High precision determination of w_0

Jana N. Guenther

for the BMW collaboration

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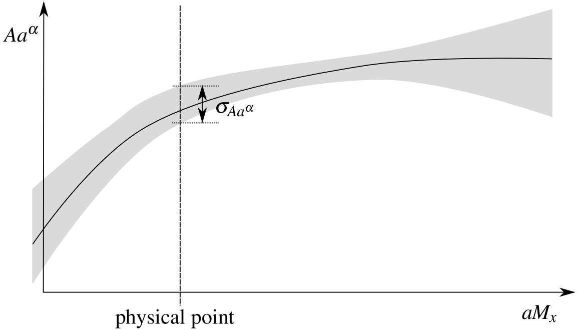
SFB TR55

BMW collaboration

Scale setting

High precision scale setting is important for any high precision calculation on the lattice, especially for dimensionfull quantities.



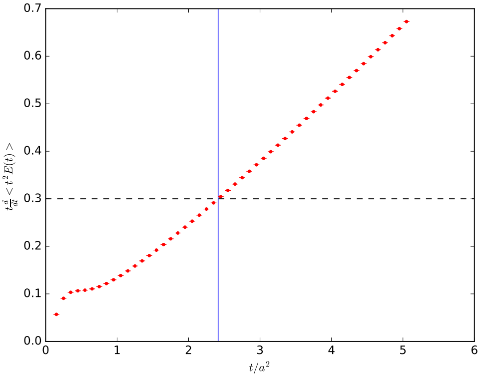
Scale setting uncertainty appears in several ways:

- Definition of the physical point
- Translation of the result in MeV.
- ...

[plot: L. Varnhorst]

w_0

Very promising for high precision scale setting: w_0.



[plot: L. Varnhorst]

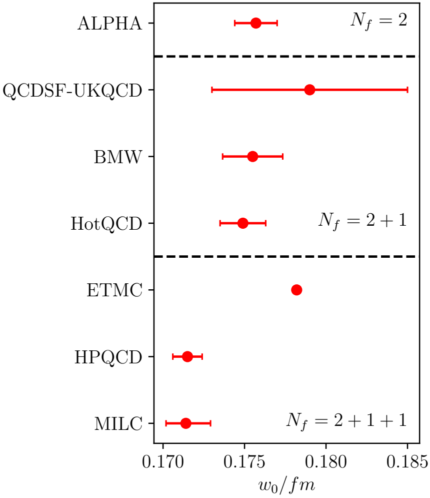
- Apply Wilson flow to "smooth out" the gauge fields and bring them closer to the classical solution. [Luscher:2010iy]
- Monitor the action density.
- Define w_0 to be the value of $\sqrt{t}$ where $t\frac{\mathrm{d}}{\mathrm{d}t}\langle t^2 E(t)\rangle = 0.3$. [Borsanyi:2012zs]
- Closely related to t_0. [Luscher:2010iy]

Recent results

w_0 can not be determined expertimentally. $\Rightarrow$ It has to be determined once on the lattice.

In this talk I will present an ongoing effort to determine w_0 with high precision in a blind analysis.

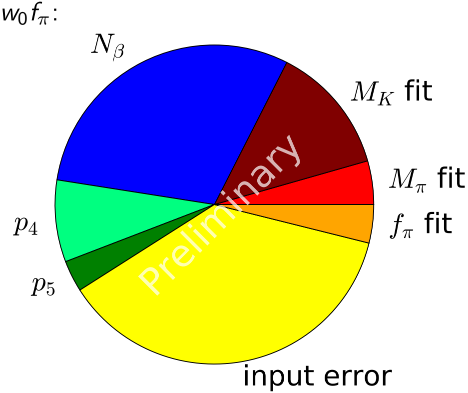
[ALPHA] M. Bruno *et al.* [ALPHA Collaboration], PoS LATTICE **2013** (2014) 321 [arXiv:1311.5585 [hep-lat]].
[QCDSF-UKQCD] R. Horsley *et al.* [QCDSF-UKQCD Collaboration], PoS LATTICE **2013** (2014) 249 [arXiv:1311.5010 [hep-lat]].
[BMW] S. Borsanyi *et al.*, "High-precision scale setting in lattice QCD," JHEP **1209** (2012) 010 [arXiv:1203.4469 [hep-lat]].
[HotQCD] A. Bazavov *et al.* [HotQCD Collaboration], Phys. Rev. D **90** (2014) 094503 [arXiv:1407.6387 [hep-lat]].
[ETMC] A. Deuzeman and U. Wenger, PoS LATTICE **2012** (2012) 162.
[HPQCD] R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, Phys. Rev. D **88** (2013) 074504 [arXiv:1303.1670 [hep-lat]].
[MILC] A. Bazavov *et al.* [MILC Collaboration], Phys. Rev. D **93** (2016) no.9, 094510 [arXiv:1503.02769 [hep-lat]].



