

# Calculation Of PCAC Mass With Wilson Fermion Using Gradient Flow

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For

WHOT-QCD Collaboration

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# PCAC mass

## Chiral Ward-Takahashi Identity

$$\langle 0 | \{ \partial_\mu A_\mu^a(x) + 2m_f P^a(x) \} \mathcal{O}(y) | 0 \rangle + \langle 0 | \delta_x^a \mathcal{O}(y) | 0 \rangle = 0$$

$$A_\mu^a(x) = \bar{\psi}(x) \gamma_5 \gamma_\mu T^a \psi(x)$$

$$P^a(x) = \bar{\psi}(x) \gamma_5 T^a \psi(x)$$

Set  $\mathcal{O}(y) = P^a(y)$  + Integrate out space direction

## PCAC relation

$$\langle \partial_0 A_0^a(x_0) P^a(0) \rangle = -2m_f \langle P^a(x_0) P^a(0) \rangle$$

$$m_f = - \frac{\langle \partial_0 A_0^a(x_0) P^a(0) \rangle}{2 \langle P^a(x_0) P^a(0) \rangle}$$

# Wilson fermion

Merit: Reasonable numerical cost  
Continuum limit is insured

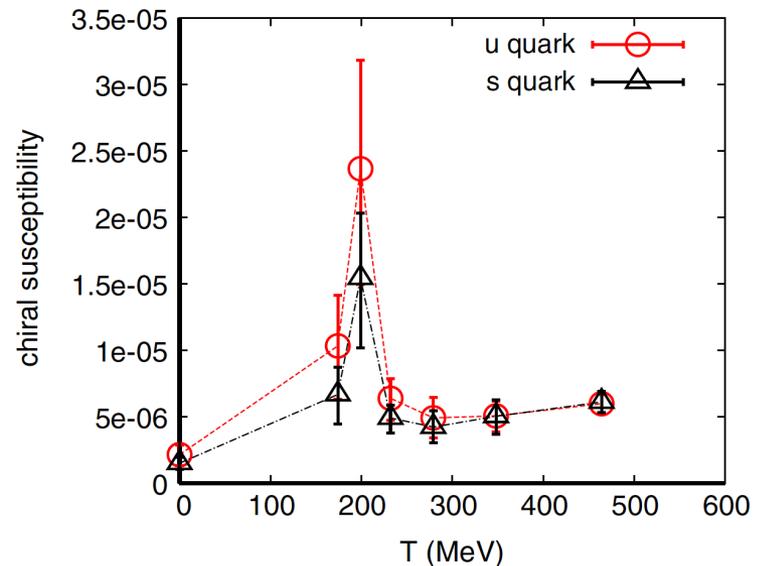


Demerit: Chiral symmetry is explicitly broken

To care this problem: **gradient flow method**

ex) chiral susceptibility

WHOT-QCD Collaboration  
PRD. 96, 014509 (2017)



# Gradient flow

## Flow equations

Gauge:  $\partial_t B_\mu(t, x) = D_\nu G_{\mu\nu}(x, t)$

Fermions:  $\partial_t \chi(t, x) = D^2 \chi(t, x)$   
 $\partial_t \bar{\chi}(t, x) = \bar{\chi}(t, x) \overleftarrow{D}^2$

