Vector current renormalisation in momentum subtraction schemes using the HISQ action

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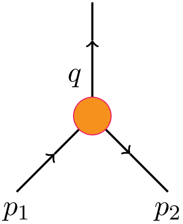
Motivations

- Vector current renormalisations are needed for various HPQCD projects
- HPQCD calculations have used form factor ($F(0)$) methods [1703.05552]
  - Precise; results on finer lattices obtained by extrapolation enabled by a comparison to lattice perturbation theory
- Computationally expensive
  - Desire faster method; SMOM offers this
- Very fine lattices now available
- QED effects to be computed
- Z_V can be examined on its own (unlike Z_m [1805.06225])
  - Good place to check systematics

Strategy

- All share a lattice perturbation theory expansion
  - $1 + c_1\alpha_s(1/a) + c_2\alpha_s(1/a)^2 + ...$
- Compare SMOM and $F(0)$ Z_V determinations
- Different discretisation effects
  - Condensates can appear in MOM calculations
- Compare merits
- Compare SMOM and $F(0)$ Z_V determinations at high precision

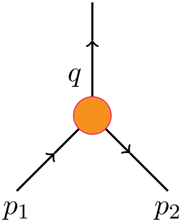
Momentum subtraction schemes



SMOM:

$$p_1^2 = p_2^2 = q^2 = \mu^2$$

Momentum subtraction schemes



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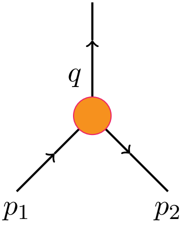
Green functions between external off-shell quark states set equal to their tree level values

Momentum subtraction schemes



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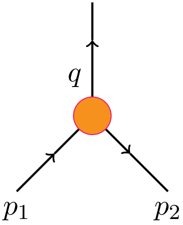
$$p_1^2 = p_2^2 = q^2 = \mu^2$$

Green functions between external off-shell quark states set equal to their tree level values

$$G_V = \langle \chi(p_1) \left(\sum_x \overline{\chi}(x) \gamma_\mu \chi(x) e^{i(p_1 - p_2)x} \right) \overline{\chi}(p_2) \rangle$$

Momentum subtraction schemes



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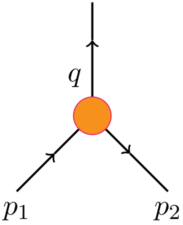
$$\Lambda_V(p) = S^{-1}(p_1) G_V S^{-1}(p_2)$$

Momentum subtraction schemes



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$$\Lambda_V(p) = S^{-1}(p_1) G_V S^{-1}(p_2)$$

$$Z_V = \frac{Z_q}{\mathrm{Tr}(P_V \Lambda_V)}$$

The Ward identity

$$\langle \partial_\mu J^\mu(x)\psi(y_1)\overline{\psi}(y_2)\rangle = \langle \delta_{y_2,x}\psi(y_1)\overline{\psi}(x) - \delta_{y_1,x}\psi(x)\overline{\psi}(y_2)\rangle$$

- Fourier transforming, this becomes

$$iq_\mu G_V^\mu(p_1, p_2) = S(p_1) - S(p_2)$$

Lattice Ward identity

- An exact vector Ward identity holds on the lattice

$$\Delta_\mu \langle J^\mu(x)\overline{\psi}(y_1)\psi(y_2)\rangle = \delta_{y_1,x}\langle\psi(y_2)\overline{\psi}(x)\rangle - \delta_{y_2,x}\langle\psi(x)\overline{\psi}(y_1)\rangle$$

- Fourier transforming

$$(1 - e^{iq_\mu}) \sum_x \langle\overline{\psi}(p_1)e^{-iq\cdot x}J^\mu(x)\psi(p_2)\rangle = \langle\psi(p_2)\overline{\psi}(p_1)\rangle - \langle\psi(p_1)\overline{\psi}(p_2)\rangle$$

RI-SMOM

- An exact vector Ward identity holds on the lattice

$$\Delta_\mu \langle J^\mu(x) \overline{\psi}(y_1) \psi(y_2) \rangle = \delta_{y_1,x} \langle \psi(y_2) \overline{\psi}(x) \rangle - \delta_{y_2,x} \langle \psi(x) \overline{\psi}(y_1) \rangle$$

