

Nucleon Charges and Form Factors using Clover Fermions

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(on behalf of NME and PNDME Collaborations)

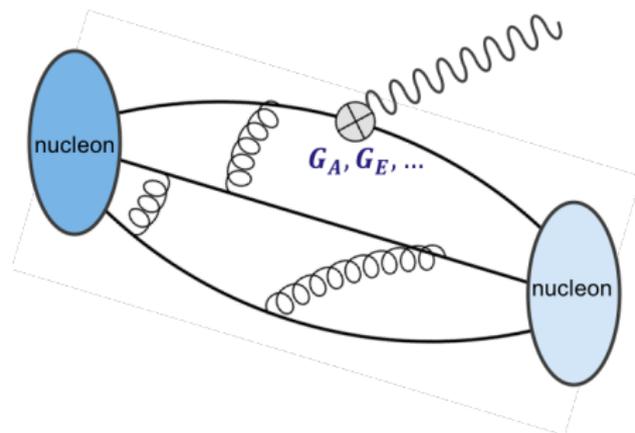
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Isovector charges on $N_f = 2 + 1$ Clover and $N_f = 2 + 1 + 1$ HISQ lattices

Physics from isovector charges and form factors: Isovector electric, magnetic and axial nucleon charges and form factors

Nucleon charges and form factors give the strength of the interaction of external probes (electrons, neutrinos, \dots) with nucleons. We have extracted the following charges and form factors that are critical inputs in experimental searches of physics beyond the standard model:



- $G_E(Q^2)$, $g_V = G_E(0)$ Electric
- $G_M(Q^2)$, $\mu = G_M(0)$ Magnetic
- $G_A(Q^2)$, $g_A = G_A(0)$ Axial
- $\tilde{G}_P(Q^2)$, Induced pseudoscalar
- $G_P(Q^2)$, Pseudoscalar
- g_S , Scalar
- g_T , Tensor

We calculate these charges and form factors using Clover fermions on five $N_f = 2 + 1$ Clover ensembles.

Statistics on Five 2+1-flavor Clover Ensembles

Five ensembles generated by JLab/W&M/LANL/MIT Collaborations

Ensemble ID	a (fm)	M_π (MeV)	$M_\pi L$	$L^3 \times T$
a127m285	0.127(2)	285(3)	5.85	$32^3 \times 96$
a094m280	0.094(1)	278(3)	4.11	$32^3 \times 64$
a091m170	0.091(1)	166(2)	3.7	$48^3 \times 96$
a091m170L	0.091(1)	172(6)	5.08	$64^3 \times 128$
a073m270	0.0728(8)	272(3)	4.8	$48^3 \times 128$

On each of ensemble, the statistics are

Ensemble ID	τ/a	N_{conf}	N_{meas}^{HP}	N_{meas}^{LP}
a127m285	8,10,12,14	2002	8008	256256
a094m280	8,10,12,14,16	2469	7407	237024
a091m170	8,10,12,14,16	1318	5272	168704
a091m170L	8,10,12,14,16	1823	9115	291680
a073m270	11,13,15,17,19	1893	7572	242304

- τ : source-sink time separation
- Valance: Clover fermion, stout smearing, covariant Gaussian smearing, TSM

[NME, PRD95,074508 (2017)]

Steps in analysis

Clover: Clover fermion study on **Clover** lattices

HISQ : Clover fermion study on **HISQ** lattices

- Excited state fits**

	Clover	HISQ (PNDME '18)
$g_{A,T}^{u-d,bare}$	3*-state	3*-state
$g_S^{u-d,bare}$	3*-state	2-state

Here, 3*-state: fix $\langle 2' | \mathcal{O} | 2 \rangle = 0$

- Chiral-continuum extrapolation**

Finite volume effect is not significant: Ignore it for now in the **clover** study.

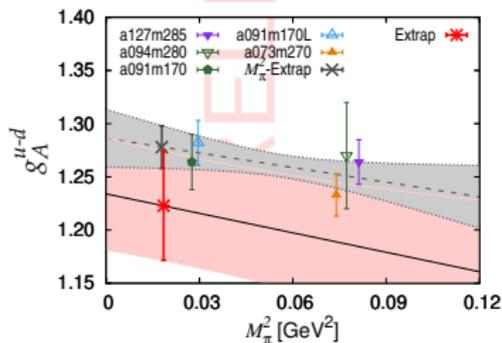
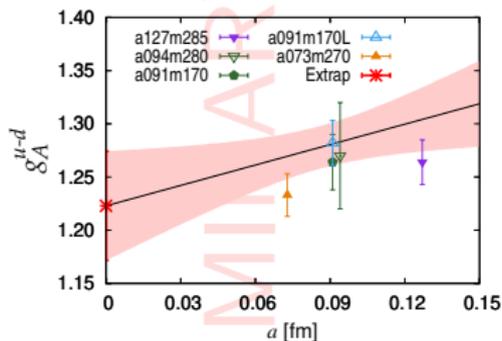
Clover
$$g_{A,S,T}^{u-d}(a, M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$$

HISQ (PNDME '18)
$$g_{A,S,T}^{u-d}(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_3^{\log} M_\pi^2 \ln\left(\frac{M_\pi}{M_\rho}\right) + c_4 M_\pi^2 \frac{e^{-M_\pi L}}{X(M_\pi L)}$$

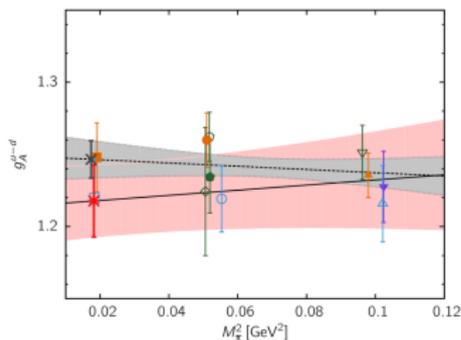
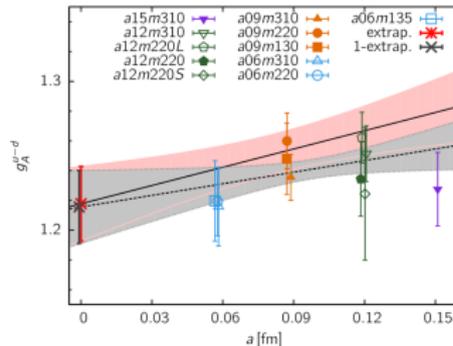
[PNDME, PRD98, 034503 (2018)]