## Initial condition

$$B_\mu(t, x) = A_\mu(x)$$

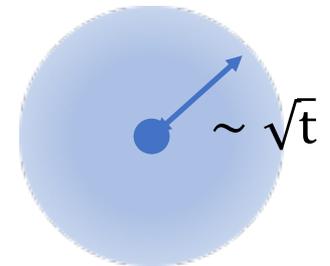
$$\chi(t, x) = \psi(x)$$

$$\bar{\chi}(0, x) = \bar{\psi}(x)$$

## Formal solution

$$B_\mu = \int d^4 y K_t(x - y) A_\mu(y) + \text{interactions}$$

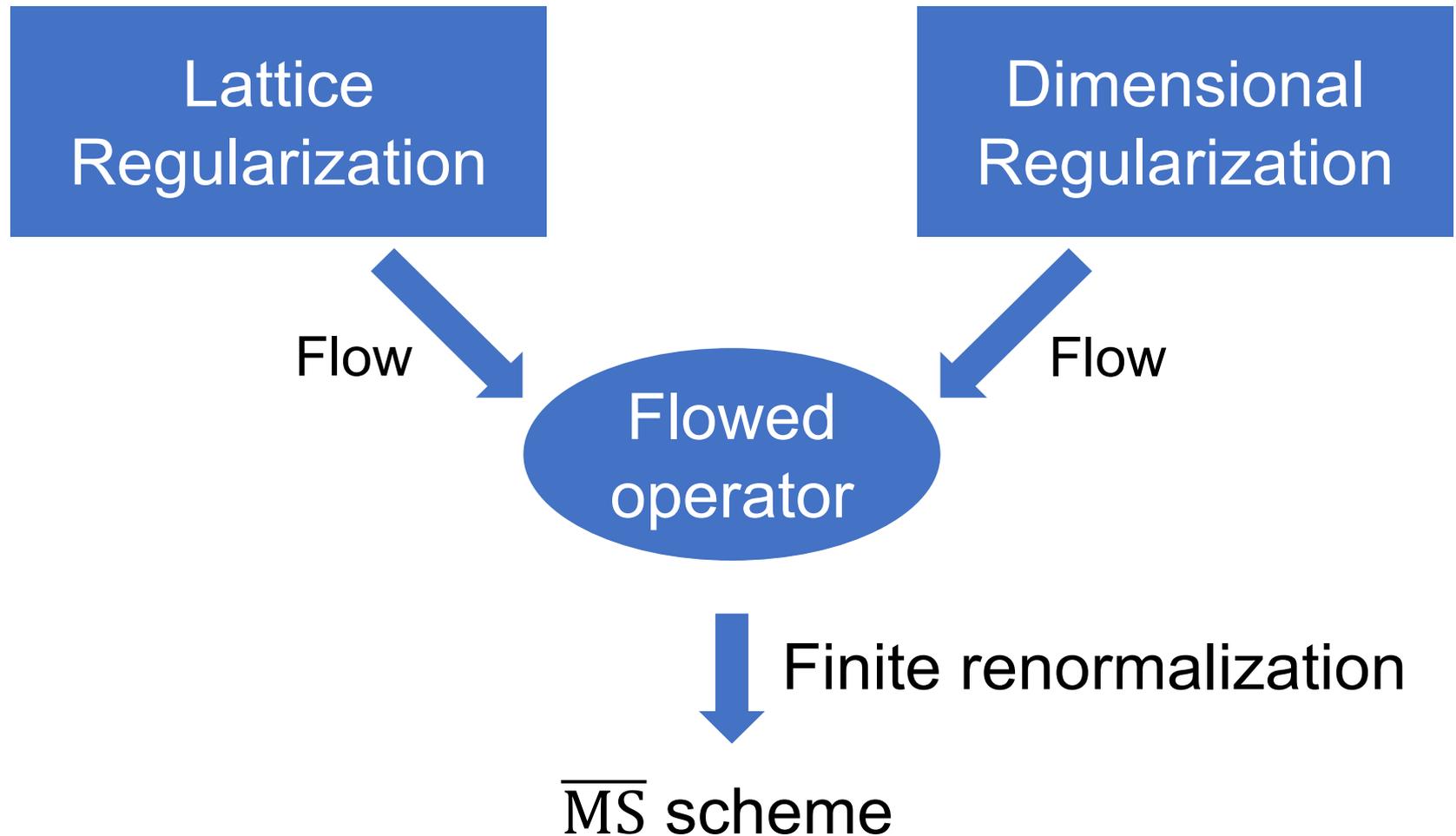
$$K_t(x) = \int_p \underbrace{e^{-ipx} e^{-tp^2}}_{\text{red arrow}} = \frac{e^{-z^2/4t}}{(4\pi t)^2}$$



This factor dumps high energy mode at  $t > 0$

# Gradient flow

Flowed fields are UV finite!



# Small flow time expansion

In  $t \rightarrow 0$  limit, we can get

$$\{\mathcal{O}\}_R = \lim_{t \rightarrow 0} Z(t) \mathcal{O}^{GF}(t)$$

To extrapolate  $\{\mathcal{O}\}_R$  from finite  $t$ ,

Mixing from higher dimensional operators

$$\begin{aligned} \mathcal{O}^{GF}(t) &= \{\mathcal{O}\}_R + \boxed{O(t)} \sim tA + t^2B + \dots \\ &\sim \{\mathcal{O}\}_R + tA \end{aligned}$$

:small flow time expansion

And extract lattice artifact,

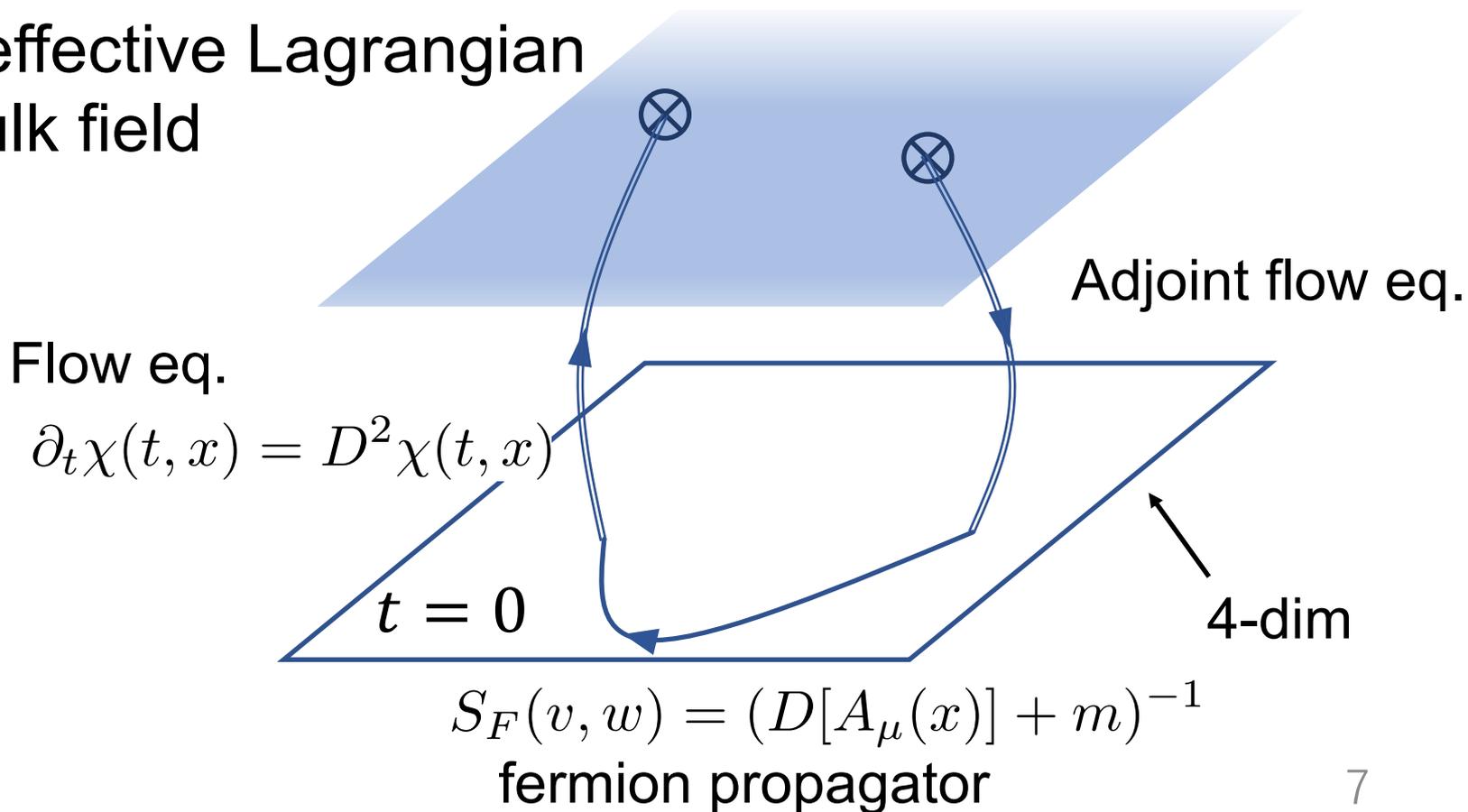
$$\sim C \frac{a^2}{t} + D a^2 \Lambda_{QCD}^2 + \dots$$

