STATIC FORCE FROM LATTICE

Nora Brambilla[1], Viljami Leino[1], Owe Philipsen[2], Christian Reisinger[2], Antonio Vairo[1], and Marc Wagner[2]

[1]Technische Universität München, Physik-Department T30F, [2]Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik

STATIC POTENTIAL

The static potential $V(r)$ is the energy between static quark and antiquark distance r and can be written as:

$$V = \lim_{T\to\infty} \frac{i}{T} \ln \langle \mathrm{Tr}\, W \rangle \ ,$$

where W is the Wilson loop, which in continuum has the form:

$$W = \mathrm{P}\, \exp\left[ig \oint_{r\times T} \mathrm{d}z^\mu A_\mu(z) \right] .$$

For short distances $r\Lambda_{\mathrm{QCD}} \ll 1$ and $\alpha_s(\frac{1}{r}) \ll 1$, the perturbative expansion V is:

$$\begin{aligned} V(r) = \Lambda - C_{\mathrm{F}} \frac{\alpha_s(\frac{1}{r})}{r} \Bigg\{ & 1 + \frac{\alpha_s(\frac{1}{r})}{4\pi}\tilde{a}_1 + \left(\frac{\alpha_s(\frac{1}{r})}{4\pi}\right)^2 \tilde{a}_2 \\ & + \left(\frac{\alpha_s(\frac{1}{r})}{4\pi}\right)^3 \left[a_3^{\mathrm{L}} \log \frac{C_{\mathrm{A}}\alpha_s(\frac{1}{r})}{2} + \tilde{a}_3 \right] \\ & + \left(\frac{\alpha_s(\frac{1}{r})}{4\pi}\right)^4 \left[a_4^{\mathrm{L2}} \log^2 \frac{C_{\mathrm{A}}\alpha_s(\frac{1}{r})}{2} + a_4^{\mathrm{L}} \log \frac{C_{\mathrm{A}}\alpha_s(\frac{1}{r})}{2} + \tilde{a}_4 \right] \Bigg\} , \end{aligned}$$

where the constants a_k can be found in [1,2]. In the lattice gauge theory, the self-energy constant Λ diverges $\propto 1/a$ (a: lattice spacing) and in the continuum Λ is affected by the renormalon ambiguity.

STATIC FORCE FROM POTENTIAL

Among different ways of eliminating the self-energy Λ and renormalons, when lattice results are matched to perturbative ones in order to determine α_s, one approach is to take the derivative of the static potential with respect to r. This defines the static force:

$$F(r) = \partial_r V(r) = \lim_{T\to\infty} \frac{\langle \mathrm{Tr}\, \partial_r W \rangle}{\langle \mathrm{Tr}\, W \rangle} .$$

A straightforward way for a lattice determination of $F(r)$ is to fit an ansatz such as the Cornell potential to the data points for $V(r)$ and to take the derivative of the resulting parameterization (the black dashed lines in the figures).

Alternatively, one can express the derivative of the Wilson loop by an insertion of a chromo-electric field [3,4]:

$$\begin{aligned} F(r) &= \lim_{T\to\infty} \frac{\langle \mathrm{Tr}\, \partial_r W \rangle}{\langle \mathrm{Tr}\, W \rangle} = \lim_{T\to\infty} \frac{\langle \mathrm{Tr}\, E \rangle_W}{\langle \mathrm{Tr}\, W \rangle} = \langle\!\langle E \rangle\!\rangle \qquad (1) \\ &= -\lim_{T\to\infty} \frac{\langle \mathrm{Tr}\, \mathrm{P}\, \hat{r} \cdot gE(t,R) \exp(ig \int \mathrm{d}z^\mu A_\mu) \rangle}{\langle \mathrm{Tr}\, \mathrm{P}\, \exp(ig \int \mathrm{d}z^\mu A_\mu) \rangle} . \end{aligned}$$

Presented with blue data points in figures.

RENORMALIZATION

The inserted chromo-electric field needs to be renormalized, which is also indicated numerically by the discrepancy of the black dashed lines and the blue data points At the moment we remove the self-energy up to order $\mathcal{O}(g^4)$ using the Huntley-Michael (HM) procedure [5]. We multiply the inserted chromo-electric field by $Z_{\mathrm{E}} = \langle\!\langle \bar{E} \rangle\!\rangle^{-1}$ with $\bar{E}$ being the chromo-electric field calculated with the symmetric definition, i.e. with plus signs in equations (2) and (3). The data points renormalized with HM procedure are presented in orange in figures.

