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ALPHA Collaboration

Towards the determination of the charm quark mass on $N_f = 2+1$ CLS ensembles

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Motivation

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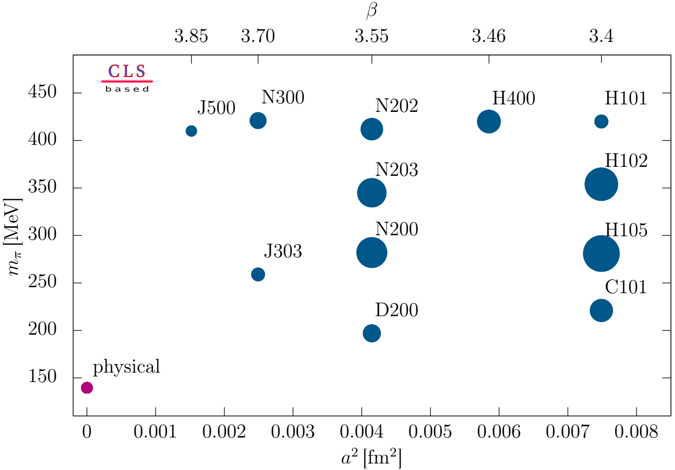
- Charm quark mass is a basic parameter of the Standard Model
- Needed as parameter in the non-perturbative matching of QCD and HQET
- Renormalization factors and improvement coefficients have been determined non-perturbatively
- Determination of the light and strange quark masses on CLS ensembles almost finished
  M. Bruno et al. [1903.04094]

Setup

CLS ensembles



- $2+1$ flavors of $\mathcal{O}(a)$ improved Wilson fermions and Lüscher-Weisz gluons.
- open boundary conditions in time direction
- Trajectory in this work: $\mathrm{Tr}[M_q] = 2m_l + m_s = \text{const.}$ to keep $\tilde{g}_0^2 = g_0^2 \left(1 + \frac{1}{N_f} b_g a \mathrm{Tr}[M_q]\right)$ constant up to $\mathcal{O}(a^2)$

- Correlation functions for all possible flavor combinations (light, strange, heavy)

$$f_O^{rs}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x},\vec{y}} \langle O^{rs}(x_0, \vec{x}) P^{rs}(y_0, \vec{y}) \rangle, \qquad O^{rs} = \bar{\Psi}^r(x)\, \Gamma\, \Psi^s(x)$$

- Sources placed at $y_0 = a$ and $y_0 = T - a$, use 16 noise sources per time slice
- Effective meson masses from pseudoscalar density and the vector current
- Bare quark masses from PCAC relation:

$$m_{rs}(x_0) = \frac{\tilde{\partial}_0 f_A^{rs}(x_0) + a c_A \partial_0^* \partial_0 f_P^{rs}(x_0)}{2 f_P^{rs}(x_0)}$$

Distance Preconditioning

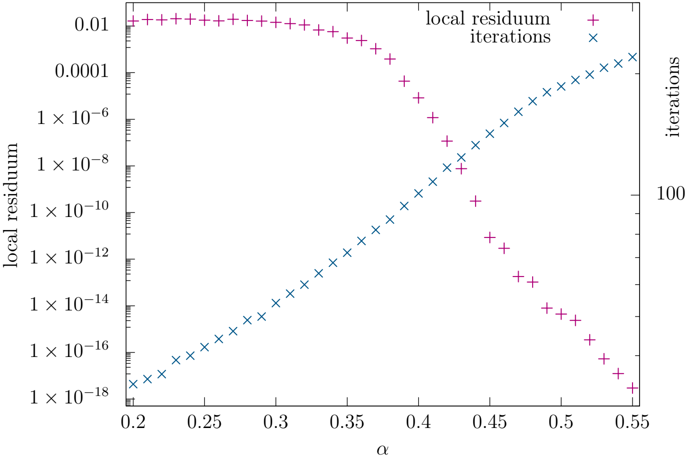
- Iterative solution of Dirac equation based on global residuum

$$\left| \sum_y D(x,y) S_h(y) - \eta(x) \right| < r_\mathrm{gl}$$

- Heavy propagator decays exponentially with m_h: $S_\mathrm{h}(y) \propto \exp(-m_\mathrm{h}(y_0 - x_0))$
  $\rightarrow$ time slices far away from the source are exponentially suppressed