- Fourier transforming

$$(1 - e^{iq_\mu}) \sum_x \langle \overline{\psi}(p_1) e^{-iq\cdot x} J^\mu(x) \psi(p_2) \rangle = \langle \psi(p_2) \overline{\psi}(p_1) \rangle - \langle \psi(p_1) \overline{\psi}(p_2) \rangle$$

$$Z_q = \frac{1}{12p^2} \mathrm{Tr}(\not{p} S^{-1}(p)) \; , \frac{Z_q}{Z_V} = \frac{1}{12q^2} \mathrm{Tr}((1 - e^{iq_\mu}) \Lambda_V^\mu \not{q})$$

$$\frac{1}{12q^2} \mathrm{Tr} \left((1 - e^{iq_\mu}) \Lambda_V^\mu \not{q} \right) = \frac{1}{12q^2} \left[\mathrm{Tr} \left(S^{-1}(p_2) \not{q} \right) - \mathrm{Tr} \left(S^{-1}(p_1) \not{q} \right) \right]$$

Different "q's" are needed which are in general different for different currents

RI-SMOM

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$$\Delta_\mu \langle J^\mu(x)\overline{\psi}(y_1)\psi(y_2)\rangle = \delta_{y_1,x}\langle \psi(y_2)\overline{\psi}(x)\rangle - \delta_{y_2,x}\langle \psi(x)\overline{\psi}(y_1)\rangle$$

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$$= \frac{1}{12q^2}\mathrm{Tr}\left(S^{-1}(q)\not{q}\right)$$

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$$= \frac{1}{12q^2}\mathrm{Tr}\left(S^{-1}(q)\not{q}\right) + \text{condensates?}$$

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$$= \frac{1}{12q^2}\mathrm{Tr}\left(S^{-1}(q)\not{q}\right) + \text{condensates?}$$

We see no condensates for the conserved current in SMOM: $Z_V = 1$ exactly

RI-SMOM / RI′-MOM

RI-SMOM:

$$\frac{Z_q}{Z_V} = \frac{1}{12q^2}\mathrm{Tr}(q_\mu \Lambda_V^\mu \not{q})$$

RI′-MOM:

$$\frac{Z_q}{Z_V} = \frac{1}{48}\mathrm{Tr}(\gamma_\mu \Lambda_V^\mu)$$

- Older RI′-MOM scheme uses different kinematics and simpler Z_V (same Z_q)
- Requires a perturbative MOM to $\overline{\text{MS}}$ matching
- Expect condensates there

Implementation

- Fix to Landau gauge
- Use momentum sources $e^{-ip\cdot x}$ ($DS = e^{-ip\cdot x}$): multiple solves per configuration
- Only 20 configurations required for good precision

Lytle & Sharpe [1306.3881] and HPQCD [1805.06225]

Set	β	a [fm]	L_s	L_t	am_l^{sea}	am_s^{sea}	am_c^{sea}
1	6.0	0.12404(66)	24	64	0.0102	0.0509	0.635
2	6.30	0.08872(47)	48	96	0.00363	0.0363	0.430
3	6.72	0.05922(33)	48	144	0.0048	0.024	0.286
4	7.00	0.04406(23)	64	192	0.00316	0.0158	0.188

Fit functions

- Calculate Z_V in RI′-MOM and RI-SMOM schemes for different a and μ
- Fit accounting for:
  - Discretisation effects $(a\mu)^n$
  - Possible condensates, i.e. $\langle A^2 \rangle / \mu^2$

Conserved current MOM Z_V fit

$$
\begin{aligned}
Z_V^{\text{con-MOM}}(a,\mu) = 1 &+ \sum_i c^{(i)}_{a^2\mu^2}(a\mu/\pi)^{2i} \\
&+ \sum_i c^{(i)}_{\alpha a^2\mu^2}(a\mu/\pi)^{2i}\alpha_{\overline{\text{MS}}}(1/a) \\
&+ \sum_j c^{(j)}_{\text{cond}}\alpha_{\overline{\text{MS}}}(\mu)\frac{(1\ \text{GeV})^{2j}}{\mu^{2j}} \\
&\times [1 + c^{(j)}_{\text{cond},a^2}(a\Lambda/\pi)^2] + c_\alpha\alpha^4_{\overline{\text{MS}}}(\mu)
\end{aligned}
$$