Isovector axial charge g_A^{u-d}

Clover (Preliminary)



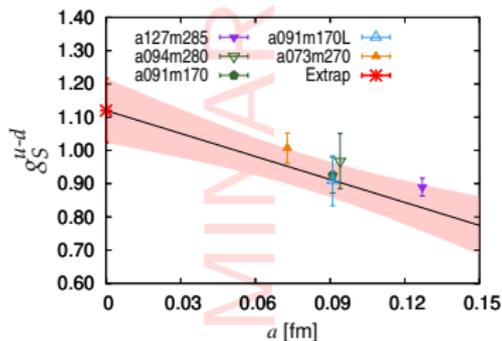
HISQ (PNDME '18)



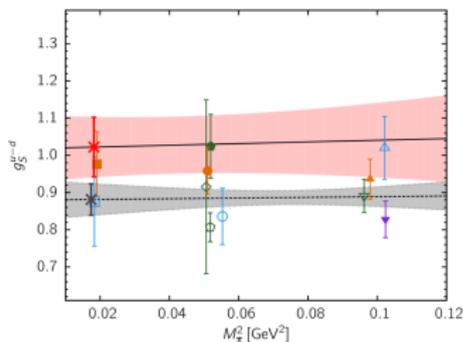
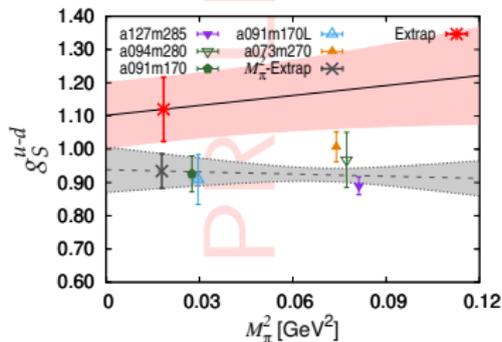
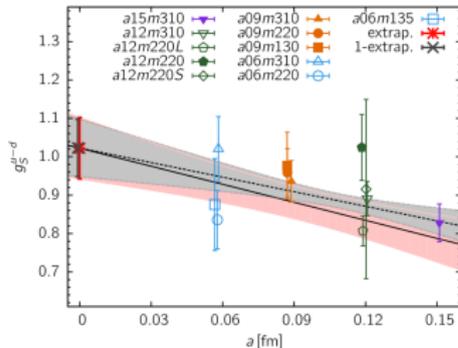
[PNDME, PRD98, 034503 (2018)]

Isovector scalar charge g_S^{u-d}

Clover (Preliminary)



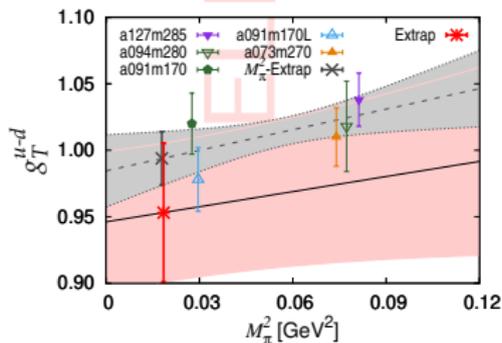
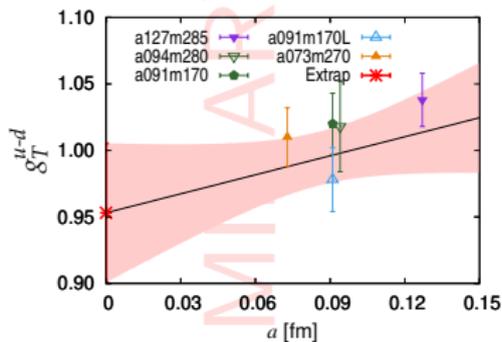
HISQ (PNDME '18)



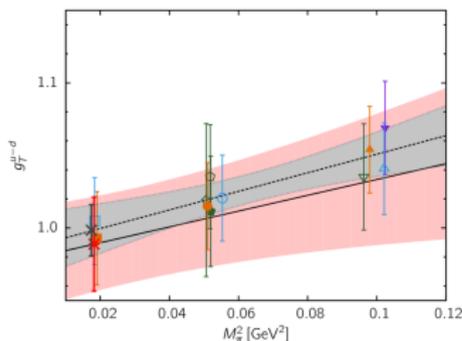
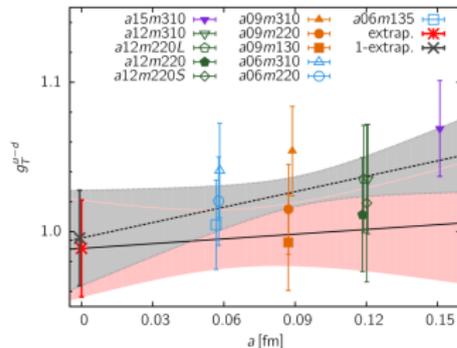
[PNDME, PRD98, 034503 (2018)]

Isovector tensor charge g_T^{u-d}

Clover (Preliminary)



HISQ (PNDME '18)



[PNDME, PRD98, 034503 (2018)]

Summary: isovector nucleon charges

- Results are consistent
- Pattern of variation versus a , M_π is similar

	Clover (Preliminary)	HISQ (PNDME '18)
g_A^{u-d}	1.247(64)	1.218(25)
g_S^{u-d}	1.15(12)	1.022(80)
g_T^{u-d}	0.967(31)	0.989(32)

clover: errors are statistical

[PNDME, PRD98, 034503 (2018)]

Isovector form factors on $N_f = 2 + 1$ Clover and $N_f = 2 + 1 + 1$ HISQ lattices

Isovector form factors

The matrix elements of isovector vector and axial vector operators

$$\langle N(\vec{p}') | V_\mu^{u-d} | N(\vec{p}) \rangle = \bar{u}(p') \left[\gamma_\mu F_1(Q^2) + \sigma_{\mu\nu} \frac{(p' - p)_\nu}{2M_N} F_2(Q^2) \right] u(p)$$

$$\langle N(\vec{p}') | A_\mu^{u-d} | N(\vec{p}) \rangle = \bar{u}(p') \left[\gamma_\mu G_A(Q^2) + \frac{(p' - p)_\mu}{2M_N} \tilde{G}_P(Q^2) \right] \gamma_5 u(p)$$

give the electric, magnetic and axial form factors G_E , G_M and G_A

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2,$$

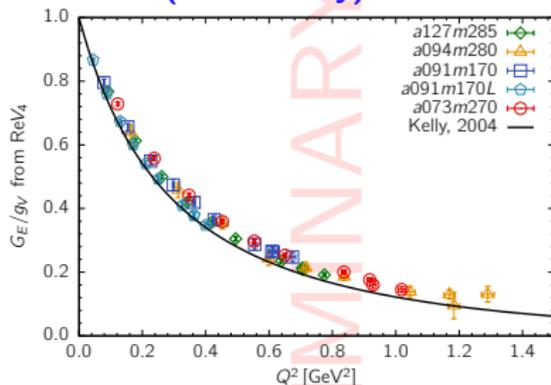
$$G_M = F_1 + F_2$$

Momentum transfer $Q^2 = -(p' - p)^2$.

