Nucleon sigma terms

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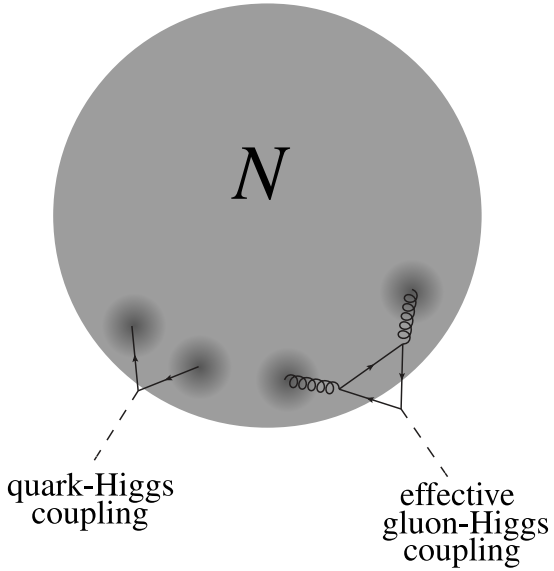
Why nucleon sigma terms?



Nucleon couples to the external Higgs field. That coupling is mediated by the couplings g_q of the quark flavors q to the Higgs field.

Elementary fermions have a mass, which is proportional to the fermion-Higgs coupling g_f. As a conseqence $m_f = g_f \frac{\partial m_f}{\partial g_f}$.

We can define theis logarithmic derivative also for non-elementary particles, like the nucleon:

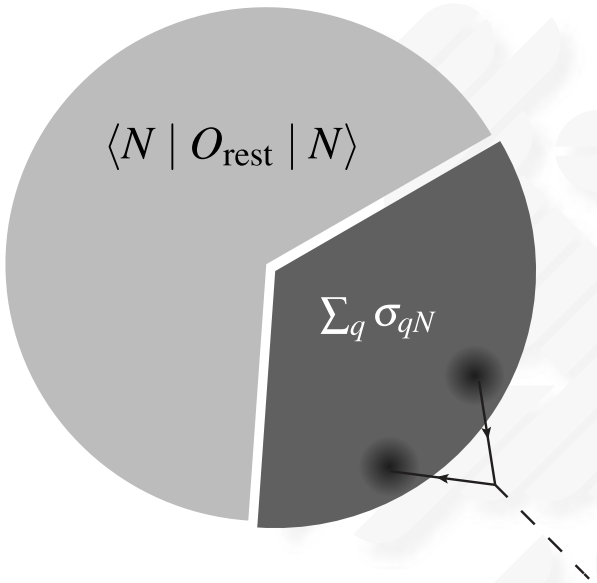
$$M_N f_{qN} = \sigma_{qN} := g_q \frac{\partial M_N}{\partial g_q} = m_q \frac{\partial M_N}{\partial m_q}$$

For the nucleon $\sum_q \sigma_{qN} < M_N$, because a large part is generated by the scale anomaly of QCD.

Sigma terms and the mass of the nucleon

The Hamiltonian of QCD can be decomposed like [1]

$$H = \sum_q m_q \bar{q} q + O_{\text{rest}} \quad , \quad \frac{\partial O_{\text{rest}}}{\partial m_q} = 0$$



The Feynman-Hellmann theorem states

$$\sigma_{qN} = m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q} q \mid N \rangle \ , \quad \langle N \mid N \rangle = 1$$

so that $M_N = \sum_q \sigma_{qN} + \langle N \mid O_{\text{rest}} \mid N \rangle$.

[1] X. D. Ji, Phys. Rev. D **52** (1995) 271 [hep-ph/9502213].

Sigma terms and the mass of the nucleon

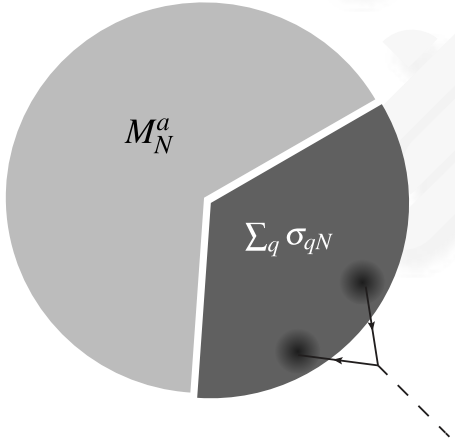
What is $\langle N \mid O_{\text{rest}} \mid N \rangle$?

We look at the trace $\theta^{\mu}{}_{\mu}$ of the enegry momentum tensor [1]:

$$\langle N \mid \theta^{\mu}{}_{\mu} \mid N \rangle =$$

$$\underbrace{\langle N \mid \sum_q m_q \bar{q} q \mid N \rangle}_{\sum_q \sigma_{qN}} + \underbrace{\langle N \mid \gamma_m \sum_q m_q \bar{q} q + \frac{\beta}{g} G^2 \mid N \rangle}_{\text{anomaly contribution } M_N^a} = M_N$$

Note: $\gamma_m \sum_q m_q \bar{q} q + \frac{\beta}{g} G^2$ does not depend on m, because the $\gamma_m m$-term cancels with a contribution in the renormalized G^2-term.



We find $\langle N \mid O_{\text{rest}} \mid N \rangle = M_N^a$ and

$$M_N = \sum_q \sigma_{qN} + M_N^a$$

[1] X. D. Ji, Phys. Rev. D **52** (1995) 271 [hep-ph/9502213].

Is $M_N = \sum_q \sigma_{qN} + M_N^a$ **the** decomposition of the nucleon mass?

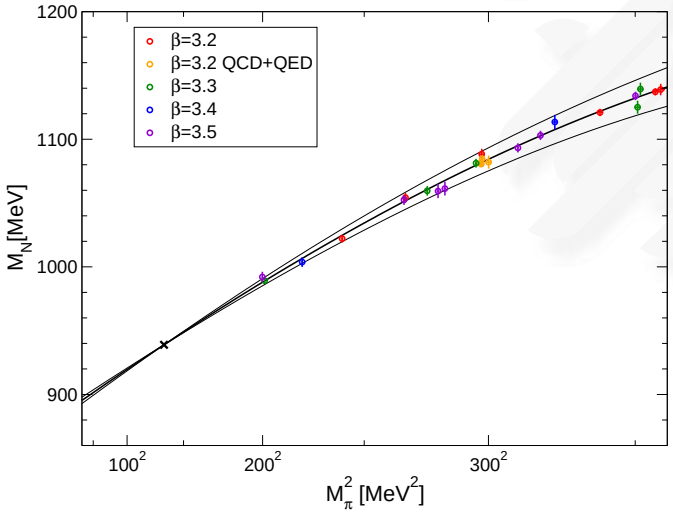
It is **a** decomposition in the following sense:

