Developments in the position-space approach to the HLbL contribution to the muon $g-2$ on the lattice

Nils Asmussen

in Collaboration with
En-Hung Chao, Antoine Gérardin, Jeremy Green, Jamie Hudspith,
Harvey Meyer, Andreas Nyffeler

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Anomalous Magnetic Moment of the Muon

gyromagnetic moment: $\boldsymbol{\mu} = g \frac{e}{2m} \mathbf{S}$

anomalous magnetic moment: $a_\mu = \frac{g-2}{2}$

$\approx$ 3 to 4 standard deviations tension between a_μ^{exp} and a_μ^{theo}

$\rightarrow$ new physics?

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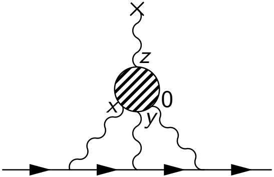
reduce uncertainties

experiment	theory for HLbL	
J-PARC Fermilab	**phenomenology** reduce model uncertainties for dominant contribution $(\pi^0, \eta, \eta'; \pi\pi)$ using experimental input Colangelo *et al.* '14,...,'17 Hoferichter *et al.* '18,'18 Pauk and Vanderhaeghen '14	**lattice QCD** model independent estimates Blum *et al.* ('05,...)'15,...,'17 Mainz lattice group '15,...,'18 Pion TFF: Gérardin *et al.* '16, '19

Euclidean position-space approach to a_μ^{HLbL}



master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int d^4y \Big[\int d^4x \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\mathrm{QED}} \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{\mathrm{QCD}} \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z\, z_\rho \left\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \right\rangle.$$

- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects from the photons
- manifest Lorentz covariance

Outline

Tests in Continuum, Infinite Volume

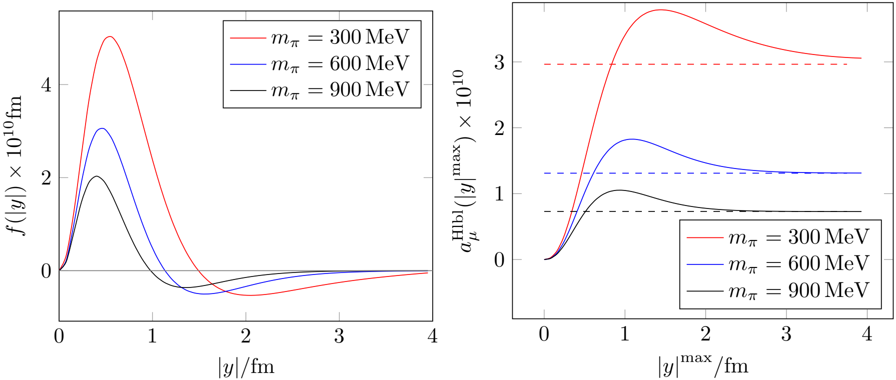
master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} 8\pi^3 \int_0^\infty d|y| |y|^3 \Big[\int_0^\infty d|x| |x|^3 \int_0^\pi d\beta \sin^2\beta \, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \, z_\rho \left\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \right\rangle.$$

- $\int d|x|$, $\int d|y|$ and $\int d\beta$ evaluated numerically
- $\int d^4z$ evaluated (semi-)analytically

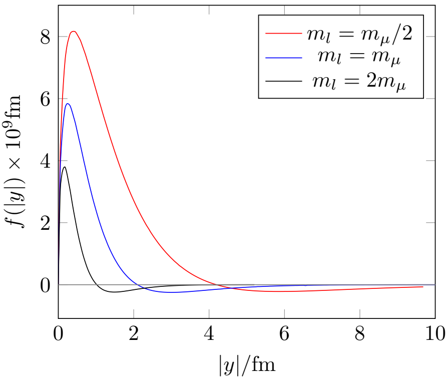
Contribution of the π^0 to a_μ^{HLbL} (VMD Model)



dashed line = result from momentum-space integration

- we reproduce the known result
- contribution is perhaps surprisingly long-range
- integrand peaked at short distances

Lepton loop integrand contribution to a_μ^{HLbL}



- we reproduce the known result
- contribution is long-range
- integrand sharply peaked at short distances

The QED kernel is correct

- we reproduce the π^0-pole in VMD model
- we reproduce the lepton loop

What next?