- Solve this via **Distance Preconditioning** *G.M. de Divitiis, R. Petronzio, N. Tantalo* [1006.4028]
  Preconditioned system:

$$(PDP^{-1})(PS) = (P\eta), \quad P = \mathrm{diag}(p_i), \quad p_i = \exp(\alpha \, |y_0 - x_0|)$$



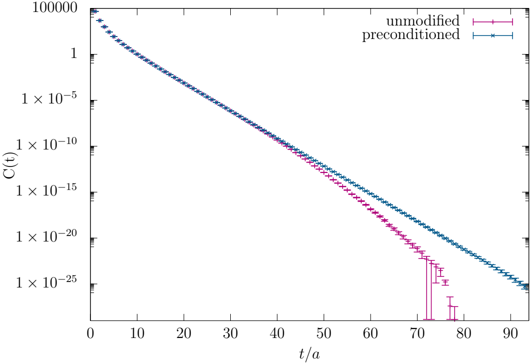
- monitor **local residuum**

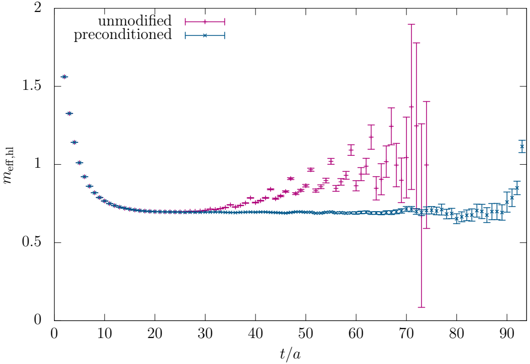
$$r_{\text{loc}}(x_0, y_0) = \frac{|D(x_0, y_0)S_h(y_0) - \eta(x_0)|}{|S_h(y_0)|}$$

at $y_0 = \dfrac{7T}{8}$

- exemplary: H400 at $y_0/a = 84$
- SAP-preconditioned GCR solver
  S. Collins et al. [1701.05502]
  T. Korzec
  https://github.com/to-ko/mesons