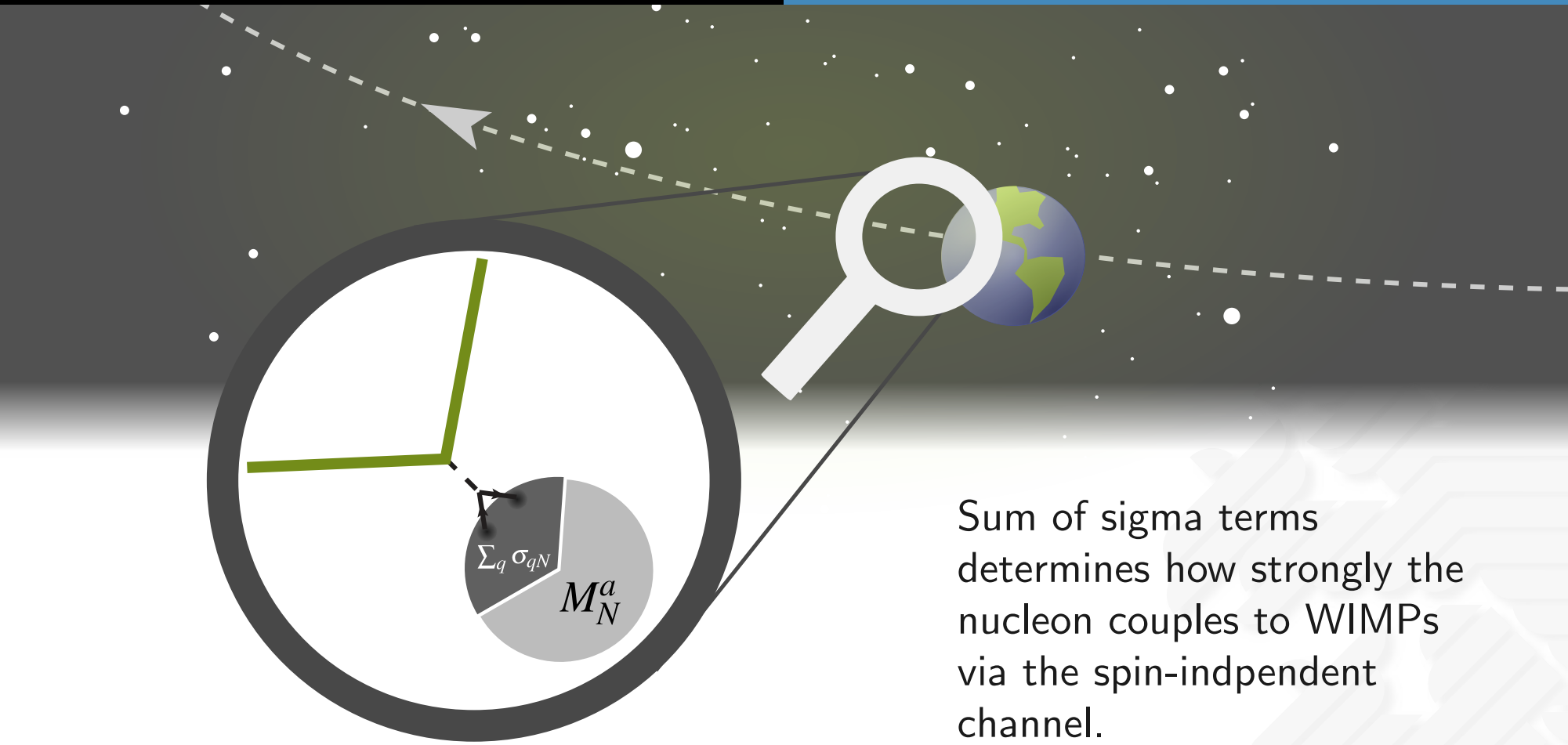
- It is a sum of *positive contributions*.
- Each contribution is a *scale and scheme independent* oberservable in QCD.
- The individual contributions have a *clear physical meaning*. (experiments!)
- The contributions are properties of the *physical nucleon state*, making no references to unphysical theories.

However, the nucleon states do also depend on the quark mass. Therefore

$$M_N = \langle N(m_q) \mid H(m_q) \mid N(m_q) \rangle,$$

$$M_N^{(\phi)} - M_N^{(\chi)} = \sigma_{qN} + \mathcal{O}(m_q^{(\phi)2}).$$





Sum of sigma terms determines how strongly the nucleon couples to WIMPs via the spin-indpendent channel.

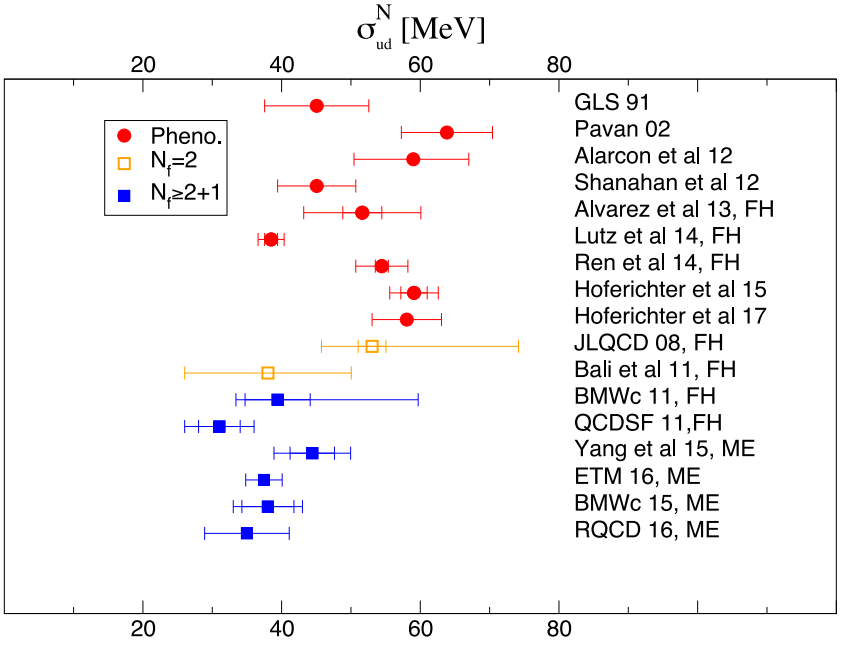
How can the sigma terms be observed?

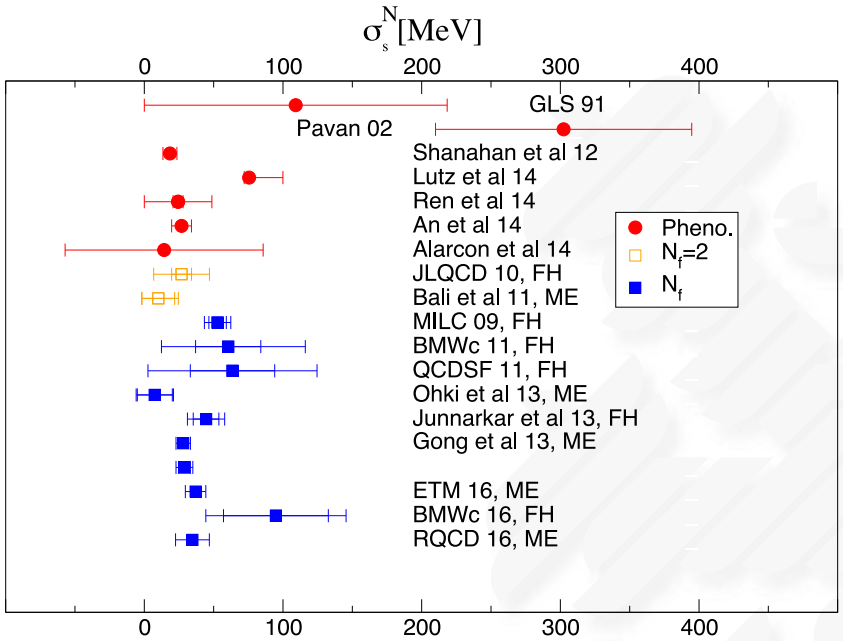
The Higgs field and WIMP dark matter interacts with the nucleon via the scalar quark density.

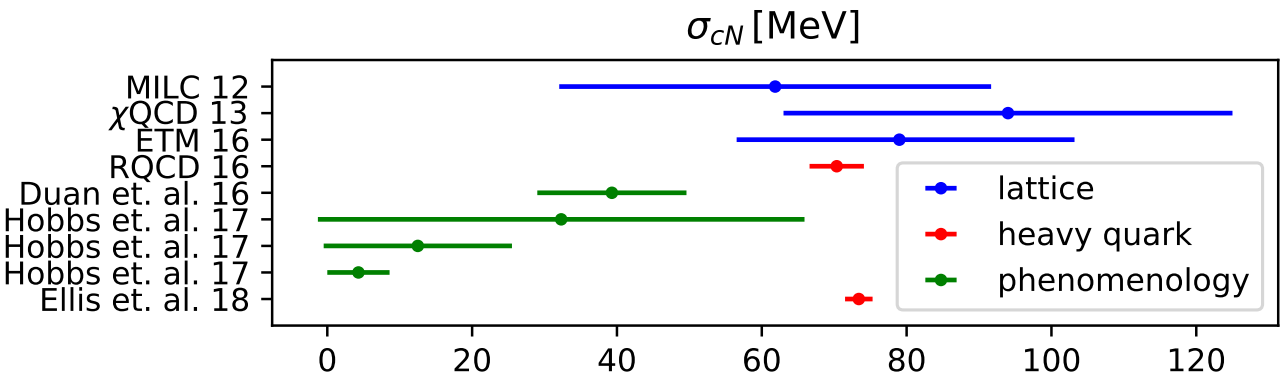
Sigma terms are required for the interpretation of direct dark matter detection experiments. [1]

[1] J. Ellis, N. Nagata and K. A. Olive, Eur. Phys. J. C **78** (2018) no.7, 569 [arXiv:1805.09795].