TREE-LEVEL IMPROVEMENT

We reduce cutoff effects by matching the $V(r)$ from lattice perturbation theory with $V(r)$ from continuum perturbation theory at tree level via a redefinition of r for the lattice results [6], $r \to r_I$. We obtain r_I from:

$$(4\pi r_I^2)^{-1} = [G(r+a) - G(r-a)]/(2a) ,$$

where G is the lattice gluon propagator.

CONCLUSIONS

We are exploring an alternative way to compute the static force with lattice gauge theory. Current crude results provide a proof-of-concept, while in future this method could be applied for precise determination of α_s. Next steps include more precise computations at larger lattice volumes and higher statistics to check, which level of precision can be obtained with the presented method.

LATTICE IMPLEMENTATION

We compute the force with equation (1) by inserting a chromo-electric field E_i into either a Polyakov or Wilson loop in a gauge invariant way. For the Polyakov loop we use the Butterfly notation for E_i:

$$ga^2 F_{\mu\nu} = \frac{1}{2i}\left(P_{\mu,\nu} - P_{\mu,\nu}^\dagger\right) , \quad ga^2 E_i = \frac{ga^2}{2}\left(F_{0i} + F_{-i0}\right) , \qquad (2)$$

where $P_{\mu,\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)$ is the Plaquette. Moreover, with Wilson loops we use the clover definition:

$$\Pi_{\mu\nu} = \frac{1}{4}\left(P_{\mu,\nu} + P_{\nu,-\mu} + P_{-\mu,-\nu} + P_{-\nu,\mu}\right) , \quad ga^2 E_i = \frac{ga^2}{2i}\left(\Pi_{i0} - \Pi_{i0}^\dagger\right) . \qquad (3)$$

We perform pure gauge SU(3) simulations. For Polyakov loops we use a multi-level algorithm with 6000 sub-updates, while the Wilson loops are APE smeared on the spatial links to maximize the ground state overlap. For the $T \to \infty$ limit, we perform a fit to a large T at $T \in [8, 12]$.