$$\begin{aligned} \mathcal{O}^{GF}(t) &= \{\mathcal{O}\}_R + O(t) + \boxed{O(a^2)} \\ &\sim \{\mathcal{O}\}_R + tA + C \frac{a^2}{t} \end{aligned}$$

# Propagator for flowed fermions

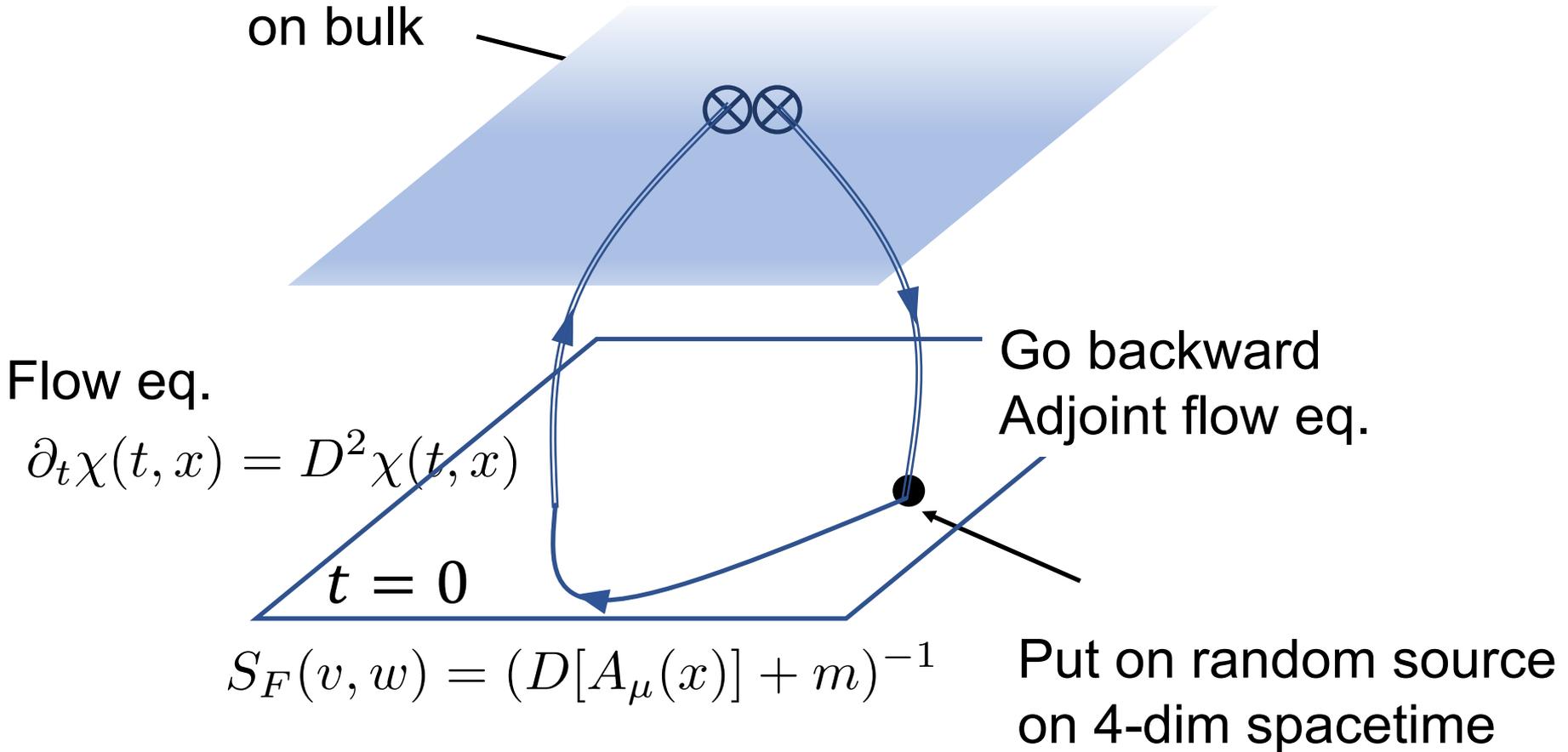
$$\overline{\chi(t, y)} \chi(t, x) \neq (D[B_\mu(t, x)] + m)^{-1}$$