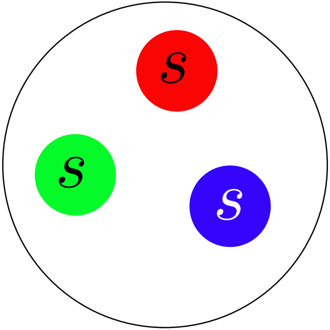
(Plot based on [MILC])

Determination from $w_0 f_\pi$

- presented at LATTICE2017 by L. Varnhorst
- determined on 2+1+1 ensembles
- accuracy $\approx$ 6‰
- 2 challenges:
  1. Input error of f_π
  2. What about QED?
- determination from $w_0 M_\Omega$



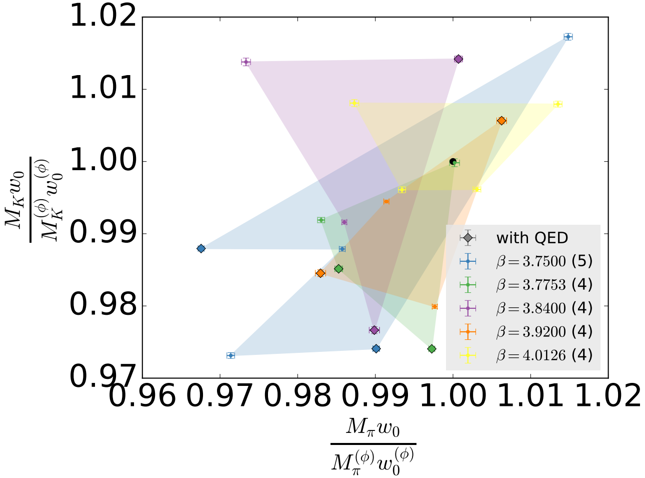
Ω-mugshot



- hadron type: baryon
- valence quarks: sss
- charge: -
- mass: (1672.45 ± 0.29) MeV, $\Delta_{m_\Omega} = 0.17‰$ ($\Delta_{f_\pi} = 1.5‰$)

[PDG 2018]

Lattice ensembles



The ensembles are generated with a staggered fermion action on stout-smeared gauge configurations. The gauge action is tree-level Symmanzik improved.

QED-Fit

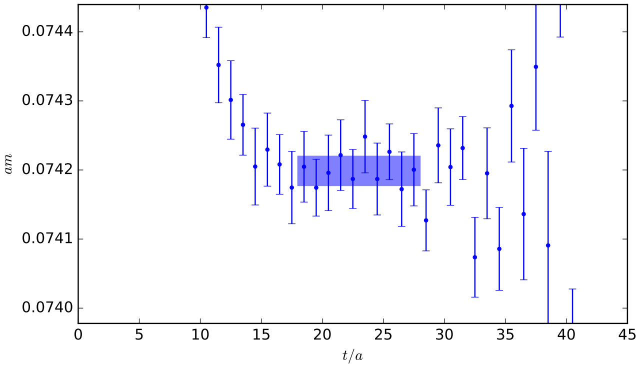
$$w_0 M_\Omega = A + B M_{\pi^0}^2 w_0^2 + C M_{K_\chi}^2 w_0^2 + E e_v^2 + F e_v e_s + G e_s^2$$

$$\begin{aligned}
M_{K_\chi}^2 &= M_{K^+}^2 + M_{K^0}^2 - M_{\pi_+}^2 \\
A &= A_0 + A_1 a^2 \\
B &= B_0 + B_1 a^2 \\
C &= C_0 + C_1 a^2 \\
E &= E_0 + E_1 a^2 \\
F &= F_0 \\
G &= G_0
\end{aligned}$$

The scale is set by w_0. To determine the final result, the equation is solved for w_0 in the continuum, at the physical point. In this way the determination is self consistent.

Mass determination

M_π, M_K and M_Ω are extracted by fitting appropriate correlators in two different fit ranges each.



