting source sink momentum to be the same to approach T1(0) Form Factors

$$\mathrm{Tr}\left[\Gamma^{unpol} G^{(f)N\mathcal{T}_{44}N}(\vec{p}, t_2; \vec{0}, t_1; \vec{p}, t_0)\right] \propto T_1(0)$$



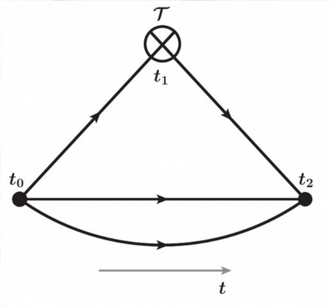


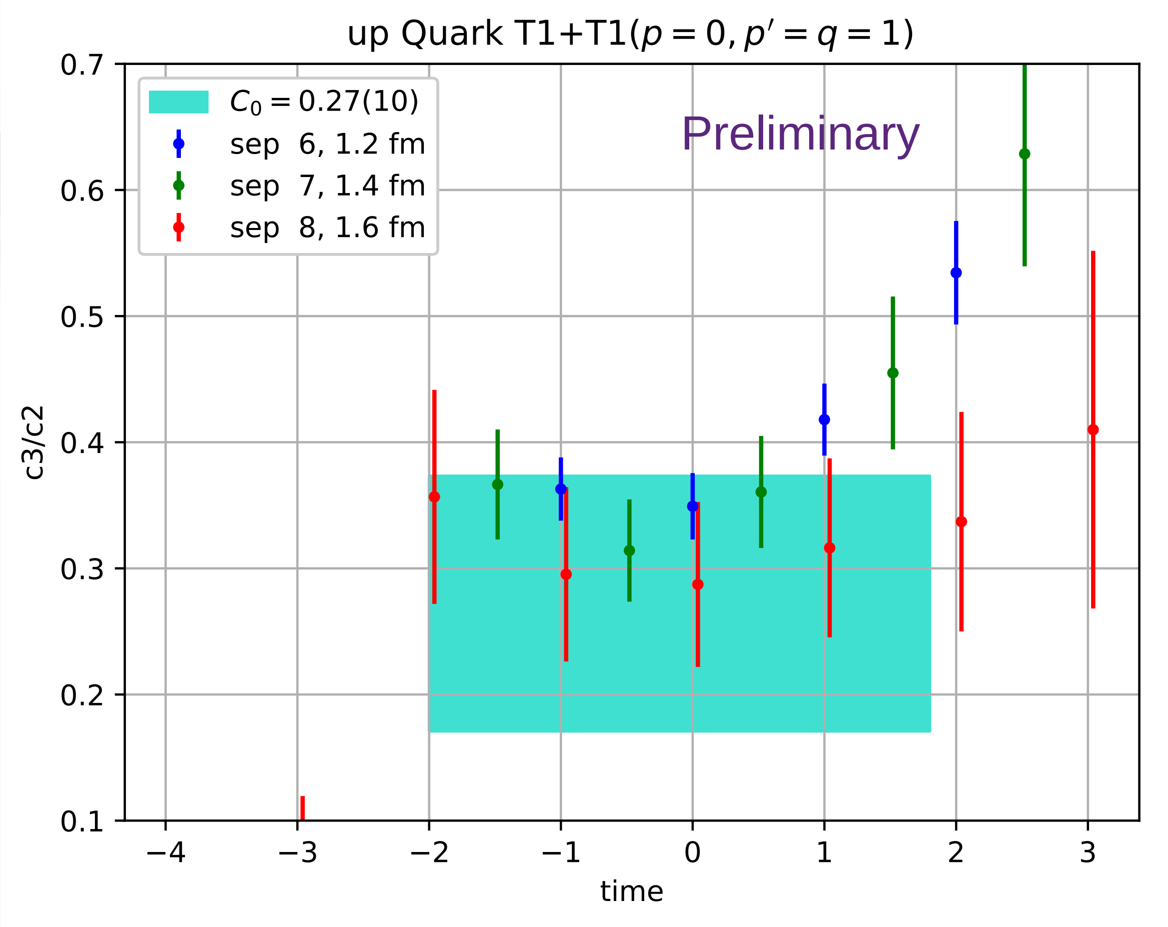


Quark [T1+T2](Q^2) from T4i connected contribution

Setting source momentum to be zero to approach [T1+T2](Q^2) Form Factors

$$\mathrm{Tr}\left[\Gamma^{pol} G^{(f)N\mathcal{T}_{4i}N}(\vec{p},t_2;\vec{p},t_1;\vec{0},t_0)\right] \propto T_1(Q^2)+T_2(Q^2)$$