We don't know  
the effective Lagrangian  
in bulk field



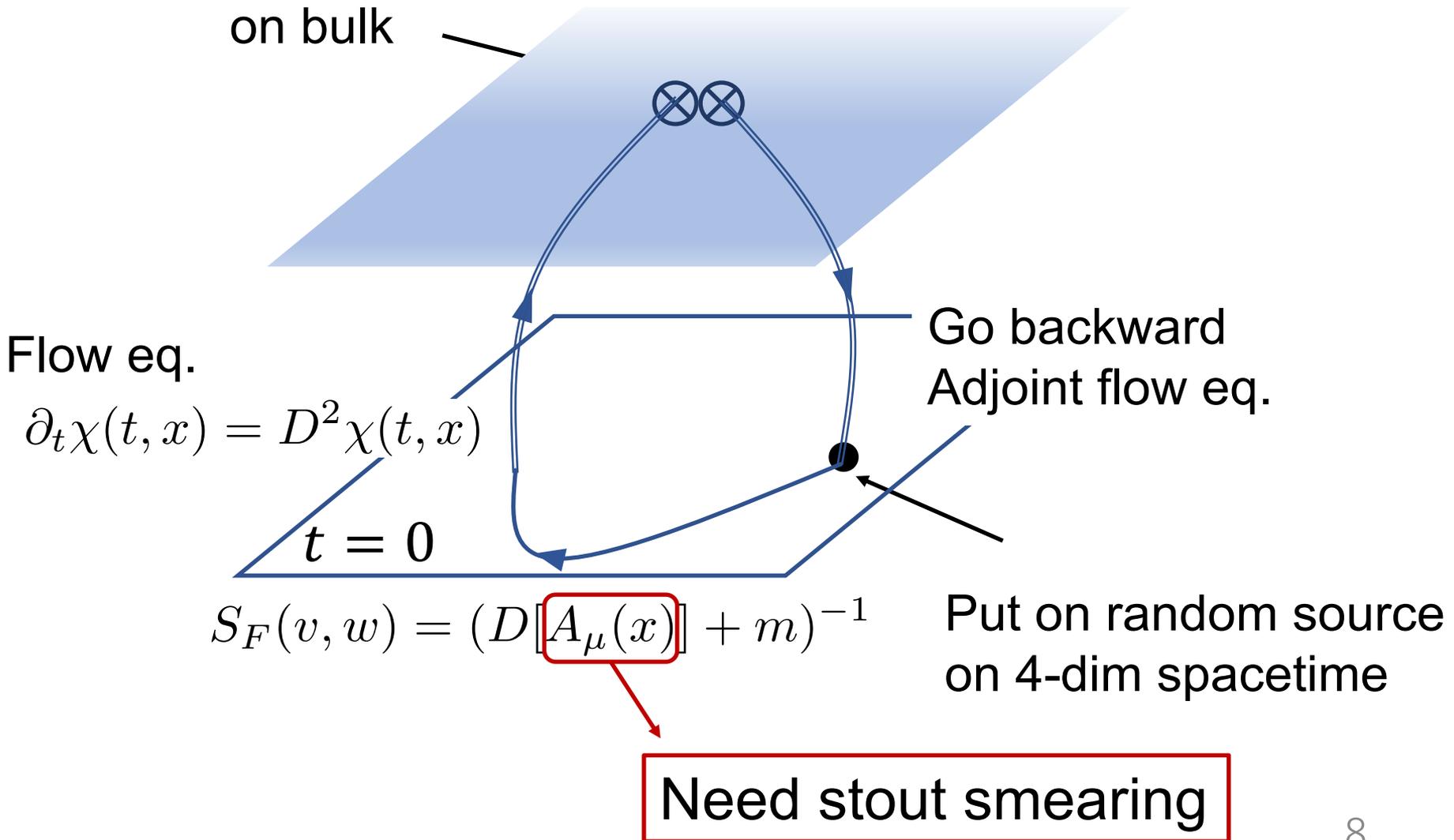
# Measurement: 1pt function

Put the sink  
on bulk



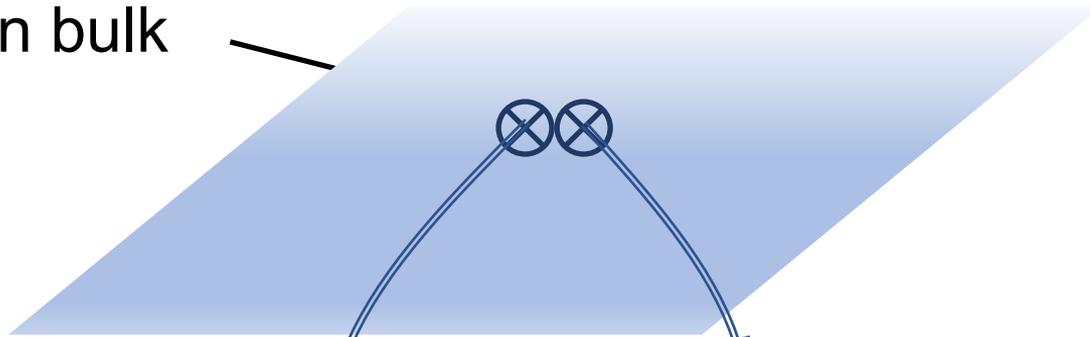
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# Measurement: 1pt function

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Flow eq.

$$\partial_t \chi(t, x) = D^2 \chi(t, x)$$

$t = 0$

Go backward  
Adjoint flow eq.