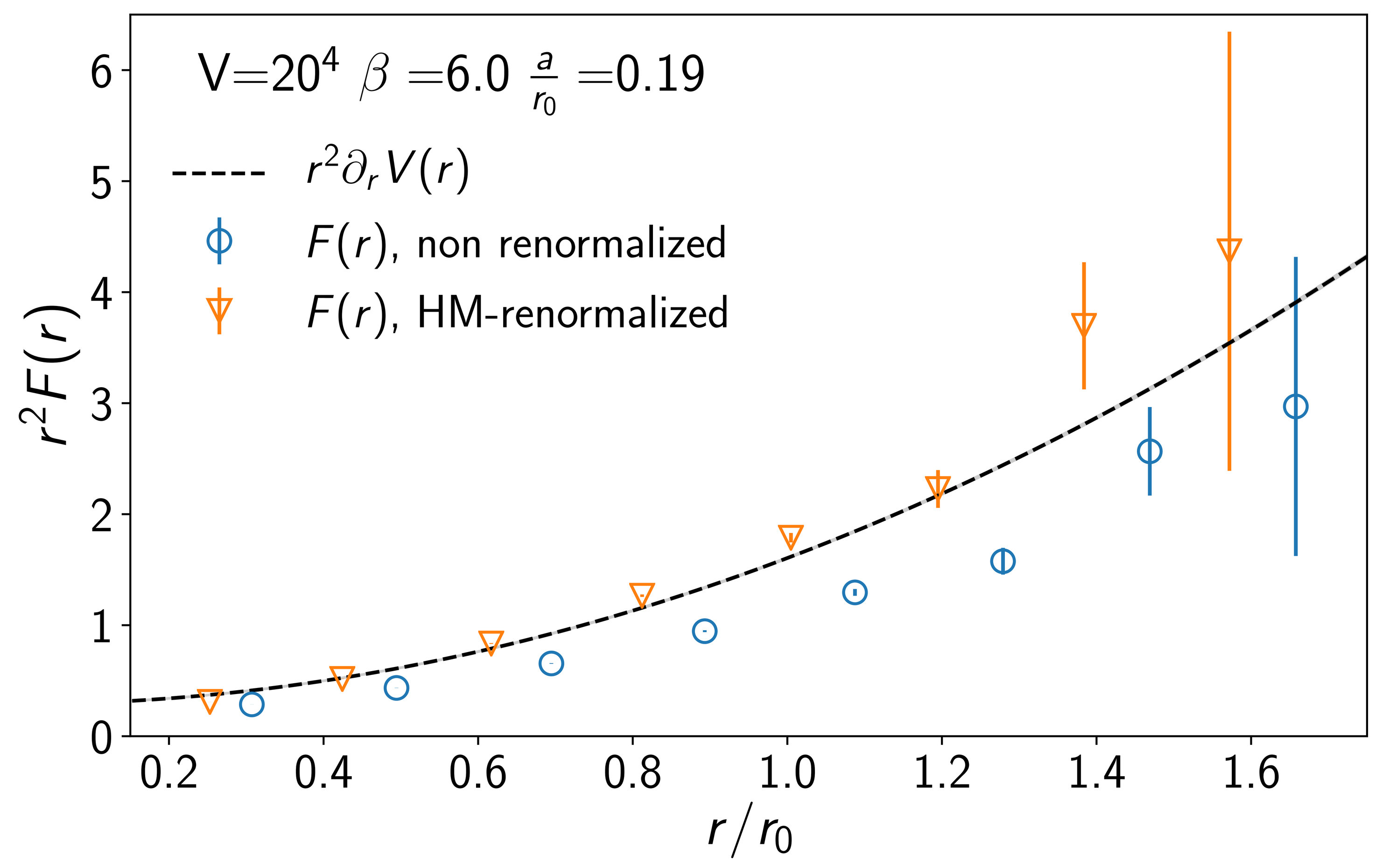

Figure 1: Force from Polyakov loop with multilevel algorithm

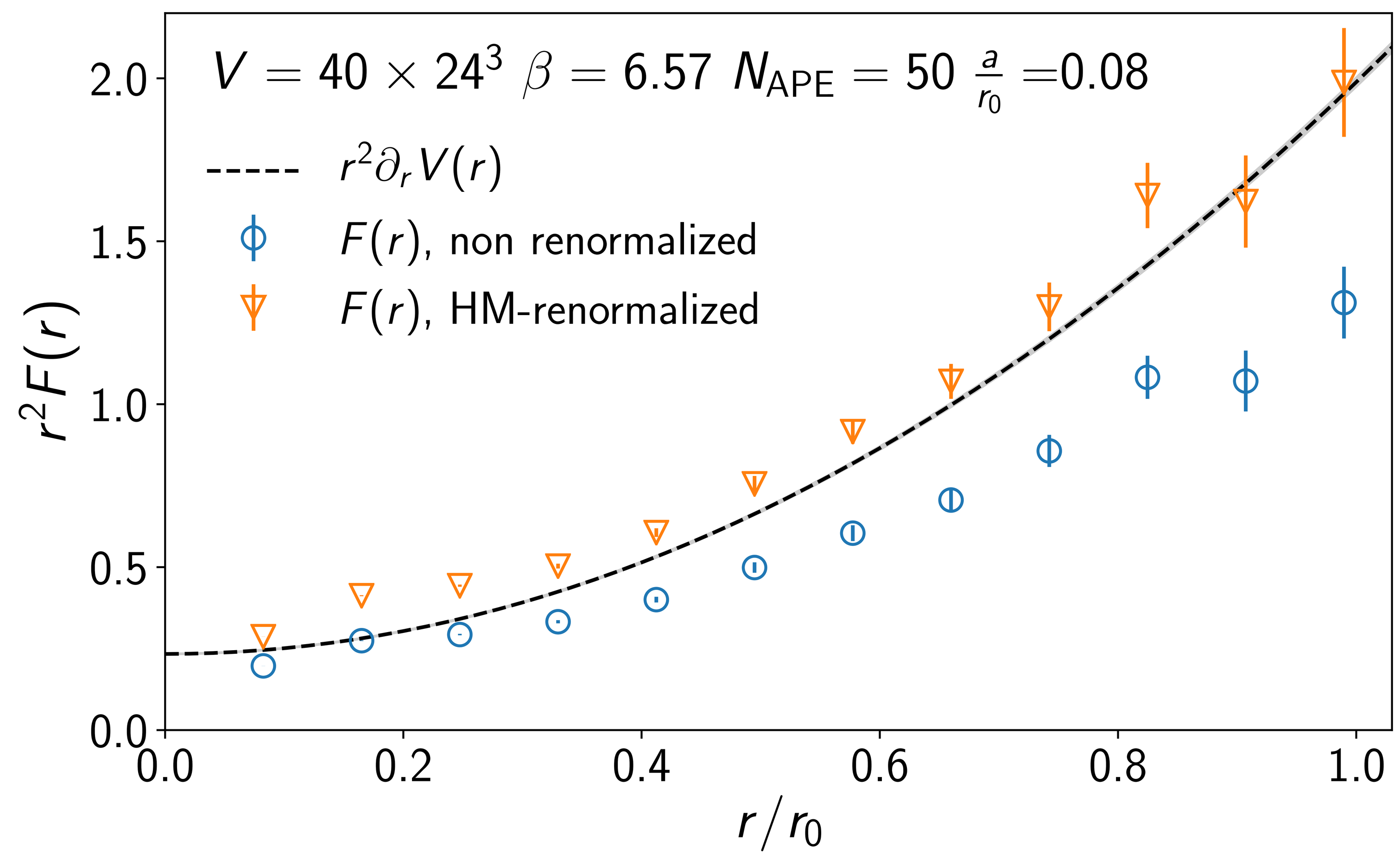

Figure 2: Force from Wilson loop with APE smearing on spatial links, no tree-level improvement