Distance Preconditioning - pseudoscalar heavy mesons on H400



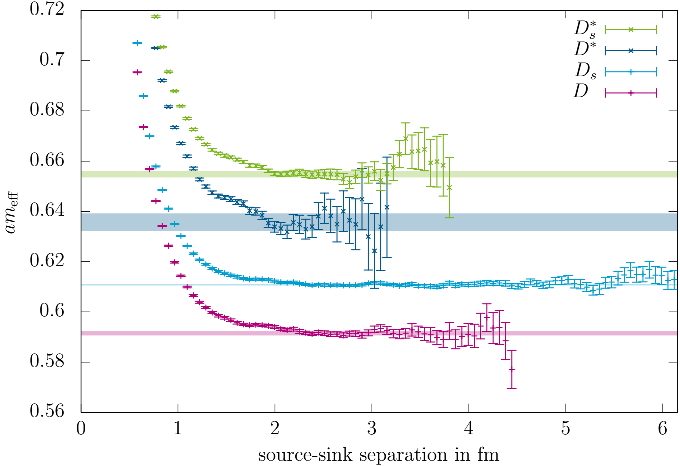


Roadmap

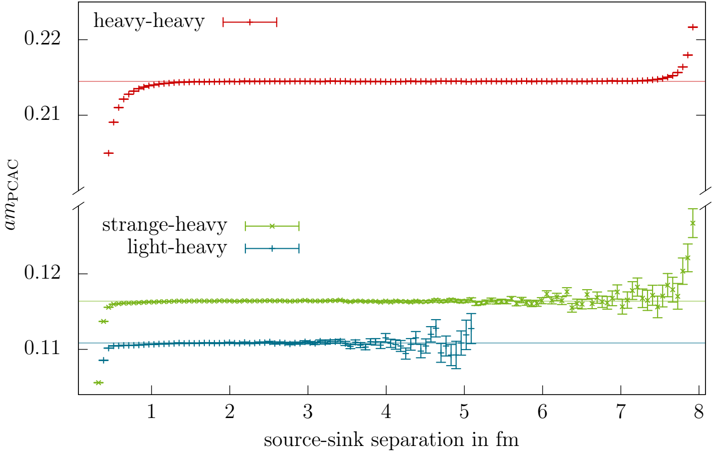
1. **Determine charm quark hopping parameter κ_c via interpolation in effective meson masses**
   - Measurements for two values of κ_h provided by RQCD-collaboration $\in$ CLS based
   - Interpolate to κ_c via matching of effective masses with physical value for
     - flavor-average $M = \frac{1}{3}(2m_D + m_{D_s})$
     - spin-flavor-average $M = \frac{1}{12}(6m_{D^*} + 2m_D + 3m_{D_s^*} + m_{D_s})$
2. **To fix the bare charm quark mass, interpolate bare heavy quark masses to κ_c**
3. **Renormalize and improve quark mass**

Preliminary results

Effective masses - heavy mesons on N200



PCAC masses on N200 - heavy quarks



Renormalized quark mass

- Compute the renormalized RGI charm quark mass from PCAC masses:

$$
\begin{aligned}
M_c^{\mathrm{RGI}} &= \frac{M}{\overline{m}(\mu_{\mathrm{had}})} m_{cc,\mathrm{R}} \\
&\equiv Z_\mathrm{M} m_{\mathrm{cc}} \left[1 + \frac{(b_\mathrm{A} - b_\mathrm{P})}{Z} a m_{\mathrm{cc}} - (b_\mathrm{A} - b_\mathrm{P})(r_\mathrm{m} - 1) a \frac{\mathrm{Tr}[M_\mathrm{q}]}{N_\mathrm{f}} + (\bar{b}_\mathrm{A} - \bar{b}_\mathrm{P}) a \mathrm{Tr}[M_\mathrm{q}] \right]
\end{aligned}
$$

  - $Z_\mathrm{M} = M/\overline{m}(\mu_{\mathrm{had}}) \cdot Z_\mathrm{A}/Z_\mathrm{P}(\mu_{\mathrm{had}})$: *I. Campos et al.* [1802.05243], including Z_A from *M. Dalla Brida et al.* [1808.09236]
  - $(b_\mathrm{A} - b_\mathrm{P})$ and $Z = Z_\mathrm{m} Z_\mathrm{P}/Z_\mathrm{A}$: *G. M. de Divitiis et al.* [1906.03445]
  - $(r_\mathrm{m} - 1) \propto \mathcal{O}(g_0^4)$, non-perturbative determination has been started
  - $(\bar{b}_\mathrm{A} - \bar{b}_\mathrm{P}) \propto \mathcal{O}(g_0^4)$, not known non-perturbatively