$$S_F(v, w) = (D[A_\mu(x)] + m)^{-1}$$

Put on random source  
on 4-dim spacetime

Smear or not smear

Need stout smearing

# 1pt function on G6464

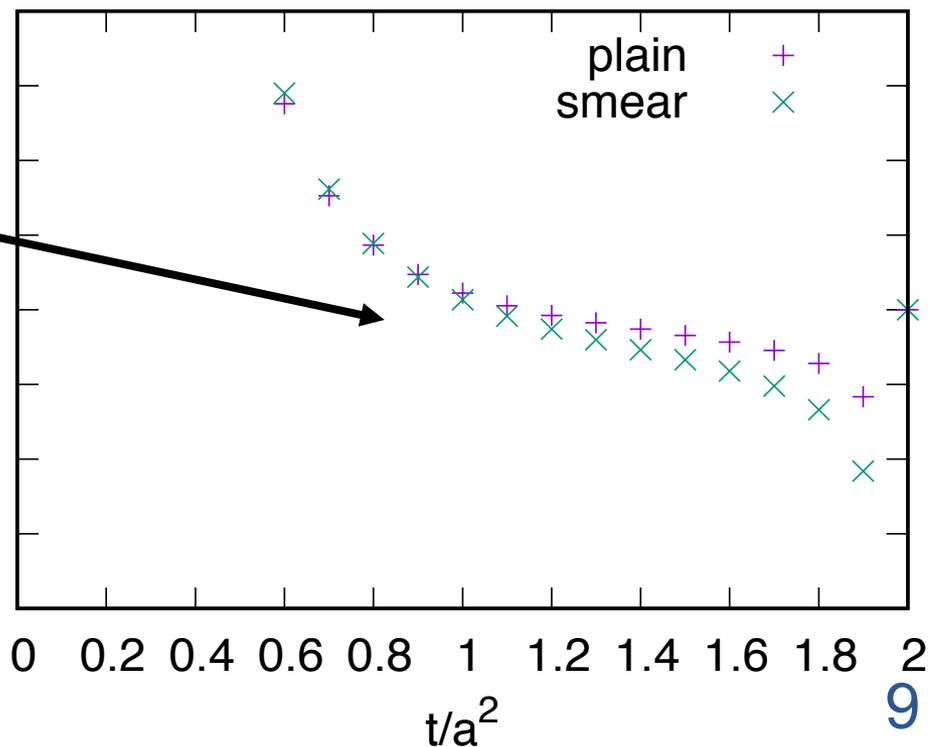
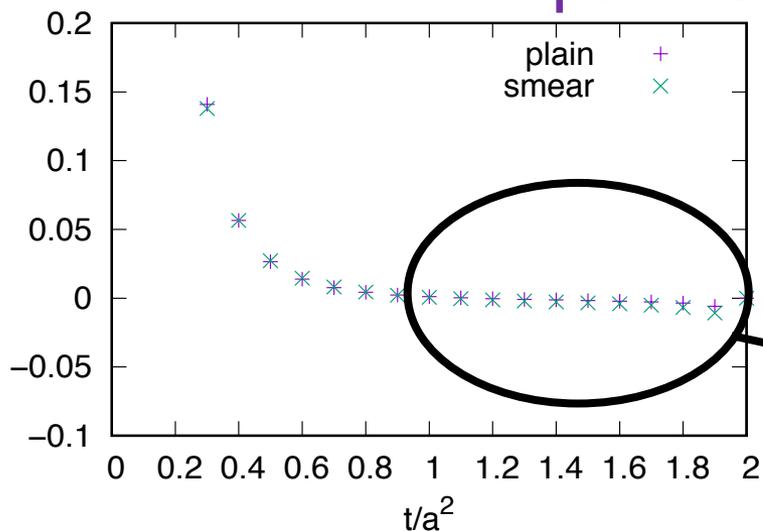
$N_f = 2 + 1$  full QCD Iwasaki + NP Wilson clover

$\beta = 1.82$  ( $a \sim 0.08$  fm)

Lattice size:  $64^4$

CP-PACS Collaboration PRD. 99, 014504 (2019)

plain link vs smeared link on 1 conf



Plain link looks  
bit well behaved

-0.01  
-0.015  
-0.02

# 1pt function on G6464

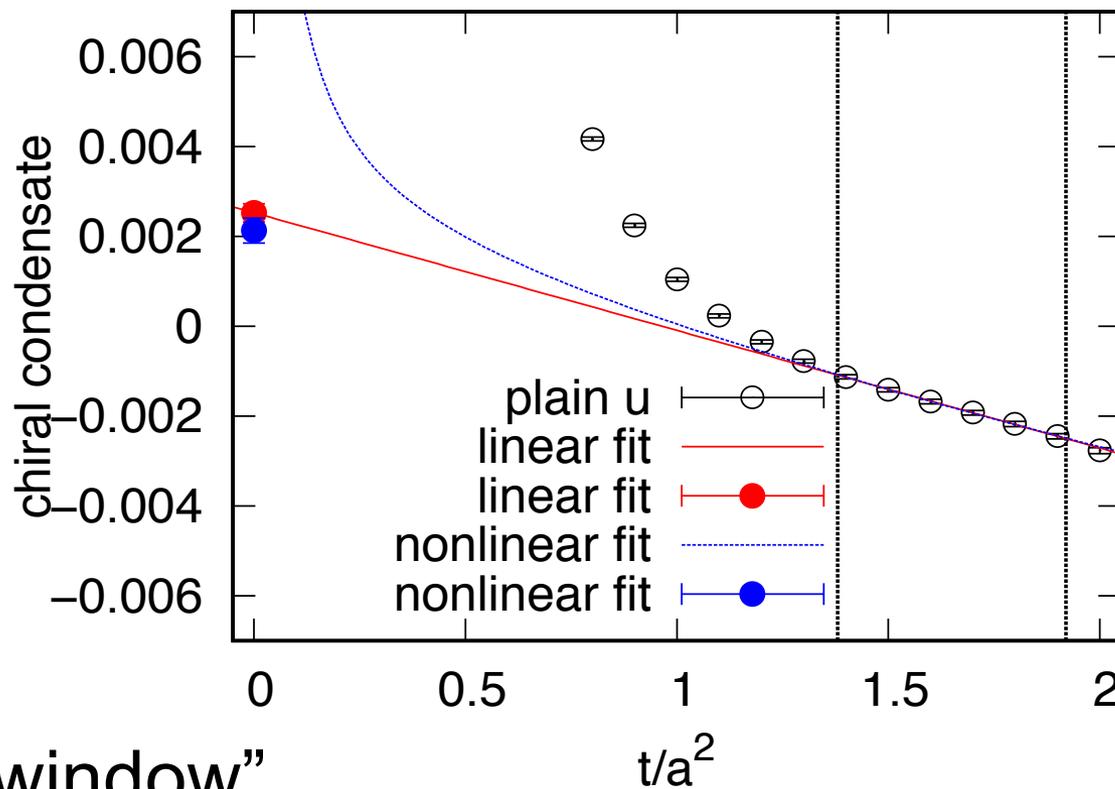
Only 8 confs (of 200 confs)

