Strange nucleon form factors and isoscalar charges with $N_f = 2 + 1$ $O(a)$-improved Wilson fermions

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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL
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Standard Model observables

- weak mixing angle Θ_W [1]

$$A_{PV} = \frac{G_\mu Q^2}{4\pi\alpha\sqrt{2}} \left\{ Q_W(p) - F(E_i, Q^2) \right\}$$

$$Q_W(p) = 1 - 4\sin^2\Theta_w$$

$\rightarrow$ $F(E_i, Q^2)$ contains G_E^s, G_M^s and G_A^s

[1] Becker et al., 1802.04759

Standard Model observables

- contribution of intrinsic spin of quark flavor f

$$G_A^f(0) = g_A^f = \Delta f$$

$$\frac{1}{2} = \sum_f \left(\frac{1}{2} \Delta f + L_f \right) + J_g$$ [1]

- neutron electric dipole moment

$$d_n = \sum_f d_f^\gamma g_T^f$$

$\Rightarrow$ appearance of quark-disconnected contributions

[1] X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), arXiv:hep-ph/9603249

Previous Work

- Strange nucleon form factors with $N_f = 2 + 1$ $\mathcal{O}(a)$-improved Wilson fermions

  D. Djukanovic, H. Meyer, K. Ottnad, G. von Hippel, J. Wilhelm, H. Wittig, arXiv:1810.10810
- Strange electromagnetic form factors of the nucleon with $N_f = 2 + 1$ $\mathcal{O}(a)$-improved Wilson fermions

  D. Djukanovic, K. Ottnad, J. Wilhelm, H. Wittig, arXiv:1903.12566
- Nucleon isovector charges and twist-2 matrix elements with $N_f = 2 + 1$ dynamical Wilson quarks

  T. Harris, G. v. Hippel, P. Junnarkar, H. Meyer, K. Ottnad, J. Wilhelm, H. Wittig, L. Wrang, arXiv:1905.01291

form factors

- construct a ratio

$$R_{J_\mu}(\vec{q};\vec{p}';\Gamma_\nu) = \frac{C^N_{3,J_\mu}(\vec{q},z_0;\vec{p}',y_0;\Gamma_\nu)}{C^N_2(\vec{p}',y_0)}\sqrt{\frac{C^N_2(\vec{p}',y_0)C^N_2(\vec{p}',z_0)C^N_2(\vec{p}'\text{-}\vec{q},y_0\text{-}z_0)}{C^N_2(\vec{p}'\text{-}\vec{q},y_0)C^N_2(\vec{p}'\text{-}\vec{q},z_0)C^N_2(\vec{p}',y_0\text{-}z_0)}}$$

$$\overset{\text{s.d.}}{=} M^1_{\nu\mu}(\vec{q},\vec{p}')G_1(Q^2) + M^2_{\nu\mu}(\vec{q},\vec{p}')G_2(Q^2)$$

- (overdetermined) system of equations at each Q^2

$$M\,\vec{G} = \vec{R}\ ;\ M = \begin{pmatrix} M^1_1 & M^2_1 \\ \vdots & \vdots \\ M^1_N & M^2_N \end{pmatrix}\ ,\ \vec{G} = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix},\ \vec{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix}$$

- minimizing the least-squares function

$$\chi^2 = (\vec{R} - M\vec{G})^T\ C^{-1}\ (\vec{R} - M\vec{G})$$

form factor parametrization

$$\left\langle N, \vec{p}, s \left| J_\mu(x) \right| N, \vec{p}', s' \right\rangle = \bar{u}^s(\vec{p}) \; \widetilde{J}_\mu \; u^{s'}(\vec{p}') e^{\mathrm{i} q \cdot x}$$

- axial vector current

$$\widetilde{A}_\mu = \gamma_\mu \gamma_5 G_\mathrm{A}(Q^2) + \gamma_5 \frac{q_\mu}{2m} G_\mathrm{P}(Q^2)$$