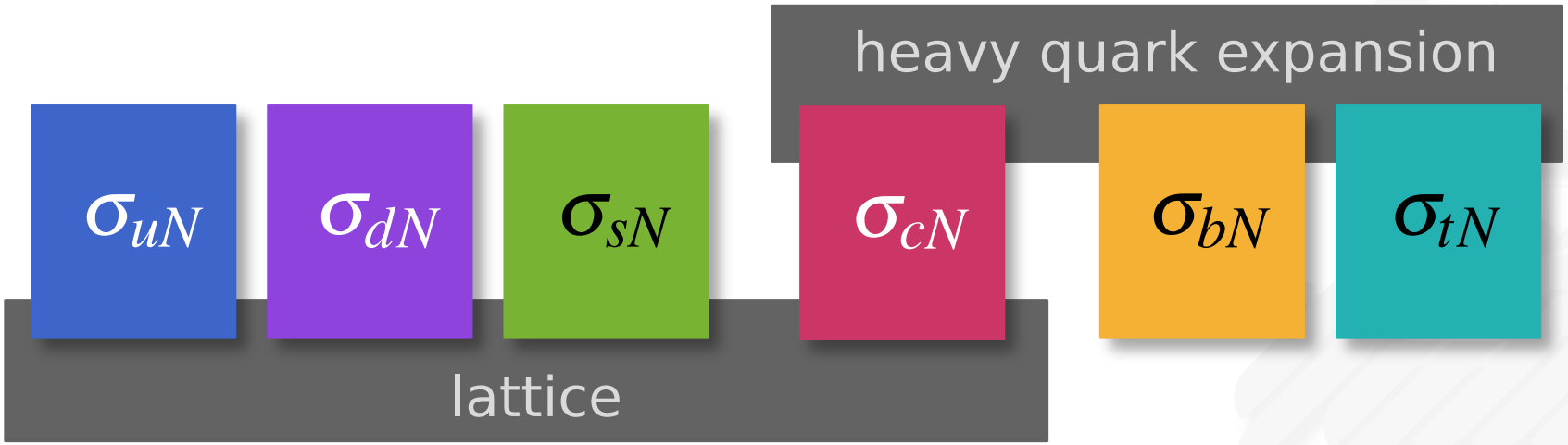
What is known about the sigma terms?







How to determine the sigma terms?

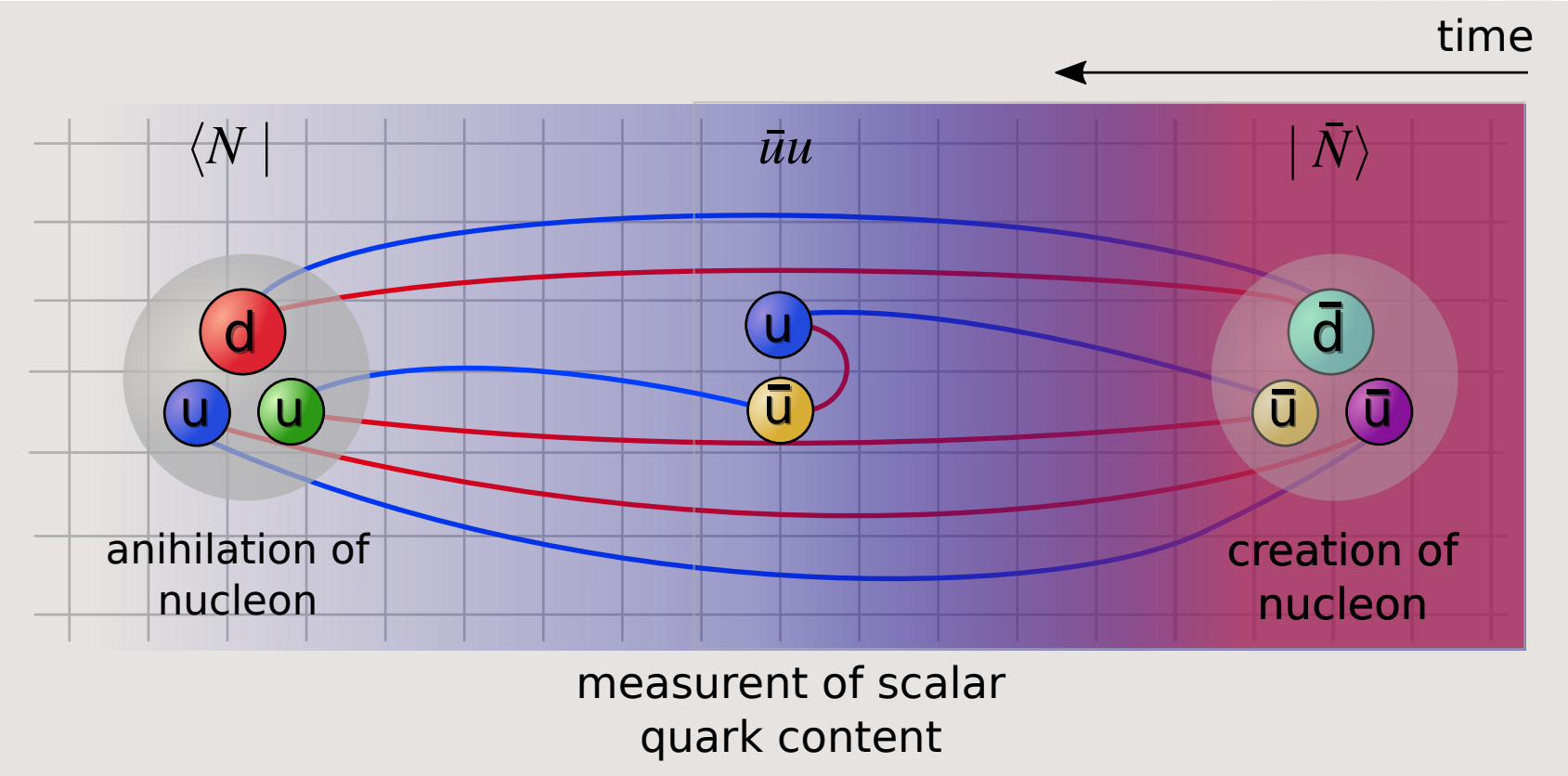


Up, down, strange, and charm sigma terms can be determined on the lattice.

For the top, bottom, and charm sigma terms an expansion in $1/m_q$ can be employed. The validity can be checked in case of σ_{cN}, where the $\mathcal{O}(m_q^{-2})$ effects are biggest.

σ_{uN} σ_{dN} σ_{sN} σ_{cN}

Direct evaluation of matrix elements $\langle N \mid m_q \bar{q} q \mid N \rangle$ requires disconnected contributions:

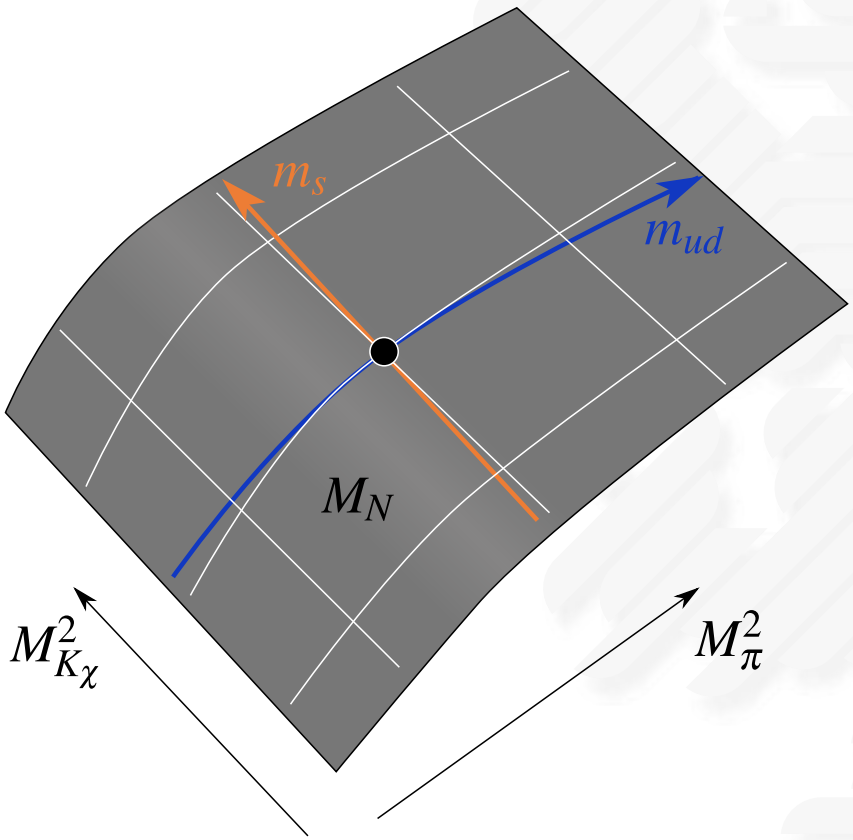


We use the Feynman-Hellmann method.