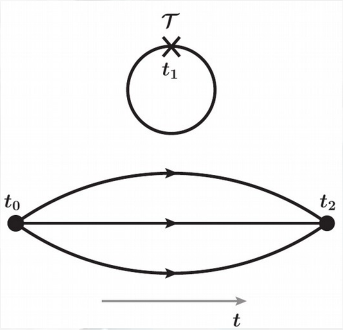


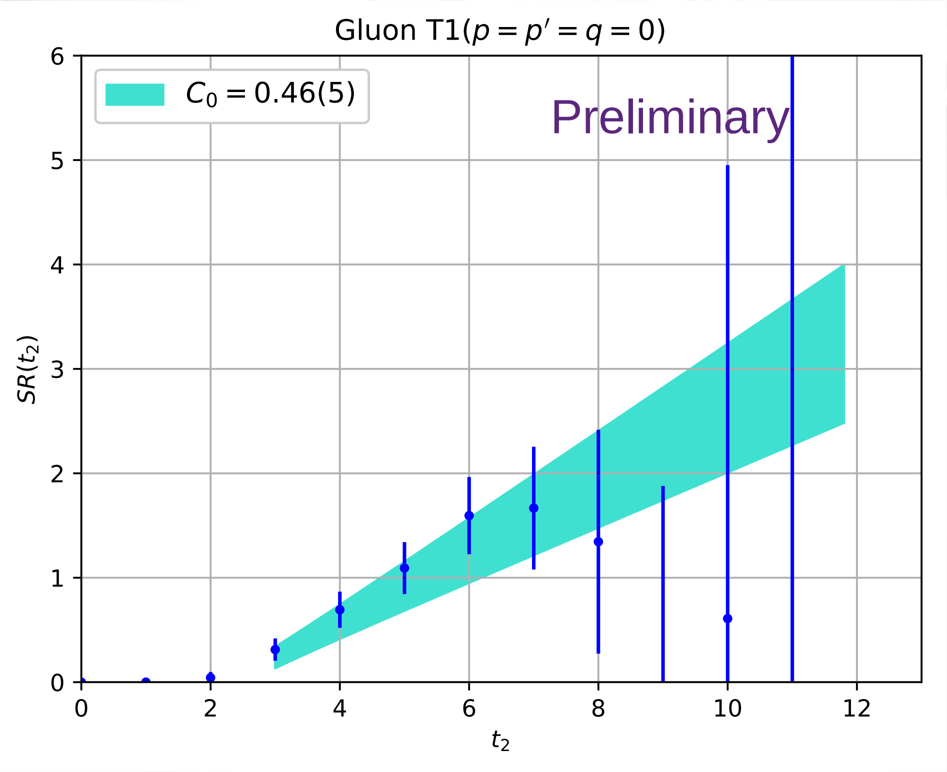
Gluon T1(0) from T44 disconnected contribution

Using summed ratio SR to fit DI matrix elements

$$R(t_2, t_1) = C_0 + C_1 e^{\Delta m(t_2 - t_1)} + C_2 e^{-\Delta m t_1} + C_3 e^{-\Delta m t_2}$$

$$SR(t_2) = \sum_{t_1 \geq 1}^{t_1 \leq (t_2 - 1)} R(t_2, t_1) = t_2 C_0 + C' + \mathcal{O}(\Delta m)$$