Renormalized quark mass

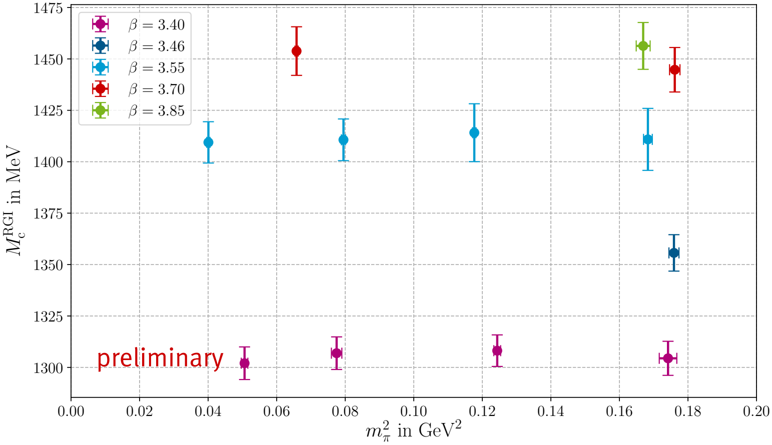
- Compute the renormalized RGI charm quark mass from PCAC masses:

$$M_c^{\mathrm{RGI}} = \frac{M}{\overline{m}(\mu_{\mathrm{had}})} m_{cc,\mathrm{R}}$$
$$\equiv Z_\mathrm{M} m_{\mathrm{cc}} \left[1 + \frac{(b_\mathrm{A} - b_\mathrm{P})}{Z} am_{\mathrm{cc}} - (b_\mathrm{A} - b_\mathrm{P})(r_\mathrm{m} - 1) a \frac{\mathrm{Tr}[M_\mathrm{q}]}{N_\mathrm{f}} + (\overline{b}_\mathrm{A} - \overline{b}_\mathrm{P}) a \mathrm{Tr}[M_\mathrm{q}]\right]$$

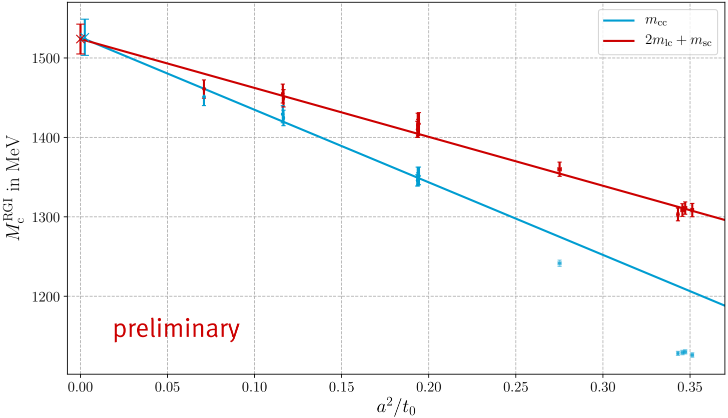
- Reduce mass dependent cut-off effects via non-degenerate PCAC masses:

$$2m_{\mathrm{lc,R}} - m_{\mathrm{ll,R}} \equiv 2\frac{m_{\mathrm{c,R}} + m_{\mathrm{l,R}}}{2} - \frac{m_{\mathrm{l,R}} + m_{\mathrm{l,R}}}{2} = m_{\mathrm{c,R}}$$

Charm quark mass: dependence on pion mass and coupling



Chiral and continuum extrapolation



- Continuum extrapolation in a^2:

  $$M_c^{RGI} = c_0(1 + c_1 t_0 \delta_M^2)(1 + c_2 \frac{a^2}{t_0})$$

  with $t_0 \delta_M^2 = t_0(m_K^2 - m_\pi^2)$
- $t_{0,\mathrm{phys.}}$ from *M. Bruno, T. Korzec, S. Schaefer* [1608.08900]
- larger cut-off effects for m_{cc}

Conclusions and outlook

- Distance preconditioning to reduce systematic uncertainties
- Preliminary extrapolations to the physical point

- Increase statistics
- Investigate finite volume effects
- Estimate systematics from different definitions
  e.g. ratio-difference method *S. Dürr et al.* [1011.2711]

Spare slides

- From HQET we know: *M. Neubert* [hep-ph/9610266]

$$m_\mathrm{H} = m_\mathrm{q} + \bar{\Lambda} + \frac{1}{2m_\mathrm{q}}\left(-\lambda_1 + 2\left[J(J+1) - \frac{3}{2}\right]\lambda_2\right) + \mathcal{O}\left(1/m_\mathrm{q}^2\right)$$