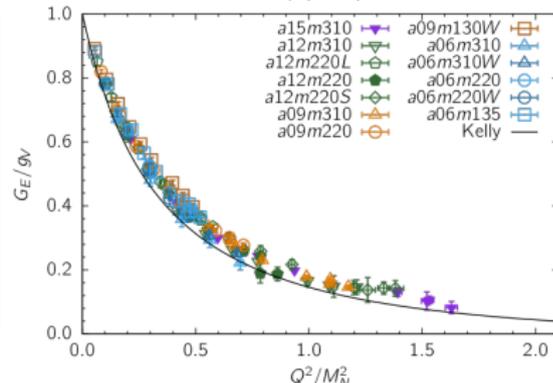
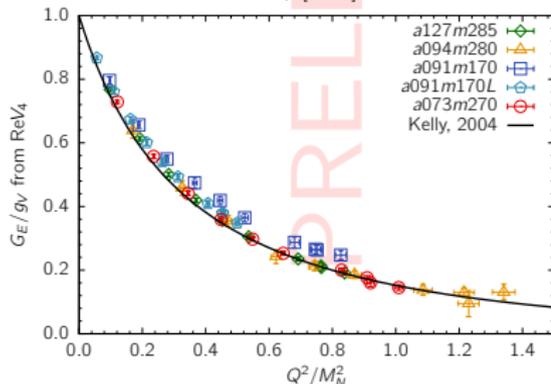
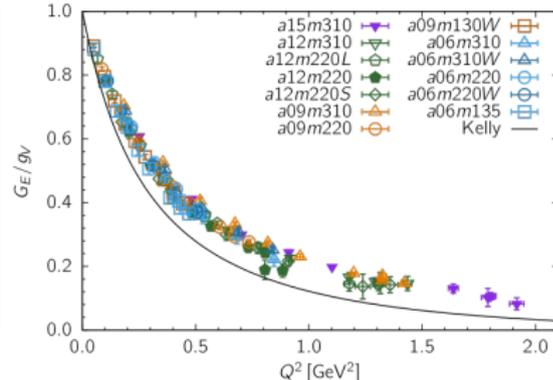
Clover calculation done on 5 ensembles

Isovector electric form factor

Clover (Preliminary)



HISQ (PNDME '19, See Jang's Talk)

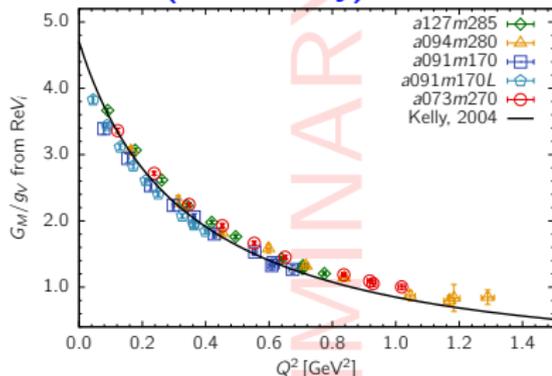


G_E and G_M are compared to the phenomenological Kelly curve.

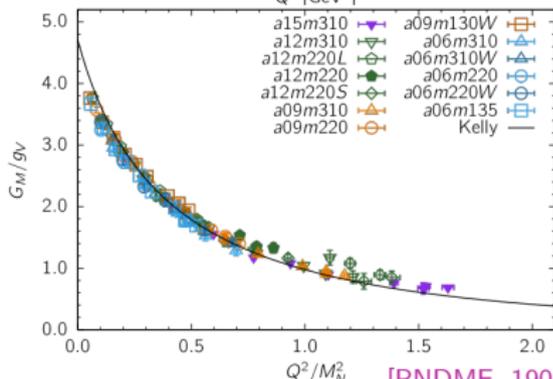
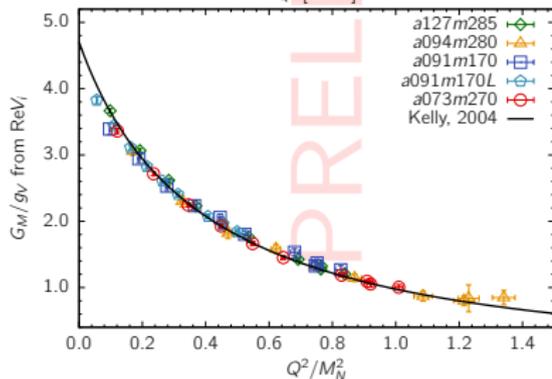
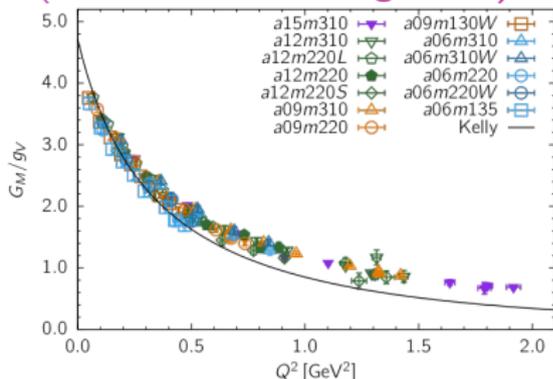
[PNDME, 1906.07217]

Isovector magnetic form factor

Clover (Preliminary)



HISQ (PNDME '19, See Jang's Talk)