Conserved current MOM Z_V fit

Discretisation errors

$$
\begin{aligned}
Z_V^{\text{con-MOM}}(a,\mu) &= 1 + \sum_i c^{(i)}_{a^2\mu^2}(a\mu/\pi)^{2i} \\
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&+ \sum_j c^{(j)}_{\text{cond}}\alpha_{\overline{\text{MS}}}(\mu)\frac{(1\ \text{GeV})^{2j}}{\mu^{2j}} \\
&\times [1 + c^{(j)}_{\text{cond},a^2}(a\Lambda/\pi)^2] + c_\alpha \alpha^4_{\overline{\text{MS}}}(\mu)
\end{aligned}
$$

Conserved current MOM Z_V fit

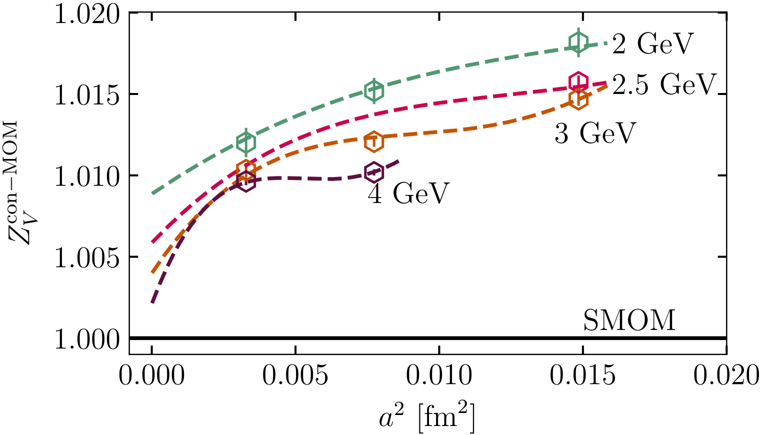
Condensates

$$Z_V^{\text{con-MOM}}(a,\mu) = 1 + \sum_i c^{(i)}_{a^2\mu^2}(a\mu/\pi)^{2i} + \sum_i c^{(i)}_{\alpha a^2\mu^2}(a\mu/\pi)^{2i}\alpha_{\overline{\text{MS}}}(1/a) + \sum_j c^{(j)}_{\text{cond}}\alpha_{\overline{\text{MS}}}(\mu)\frac{(1\ \text{GeV})^{2j}}{\mu^{2j}} \times [1 + c^{(j)}_{\text{cond},a^2}(a\Lambda/\pi)^2] + c_\alpha \alpha^4_{\overline{\text{MS}}}(\mu)$$

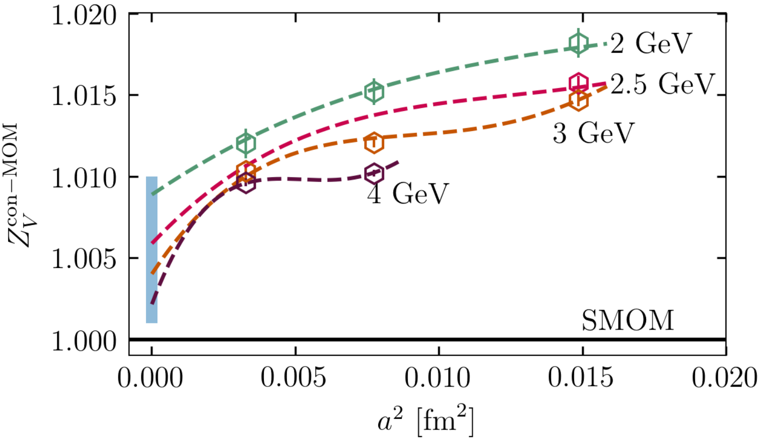
Conserved current MOM Z_V fit

Higher order perturbation theory in $\overline{\text{MS}}$ matching

$$
\begin{aligned}
Z_V^{\text{con-MOM}}(a,\mu) = 1 &+ \sum_i c^{(i)}_{a^2\mu^2}(a\mu/\pi)^{2i} \\
&+ \sum_i c^{(i)}_{\alpha a^2\mu^2}(a\mu/\pi)^{2i}\alpha_{\overline{\text{MS}}}(1/a) \\
&+ \sum_j c^{(j)}_{\text{cond}}\alpha_{\overline{\text{MS}}}(\mu)\frac{(1\text{ GeV})^{2j}}{\mu^{2j}} \\
&\times [1 + c^{(j)}_{\text{cond},a^2}(a\Lambda/\pi)^2] + c_\alpha \alpha^4_{\overline{\text{MS}}}(\mu)
\end{aligned}
$$