σ_{uN} σ_{dN} σ_{sN}

Strategy for the light and strange sigma terms:

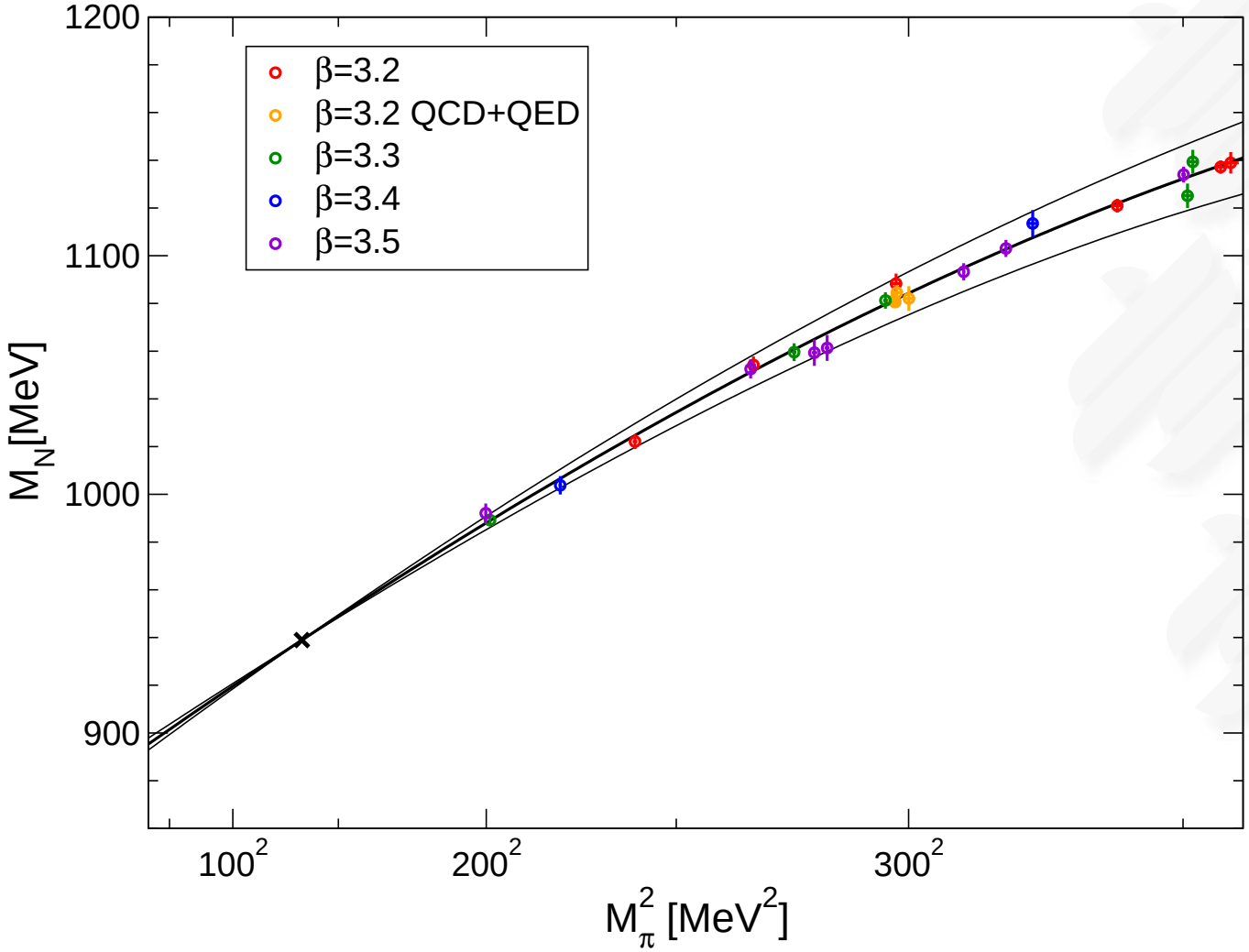
1. Determine $M_N(M_\pi^2, M_{K_\chi}^2)$ from fit to lattice data. Use Wilson fermions.
2. Calculated $J = \partial \log M_{\text{meson}}^2 / \partial \log m_{\text{quarks}}$ to get light and strange sigma term. Use staggered fermions.
3. Use isospin splitting relationships to disentangle up and down sigma terms.



For details, see last lattice conference.

