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SAPIENZA UNIVERSITÀ DI ROMA

INFN Istituto Nazionale di Fisica Nucleare

Non-perturbative renormalization in QCD+QED
and its application to weak decays

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in collaboration with:

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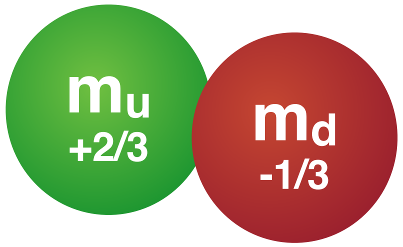
Isospin Breaking (IB) Effects



$SU(2)_V$ **isospin symmetry** is an almost exact property of **QCD**, but it is mildly broken by:

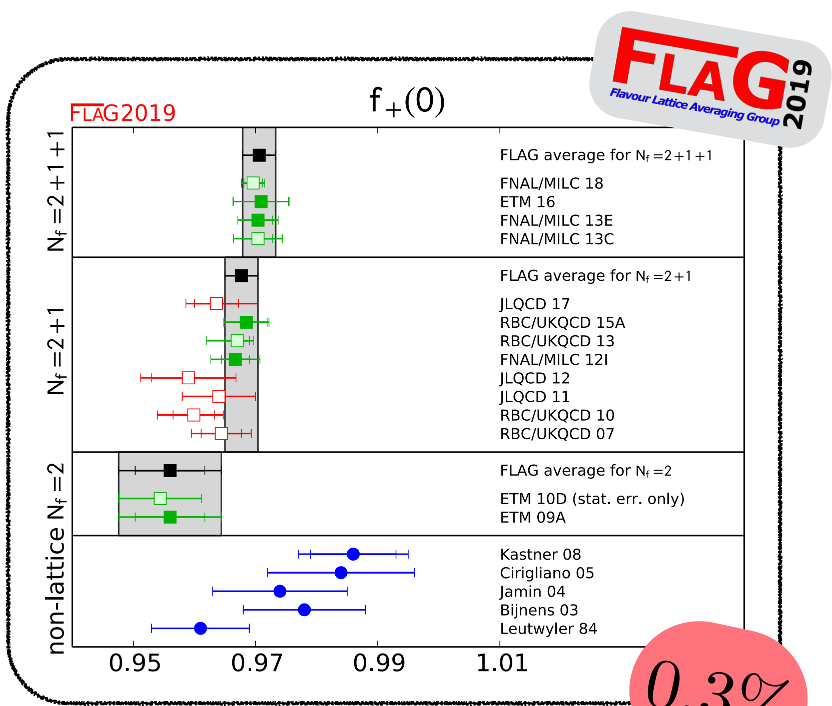
★ $m_u \neq m_d$ $\quad \mathcal{O}\left(\frac{m_d - m_u}{\Lambda_{QCD}}\right) \sim 1/100$

Strong

★ $e_u \neq e_d$ $\quad \mathcal{O}(\alpha_{em}) \sim 1/137$

Electromagnetic

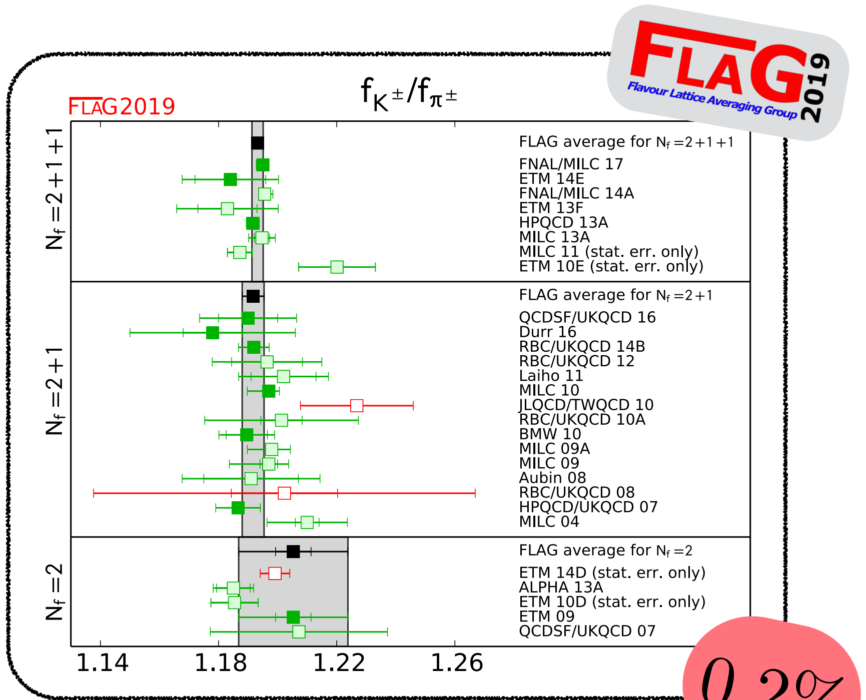
The precision reached in lattice calculations of some flavour observables is such that **EM corrections** and **strong IB effects** **cannot be neglected anymore**.



