Nucleon isovector couplings from 2+1 flavor lattice QCD at the physical point

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Previous results

The PACS collaboration performed realistic lattice QCD simulations using the PACS10 conf.

**PACS10 conf.**

- stout-smeared $O(a)$ improved Wilson action
- Iwasaki gauge action


- $m_\pi = 135\text{MeV}$
- $L^3T = 128^4$, $aL \sim 10.8\text{fm}$
- $a^{-1} \approx 2.3[\text{GeV}]$  

**Achievement**

a few-percent determination of the axial-vector coupling and proton charge radius

- $g_A = 1.273(24)_{\text{stat}}(5)_{\text{sys}}(9)_{\text{ren}}$
- $\sqrt{\langle r_E^2 \rangle} = 0.858(13)_{\text{stat}}(35)_{\text{sys}}$

E. Shintani et al., PRD 99 014510(2019).

N. Tsukamoto, Y. Aoki, K.-I. Ishikawa, Y. KiNucleon isovector couplings from 2+1 flavor I
Previous results

We had computed nucleon 2pt and 3pt correlators using the all-mode-averaging (AMA) technique with the 4 different source-sink separations ($t_{\text{sep}}$).

![Graph](image)

<table>
<thead>
<tr>
<th>Source</th>
<th>$N_{\text{conf}}$</th>
<th>$t_{\text{sep}}/a$</th>
<th># of meas.</th>
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<tbody>
<tr>
<td>Exp</td>
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<td>10</td>
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our next goal

- 1% level precision on $g_A$ and $\sqrt{\langle r^2 \rangle_E}$
- Accurate determination of $g_S$ and $g_T$
Update of $g_A$

Using the gauge-covariant Gauss-smeared source, the statistical errors can be efficiently reduced.

$g_A$ with $t_{\text{sep}}/a = 16$ as an example

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5-6 times lower cost than the case of the Exp source.
A few %-level precision on $g_A$ with $t_{\text{sep}}/a = 16$ will be easily achieved.

Combined with the result obtained with $t_{\text{sep}}/a = 13$, our final result of $g_A$ will be able to reach 1%-level precision.

Analysis on $\langle r_E^2 \rangle$ is now in progress.

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Determination of bare couplings $g_S$ and $g_T$

We also measured bare coupling constants in the tensor and scalar channels with the several different source-sink separations.

The systematic uncertainties stemming from the excited state contamination are enough small for $t_{sep}/a \geq 12$

To be compared with the other simulations or experiments, the renormalization constants are required.
Renormalization

- calculate the renormalization constants in the RI/SMOM scheme at the scale of $\mu$ (1 < $\mu$ < 5 [GeV]).
- then convert it into the $\overline{\text{MS}}$ scheme at the scale of 2GeV.
- The scale dependence in $Z_{\text{MS}}^{-1}(2\text{GeV})$ is observed in both cases.

- lattice artifact (the power function of $\mu$) at large $\mu$
- non-perturbative effect (e.g. $\langle A^2 \rangle_{\mu^2}$) at small $\mu$

\(^1\)Ph. Boucaud et al., PRD74 034505 (2006).
Residual scale dependence

To eliminate the residual scale dependence, we performed two types of the fits

\[ Z_{\overline{\text{MS}}}^{\text{MS}}(2\text{GeV}, \mu) = \begin{cases} \frac{c_{-1}}{\mu^2} + c_0 + c_1 \mu + c_2 \mu^2 & \text{pole + power} \\ \frac{c_{-1}}{\mu^2} + c_0 + c_2 \mu^2 + c_4 \mu^4 & \text{pole + even power} \end{cases} \]  

(1)

with \( c_0 \) being the \( \mu \)-independent value of \( Z \).

The difference between the results from two types of the fits is treated as the systematic uncertainty.

\[ Z_{\text{S}}^{\overline{\text{MS}}}(2\text{GeV}) = 1.00 \pm 0.02_{\text{stat}} \pm 0.06_{\text{sys}} \]

\[ Z_{\text{T}}^{\overline{\text{MS}}}(2\text{GeV}) = 1.05 \pm 0.01_{\text{stat}} \pm 0.02_{\text{sys}} \]

\(^2\text{pole + even power}\) N. Hasan et al., arXiv:1903.06487 [hep-lat].
Residual scale dependence

To eliminate the residual scale dependence, we performed two types of the fits \(^2\)

\[
Z_{\overline{\text{MS}}}^{\text{MS}}(2\text{GeV}, \mu) = \begin{cases} 
\frac{c_{-1}}{\mu^2} + c_0 + c_1 \mu + c_2 \mu^2 & \text{pole + power} \\
\frac{c_{-1}}{\mu^2} + c_0 + c_2 \mu^2 + c_4 \mu^4 & \text{pole + even power}
\end{cases}
\]

(1)

with \(c_0\) being the \(\mu\)-independent value of \(Z\).

\[
Z_{\overline{\text{MS}}}^{\text{MS}}(2\text{GeV}) = 1.00 \pm 0.02_{\text{stat}} \pm 0.06_{\text{sys}}
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\]

The difference between the results from two types of the fits is treated as the systematic uncertainty.

\(^2\)pole + even power N. Hasan et al., arXiv:1903.06487 [hep-lat].
Renormalized coupling constants

We finally obtain renormalized coupling constants $g_S, g_T$ which are consistent with the FLAG average values, while the axial-vector coupling $g_A$ is in good agreement with the experiment.

![Graph showing renormalized coupling constants $g_S$ and $g_T$ with error bars for different source types and comparison with experimental data.](image)

![Graph showing renormalized coupling constant $g_A$ with error bars for different source types and comparison with experimental data.](image)
Summary

We reported the recent progress on the nucleon iso-vector couplings.

- Using the Gauss-smeared source, the statistical errors are efficiently reduced.
  - A few % precision on $g_A$ with $t_{sep}/a = 16$ will be easily achieved.
  - Combining the results with $t_{sep}/a = 13, 16$, our final result of $g_A$ will be able to reach 1%-level precision.
- We measured the bare couplings in the scalar and tensor channels with different source-sink separations.
  - Excited state contamination is well under control.
  - We also non-perturbatively estimated the renormalization constants for the scalar and tensor current using the RI/SMOM scheme.
    - We obtain the renormalized $g_A, g_S, g_T$ which are consistent with the recent FLAG averages (2019).
      - $g_S$ 54%(stat), 6%(sys) uncertainties
      - $g_T$ 4%(stat), 2%(sys) uncertainties
    with $t_{sep}/a = 16$, using the Gauss-smeared source.
- (Future work) Measurements on $160^4$ lattice with a different lattice spacing → Free from systematic errors, lattice artifact, chiral extrapolation, and finite volume effect.
effective mass

\[ a_M \]

Exp-Exp vs Exp-Local

\[ t/a \]

0.3

0.32

0.34

0.36

0.38

0.4

0.42

0.44

0.46

0.48

0.5

\[ a_M \]

Gauss-Gauss vs Gauss-Local

\[ t/a \]

0.3

0.32

0.34

0.36

0.38

0.4

0.42

0.44

0.46

0.48

0.5

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insertion time dependence in the scalar and tensor channels