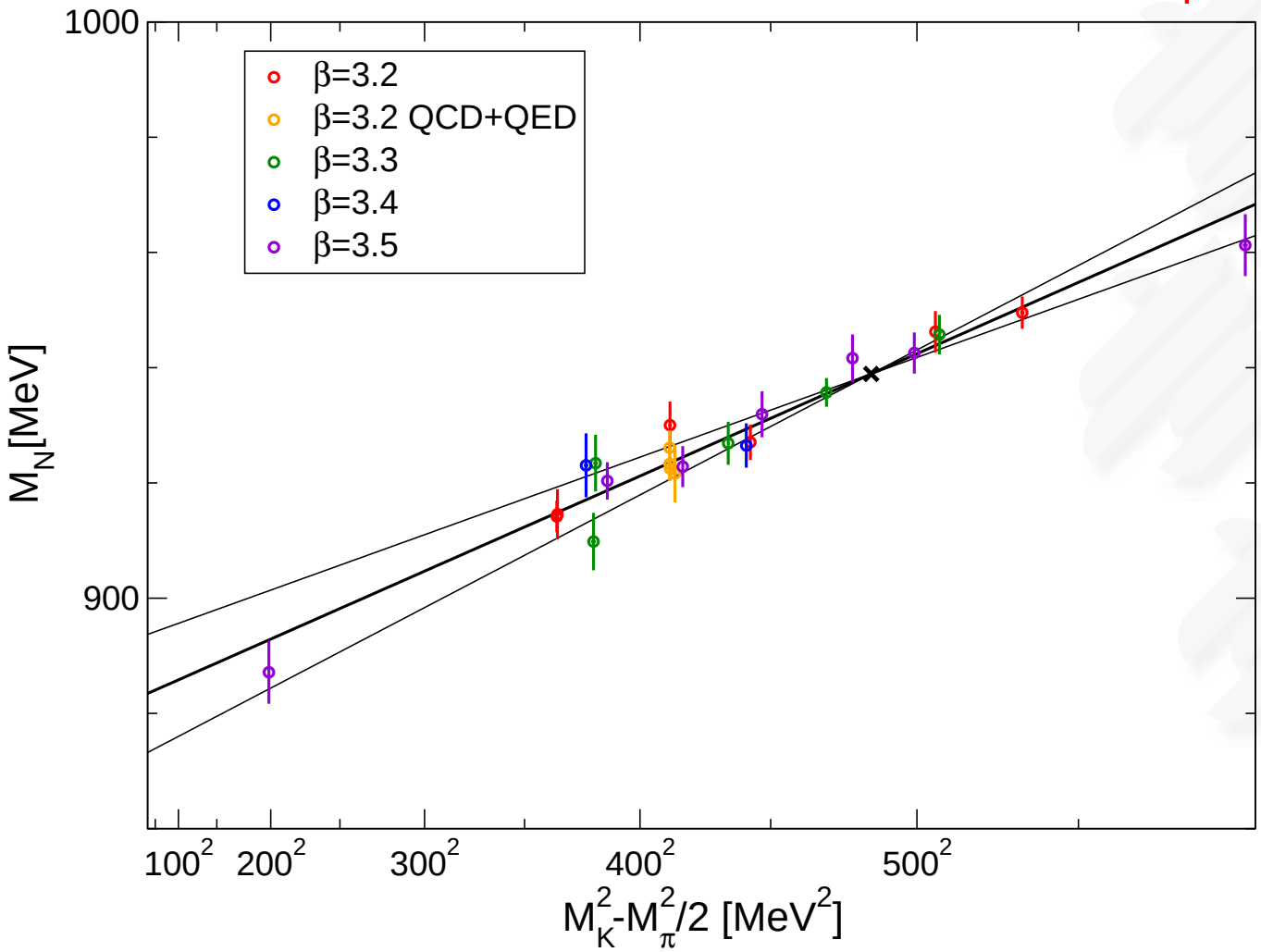
σ_{uN} σ_{dN} σ_{sN}

preliminary



σ_{uN} σ_{dN} σ_{sN}

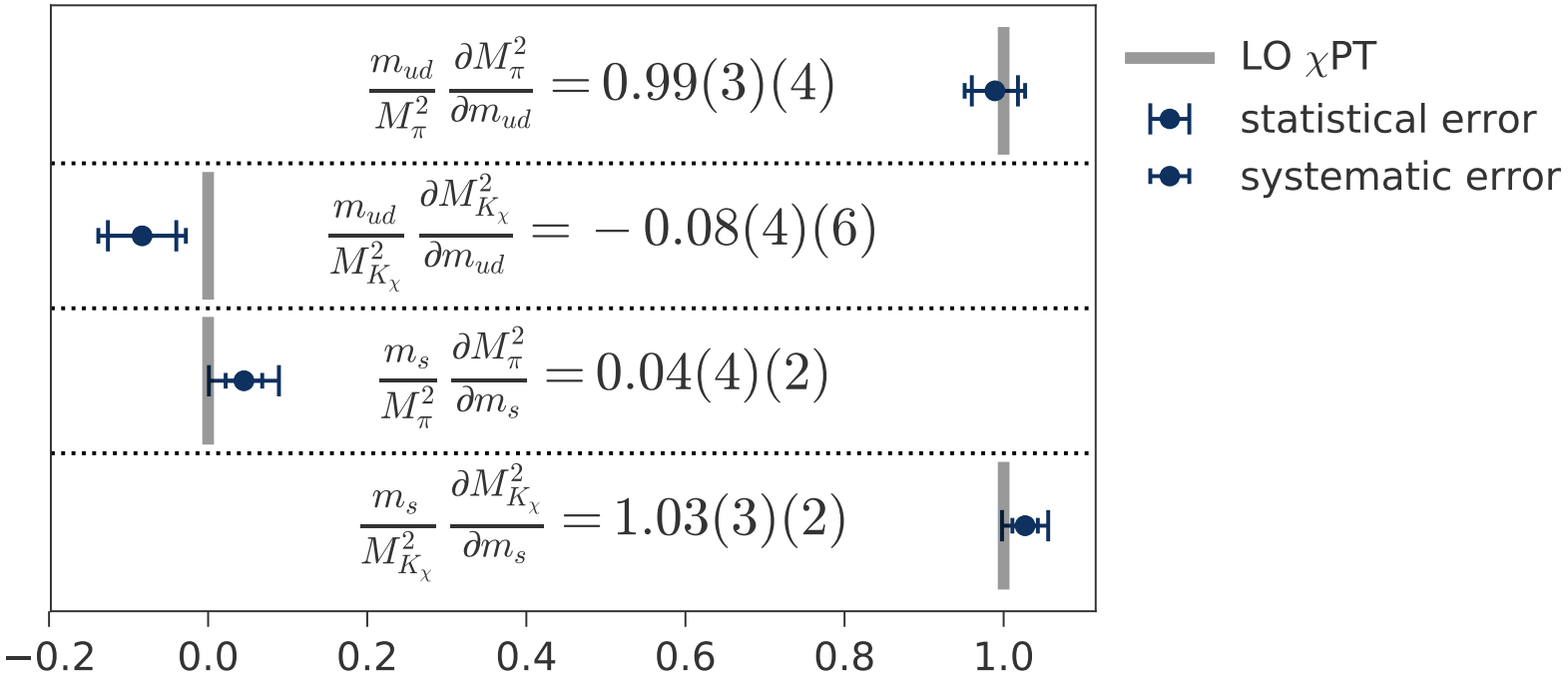
preliminary



σ_{uN} σ_{dN} σ_{sN}

preliminary

The results for the mixing matrix are:



The result for the strange to light quark mass ratio are

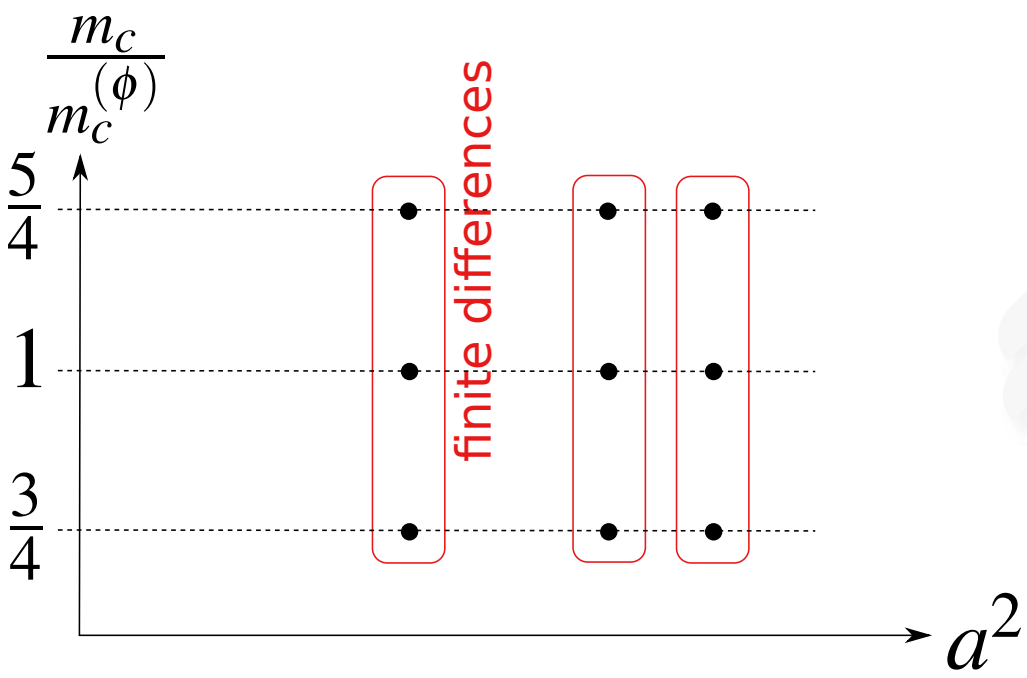
$$\frac{m_s}{m_{ud}} = 27.293(33)(08)$$

(FLAG result: $\frac{m_s}{m_{ud}} = 27.30(34)$).

Strategy for the charm sigma term:

σ_{cN}

We measured the nucleon mass on 9 staggered, Symmanzik improved, 4 times stout smeared ensambles:



We cover

- Three lattice spacings: $\beta = 3.75$, $\beta = 3.7753$, and $\beta = 3.84$.
- Three charm masses: $\frac{3}{4} m_c^{(\phi)}$, $m_c^{(\phi)}$, $\frac{5}{4} m_c^{(\phi)}$.

We use finite differences to determine the charm sigma term.

σ_{cN}

Determination of the nucleon mass:

With staggered fermions the nucleon propagator behaves like:

$$C_O(t) = \sum_{i=0}^{N_{\text{states}}-1} a_i p_i^{t+1} (\exp(-m_i t) + (-1)^{t+1} \exp(-m_i (N_t - t)))$$

We need not only the mass, but a propagator with the parity partner removed. To that end we construct operators of the form

$$O' = \sum_{\tau=0}^{m} b_j \exp(H\tau) O \exp(-H\tau)$$

and apply a variational method.

σ_{cN}

We construct the matrix [1-3]

$$M(t) = \begin{pmatrix} C_O(t) & C_O(t+1) & \dots & C_O(t+m) \\ C_O(t+1) & C_O(t+2) & \dots & C_O(t+m+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_O(t+m) & C_O(t+m+2) & \dots & C_O(t+2m) \end{pmatrix}.$$

and solve the GEVP [4]

$$M(t_0)\vec{v}_i(t_0,t_1) = M(t_1)\lambda_i(t_0,t_1)\vec{v}_i(t_0,t_1).$$

We get the correlation functions of the O' via

$$C'_i(t;t_0,t_1) = \vec{v}_i^\dagger(t_0,t_1)M(t)\vec{v}_i(t_0,t_1)$$

[1] C. Aubin and K. Orginos, "A new approach for Delta form factors," AIP Conf. Proc. **1374** (2011) 621 doi:10.1063/1.3647217 [arXiv:1010.0202 [hep-lat]].