0.3%

$f_+^{K\pi}(0) = 0.9706(27)$

Lattice simulations in the "full" **QCD+QED** theory



0.2%

$f_{K^\pm}/f_{\pi^\pm} = 1.1932(19)$

The RM123 method

QED and **strong IB (SIB)** effects treated as **perturbations** to the iso-symmetric theory:

$$\mathcal{L} = \mathcal{L}^{\text{iso-symm}} + e\,\mathcal{L}_{\text{QED}} + \delta m\,\mathcal{L}_{\text{mass}}$$

$$e^2 = 4\pi/137.036 \quad \delta m = (m_d - m_u)/2$$

Pioneering papers:

G.M. de Divitiis et al., JHEP 04 (2012) 124 ★

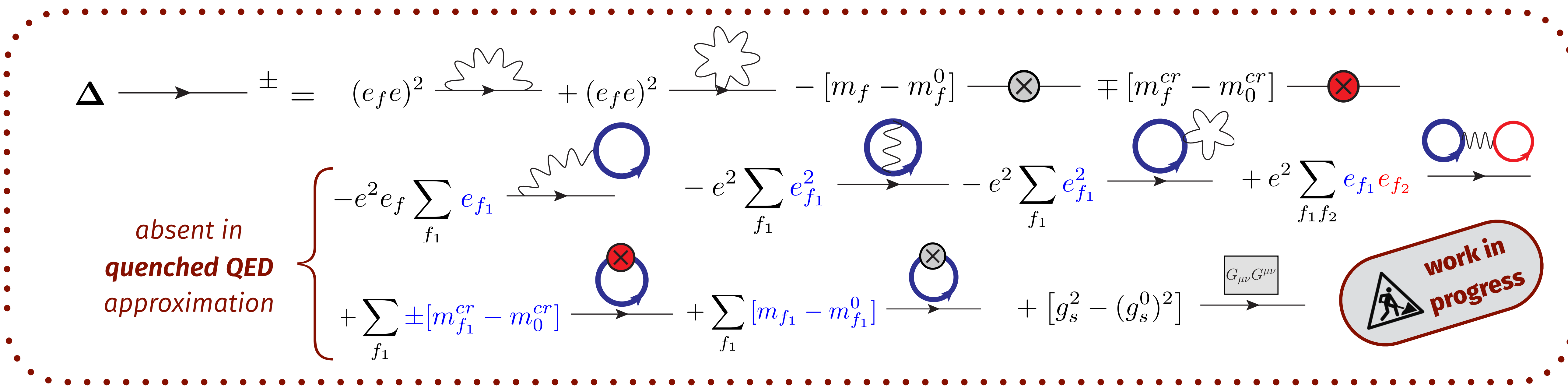
G.M. de Divitiis et al., PRD 87 (2013) 114505 ★★

✔ no need to generate new gauge ensembles, no α_{em} extrapolation required

✘ more vertices and correlation functions to be computed (also disconnected diagrams!)