M_Ω is also determined by solving the generalized eigen value problem (GEVP) $\longrightarrow$ talk by L. Varnhorst, 20.06.2019, 17:10

QED-effects

$$w_0 M_\Omega = A + B M_{\pi^0}^2 w_0^2 + C M_{K_\chi}^2 w_0^2 + E e_v^2 + F e_v e_s + G e_s^2$$

$$[w_0 M_\Omega]_{\text{iso}} = A + B \left[M_{\pi^0}^2 w_0^2\right]_{\text{iso}} + C \left[M_{K_\chi}^2 w_0^2\right]_{\text{iso}}$$
$$\frac{\partial^2}{\partial e_v^2} [w_0 M_\Omega] = B \frac{\partial^2}{\partial e_v^2} \left[M_{\pi^0}^2 w_0^2\right] + C \frac{\partial^2}{\partial e_v^2} \left[M_{K_\chi}^2 w_0^2\right] + E$$
$$\frac{\partial}{\partial e_s e_v} [w_0 M_\Omega]_{q11} = B \frac{\partial}{\partial e_s e_v} \left[M_{\pi^0}^2 w_0^2\right] + C \frac{\partial}{\partial e_s e_v} \left[M_{K_\chi}^2 w_0^2\right] + F$$
$$\frac{\partial^2}{\partial e_s^2} [w_0 M_\Omega] = B \frac{\partial^2}{\partial e_s^2} \left[M_{\pi^0}^2 w_0^2\right] + C \frac{\partial^2}{\partial e_s^2} \left[M_{K_\chi}^2 w_0^2\right] + G$$

Perform a combined, fully correlated fit to all components, which takes x- and y-Errors into account

For more details on QED effects $\longrightarrow$ talk by B. Toth, 18.06.2019, 17:10

Systematic error estimation

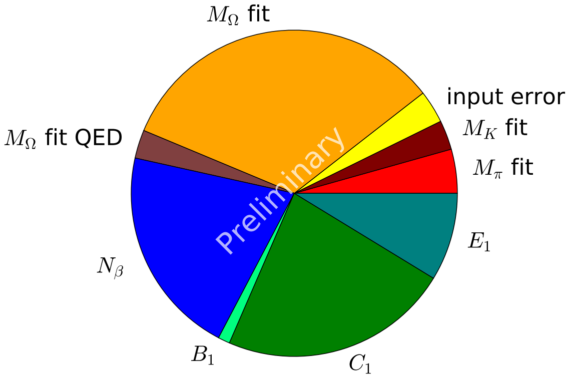
$$w_0 M_\Omega = A + B M_{\pi^0}^2 w_0^2 + C M_{K_\chi}^2 w_0^2 + E e_v^2 + F e_v e_s + G e_s^2$$

$$M_{K_\chi}^2 = m_{K^+}^2 + M_{K^0}^2 - M_{\pi_+}^2$$
$$A = A_0 + A_1 a^2$$
$$B = B_0 + B_1 a^2$$
$$C = C_0 + C_1 a^2$$
$$E = E_0 + E_1 a^2$$
$$F = F_0$$
$$G = G_0$$

$$\beta = 3.7500, 3.7753, 3.8400, 3.9200, 4.0126$$

Everything colored in red is varied. If there are only three βs, E_1 is excluded. Also the different results of the omega mass fits are varied. There is a total of 1920 fits.

Final systematic error budged



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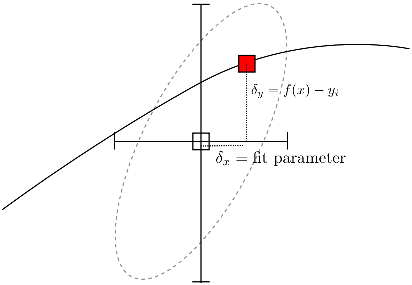
Correlation and x-errors

The statistic error is estimated by the Jackknife method.



[plot: L.Varnhorst]

For one quantity:

$$\vec{\delta} = \begin{pmatrix} f(x_i) - y_i \\ \delta_{x,i} \end{pmatrix} \quad \chi^2 = \sum_i \vec{\delta}^T C^{-1} \vec{\delta}$$

where C is the covariance matrix.

When generalized to several quantities, the number of fit parameters and the size of C increases. This can be numerically difficult.