Linear fit

$$a + bt$$

Nonlinear fit

$$a + bt + ct^2 + \frac{d}{t}$$



Choice the “linear window”

And

Suspect systematic error

# Lattice Setup

$N_f = 2 + 1$  full QCD

Iwasaki gauge action +

NP improved Wilson clover  $c_{SW}$

$\beta = 2.05$  ( $a \sim 0.07$  fm),

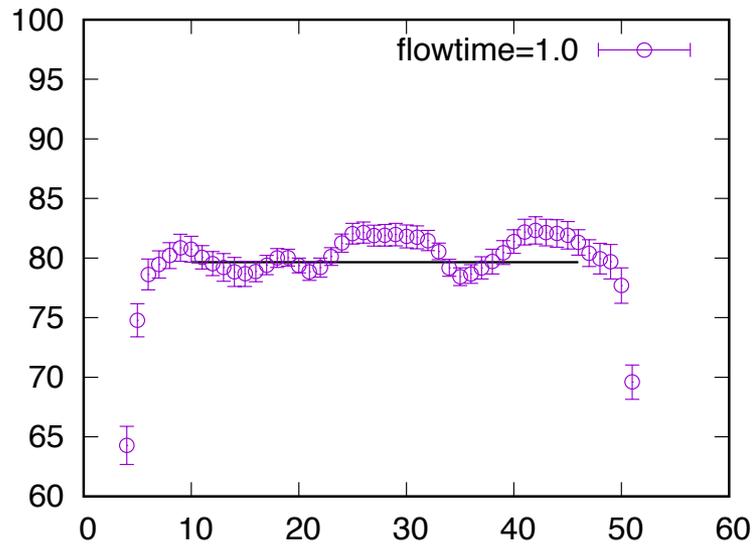
Heavy ud quarks:  $m_\pi/m_\rho \sim 0.63$ ,

Almost physical s quark:  $m_{\eta_{ss}}/m_\phi \sim 0.74$

Lattice size:  $28^3 \times 56$

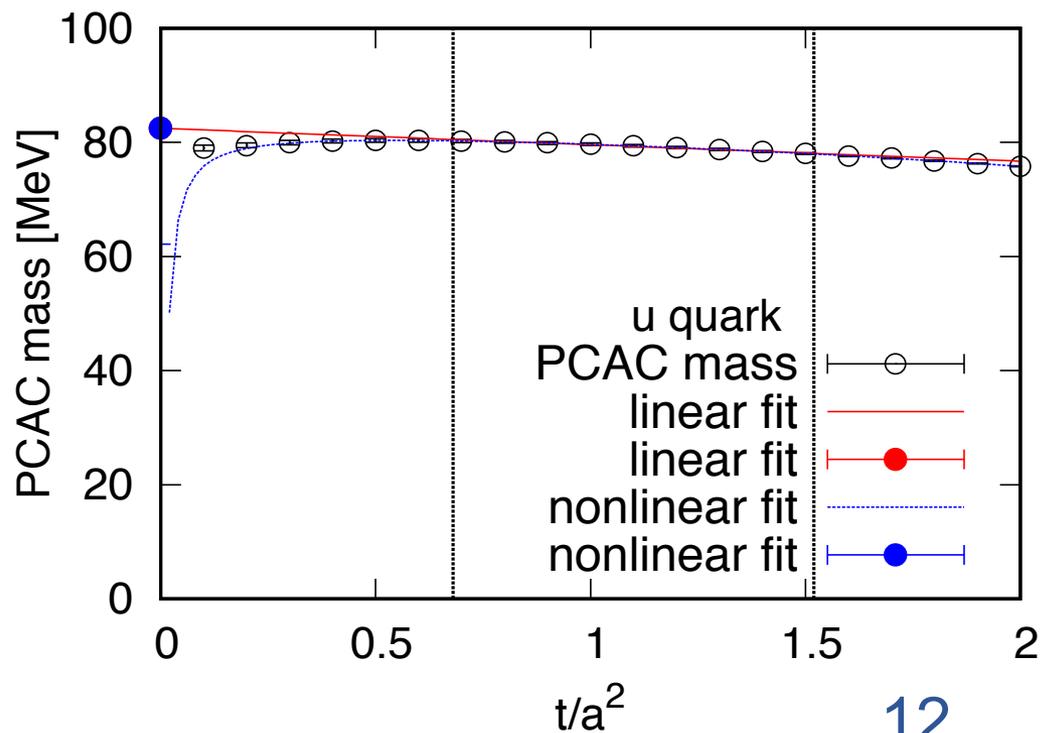
(CP-PACS and JLQCD Collaborations 2008)