- Expand the QCD+QED path integral with respect to isosymmetric QCD:

$$\langle \mathcal{O} \rangle^{\vec{g}} = \langle \mathcal{O} \rangle^{\vec{g}^0} + \mathbf{\Delta}\mathcal{O}$$

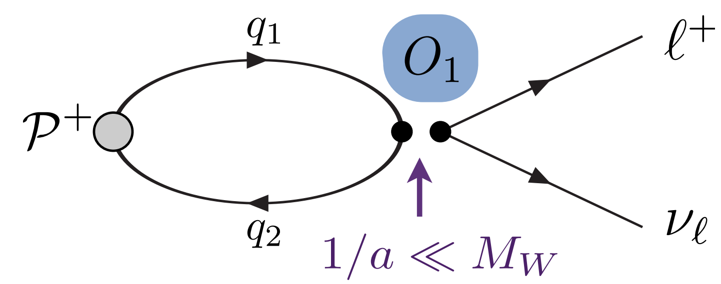


Need for Renormalization corrections

We need to **renormalize the operators** mediating the physical process of interest!

Light-meson leptonic decay

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V^*_{q_1 q_2} \left[\, \bar{q}_2\, \gamma_\mu (1-\gamma_5)\, q_1 \,\right] \left[\, \bar{\nu}_\ell\, \gamma^\mu (1-\gamma_5)\, \ell \,\right]$$



1st step

$$\frac{\Gamma_{K^\pm \to \mu^\pm \nu_\mu [\gamma]}}{\Gamma_{\pi^\pm \to \mu^\pm \nu_\mu [\gamma]}} = \frac{\Gamma_{K^\pm \to \mu^\pm \nu_\mu}}{\Gamma_{\pi^\pm \to \mu^\pm \nu_\mu}} (1 + \delta R_{K\pi})$$

- Large cancellation of renormalization corrections
- NP renormalization in QCD + perturbation theory for QED

$\mathcal{O}(\alpha_s^n)$ $\mathcal{O}(\alpha_{\rm em})$ ~~$\mathcal{O}(\alpha_s^n \alpha_{\rm em})$~~

N. Carrasco et al., PRD 91 (2015) 074506
D. Giusti et al., PRL 120 (2018) 072001

now

$$\Gamma_{K^\pm \to \mu^\pm \nu_\mu [\gamma]} = \Gamma_{K^\pm \to \mu^\pm \nu_\mu} (1 + \delta R_K)$$

$$\Gamma_{\pi^\pm \to \mu^\pm \nu_\mu [\gamma]} = \Gamma_{\pi^\pm \to \mu^\pm \nu_\mu} (1 + \delta R_\pi)$$

- Renormalization corrections must be included
- NP renormalization in QCD **and** in QED (at first order)

$\mathcal{O}(\alpha_s^n)$ $\mathcal{O}(\alpha_{\rm em})$ $\mathcal{O}(\alpha_s^n \alpha_{\rm em})$

NEW **MDC et al., arXiv:1904.08731 [hep-lat]**

Leptonic decay rate at $\mathcal{O}(\alpha_{\rm em})$

IR finite — IR divergent — IR divergent

$$\Gamma(P_{\ell 2}) = \Gamma_P^{(0)}\,(1+\delta R_P) = \Gamma_0(P\to\ell\nu_\ell) + \Gamma_1(P\to\ell\nu_\ell\gamma)$$

F. Bloch & A. Nordsieck, PR 52 (1937) 54

RM123 strategy:

- $\Gamma_1(P\to\ell\nu_\ell\gamma) \sim \Gamma_1^{\rm pt}(\Delta E_\gamma)$ for sufficiently **soft photons** $\Delta E_\gamma \sim \mathcal{O}(20\ \mathrm{MeV})$
- $\Gamma(P_{\ell 2}) = \lim_{L\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\rm pt}(L)\right) + \lim_{m_\gamma\to 0}\left(\Gamma_0^{\rm pt}(m_\gamma) + \Gamma_1^{\rm pt}(\Delta E_\gamma, m_\gamma)\right)$ (IR finite + IR finite)