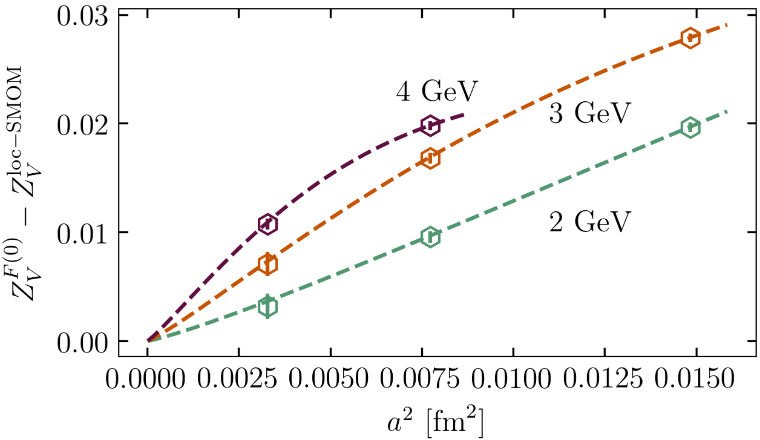
1.020
1.015
1.010
1.005
1.000
$Z_V^{\text{con}-\text{MOM}}$
0.000
0.005
0.010
0.015
0.020
a^2 [fm^2]
2 GeV
2.5 GeV
3 GeV
4 GeV
SMOM



Condensates appear at 1% level in RI′-MOM but absent from SMOM

Local current SMOM Z_V fit

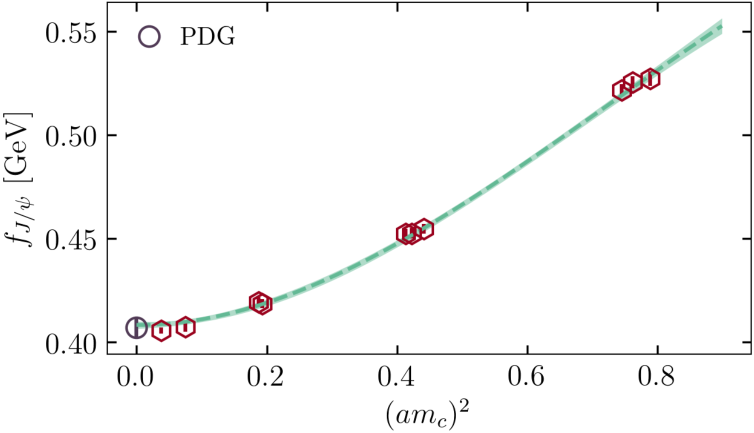
- SMOM conserved has no visible condensates
- Does SMOM local?
- $Z_V^{F(0)} - Z_V^{\mathrm{loc-SMOM}}$ should be pure discretisation effects if there are no condensates
  - Lattice perturbation theory component cancels



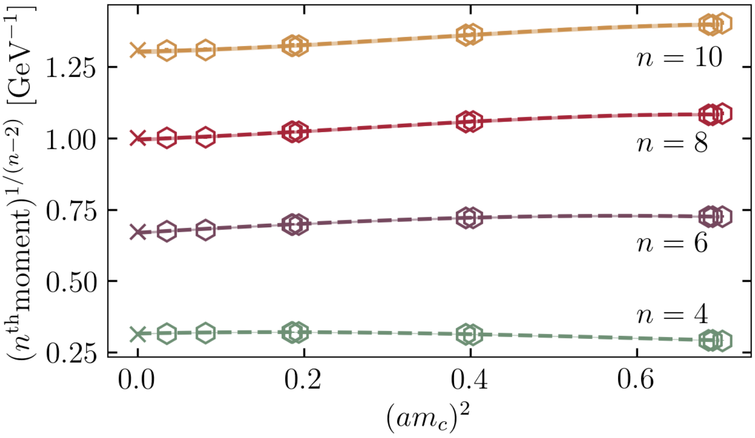
Fit with just $(a\mu)^n$ and $\alpha(a\mu)^n$; no visible condensates

Applications to charm physics

- Z_V can be used in conjunction with already available charmonium data
- Obtain very precise J/ψ decay constant
- Calculate vector correlator time moments
  - $Z_V^2 \sum_t t^n C_{J/\psi}(t)$
- $\Rightarrow a_\mu^c$