# Numerical result



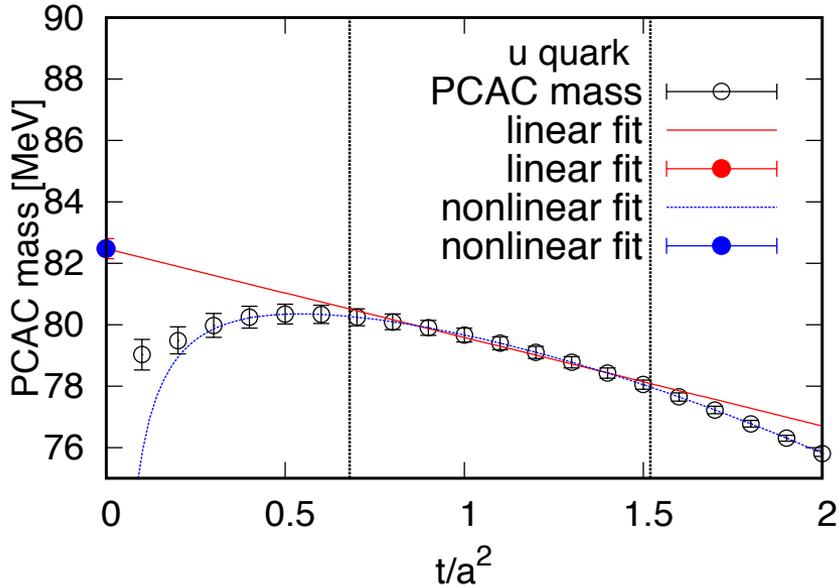
Linear/Nonlinear fit

Constant fit for each flow time



# Numerical result

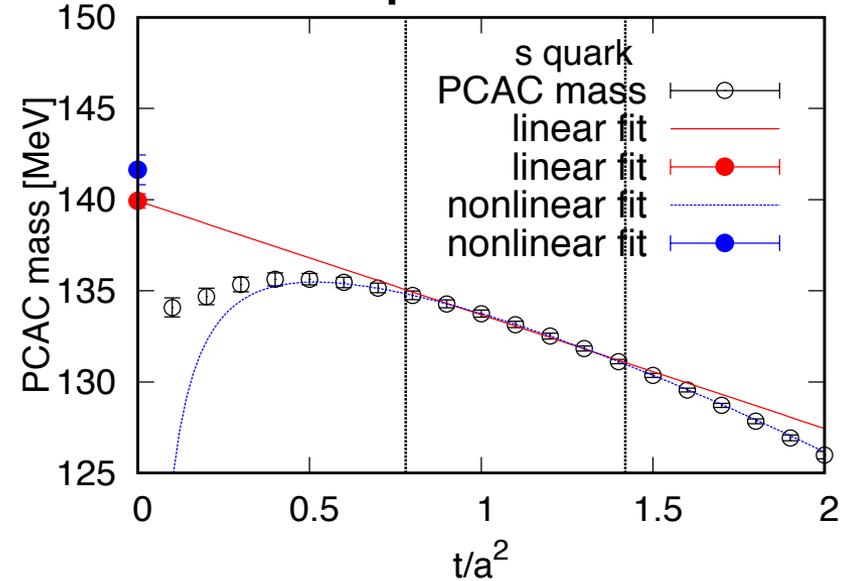
u/d quark



$$m_u^{GF} = 82.5 \pm 0.3 \text{ MeV}$$

$$m_u^{SF} = 82.3 \pm 4.1 \text{ MeV}$$

s quark



$$m_s^{GF} = 139.642^{1.6}_{0.5} \text{ MeV}$$

$$m_s^{SF} = 137.9 \pm 6.8 \text{ MeV}$$

Good consistent with Schrödinger functional method!

# Conclusion

We calculated PCAC mass using GF method under Wilson fermion

Our result is well consistent with renormalized in SF scheme.

We can challenge other operators which relate to Chiral symmetry for example:  $B_K$  Next talk!

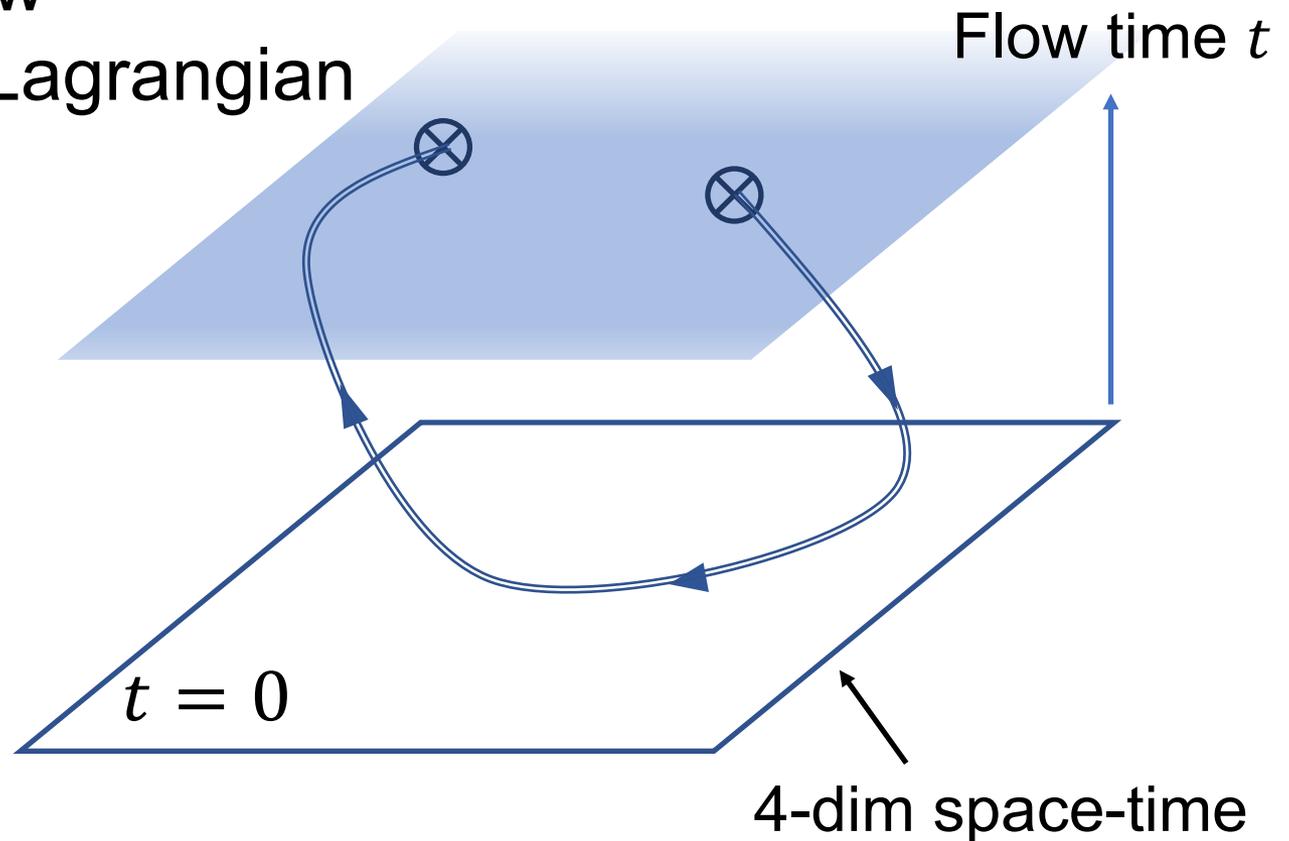
Our on going project, smeared link, is under measurement