- Remove λ_2 via spin average and tame short-distance effects

$$M_\mathrm{X} = \frac{1}{4}(3m_\mathrm{V} + m_\mathrm{PS}) = m_\mathrm{q} + \bar{\Lambda} - \frac{\lambda_1}{m_\mathrm{q}}$$

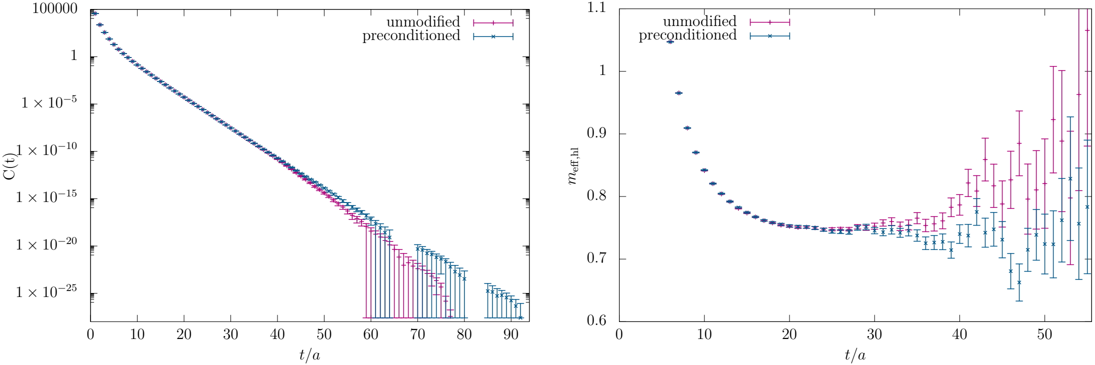
- $\mathrm{Tr}[M_q] = \text{const.} \to$ use flavor average

$$\frac{1}{3}\left(2m_\mathrm{D}^{(*)} + m_{\mathrm{D_s}}^{(*)}\right)$$

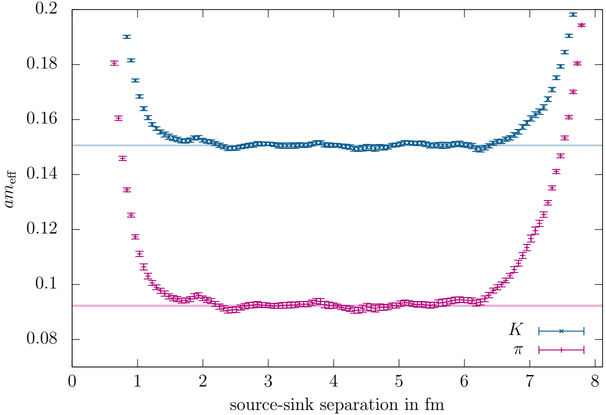
- 1S spin-flavor-average

$$M = \frac{1}{12}(6m_{\mathrm{D}^*} + 2m_\mathrm{D} + 3m_{\mathrm{D}_s^*} + m_{\mathrm{D_s}})$$

Distance Preconditioning - vector mesons



Effective masses - light mesons on N200



- Light mesons affected by open boundaries:

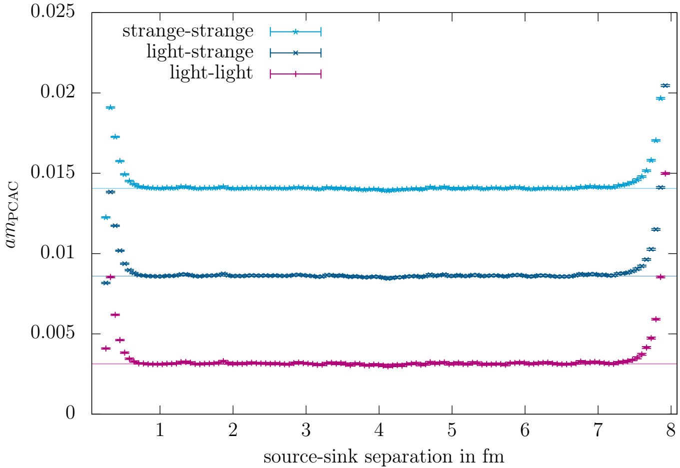
$$am_{\text{eff}}(x_0) = \log \frac{f_P(x_0)}{f_P(x_0+a)}$$

$$= am_{PS}(1 + c_1 e^{E_1 x_0} + c_2 e^{E_{2PS}(T-x_0)} + \dots)$$

$$E_1 = m' - m_{PS} \quad E_{2PS} \approx 2m_{PS}$$

M. Lüscher, S. Schaefer [1206.2809]
M. Bruno, T. Korzec, S. Schaefer [1608.08900]

PCAC masses on N200 - light quarks



Mass corrections

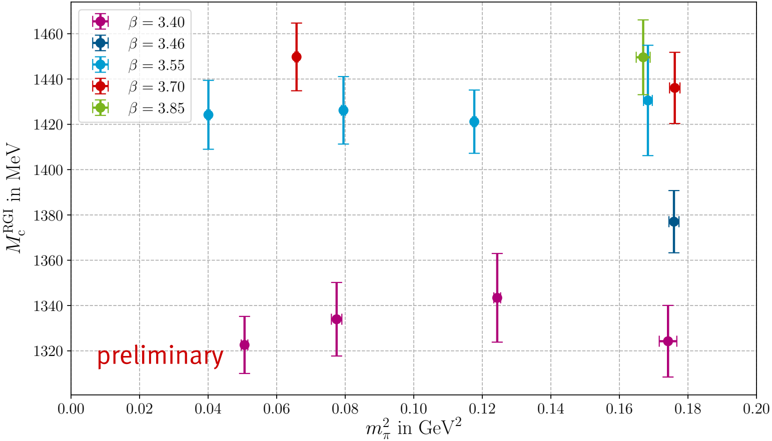
- Constant sum of renormalized quark masses violated to $\mathcal{O}(a)$
- Chiral trajectory defined by

$$\phi_2 = 8t_0 m_\pi^2 \quad \phi_4 = 8t_0 \left(m_K^2 + \frac{1}{2} m_\pi^2 \right)$$

- $\phi_4^{\text{phys.}} = 1.11(2)$ *M. Bruno, T. Korzec, S. Schaefer* [1608.08900]
- Shift observables to $\phi_4^{\text{phys.}}$ via Taylor expansion:

$$f(m') \to f(m) + (m - m') \frac{\mathrm{d}}{\mathrm{d}m} f(m)$$

Spin-flavor average



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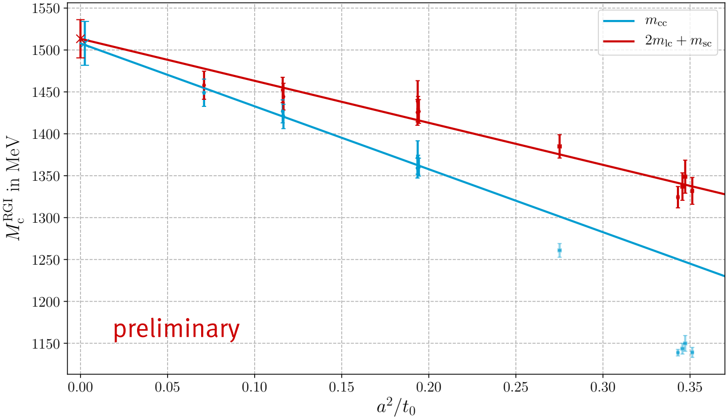

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