- vector current

$$\widetilde{V}_\mu = \gamma_\mu F_1(Q^2) + \mathrm{i}\sigma_{\mu\nu} \frac{q^\nu}{2m} F_2(Q^2)$$

$$F_1, F_2 \leftrightarrow G_\mathrm{E}, G_\mathrm{M}$$

- tensor current at $Q^2 = 0$

$$\widetilde{T}_{\mu\nu} = \mathrm{i}\sigma_{\mu\nu} g_\mathrm{T}$$

ensembles

- **C**oordinated **L**attice **S**imulations (CLS)[1,2]
  - $N_f = 2 + 1$ $\mathcal{O}(a)$-improved Wilson fermions
  - tree-level improved Lüscher-Weisz gauge action
  - open boundary conditions in time
  - $\mathrm{tr}\ M_q = \text{const}$

	β	a [fm]	$N_s^3 \times N_t$	m_π[MeV]	m_K[MeV]	$N_{\text{cfg}}^{\text{dis}}$	$N_{\text{cfg}}^{\text{con}}$
H102	3.40	0.08636	$32^3 \times 96$	352	436	997	997
H105	3.40	0.08636	$32^3 \times 96$	278	461	1019	1019
C101	3.40	0.08636	$48^3 \times 96$	223	473	489	1956
N401	3.46	0.07634	$48^3 \times 128$	289	462	701	701
N203	3.55	0.06426	$48^3 \times 128$	345	441	768	1536
N200	3.55	0.06426	$48^3 \times 128$	282	462	852	852
D200	3.55	0.06426	$64^3 \times 128$	203	482	234	936
N302	3.70	0.04981	$48^3 \times 128$	359	458	1177	1177

[1] CLS, 1411.3982
[2] CLS, 1608.08900

setup

- nucleon interpolator

$$N_\alpha(x) = \epsilon_{abc} \left(u^a_\beta(x) \; (C\gamma_5)_{\beta\gamma} \; d^b_\gamma(x) \right) \; u^c_\alpha(x)$$

- smeared-smeared quark propagators
- truncated solver method for two- and three-point functions[1,2]

$$C^N_{2/3} = \underbrace{\frac{1}{N^{LP}_{\text{src}}} \sum_{n=1}^{N^{LP}_{\text{src}}} C^N_{2/3}(x_n)^{LP}}_{\text{biased estimate}} + \underbrace{\left(\frac{1}{N^{HP}_{\text{src}}} \sum_{n=1}^{N^{HP}_{\text{src}}} \left(C^N_{2/3}(x_n)^{HP} - C^N_{2/3}(x_n)^{LP} \right) \right)}_{\text{bias correction}}$$

[1] G. Bali et al., PoS (LATTICE 2015) 350

[2] E. Shintani et al., Phys. Rev. D91 114511 (2015)

lattice currents

- improved local axial current

$$A_\mu(\vec{z}, z_0)^{Imp.} = A_\mu(\vec{z}, z_0) + ac_A\ \partial_\mu P(\vec{z}, z_0)$$

- improved conserved vector current

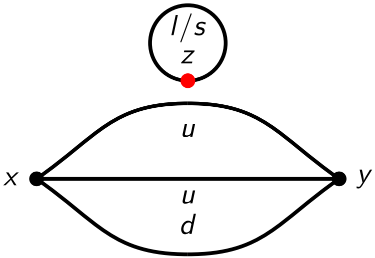
$$V_\mu(\vec{z}, z_0)^{Imp.} = V_\mu(\vec{z}, z_0) + ac_V\ \partial_\nu T_{\nu\mu}(\vec{z}, z_0)$$