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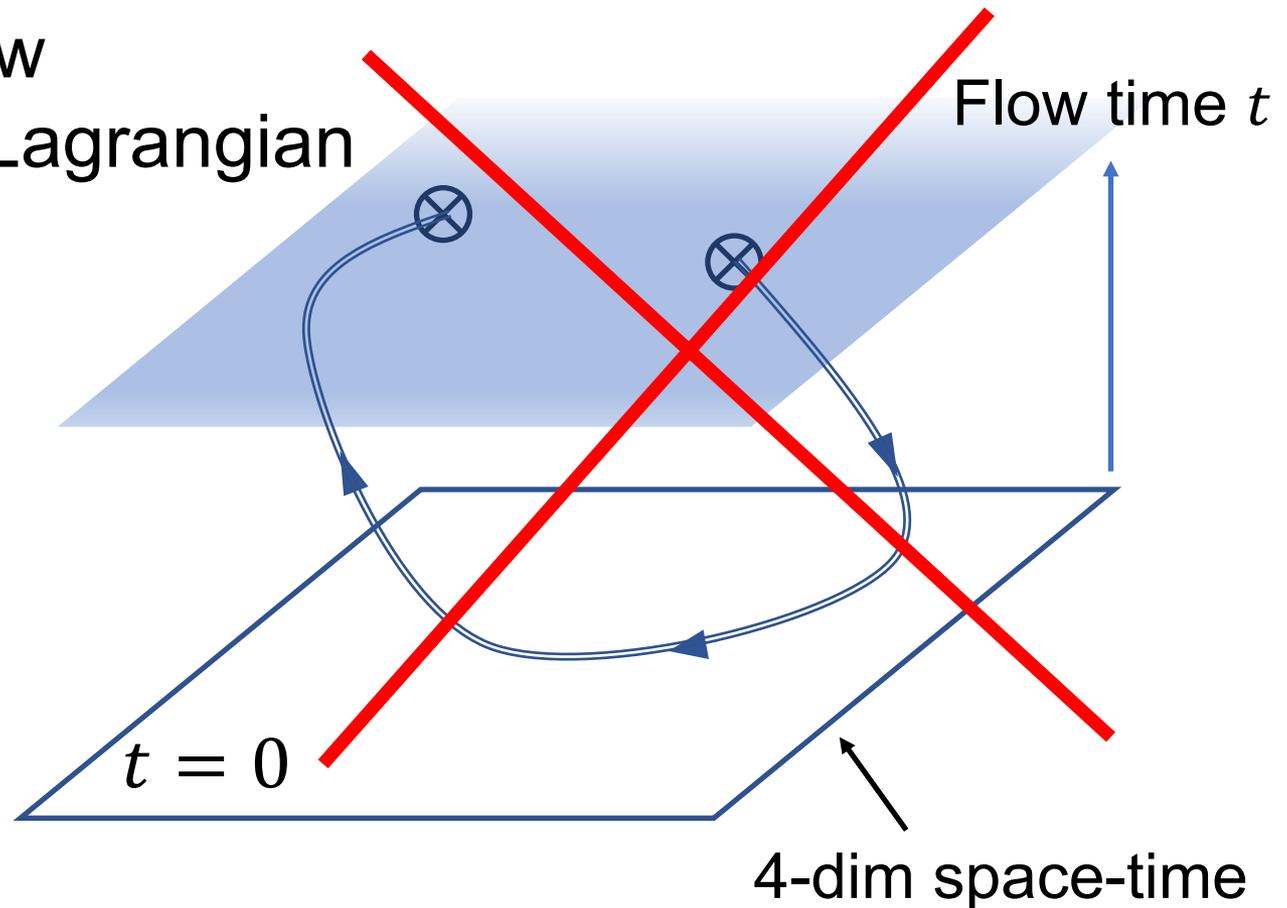
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What we know?  
4-dim Lagrangian  
flow eq