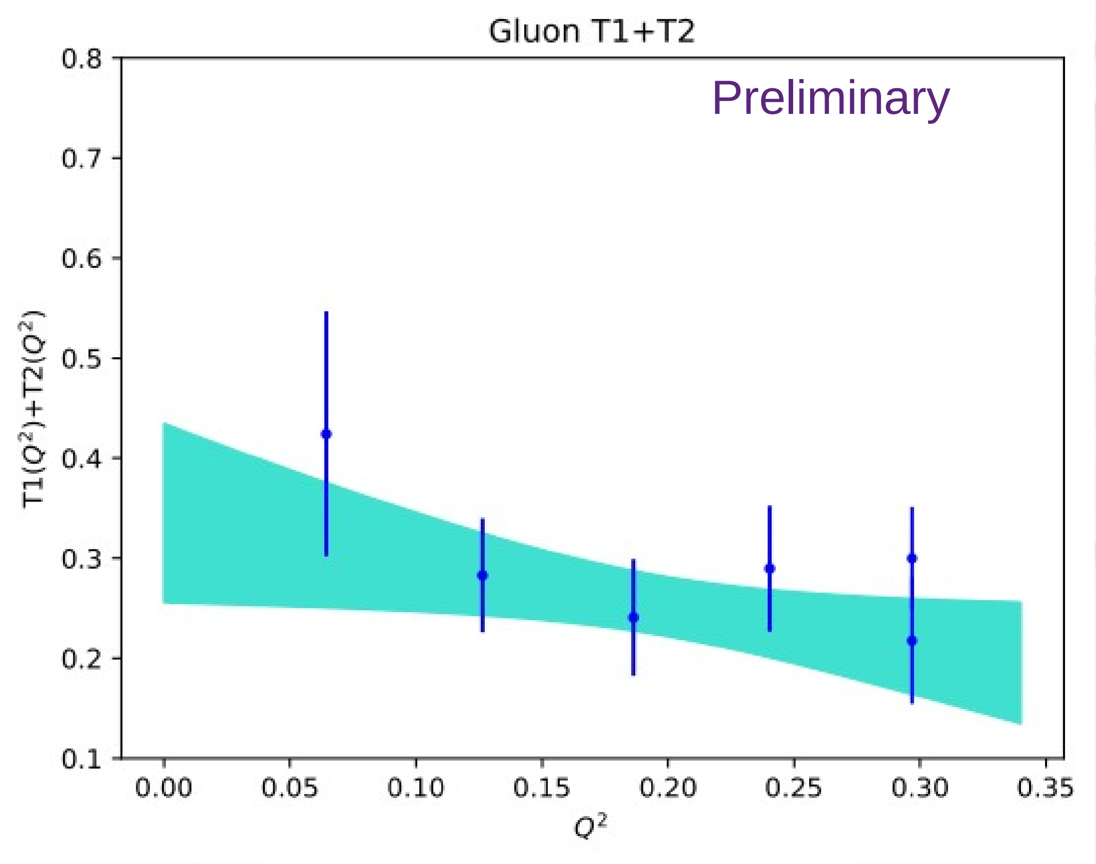
Gluon [T1+T2](Q^2) from T4i disconnected contribution

$$f(Q^2) = 1 + \sum_{k=1}^{k_{max}} a_k z^k$$

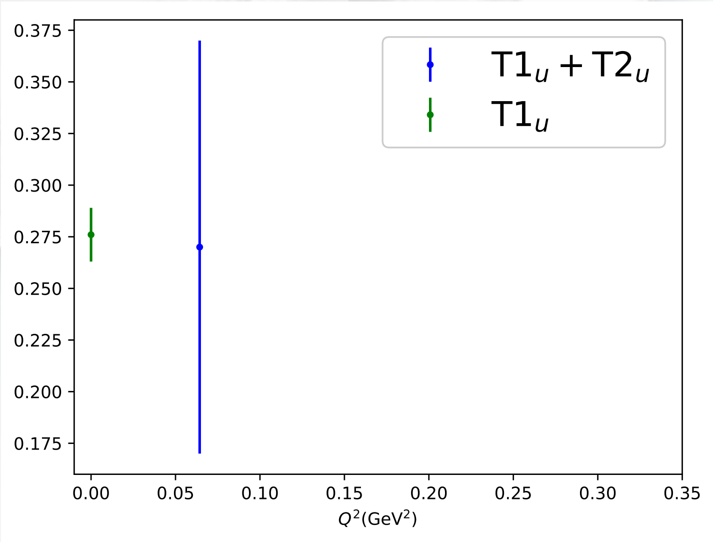
$$z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - t_0} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} - t_0} + \sqrt{t_{cut} - t_0}}$$