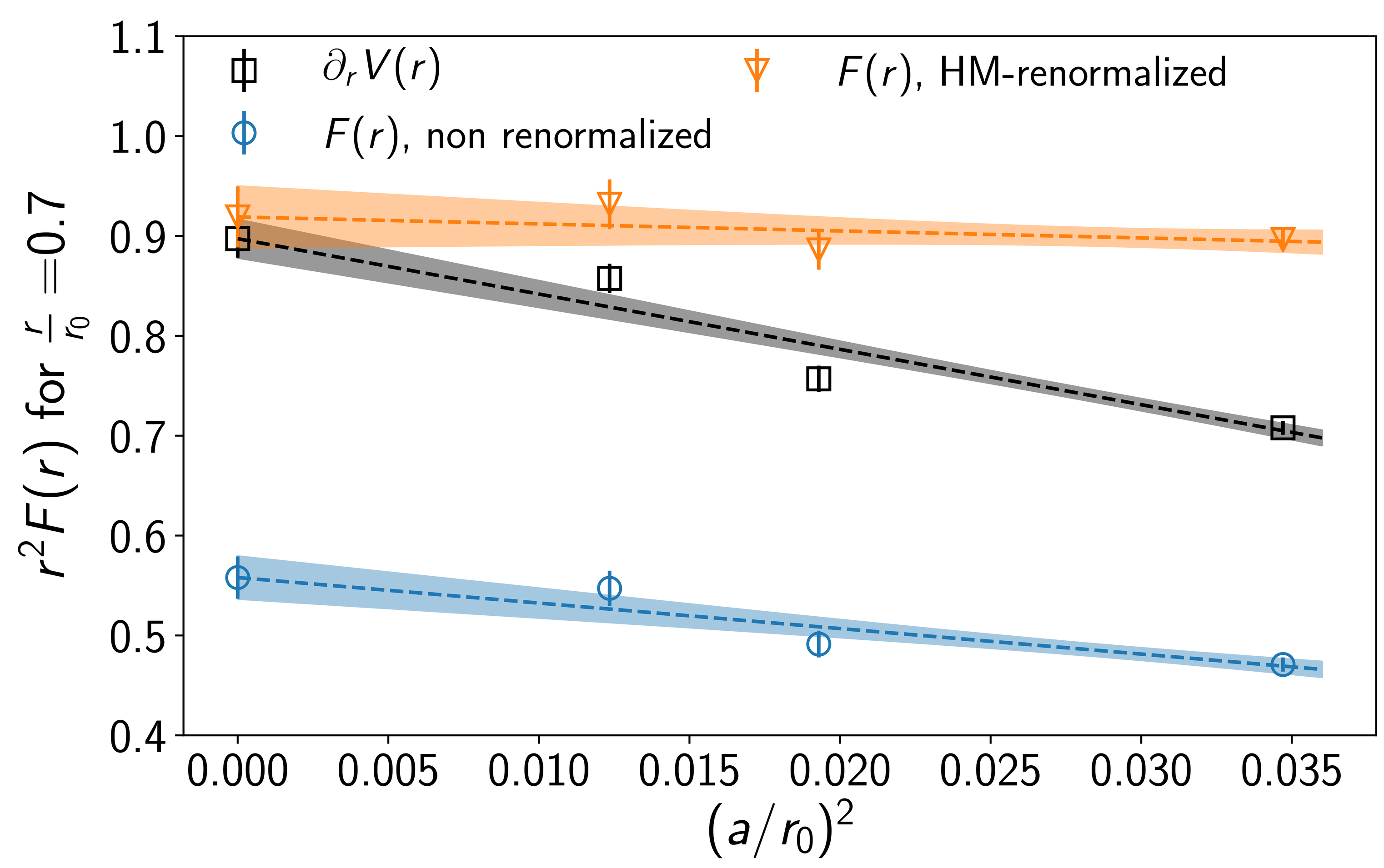

Figure 3: Continuum limit of $r^2F(r)$ for $\frac{r}{r_0} = 0.7$ at constant physical volume of 1.07fm

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