achievements

- method for a_μ^{HLbL} on the lattice
- verified the QED kernel
- learned about the integrand

challenges in the view of lattice computations

- contributions are quite long range
- integrand peaked at small distances

What next?

achievements

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challenges in the view of lattice computations

- contributions are quite long range
- integrand peaked at small distances

a way to improve

- do subtractions on the kernel (first proposed by Blum *et al.* '17)
- exploit $\int_x i\hat{\Pi}(x,y) = \int_y i\hat{\Pi}(x,y) = 0$
- example:
  - $\mathcal{L}^{(0)} = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$
  - $\mathcal{L}^{(1)} = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) - \frac{1}{2}\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,x) - \frac{1}{2}\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(y,y)$
  - $a_\mu^{\text{HLbL}} = \frac{me^6}{3}\int_{x,y}\mathcal{L}^{(0)}(x,y)i\hat{\Pi}(x,y) = \frac{me^6}{3}\int_{x,y}\mathcal{L}^{(1)}(x,y)i\hat{\Pi}(x,y)$

Subtractions

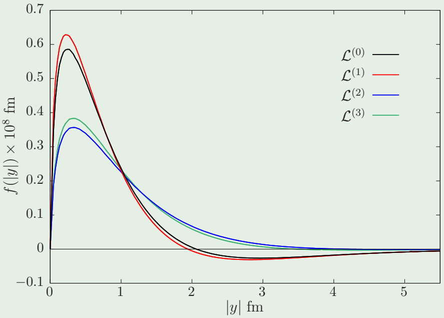
master formula

$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} 8\pi^3 \int_0^\infty d|y||y|^3 \left[\int_0^\infty d|x||x|^3 \int_0^\pi d\beta \sin^2\beta \, \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \; i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \right].$$

subtractions on the kernel

- we try (short notation):
  $\mathcal{L}^{(0)} = \bar{\mathcal{L}}(x,y)$ (standard kernel)
  $\mathcal{L}^{(1)} = \bar{\mathcal{L}}(x,y) - \frac{1}{2}\bar{\mathcal{L}}(x,x) - \frac{1}{2}\bar{\mathcal{L}}(y,y)$
  $\mathcal{L}^{(2)} = \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,0)$
  $\mathcal{L}^{(3)} = \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,x) + \bar{\mathcal{L}}(0,x)$
- $\mathcal{L}^{(0)}(0,0) = 0$
  $\mathcal{L}^{(1)}(x,x) = 0$
  $\mathcal{L}^{(2)}(0,y) = \mathcal{L}^{(2)}(x,0) = 0$
  $\mathcal{L}^{(3)}(x,x) = \mathcal{L}^{(3)}(0,y) = 0$

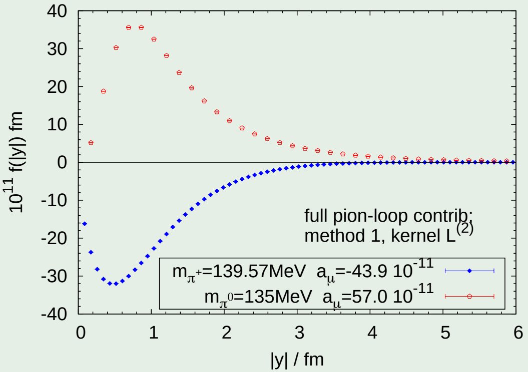
y integrand lepton loop $m_l = m_\mu$



- with all kernels $\mathcal{L}^{(0,1,2,3)}$ we can reproduce the known result
- we expect $\mathcal{L}^{(2,3)}$ to be advantageous on the Lattice

Charged Pion Loop

y integrand π^+-loop and π^0-pole



- charged pion loop from scalar QED, $\mathcal{L} = (\partial_\mu + ieA_\mu)\phi^*(\partial_\mu - ieA_\mu)\phi$ (note that we did not apply form factors)
- the π^+-loop contribution is successfully reproduced