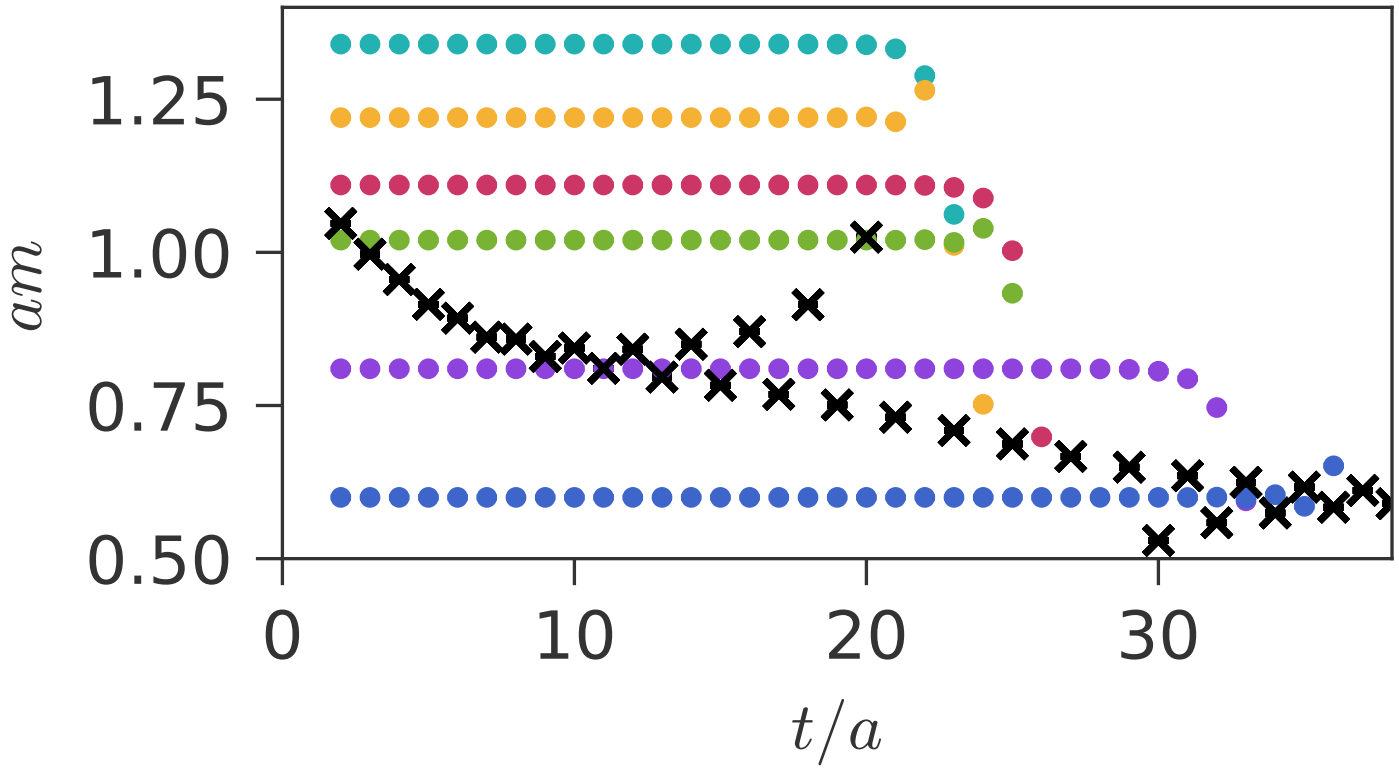
[2] Y. Hua, and T. Sarkar, IEEE transactions on antennas and propagation **37**, 229234 (1989)

[3] T. Sarkar, and O. Pereira, IEEE Antennas and Propagation Magazine **37**, 4855 (1995)

[4] C. DeTar and S. H. Lee, Phys. Rev. D **91** (2015) no.3, 034504 [arXiv:1411.4676 [hep-lat]].

σ_{cN}

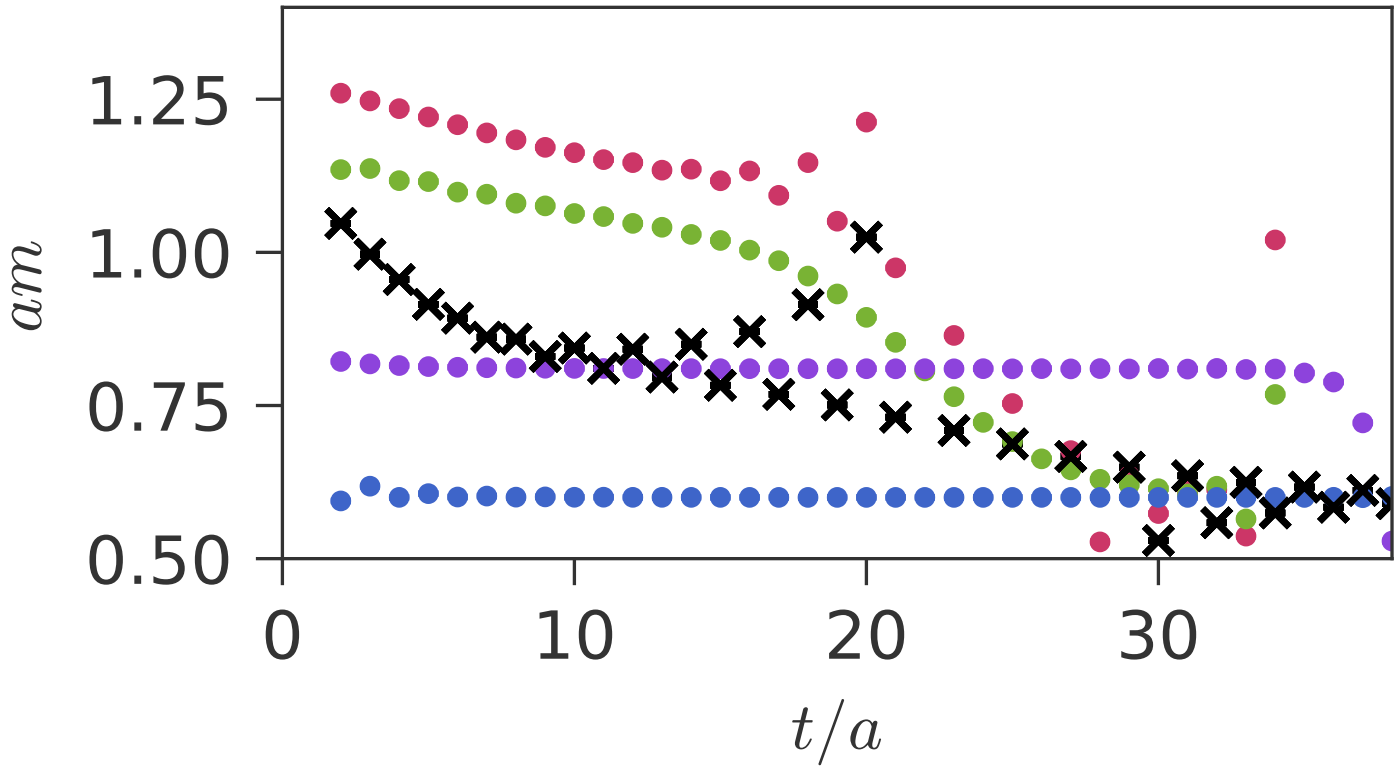
We performed a study with mock data:



Black: Original correlation function. Colored: "Projected" correlation functions.

σ_{cN}

We performed a study with mock data:

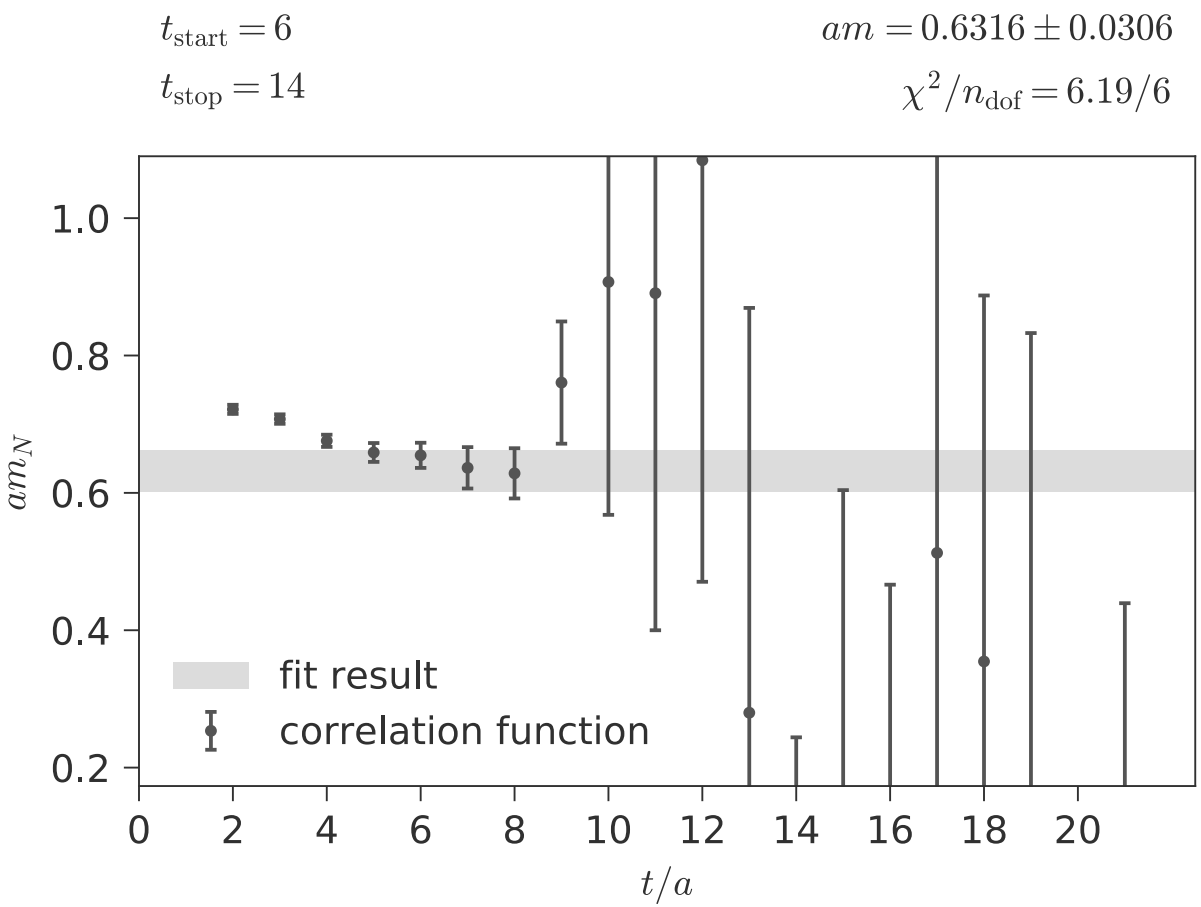


Black: Original correlation function. Colored: "Projected" correlation functions.

σ_{cN}

This method is well suited for the extraction of ground states in the presence of many excited (oscillating) states. It is also applied for the extraction of Ω masses in
"High precision determination of w_0" [Jana N. Guenther].

For the nucleon the "projected" correlation function behaves like



σ_{cN}

As an alternative we used

$$D_O(t) = C_O(t) + e^{\tilde{m}} C_O(t+1)$$

We defined the effective mass

$$m^{\text{eff}}(t) = -\log\left(\frac{D_O(t)}{D_O(t+1)}\right)$$

and its average between the times t_a and t_b. We then minimized the deviation of the effective mass from the averaged effective mass.

The results agree very well with the GEVP based method.

σ_{cN}

Finite difference approximation: For each lattice spacing we used finite differences to estimate the charm sigma term. We used two differences:

$$\Delta^+ M_N = M_N(m_c = {}^5\!/_4 m_c^{\text{central}}) - M_N(m_c = m_c^{\text{central}})$$
$$\Delta^- M_N = M_N(m_c = m_c^{\text{central}}) - M_N(m_c = {}^3\!/_4 m_c^{\text{central}})$$

We combined them in two ways:

- The standard finite difference formula (error: $\mathcal{O}((\delta m_c/m_c)^2) = \mathcal{O}(1/16)$)

$$\sigma_{cN} = m_c \frac{\partial M_N}{\partial m_c} = 2\frac{\Delta^+ M_N + \Delta^- M_N}{M_N^{(\phi)}}.$$

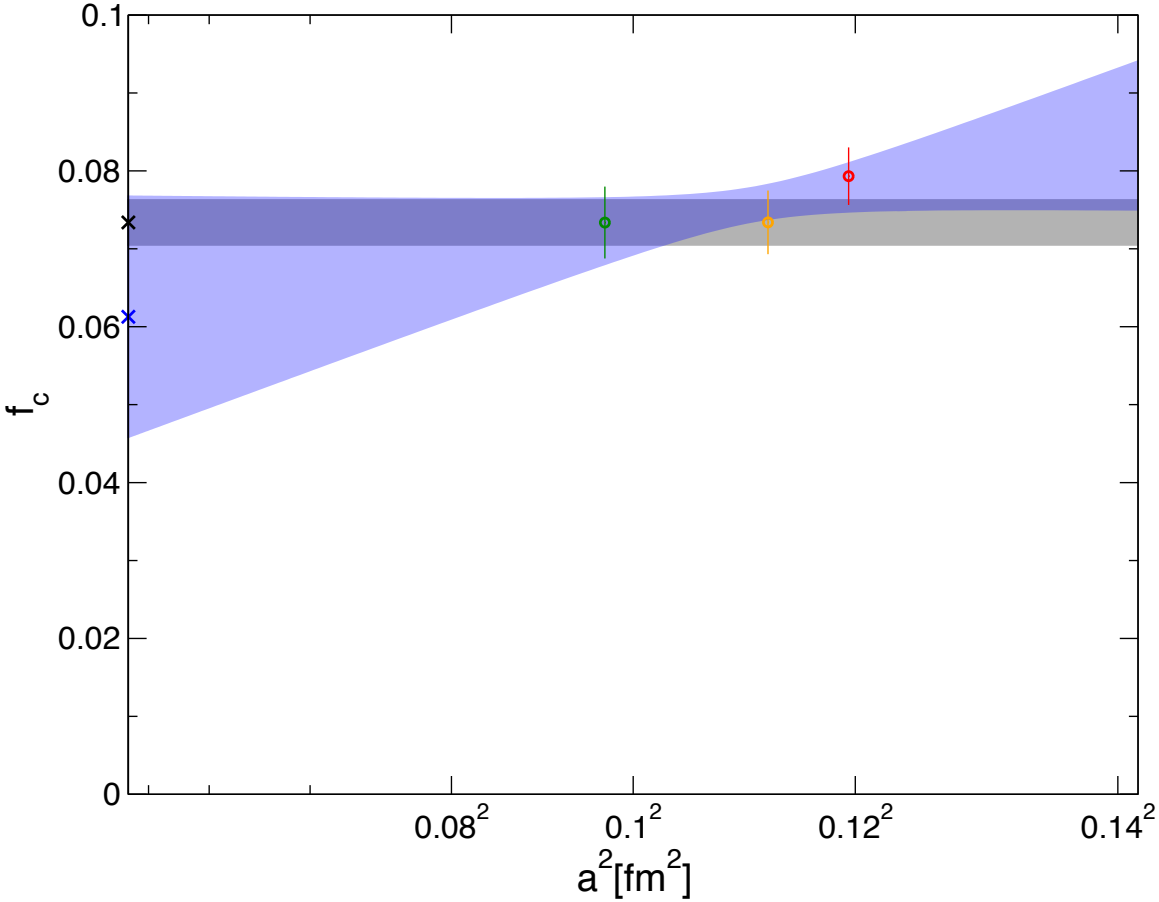
- Based on the HQ behaviour of sigma terms (error: $\mathcal{O}((\sigma_{cN}/M_N^{(\phi)})^3) = \mathcal{O}(3 \times 10^{-4})$)

$$\sigma_{cN} = \frac{1}{\log\frac{5}{4}\log\frac{4}{3}\log\frac{5}{3}}\left(\log^2\frac{4}{3}\Delta^+ M_N + \log^2\frac{5}{4}\Delta^- M_N\right)$$