Statistic error estimation

Computationally easier, but leading to the same minimum:

Minimizing:

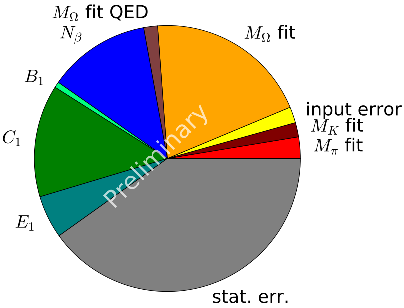
$$\chi^2 = \sum_i \frac{y_i^{(k)} - f^{(k)}(x_i)}{\sigma(y_i^{(k)} - f^{(k)}(x_i))} C_{kl}^{-1} \frac{y_i^{(l)} - f^{(l)}(x_i)}{\sigma(y_i^{(l)} - f^{(l)}(x_i))}$$

$$C_{lm} = \sum_k \left(y_l^{(k)} - f^{(k)}(x_l) - (y_l - f(x_l))\right) \left(y_m^{(k)} - f^{(k)}(x_m) - (y_m - f(x_m))\right)$$

σ: Jackknife error, f: fit function for $w_0 M_\Omega$

Full error

$w_0 M_\Omega$:



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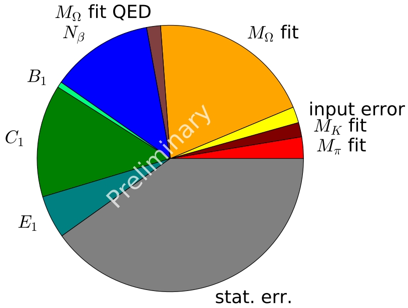
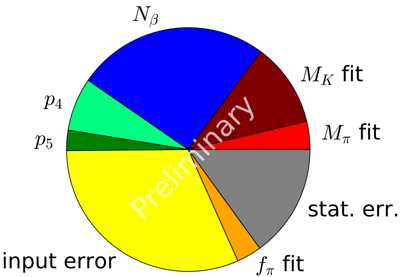
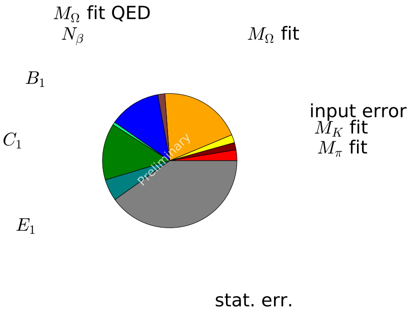


$w_0 f_\pi$:



Full error

$w_0 M_\Omega$:



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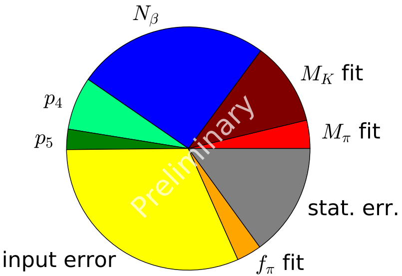
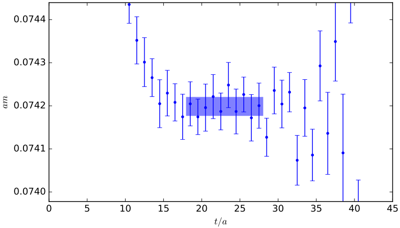


Analysis overview

1. Determine M_Ω, M_π, and M_K either by fitting the correlator or solving GEVP.



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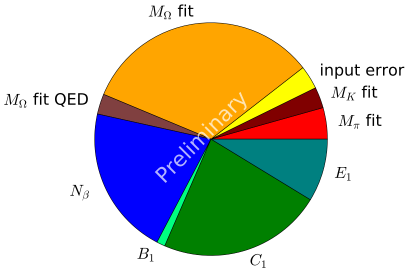
4. Estimate the systematic error via the Histogram method by various analyses.



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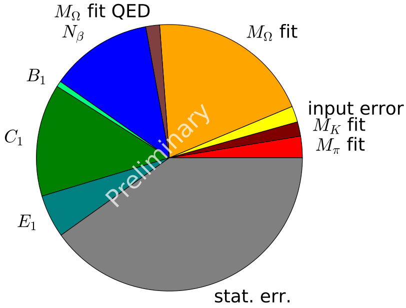
4. Estimate the systematic error via the Histogram method by various analyses.
5. Determine the statistic error via the Jackknife method.



Summary

- $w_0 M_\Omega$ is a suitable quantity to reduce the systematic error on w_0, due to the precise determination in experiment.
- The overall error can be reduced below 2‰

$w_0 M_\omega$: