Nucleon scalar charge with overlap fermions

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in collaboration with
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Outline

- Introduction.
- Numerical details.
- Results.
- Summary and outlook.
Motivation

- Fundamental property of hadron structure.
- Probe new physics beyond standard model.
- Not well known experimentally.
- Need accurate lattice determination.
Gauge configurations generated by the RBC/UKQCD collaboration with 2+1 flavor domain-wall fermion.

<table>
<thead>
<tr>
<th>Ensemble ID</th>
<th>$L^3 \times T$</th>
<th>$a^{-1}$ (GeV)</th>
<th>$m_{\pi}^{sea}$ (MeV)</th>
<th>$N_{conf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24I</td>
<td>$24^3 \times 64$</td>
<td>1.747</td>
<td>337</td>
<td>203</td>
</tr>
<tr>
<td>32I</td>
<td>$32^3 \times 64$</td>
<td>2.310</td>
<td>302</td>
<td>309</td>
</tr>
<tr>
<td>48I</td>
<td>$48^3 \times 96$</td>
<td>1.730</td>
<td>139</td>
<td>81</td>
</tr>
<tr>
<td>32ID</td>
<td>$32^3 \times 64$</td>
<td>1.3709</td>
<td>171</td>
<td>200</td>
</tr>
<tr>
<td>32Ifine</td>
<td>$32^3 \times 64$</td>
<td>3.148</td>
<td>371</td>
<td>459</td>
</tr>
</tbody>
</table>
Overlap fermions

- Overlap fermion action for the valence quark.
- Chiral symmetry at finite lattice spacing via the Ginsparg-Wilson relation.
- Has better control of the systematic error, but more costly.

The overlap Dirac operator is

\[ D(m) = \rho D_{ov}(\rho) + m \left( 1 - \frac{D_{ov}(\rho)}{2} \right), \]

where \( D_{ov} = 1 + \gamma_5 \epsilon (\gamma_5 D_w(\rho)) \), we use the parameter \( \rho = -1.5 \).

The effective propagator is

\[ D^{-1}_{\text{eff}} = \frac{1}{D_c + m'}, \]

where \( D_c = \frac{\rho D_{ov}}{1 - D_{ov}/2} \) is exactly chiral, i.e. \( \{ \gamma_5, D_c \} = 0 \).
Computational strategies

- **Low-mode substitution (LMS):** Use the low lying eigenvalues and eigenvectors of $D_c$ to speed up the inversion and separate the propagators into its low-mode and high-mode parts.

  \[ S(y, x) = S_L(y, x) + S_H(y, x), \]
  \[ S_L(y, x) = \sum_{|\lambda_i| < \epsilon} \frac{1}{\lambda_i + m} |i\rangle_y \langle i|_x \]

- **$Z_3$ noise grid source:** Tie the sources of the three quark propagators stochastically to each point so that one can have multi-to-all propagator from one inversion.

- **Stochastic sandwich method with LMS:** all-to-all low mode + noise many-to-all high mode
To extract the scalar charge, we compute the two- and three-point functions:

\[ C_{2pt}(t_2) = \sum_{\vec{x}} \Gamma^e_{\alpha\beta} < 0|\chi_\alpha(t_2, \vec{x})\bar{\chi}_\beta(0, 0)|0>, \]

\[ C_{3pt}^{u,d}(t_2, t) = \sum_{\vec{x}, \vec{x}'} \Gamma^e_{\alpha\beta} < 0|\chi_\alpha(t_2, \vec{x})\mathcal{O}_{u,d}^{t, t'}(t, \vec{x}')\chi_\beta(0, 0)|0>, \]

where \( \chi \) is the proton interpolating operator \( \chi_\alpha = \epsilon^{abc}[u^aT \gamma_5 d^b]u^c \).

Only the connected insertion is computed for isovector scalar charge.
For each ensemble, the correlation function are computed for several values of valence quark mass (use multi-mass algorithm) and source-sink separation.

<table>
<thead>
<tr>
<th>Ensemble ID</th>
<th>$m_{\pi}^{sea}$ (MeV)</th>
<th>$m_{\pi}^{valence}$ (MeV)</th>
<th>$t_{sink} - t_{source}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24I</td>
<td>337</td>
<td>282 ~ 389</td>
<td>8, 10, 12</td>
</tr>
<tr>
<td>32I</td>
<td>302</td>
<td>295 ~ 410</td>
<td>12, 14, 15</td>
</tr>
<tr>
<td>48I</td>
<td>139</td>
<td>134 ~ 372</td>
<td>8, 10, 12</td>
</tr>
<tr>
<td>32ID</td>
<td>171</td>
<td>146 ~ 325</td>
<td>7, 8, 9, 10, 11</td>
</tr>
<tr>
<td>32Ifine</td>
<td>371</td>
<td>344 ~ 485</td>
<td>14, 16, 18</td>
</tr>
</tbody>
</table>
Two-state fit

\[ R^q(t_2, t) = \frac{C_{3pt}(t_2, t)}{C_{2pt}(t_2)} = g^q_S + C_1^q e^{-\Delta M t_2} + C_2^q e^{-\Delta M (t_2-t)} + C_3^q e^{-\Delta M t}, \]

Joint fit of \( R^u(t, t_s) \) and \( R^d(t, t_s) \):

Renormalization factor \( Z_S \) (Z. Liu et al. Phys. Rev. D 90, 034505 (2014)):

<table>
<thead>
<tr>
<th>( Z_S )</th>
<th>24I</th>
<th>32I</th>
<th>32ID</th>
<th>48I</th>
<th>32I_{\text{fine}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>1.1272</td>
<td>1.0563</td>
<td>1.2355</td>
<td>1.1272</td>
<td>0.9539</td>
</tr>
</tbody>
</table>
Results

\[ g_S(m_{\pi, \nu\nu}, m_{\pi, \nu S}, \alpha, L) = c_0 + c_1 m_{\pi, \nu\nu}^2 + c_2 m_{\pi, \nu S}^{mix} + c_3 (c_4) \alpha^2 + c_5 e^{-m_{\pi, \nu\nu} L} \]

<table>
<thead>
<tr>
<th>c_0</th>
<th>c_1</th>
<th>c_2</th>
<th>c_3</th>
<th>c_4</th>
<th>c_5</th>
<th>\chi^2/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88(0.10)</td>
<td>4.03(0.62)</td>
<td>-4.71(1.00)</td>
<td>-1.11(5.42)</td>
<td>3.13(3.79)</td>
<td>4.83(2.05)</td>
<td>1.06</td>
</tr>
</tbody>
</table>

\[ g_S^{u-d} = 0.87(9) \]
This work

JLQCD'18

PNDME'18

ETMC'17

RQCD'14

LHPC'12

$g_{u-d}$
Outlook

- More statistics on the physical ensemble.
- Systematic error needs further investigation.
- Axial, tensor charges, disconnected insertion.
- $\sigma$-terms, Form factors.