- non-perturbative determination of c_A and c_V [1,2]
- local tensor current

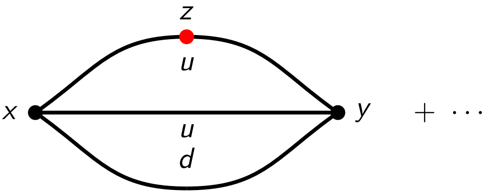
[1] Alpha Collaboration, Nucl. Phys. B 896 (2015) 555
[2] Gérardin et al., Phys. Rev. D 99, 014519 (2019)

contributions

- disconnected: $G^{l}(Q^2)$ and $G^{s}(Q^2)$



- connected: $G^{u+d}_{\text{con.}}(Q^2)$



contributions

- disconnected:
  - three-point function factorizes

$$C_{3,J_\mu}^{N,l/s}(\vec{q},z_0;\vec{p}',y_0;\Gamma_\nu) = \left\langle \mathcal{L}_{J_\mu}^{l/s}(\vec{q},z_0)\cdot \mathcal{C}_2^N(\vec{p}',y_0;\Gamma_\nu)\right\rangle_G$$

  - parity and spin projectors

$$\Gamma_0 = \frac{1}{2}(1+\gamma_0) \quad , \quad \Gamma_k = \Gamma_0 \; \mathrm{i}\gamma_5\gamma_k$$

- connected:
  - sequential propagators for three-point function
  - drop A_0 due to large excited-state contamination
  - parity and spin projector

$$\Gamma = \frac{1}{2}(1+\gamma_0)(1+\mathrm{i}\gamma_5\gamma_3)$$

source positions

- disconnected:
  - $N_{\rm src}^{HP} = 1$ or 4, $N_{\rm src}^{LP} = 32$
  - on 7 timeslices evenly distributed around $N_t/2$ separated by seven timeslices without sources
- connected:
  - $N_{\rm src}^{HP} = 1$ or 4, $N_{\rm src}^{LP} = 16 - 48$
  - on 1 timeslices
  - same sources for two- and three-point functions
  - already used in Mainz isovector charges paper[1]

[1] K. Ottnad et al., 1905.01291

quark loops

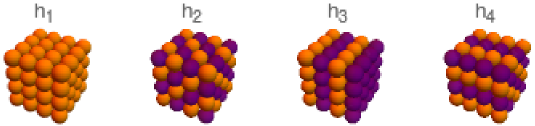
$$L^{l/s}_{\Gamma_\mu}(\vec{q}, z_0) = -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \left\langle tr\left[S^{l/s}(z;z)\,\Gamma_\mu\right]\right\rangle_G$$

$$= -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \left\langle \eta^\dagger(z)\,\Gamma_\mu\, s^{l/s}(z)\right\rangle_{G,\eta}$$

- hierarchical probing[1]

$$\eta_n \rightarrow h_n \odot \eta$$

- 4D noise vectors, 2×512 Hadamard vectors/configuration



[1] Stathopoulos et al., 1302.4018

excited-state contamination

$$G^{\text{eff}}(Q^2, z_0, y_0) = G(Q^2) + A_1(Q^2)\; e^{-\Delta z_0} + B_1(Q^2)\; e^{-\Delta'(y_0 - z_0)} + C_1(Q^2)\; e^{-\Delta z_0 - \Delta'(y_0 - z_0)}$$

- disconnected:
  - plateau method $y_0 \sim 1\,\text{fm}$
  - summation method $y_0 \in [0.5, 1.3]$ fm

$$S(Q^2, y_0) = \sum_{z_0=1}^{y_0-1} G^{\text{eff}}(Q^2, z_0, y_0) = K(Q^2) + y_0 G(Q^2) + O\left(e^{-\Delta y_0}, e^{-\Delta' y_0}\right)$$

- connected + disconnected:
  - summation method with 3 or 4 $y_0 \in [1.0, 1.5]$ fm
  - two-state fits

renormalization

- $N_f = 3$ periodic boundary condition ensembles
  - $\beta \in \{3.40, 3.46, 3.55\}$
- RI′-MOM scheme in Landau gauge

$$Z_O \left.\langle p \,|O_\Gamma|\, p\rangle\right|_{p^2=\mu^2} = \left.\langle p \,|O_\Gamma|\, p\rangle_{\mathrm{tree}}\right|_{p^2=\mu^2} \;.$$

  - extrapolation to the chiral limit
  - perturbative subtraction of leading-order lattice artifacts[1]
- conversion to RGI scheme
  - $\overline{\mathrm{MS}}$ as intermediate scheme
  - $\overline{\mathrm{MS}}$ β- and γ-functions
  - fit residual μ-dependence
- convert to $\overline{\mathrm{MS}}$ at $\mu = 2.0\,\mathrm{GeV}$