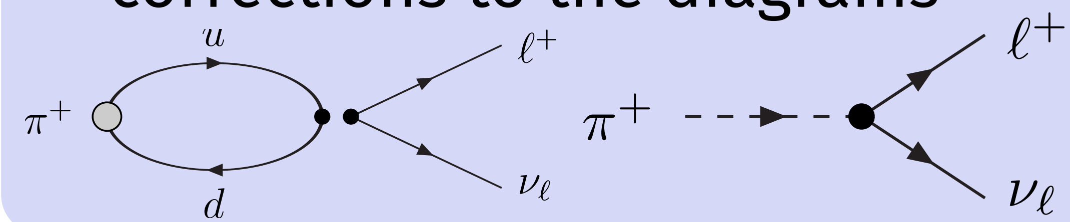
N. Carrasco et al., PRD 91 (2015) 074506 V. Lubicz et al., PRD 95 (2017) 034504 D. Giusti et al., PRL 120 (2018) 072001

$$\delta R_P = \delta R_P^{ren} + \delta R_P^{ampl}$$

contribution from **QED** corrections to:

- **matching** between lattice and W-renormalization scheme
- **mixing** between lattice operators

contributions from **SIB** and **QED** corrections to the diagrams



From Standard Model to Lattice

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V^*_{q_1 q_2}\left(1 + \frac{\alpha_{\text{em}}}{\pi}\ln\left(\frac{M_Z}{M_W}\right)\right) \underbrace{\left[\bar{q}_2\,\gamma_\mu(1-\gamma_5)\,q_1\right]\left[\bar{\nu}_\ell\,\gamma^\mu(1-\gamma_5)\,\ell\right]}_{O_1^{\text{W-reg}}(M_W)}$$

W-Regularization:

$$\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} - \frac{M_W^2}{k^2 - M_W^2}\frac{1}{k^2}$$

A. Sirlin, NP B196 (1982) 83   E. Braaten & C.S. Li, PRD 42 (1990) 3888

OUR GOAL

$$O_1^{\text{W-reg}}(M_W) = Z^{\text{W-RI'}}\left(\frac{M_W}{\mu}, \alpha_s(\mu), \alpha_{\text{em}}\right) O_1^{\text{RI'}}(\mu)$$

$$Z^{\text{W-RI'}}\left(\frac{M_W}{\mu}, \alpha_s(\mu), \alpha_{\text{em}}\right) = Z^{\text{W-RI'}}\left(\alpha_s(M_W), \alpha_{\text{em}}\right) U^{\text{RI'}}\left(M_W, \mu, \alpha_{\text{em}}\right)$$

$$\left[\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s,\alpha_{\text{em}})\frac{\partial}{\partial\alpha_s}\right] U^{\text{RI'}} = \gamma(\alpha_s,\alpha_{\text{em}})\,U^{\text{RI'}}$$

up to order $\alpha_{\text{em}}\alpha_s(\mu)\ln\left(M_W^2/\mu^2\right)$

two-loops calculation

Renormalized operator:

- *on the lattice: all orders in* α_s *& first order in* α_{em}
- *takes into account the (possible) mixing of lattice operators*

RI’-MOM in QCD+QED

For Wilson-like fermions and left-handed neutrinos:

$$O_1^{\text{RI'}}(\mu) = \sum_{i=1}^{5} \left[Z_O(a\mu) \right]_{1j} O_j^{\text{bare}}(a)$$

$$\begin{aligned} O_{1,2}^{\text{bare}} &= \left[\gamma_\mu(1 \mp \gamma_5)\right]_q \otimes \left[\gamma^\mu(1-\gamma_5)\right]_\ell \\ O_{3,4}^{\text{bare}} &= (1 \mp \gamma_5)_q \otimes (1+\gamma_5)_\ell \\ O_5^{\text{bare}} &= \left[\sigma_{\mu\nu}(1+\gamma_5)\right]_q \otimes \left[\sigma^{\mu\nu}(1+\gamma_5)\right]_\ell \end{aligned}$$

$$Z_O = \left(1 + \frac{\alpha_{\text{em}}}{4\pi} \Delta Z_O\right) Z_O^{\text{QCD}}$$

$$\Delta Z_O = -Z_{\Gamma_O}^{\text{QCD}} \Delta\Gamma_O + \frac{1}{2} \sum_f \Delta Z_f$$