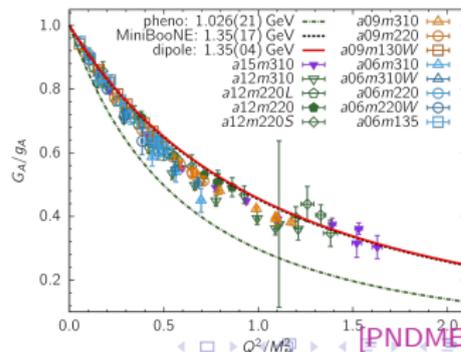
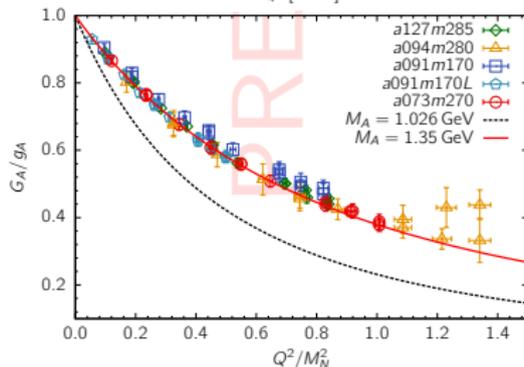
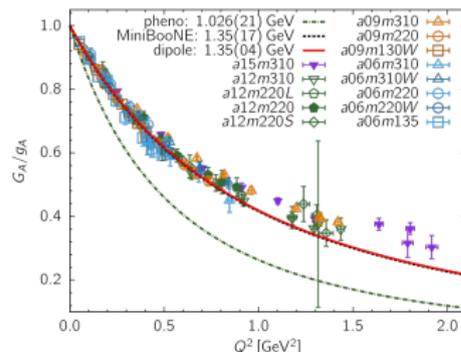
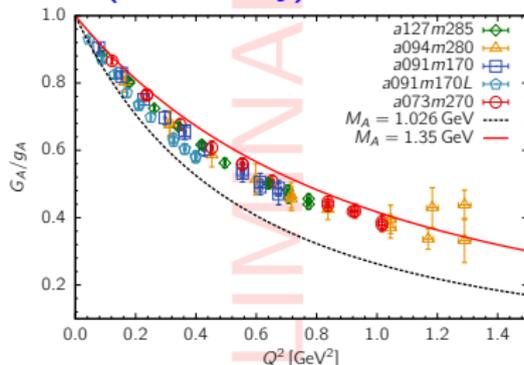
[PNDME, 1906.07217]

Isovector axial form factor

Dipole $G_A(Q^2) = \frac{G_A(0)}{(1+Q^2/M_A^2)^2}$ with $M_A = 1.35$ (red) and 1.026 (black) for comparison

Clover (Preliminary)

HISQ (PNDME '19, See Jang's Talk)



Flavor diagonal nucleon charges on $N_f = 2 + 1 + 1$ HISQ lattices

Physics from flavor diagonal nucleon charges

Quark contributions to the nucleon spin

[PNDME, PRD98,094512 (2018)]

$$\frac{1}{2} = \sum_{u,d,s,\dots} \left(\frac{1}{2} \Delta q + L_q \right) + J_g$$

- L_q : orbital angular momentum of the quark
- J_g : total angular momentum of the gluons
- $\Delta q = g_A^q = g_A^{q,conn} + g_A^{q,disc}$

Quark EDM

[PNDME, PRD98,094512 (2018)]

Assuming only quark EDM contributes to neutron EDM d_n ,

$$|d_n| = |d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s + \dots| \leq 2.9 \times 10^{-26} e \cdot cm.$$

Nucleon σ term:

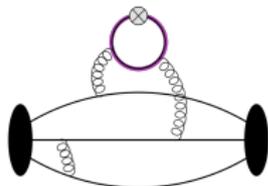
Flavor diagonal scalar charges give the slope of the nucleon mass with respect to the quark masses.

$$\sigma_{\pi N} = m_l \langle N | \bar{u}u + \bar{d}d | N \rangle = m_l g_S^{u+d},$$

$$\sigma_s = m_l \langle N | \bar{s}s | N \rangle = m_s g_S^s$$

$$m_l = m_l^{\text{Wilson}} - m_{\text{crit}}, \quad (m_l = 0 \text{ at } M_\pi \rightarrow 0).$$

Calculation of flavor diagonal nucleon charges



Calculation of flavor diagonal charges are complicated due to the additional contribution of the disconnected diagrams.

$$g_{\Gamma}^{f,bare} = g_{\Gamma}^{f,conn,bare} + g_{\Gamma}^{f,disc,bare} \quad g_{\Gamma}^f = \sum_{f'} (Z_{\Gamma}^{\overline{MS},2GeV})^{ff'} g_{\Gamma}^{f',bare}$$

Disconnected diagram contribution: $g_{\Gamma}^{q,disc,bare}$

$$g_{\Gamma}^{q,disc,bare} \leftarrow \left\langle \sum_{\vec{x}} \text{Tr}[M^{-1}(\tau, \vec{x}; \tau, \vec{x})\Gamma] \right\rangle - \frac{\langle \text{Tr}[P_{\Gamma} C^{2pt}(t)] \sum_{\vec{x}} \text{Tr}[M^{-1}(\tau, \vec{x}; \tau, \vec{x})\Gamma] \rangle}{\langle \text{Tr}[P_{\Gamma} C^{2pt}(t)] \rangle}$$

$M^{-1}(\tau, \vec{x}; \tau, \vec{x})$: all-to-all quark propagator estimated by stochastic method accelerated with the truncated solver method, hopping parameter expansion, and the dilution technique. [PNDME, PRD92,094511 (2015)]

Operator mixing with nonperturbative renormalization in RI-(s)MOM scheme.

$$(Z_{\Gamma}^{-1})^{ff'} = \sum_{f'} \frac{1}{Z_{\psi}^f} \text{Tr} \left[\begin{array}{c} \text{Diagram 1: Triangle with wavy gluon and quark lines} \\ \times \delta^{ff'} \mathbb{P}(p', p) - \text{Diagram 2: Disconnected quark loop} \\ \times \mathbb{P}(p', p) \end{array} \right],$$

$$Z_{\Gamma}^{\overline{MS},2GeV} = c^{\overline{MS}}(2GeV, \mu) c^{RI \rightarrow \overline{MS}}(\mu) Z_{\Gamma}(\mu)$$

Calculate quark propagators using momentum source.