[1] G. von Hippel et al., PoS LATTICE2016 (2016) 194

renormalization

- start with flavor-diagonal basis

$$O^a_\Gamma(x) = \bar{\psi}(x)\Gamma\lambda^a\psi(x) \quad , \quad \psi = (u,d,s)^T \quad , \quad a \in \{3,8,0\} \ ,$$

$$Z_\Gamma = \begin{pmatrix} Z^{33}_\Gamma & 0 & 0 \\ 0 & Z^{88}_\Gamma & 0 \\ 0 & 0 & Z^{00}_\Gamma \end{pmatrix} \quad , \quad Z^{33}_\Gamma = Z^{88}_\Gamma \quad , \quad Z^i_q = Z_q \ .$$

- basis transformation

$$\begin{pmatrix} O^{u-d}_\Gamma(x)_R \\ O^{u+d}_\Gamma(x)_R \\ O^s_\Gamma(x)_R \end{pmatrix} = \begin{pmatrix} Z^{u-d,u-d}_\Gamma & 0 & 0 \\ 0 & Z^{u+d,u+d}_\Gamma & Z^{u+d,s}_\Gamma \\ 0 & Z^{s,u+d}_\Gamma & Z^{s,s}_\Gamma \end{pmatrix} \begin{pmatrix} O^{u-d}_\Gamma(x) \\ O^{u+d}_\Gamma(x) \\ O^s_\Gamma(x) \end{pmatrix}$$

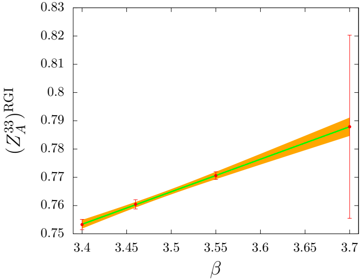
- $Z^{u-d,u-d}_\Gamma$ already used in Mainz isovector charges paper[1]

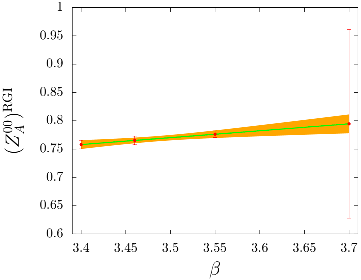
[1] K. Ottnad et al., 1905.01291

renormalization

- linear extrapolation to $\beta = 3.7$
  - error multiplied with factor 10
  - checks performed for g_A^{u-d} in Mainz isovector charges paper[1]

$\Rightarrow$ consistent results compared to using Z_A^{SF}





[1] K. Ottnad et al., 1905.01291

z-expansion

- fit the Q^2-dependence

$$G(Q^2) = \sum_{k=0}^{5} a_k z(Q^2)^k$$

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut}}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut}}} \quad , \quad t_{cut} = (2m_K)^2$$

- Gaussian priors

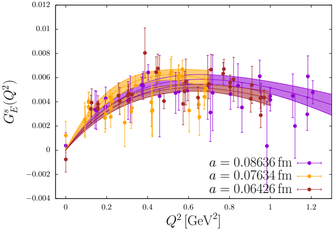
$$\tilde{a}_k = 0 \pm 5\,\max\{|a_0|, |a_1|\}\ \forall\ k > 1 \tag{1}$$

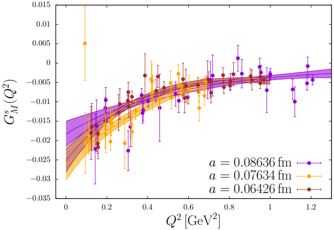
- for electric form factor: $a_0 = 0$
- charge radius

$$r^2 \equiv -6\ \frac{dG}{dQ^2}\bigg|_{Q^2=0} = -\frac{3}{2t_{cut}} a_1 \tag{2}$$