RI’-MOM SCHEME

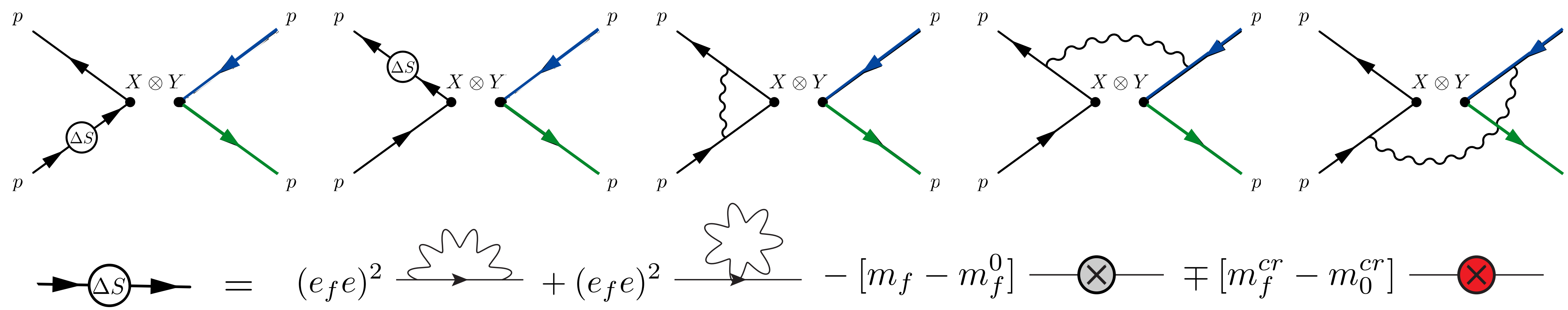
$$Z_{\Gamma_O}(a\mu)\,\Gamma_O(ap)\Big|_{p^2=\mu^2} = \hat{1}$$

$$\Gamma_O(ap) = \text{Tr}\left[\Lambda_O(ap) P_O\right]$$

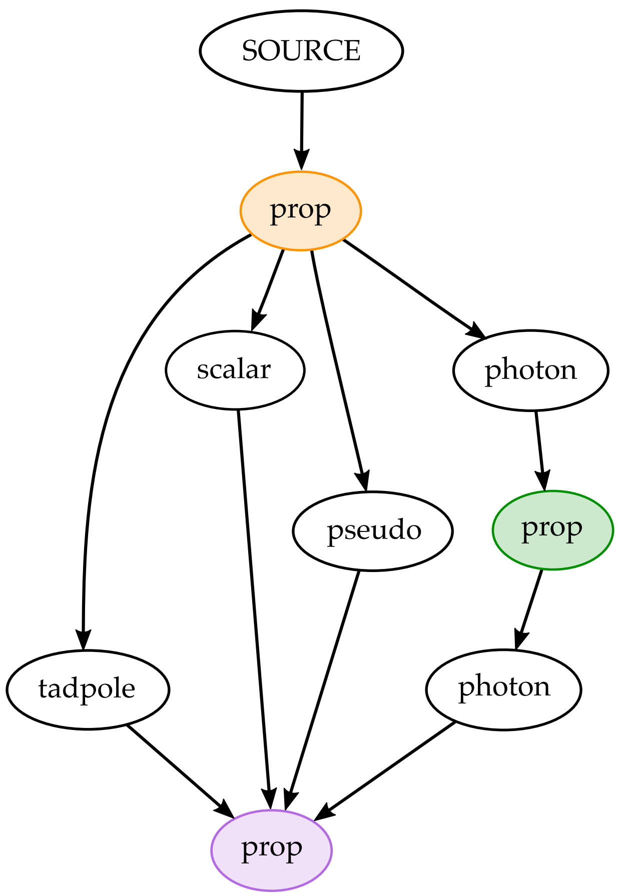
$$Z_{\Gamma_O}(a\mu) = Z_O(a\mu) \prod_f Z_f^{-1/2}(a\mu)$$

$$Z_f(a\mu) = -\frac{i}{12} \text{Tr}\left[\frac{\not{p}\, S_f^{-1}(ap)}{p^2}\right]\Bigg|_{p^2=\mu^2}$$