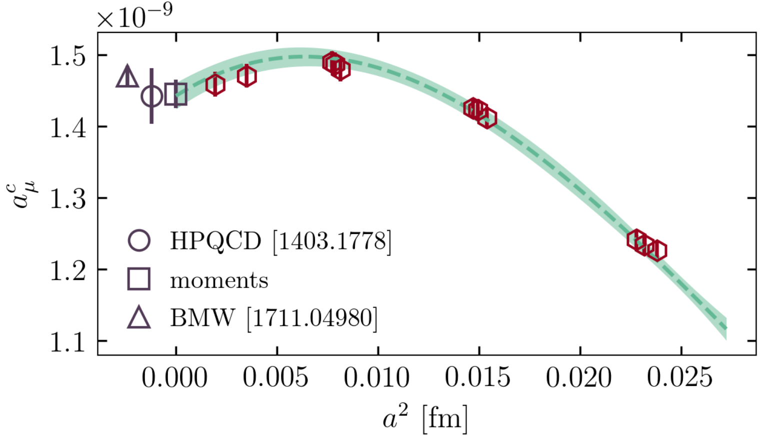
$$f_{J/\psi} = 0.4083(18) \text{ GeV (preliminary)}$$
PDG: 0.407(5) GeV



for $n = 4$

$\mathcal{M}_4 = 0.3155(22)$ GeV^{-1} (preliminary)

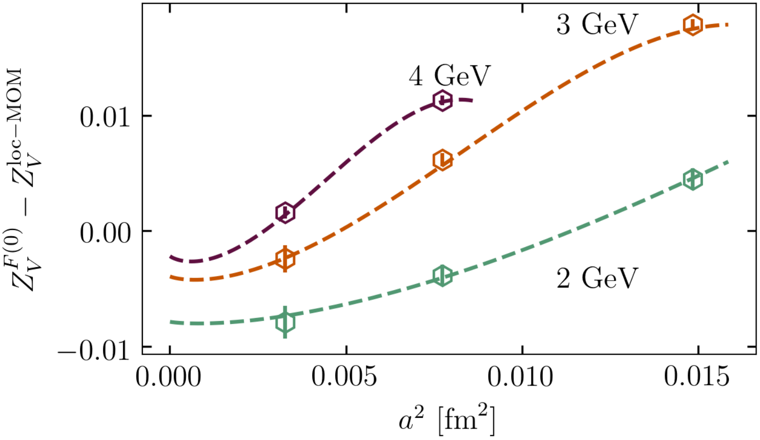
0.3142(22) GeV^{-1} expt. [0907.2110]



$a_\mu^c = 1.443(18) \times 10^{-9}$ (preliminary)
$1.470(15) \times 10^{-9}$ BMW [1711.04980]
$1.442(39) \times 10^{-9}$ HPQCD [1403.1778]

Conclusions

- HISQ implementation of RI-SMOM Z_V
- Showed $Z_V = 1$ for the conserved current in SMOM
  - Condensate effects negligible
- Nonperturbative effects are $\sim$1% in RI′-MOM
- Consistency between Z_V from SMOM and $F(0)$ (and lattice pert. theory) demonstrated
- There are many future applications of these Z_V's


3 GeV
4 GeV
2 GeV
0.01
0.00
−0.01
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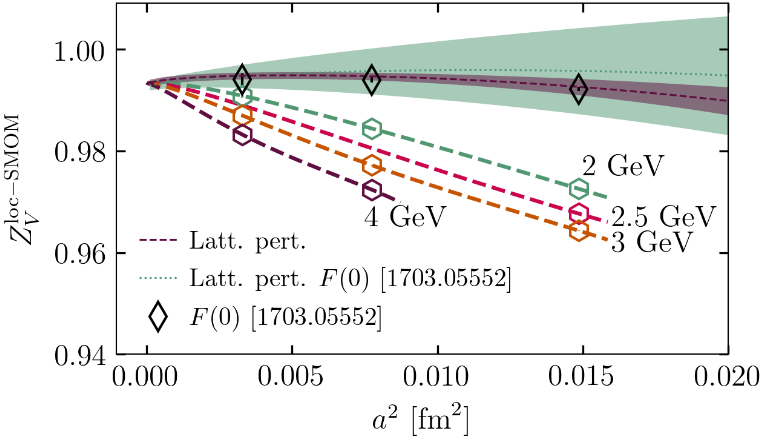
a^2 [fm^2]



1.00
0.98
0.96
0.94
0.000
0.005
0.010
0.015
0.020
$Z_V^{\text{loc}-\text{SMOM}}$
a^2 [fm^2]
2 GeV
2.5 GeV
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4 GeV
Latt. pert.
Latt. pert. $F(0)$ [1703.05552]
$F(0)$ [1703.05552]