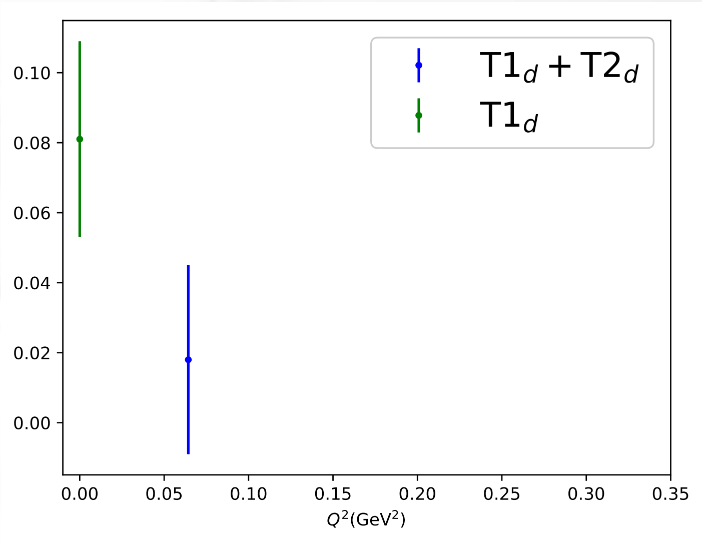
$$t = -Q^2, t_{cut} = 4M_K^2$$

By treating [T1+T2](Q^2) as one form factors, model-independent z-expansion fitting can by used to extract T1+T2 form factors at zero momentum transfer

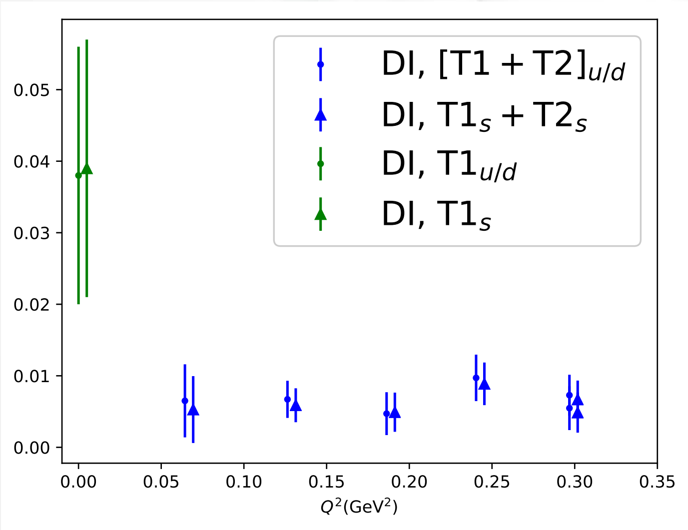


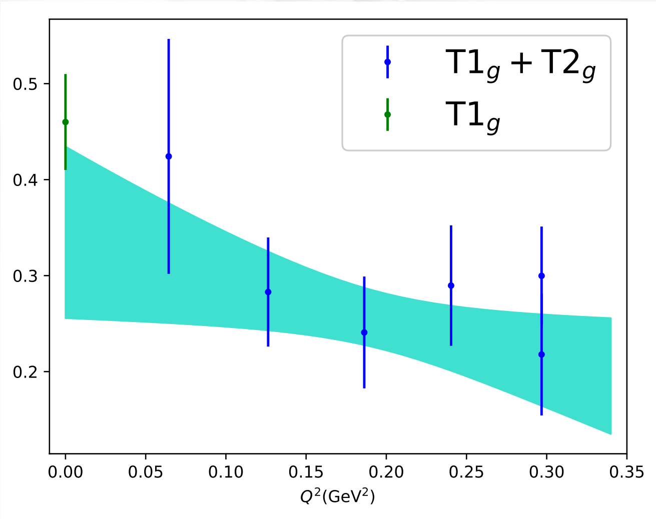
Summary plots of T1(0) and [T1+T2](Q^2)





Preliminary





Further Calculations

- We have done a testing and preliminary calculation of T1 and T1+T2 form factors at physical pion mass
- More statistic are coming for connect diagrams get quark T1+T2 form factors
- Cluster decomposition will be used for all disconnected diagrams[1]
- Heavier valence quark masses will be calculated to have better control of results at physical pion mass
- Non-perturbative renormalization and mixings between quark and gluon operators should be applied
- Calculations with other volumes and lattice spacing will be needed to reach physical limit

[1] K-F. Liu,Y-B. Yang, J. Liang, “Variance Reduction and Cluster Decomposition”, Phys. Rev. D 97, 034507 (2018)