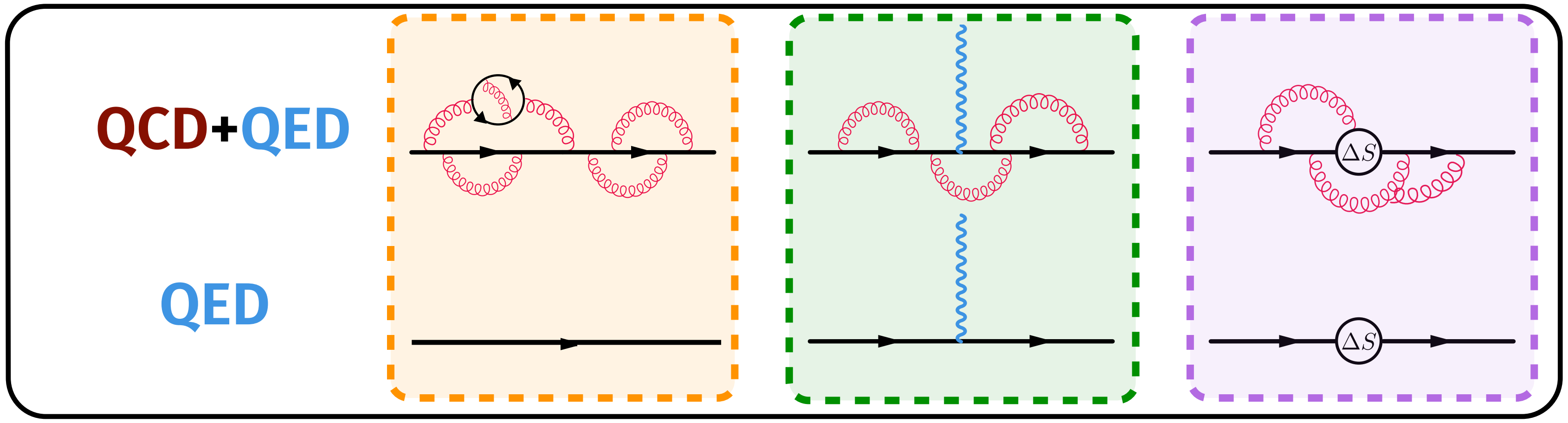
Construction of diagrams



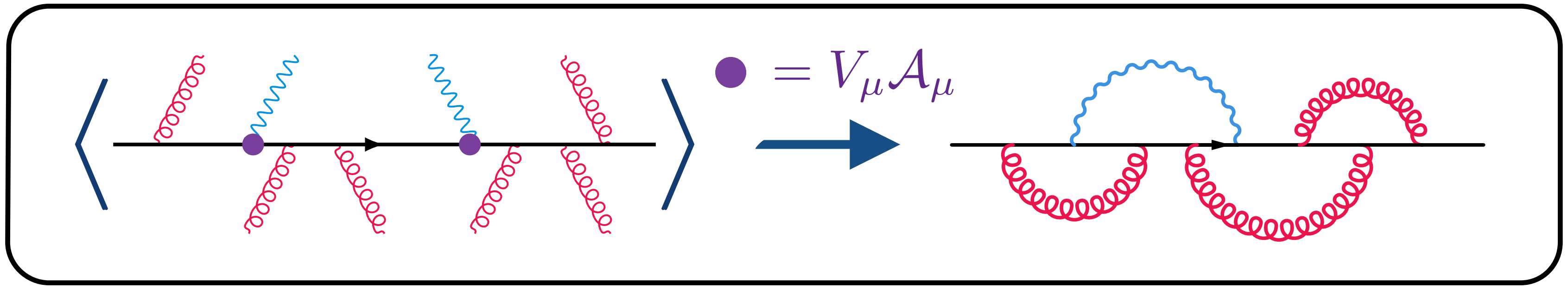
Basic Ingredients:



- **Two simulations:**



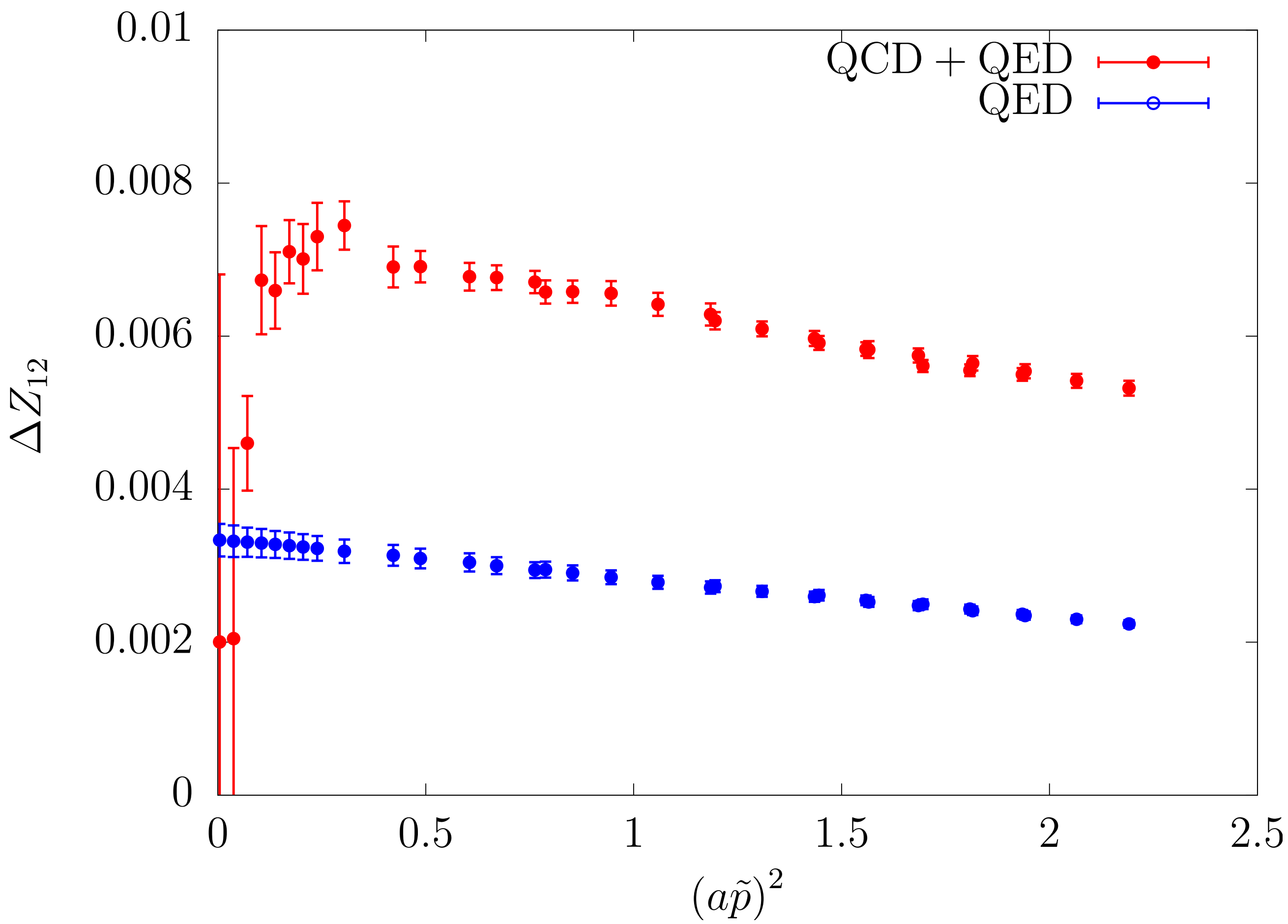
- **Sequential propagators** with **stochastic photon** fields (the **<u>same</u>** in **QCD+QED** and **QED**):