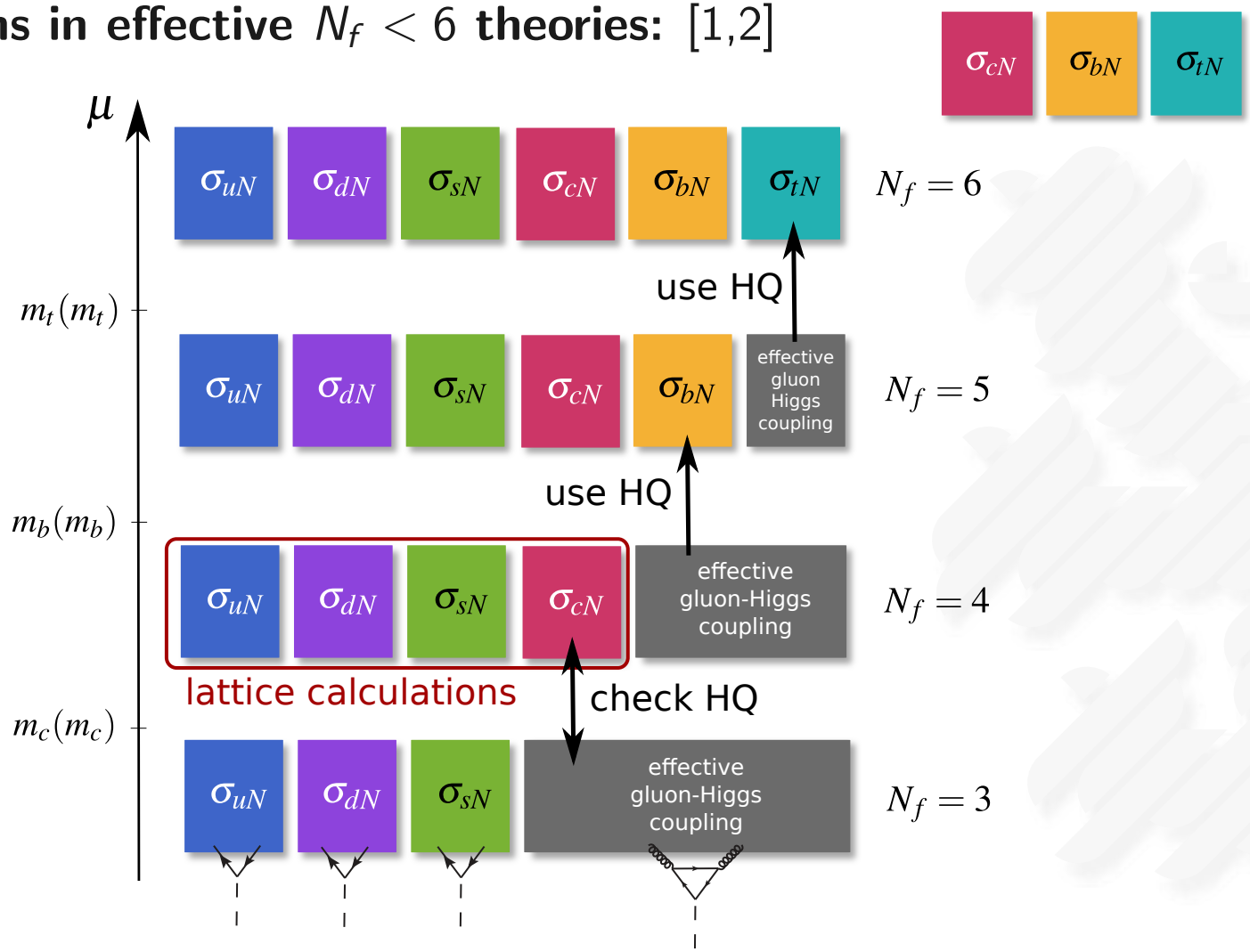
σ_{cN}

For the continuum extrapolation we used:

1. A constant fit in a^2 with only the two finest lattice spacings included.
2. A linear fit in a^2 with all lattice spacings included.
3. A quadratic fit in a^2 with all lattice spacings included.



Sigma terms in effective $N_f < 6$ theories: [1,2]



[1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. **78B** (1978) 443.

[2] R. J. Hill and M. P. Solon, Phys. Rev. D **91** (2015) 043505 [arXiv:1409.8290].

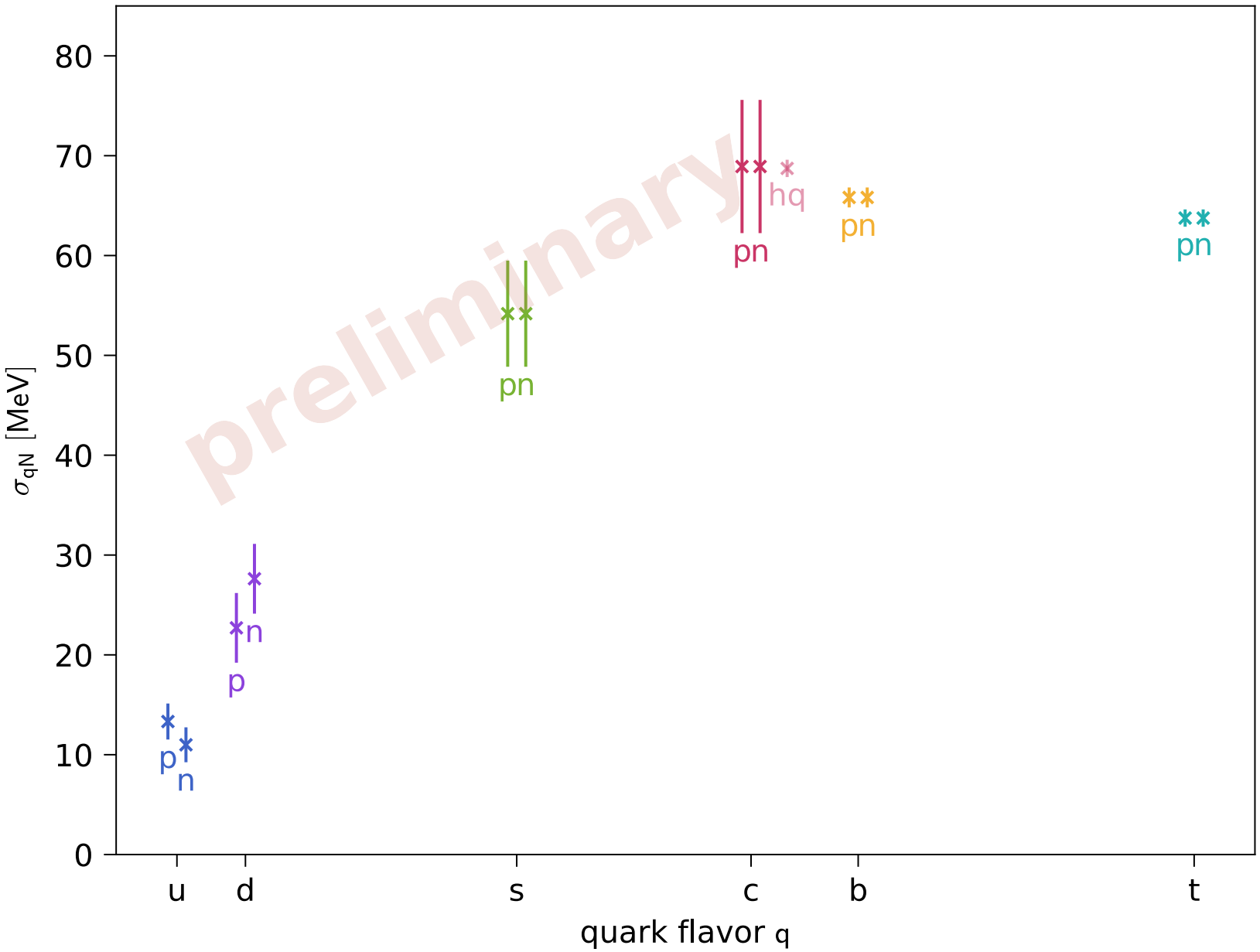
σ_{uN} σ_{dN} σ_{sN} σ_{cN} σ_{bN} σ_{tN}

Systematic uncertainties We use the histogram method to determine our systematic uncertainties.

All uncertainties from the different parts are combined.

We will provide a small programm, that allows to calculate fully correlated statistical and systematic errors for a arbitrary linear combinations of sigma terms.

Results


preliminary
80
70
60
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40
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0
σ_{qN} [MeV]
p
n
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n
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pn
hq
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b
t
quark flavor q