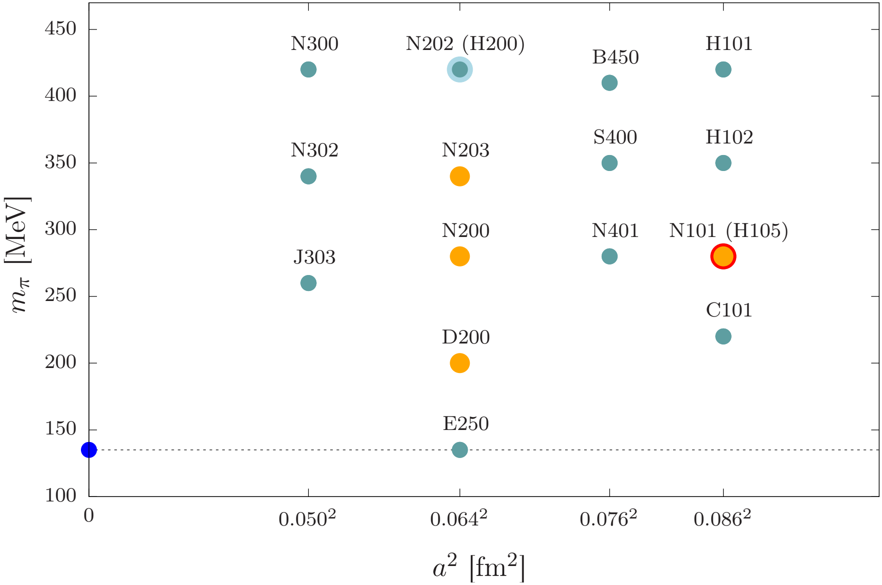
Lattice Setup

CLS $N_f = 2 + 1$ ensembles

CLS	$L^3 \times T$	a [fm]	m_π [MeV]	$m_\pi L$	L [fm]	#confs
H105	$32^3 \times 96$	0.086	285	3.9	2.7	1000
N101	$48^3 \times 128$		285	5.9	4.1	400
N203	$48^3 \times 128$	0.064	340	5.4	3.1	750
N200	$48^3 \times 128$		285	4.4	3.1	800
D200	$64^3 \times 128$		200	4.2	4.2	1100

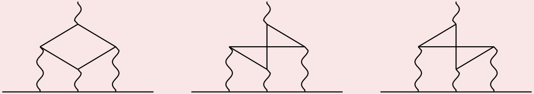
- $\mathcal{O}(a)$ improved Wilson fermions

$N_f = 2 + 1$ CLS Ensembles



Lattice Setup

method 1 (presented so far)

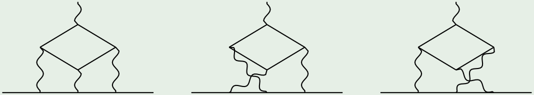


computational cost

- (1+N) forward propagators
- 6(1+N) sequential prop.

$$\int_{y,x,z} \mathcal{L}(x,y)(-z_\rho)[\Pi^{(1)}(x,y,z) + \Pi^{(1)}(y,x,z) + \Pi^{(1)}(x-y,-y,z-y)]$$

method 2



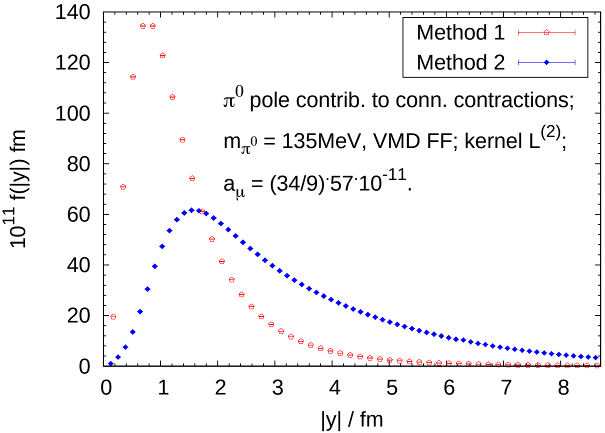
computational cost

- (1+N) forward propagators

$$\int_{y,x}\Big([\mathcal{L}(x,y) + \mathcal{L}(y,x) - \mathcal{L}(x,x-y)] i\widehat{\Pi}^{(1)}(x,y) + \int_z \mathcal{L}(x,x-y) x_\rho \Pi^{(1)}(x,y,z)\Big)$$