UPDATE on 2+1+1-flavor HISQ Calculation

Seven ensembles generated by MILC Collaborations

Ensemble ID	a (fm)	M_π (MeV)	$M_\pi L$	$L^3 \times T$
a15m310	0.1510(20)	320(5)	3.93	$16^3 \times 48$
a12m310	0.1207(11)	310(3)	4.55	$24^3 \times 64$
a12m220	0.1184(10)	228(2)	4.38	$32^3 \times 64$
a09m310	0.0888(8)	313(3)	4.51	$32^3 \times 96$
a09m220	0.0872(7)	226(2)	4.79	$48^3 \times 96$
a06m310	0.0582(4)	320(2)	3.90	$48^3 \times 144$
a06m220	0.0578(4)	235(2)	4.41	$64^3 \times 144$

The statistics for disconnected diagrams are [PNDME, PRD98,094512, PRD98,094512 (2018)]

Ensemble ID	N_{conf}^l	N_{src}^l	N_{conf}^s	N_{src}^s	N_{LP}/N_{HP}
a15m310	1917	2000	1919	2000	50
a12m310	1013	5000	1013	1500	30
a12m220	958	11000	958	4000	30
a09m310	1081	4000	1081	2000	30
a09m220	712	8000	847	10000	30/50
a06m310	830	4000	200 + 340	5000 + 10000	30/50
a06m220	593	10000	554	10000	50

- New **a06m220** data points (half of the data analyzed)
- Valance: Clover fermion, HYP smearing, covariant Gaussian smearing

Steps in analysis

- Excited state fits

$$g_{A,T}^{u+d, \text{conn}, \text{bare}} : \quad 3^* \text{-state} \quad [\text{PNDME, PRD98, 034503 (2018)}]$$

$$g_S^{u+d, \text{conn}, \text{bare}} : \quad 2 \text{-state} \quad [\text{PNDME, PRD98, 034503 (2018)}]$$

$$g_{A,S,T}^{l, \text{disco}, \text{bare}}, g_{A,S,T}^{s, \text{disco}, \text{bare}} : \quad 2 \text{-state} \quad [\text{PNDME, PRD98,094512, PRD98,094512 (2018)}]$$

- Renormalization

$$g_{A,T}^{u+d} = Z_{A,T}^{u-d} g_{A,T}^{u+d, \text{bare}} \quad [\text{PNDME, PRD98,094512, PRD98,094512 (2018)}]$$

$$g_S^{u+d} = Z_S^{u+d, u+d} g_S^{u+d, \text{bare}} + Z_S^{u+d, s} g_S^{s, \text{bare}}$$

- Chiral-continuum extrapolation

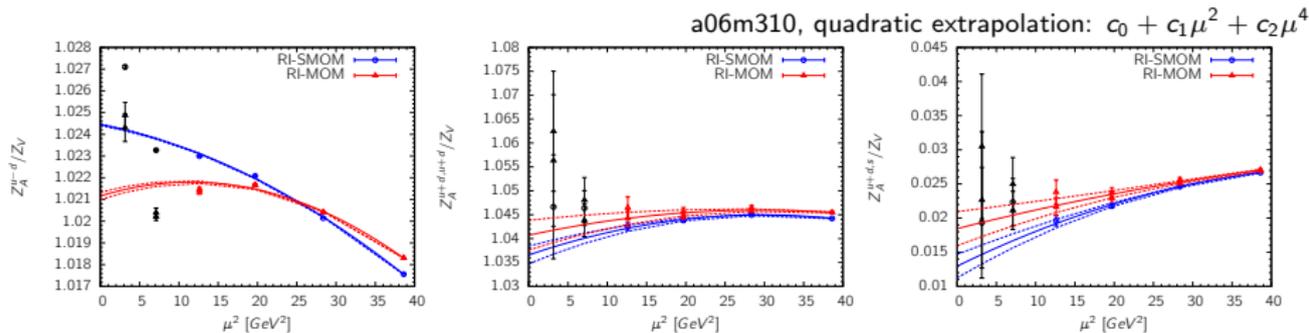
$$g_{A,T}^{u+d}(a, M_\pi) = c_1 + c_2 a + c_3 M_\pi^2 \quad [\text{PNDME, PRD98,094512, PRD98,094512 (2018)}]$$

$$g_S^{u+d}(a, M_\pi) = c_1 + c_2 a + c_3 M_\pi$$

$$\sigma_{\pi N}(a, M_\pi) = c_1 + c_2 a + c_3 M_\pi^2$$

Comparing Z_A^{u+d} and Z_A^{u-d} (\overline{MS} 2GeV)

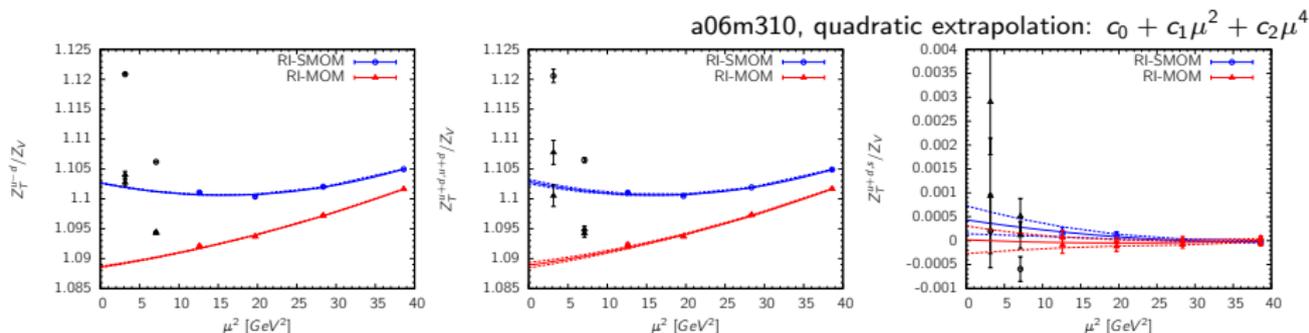
$$g_A^{u+d} = \frac{g_A^{u+d,bare}}{g_V^{u-d,bare}} \times \frac{Z_A^{u+d,u+d}}{Z_V^{u-d}} + \overbrace{\frac{g_A^{s,bare}}{g_V^{u-d,bare}} \times \frac{Z_A^{u+d,s}}{Z_V^{u-d}}}^{\approx 0} \quad (g_V^{u-d,bare} Z_V^{u-d} = 1)$$



a	Z_A^{u-d}/Z_V^{u-d}	$Z_A^{u+d,u+d}/Z_V^{u-d}$
0.15	1.080(13)	1.0856(11)
0.12	1.061(11)	1.073(11)
0.09	1.0380(40)	1.0484(32)
0.06	1.0227(19)	1.0383(28)

preliminary, errors given by the larger of the two—half the spread or the largest statistical error.