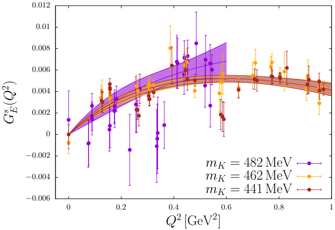
strange vector form factors

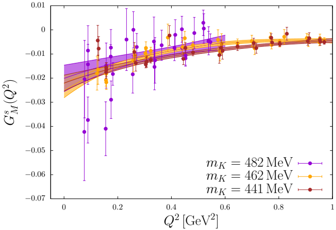
- $m_K \approx 460\,\text{MeV}$





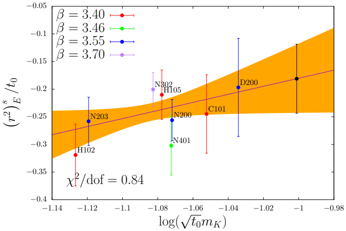
- $\beta = 3.55$

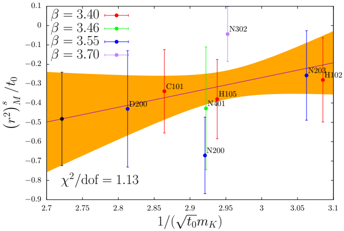


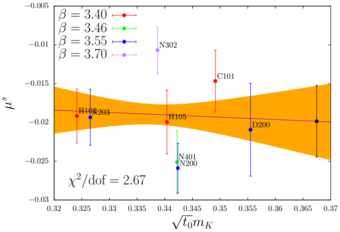


extrapolation

- SU(3) HBChPT[1]







$$(r^2)^s_E(m_K) = a_0 + a_1 \log(m_K)$$
$$\mu^s(m_K) = a_2 + a_3 m_K$$
$$(r^2)^s_M(m_K) = a_4 + a_5/m_K$$

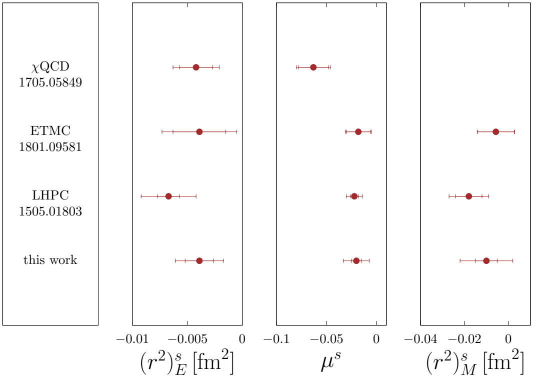
[1] T. R. Hemmert et al., nucl-th/9904076

error budget

	$(r^2)^s_E$ [fm^2]	μ^s	$(r^2)^s_M$ [fm^2]	χ^2/DOF
Main result	-0.0039(13)	-0.020(5)	-0.010(5)	0.84, 2.67, 1.13
Variations:				
Doubling prior width	-0.0049(18)	-0.022(6)	-0.015(9)	0.68, 1.83, 0.69
Plateau method $\sim 1\,\mathrm{fm}$	-0.0040(9)	-0.009(6)	-0.0026(53)	1.12, 2.54, 1.58
Including $\mathcal{O}(a^2)$	-0.0030(16)	-0.016(6)	-0.007(6)	0.75, 2.98, 2.05
No cut in Q^2	-0.0034(9)	-0.016(4)	-0.007(4)	2.36, 2.77, 1.69

$$(r^2)^s_E = -0.0039(13)(18)\,\mathrm{fm}^2$$
$$\mu^s = -0.020(5)(12)$$
$$(r^2)^s_M = -0.010(5)(11)\,\mathrm{fm}^2$$

comparison



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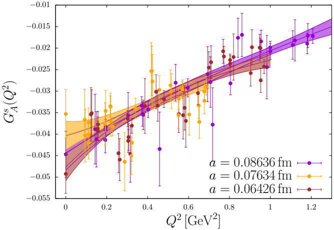
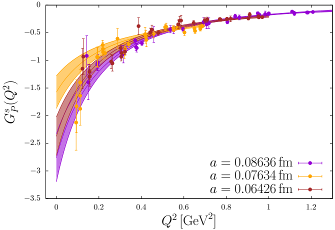
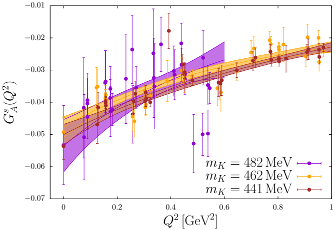
strange axial vector form factors

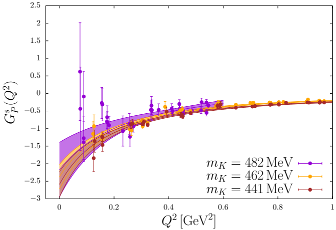
- $m_K \approx 460\,\mathrm{MeV}$





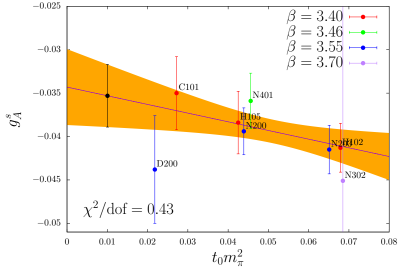
- $\beta = 3.55$

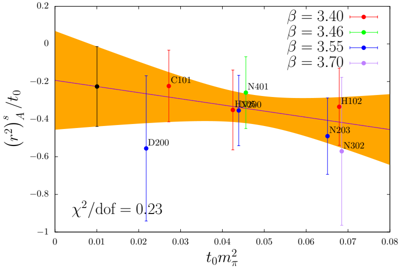




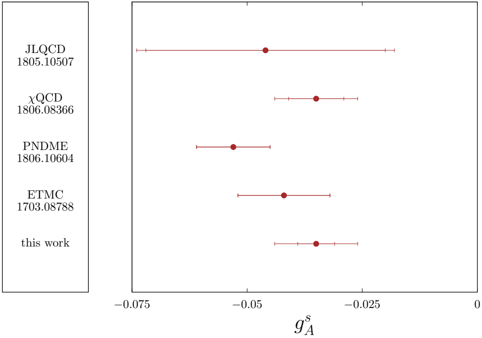
extrapolation

- linear in m_π^2





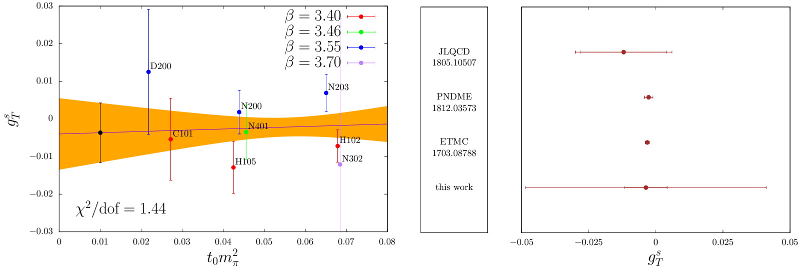
comparison



$$g_A^s = -0.035(4)(8)$$
$$(r^2)_A^s = -0.005(5)(10)\,\mathrm{fm}^2$$

- variations: same as for strange vector form factors, but $\mathcal{O}(a)$ lattice artifacts

strange tensor charge



$$g_T^s = -0.0037(79)(441)$$

- variations: plateau method $\sim 1\,\mathrm{fm}$, including $\mathcal{O}(a)$ lattice artifacts

quark flavor contributions to g_A and g_T

$$g^u_{A/T} = \frac{1}{2}\left(g^{u+d-2s}_{A/T} + 2g^s_{A/T} + g^{u-d}_{A/T}\right)$$
$$g^d_{A/T} = \frac{1}{2}\left(g^{u+d-2s}_{A/T} + 2g^s_{A/T} - g^{u-d}_{A/T}\right)$$

- $u + d - 2s$ renormalizes as $u - d$
  - additional noise cancellation for disconnected $l - s$[1]
- $g^{u-d}_{A/T}$ determined in Mainz isovector charges paper[2]
- $g^s_{A/T}$ already determined in this work
- determine $g^{u+d-2s}_{A/T}$ with two-state fit

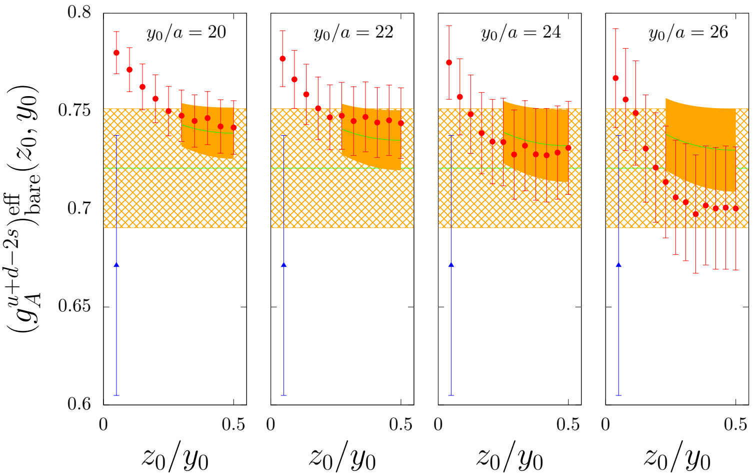
$$g_{\mathcal{O}}(z_0, y_0) = g_{\mathcal{O}} + A_{\mathcal{O}}\left(e^{-\Delta z_0} + e^{-\Delta(y_0 - z_0)}\right) + B_{\mathcal{O}}e^{-\Delta y_0}$$

  - gap Δ determined as in Mainz isovector charges paper[1]

[1] V. Gülpers et al., PoS (LATTICE2014) 128
[2] K. Ottnad et al., 1905.01291

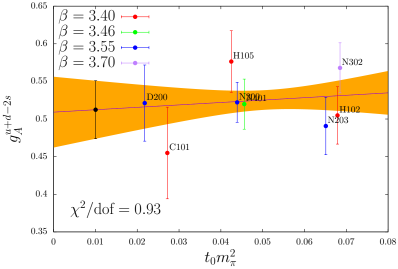
two-state fit

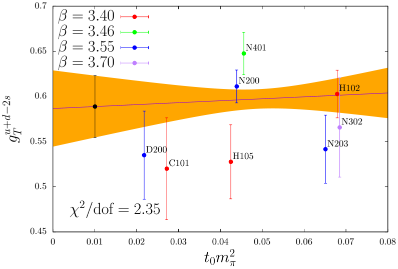
- N302



extrapolation

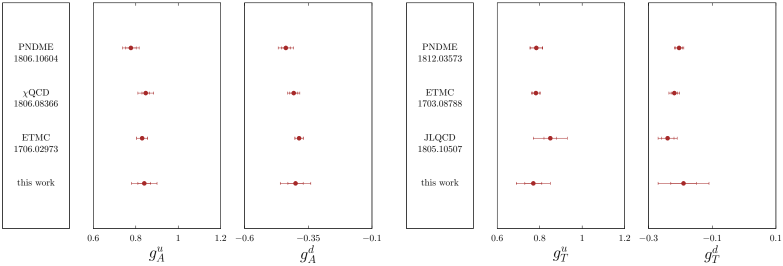
- linear in m_π^2





- variations: summation method, including $\mathcal{O}(a)$ lattice artifacts

comparison



$$g_A^u = 0.84(3)(5) \qquad g_T^u = 0.77(4)(5)$$

$$g_A^d = -0.40(3)(5) \qquad g_T^d = -0.19(4)(5)$$

outlook

- improve renormalization at $\beta = 3.7$
- more configurations and ensembles
- electric and magnetic charge radius and magnetic moment of proton and neutron

$$(r^2)^{p/n} = \frac{1}{2}\left[\frac{1}{3}(r^2)^{u+d-2s} \pm (r^2)^{u-d}\right]$$

$$\mu^{p/n} = \frac{1}{2}\left[\frac{1}{3}\mu^{u+d-2s} \pm \mu^{u-d}\right]$$