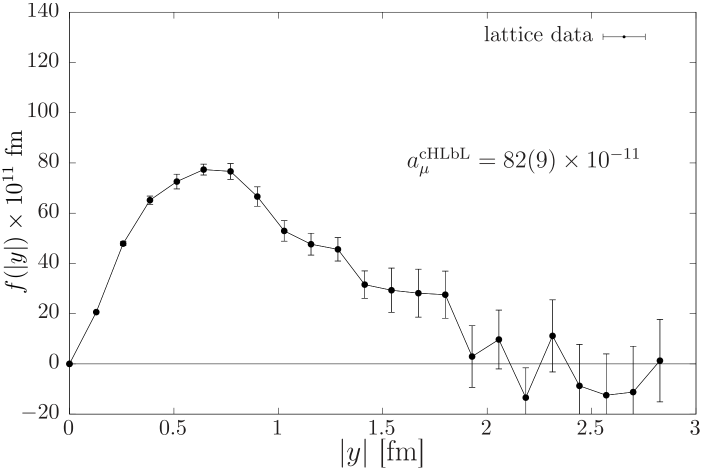
- If the 1-dim. integral over $|y|$ is done with N evaluations of the integrand.
- we sum over x and z explicitly over the whole lattice

Comparison of the Method 1 and 2



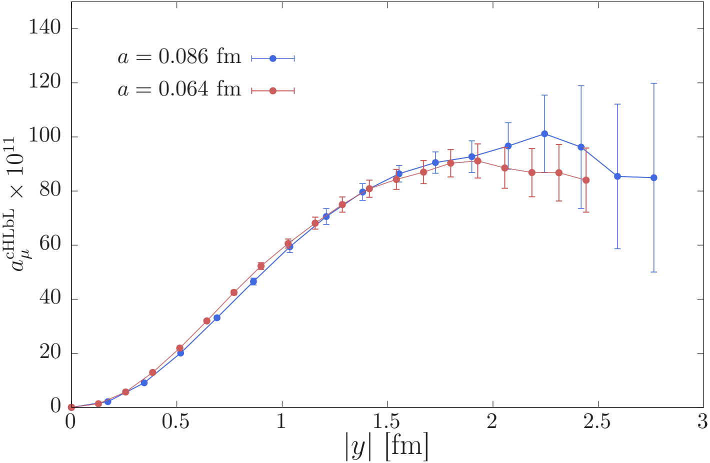
- method 2: reduced computational cost but longer range

Integrand of a_μ^{cHLbL} with $\mathcal{L}^{(2)}$, $m_\pi = 340\,\text{MeV}$, $a = 0.064\,\text{fm}$



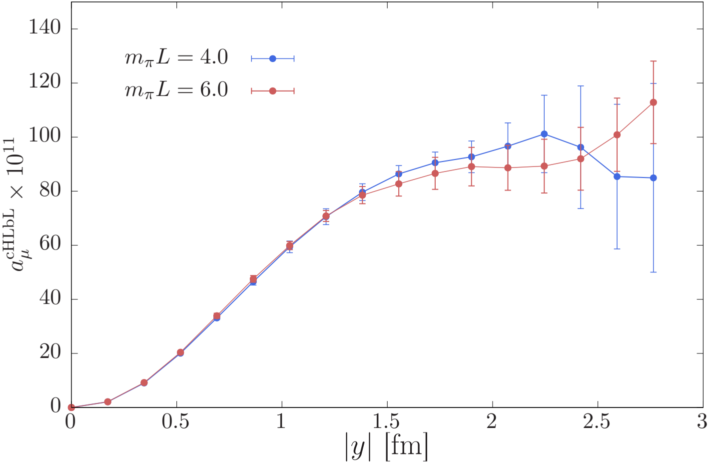
- fully connected contribution only
- we already observe a good signal
- integrand non-zero up to 2 fm

Discretisation Effects, $m_\pi = 285\,\text{MeV}$



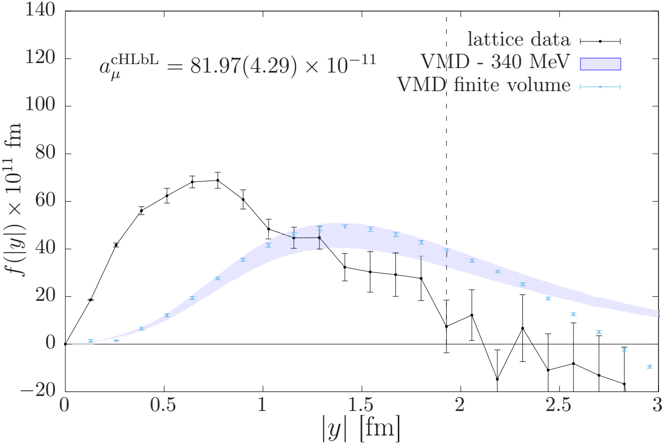
- discretisation effects seem to be small (we are increasing statistics)

Finite Size Effects, $a = 0.086\,\mathrm{fm}$



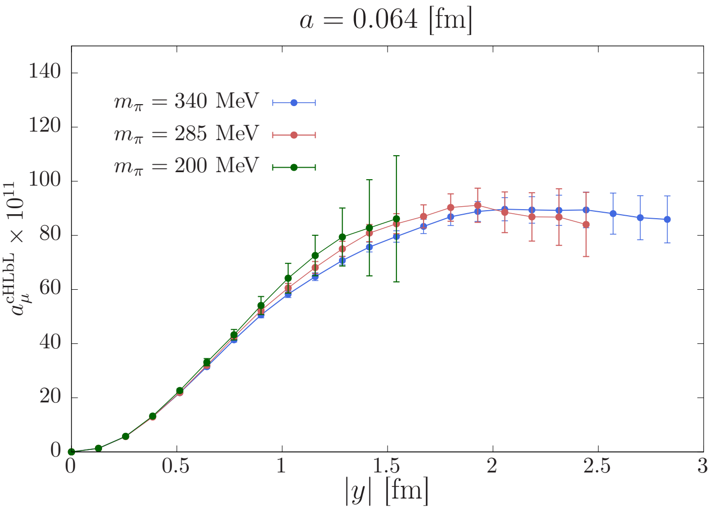
- need more statistics to test for finite-size effects

Integrand of a_μ^{cHLbL} with $\mathcal{L}^{(2)}$, $m_\pi = 340\,\text{MeV}$, $a = 0.064\,\text{fm}$



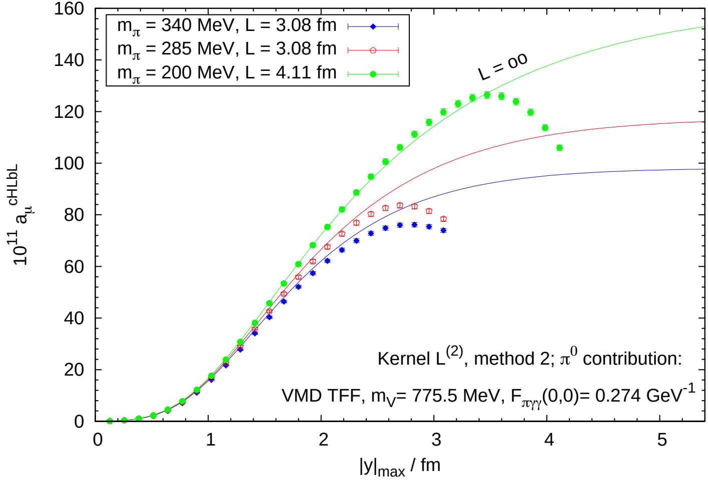
- observe effects not covered by the π^0-exchange prediction at short distances
- more statistics needed to test for agreement with prediction at long distances
- π^0-exchange prediction of the finite-size effects might be valuable to improve the accuracy of a_μ^{HLbL}

Pion Mass Dependence of a_μ^{cHLbL}



- the results show an upward trend for decreasing pion mass
- currently collecting more statistics in long distance regime

Pion Mass Dependence of a_μ^{cHLbL}



- effect of the mass for small distances is small
- deviation more pronounced for larger distances

Conclusions

- Explicit formula for a_μ^{HLbL}
  - QED kernel function multiplying the position-space QCD correlation function
- Tests
  - QED kernel: reproduce known results for π^0 pole, π^+-loop and lepton-loop in the continuum for the standard kernel $\mathcal{L}^{(0)}$ and subtracted kernels $\mathcal{L}^{(1,2,3)}$
- Lattice QCD
  - Preliminary results for the fully connected contribution (in QED_∞)
  - The discretisation effects seem to be small
  - VMD resuts suggest to further investigate finite-size effects
- Future
  - Collect more statistics
  - Study finite-size effects in VMD to complement lattice computation
  - Perform chiral and continuum extrapolations
  - Implement disconnected contribution