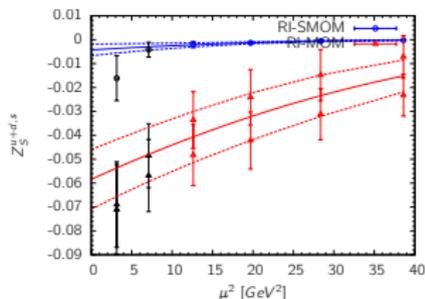
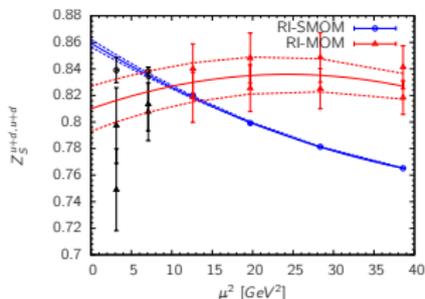
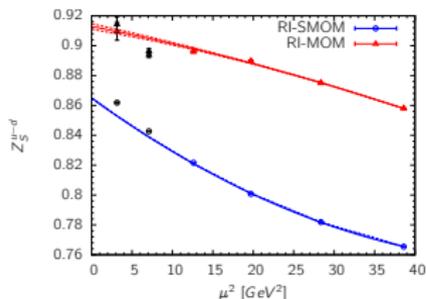
$\approx 1\%$ correction to $Z_A^{u+d,u+d}$
for all ensembles

Comparing Z_T^{u+d} and Z_T^{u-d} (\overline{MS} 2GeV)

a	Z_T^{u-d}/Z_V^{u-d}	$Z_T^{u+d,u+d}/Z_V^{u-d}$
0.15	1.032(18)	1.031(19)
0.12	1.0538(76)	1.0541(68)
0.09	1.0795(34)	1.0796(34)
0.06	1.0956(70)	1.0959(70)

preliminary, errors given by the larger of the two—half the spread or the largest statistical error.

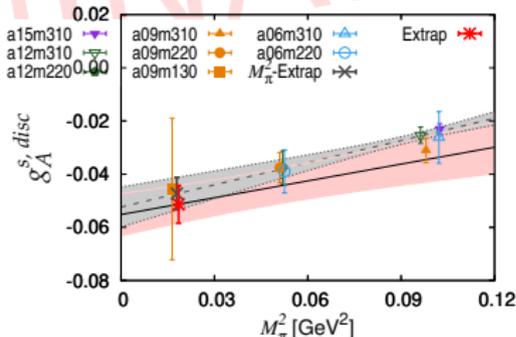
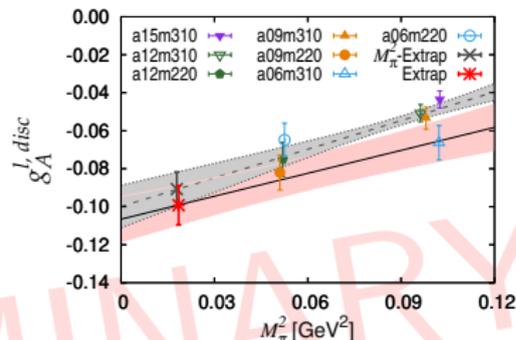
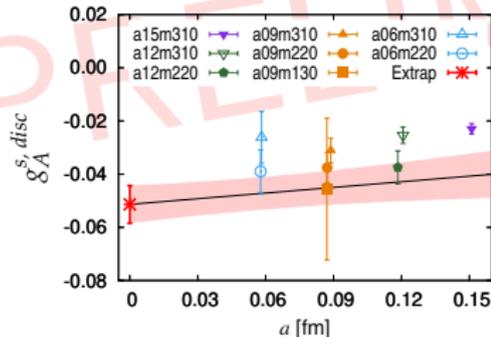
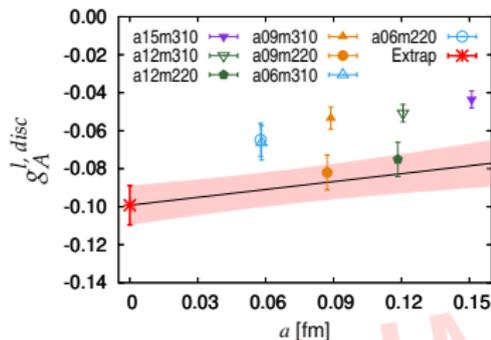
$\lesssim 0.1\%$ correction to $Z_T^{u+d,u+d}$
for all ensembles

Comparing Z_S^{u+d} and Z_S^{u-d} (\overline{MS} 2GeV)a06m310, quadratic extrapolation: $c_0 + c_1\mu^2 + c_2\mu^4$ 

- Large difference between RI-MOM and RI-SMOM schemes
 - Z_S^{u-d} (5% ~ 30%)
 - $Z_S^{u+d,u+d}$ (5% ~ 10%)
- $Z_S^{u-d} / Z_S^{u+d,u+d} \rightarrow 1$ as $a \rightarrow 0$

a	$Z_S^{u-d} / Z_S^{u+d,u+d}$	
	RI-SMOM	RI-MOM
0.15	1.0401(93)	1.238(56)
0.12	1.0127(27)	1.30(10)
0.09	1.0151(45)	1.190(32)
0.06	1.0065(33)	1.127(24)

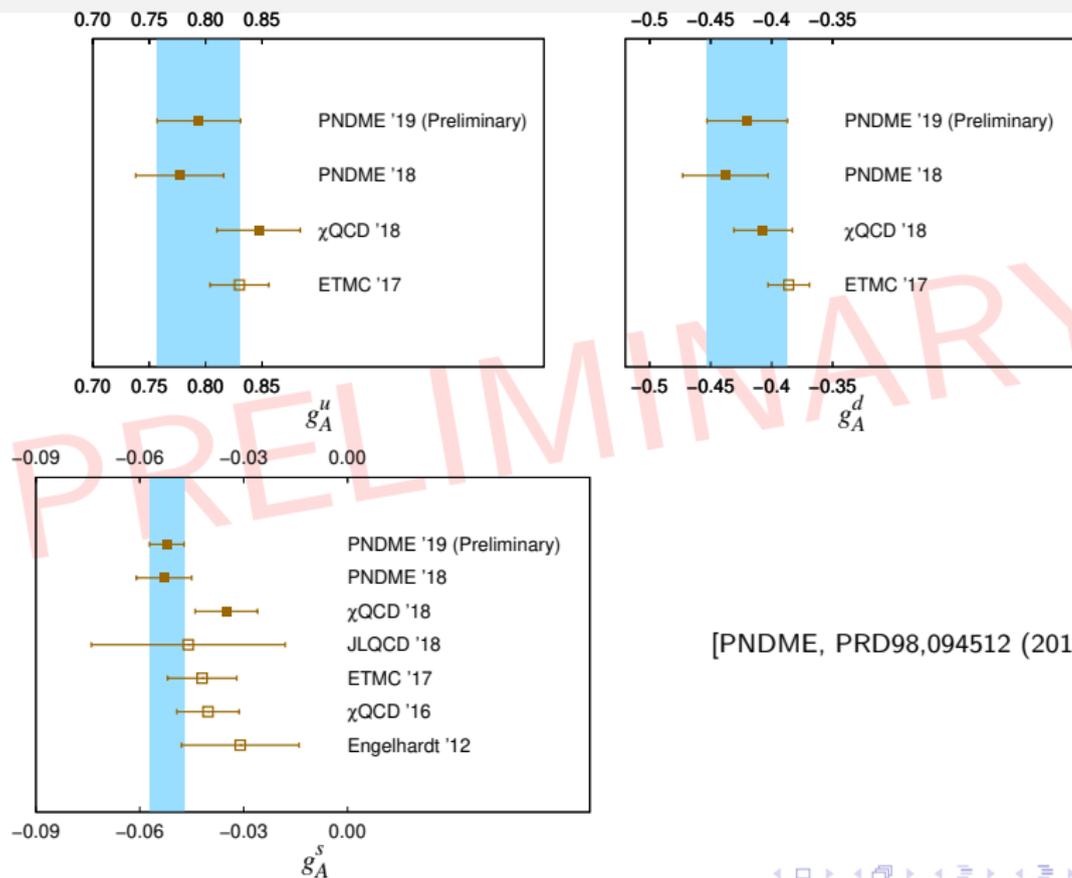
preliminary, errors are statistical

Quark contribution to the proton spin: $\Delta q = g_A^q$ 

- New a06m220 data point
- Checked: $Z_A^{u-d} \approx Z_A^{u+d}$

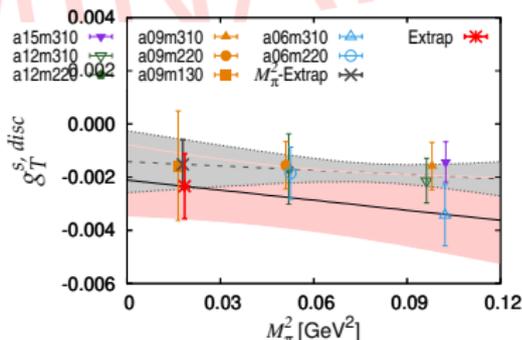
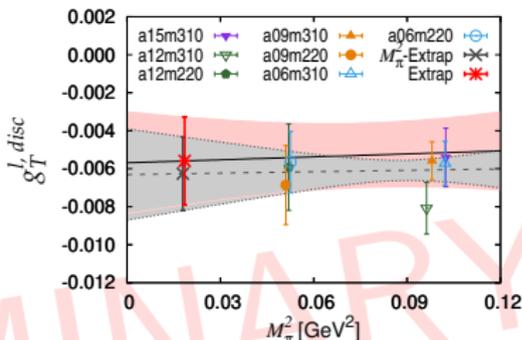
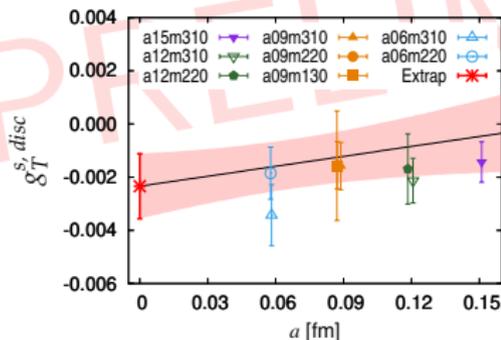
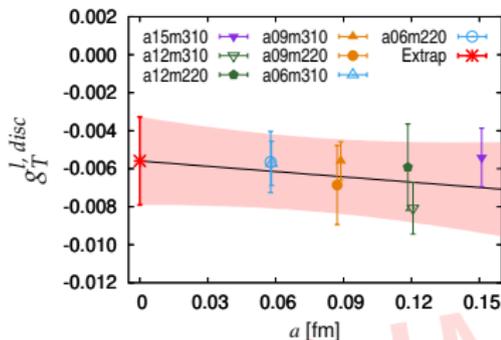
[PNDME, PRD98,094512 (2018)]

Quark contribution to the proton spin



[PNDME, PRD98,094512 (2018)]

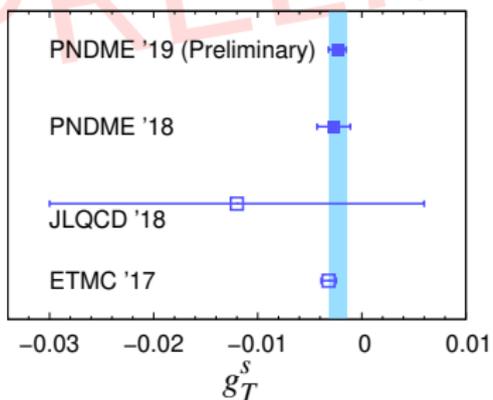
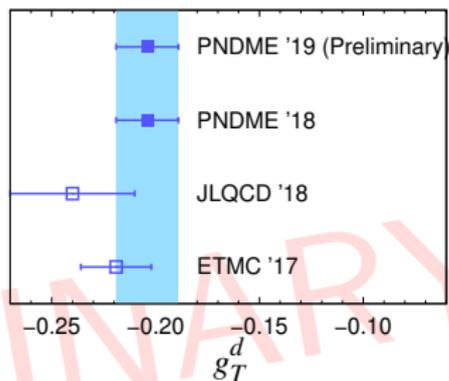
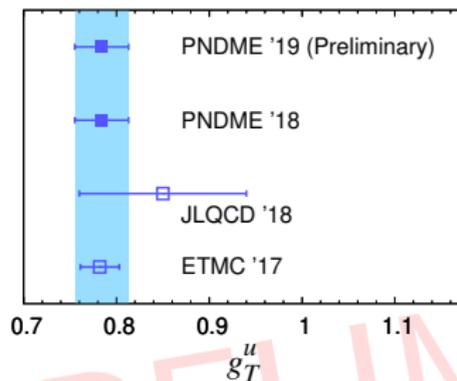
Flavor diagonal tensor charges



- New a06m220 data point
- Checked: $Z_T^{u-d} \approx Z_T^{u+d}$

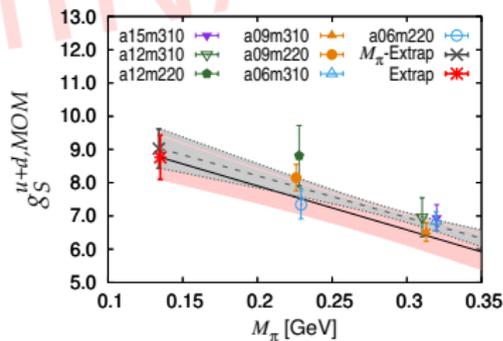
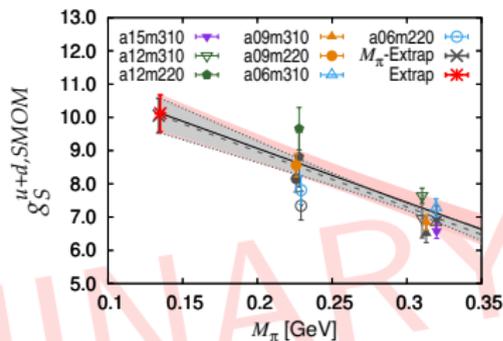
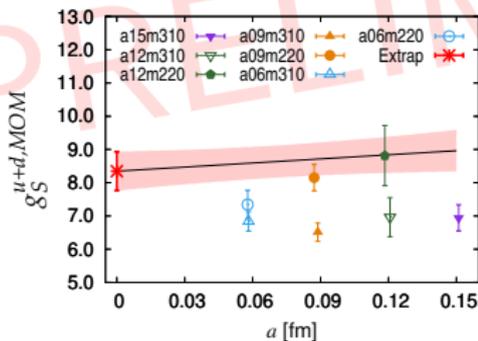
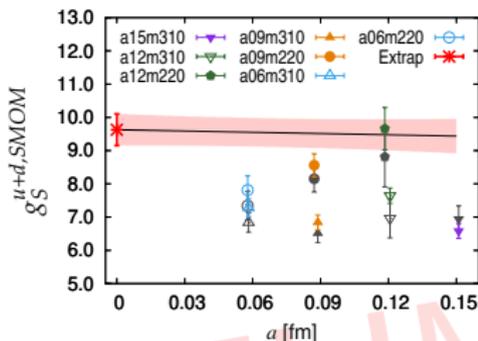
[PNDME, PRD98, 091501 (2018)]

Flavor diagonal tensor charges

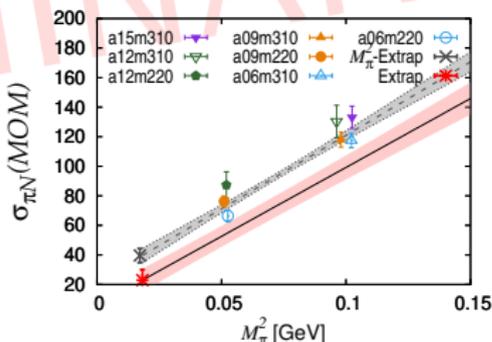
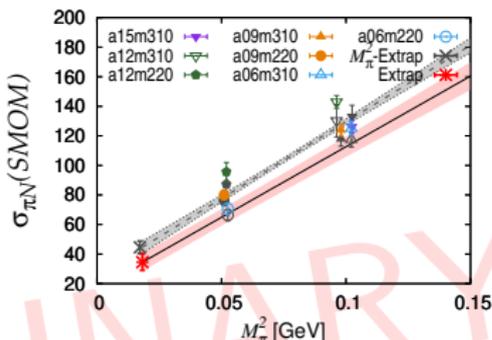
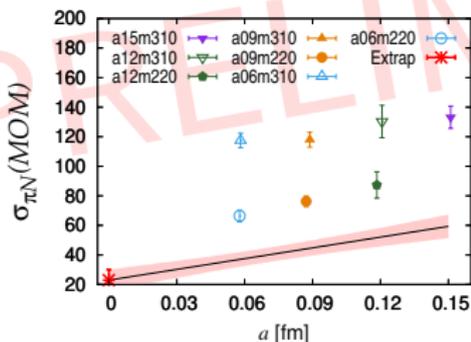
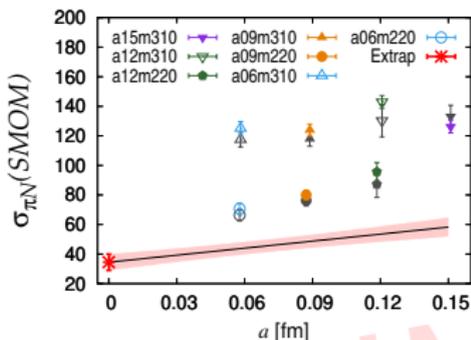


[PNDME, PRD98, 091501 (2018)]

Flavor diagonal scalar charges



- SMOM, $g_S^{\mu+d} = 10.12(55)$ [3.995]
- MOM, $g_S^{\mu+d} = 8.76(67)$ [0.961]

Nucleon $\sigma_{\pi N}$ term

- SMOM, $\sigma_{\pi N} = 34.5(5.5)$ [6.373]
- MOM, $\sigma_{\pi N} = 22.9(7.2)$ [0.426]
- FLAG 19', $N_f = 2 + 1 + 1$, $\sigma_{\pi N} = 64.9(1.5)$ (13.2)
- FLAG 19', $N_f = 2 + 1$, $\sigma_{\pi N} = 39.7(3.6)$

Conclusion

Conclusion

- High statistics results for nucleon charges and form factors for both clover-on-clover and clover-on-HISQ lattices
- Find consistent results for isovector nucleon charges and form factors between clover-on-HISQ and clover-on-clover studies.
- Update on flavor diagonal charges from clover-on-HISQ calculation with additional $a06m220$ ensemble
- Need to understand the renormalization scheme dependence in g_S^{u+d} and g_S^s
- Need more disconnected diagram data (especially for $M_\pi \approx 135$ MeV) to
 - understand g_S^{u+d} , g_S^s . Improve results for $\sigma_{\pi N}$ and σ_s
 - improve the precision on $g_A^{u,d,s}$ and $g_T^{u,d,s}$.

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