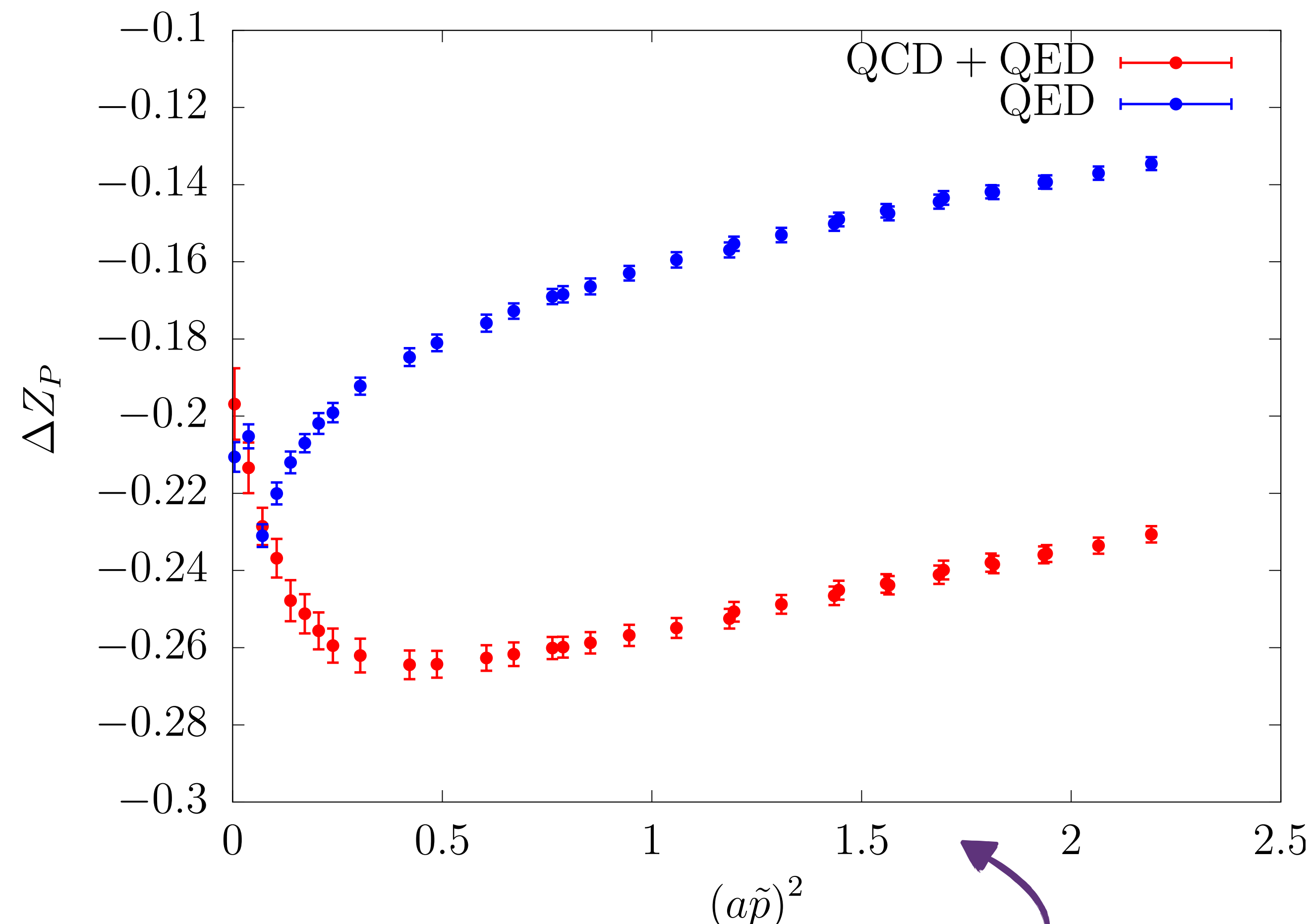


Numerical procedure: creation of vertices [1]

1. Create vertices from the ingredients in **QCD+QED** and pure **QED**, compute the correction to the RC's according to the RI'-MOM condition

$$\Delta Z_O = -Z_{\Gamma_O}^{\mathrm{QCD}} \Delta\Gamma_O + \frac{1}{2}\sum_f \Delta Z_f$$



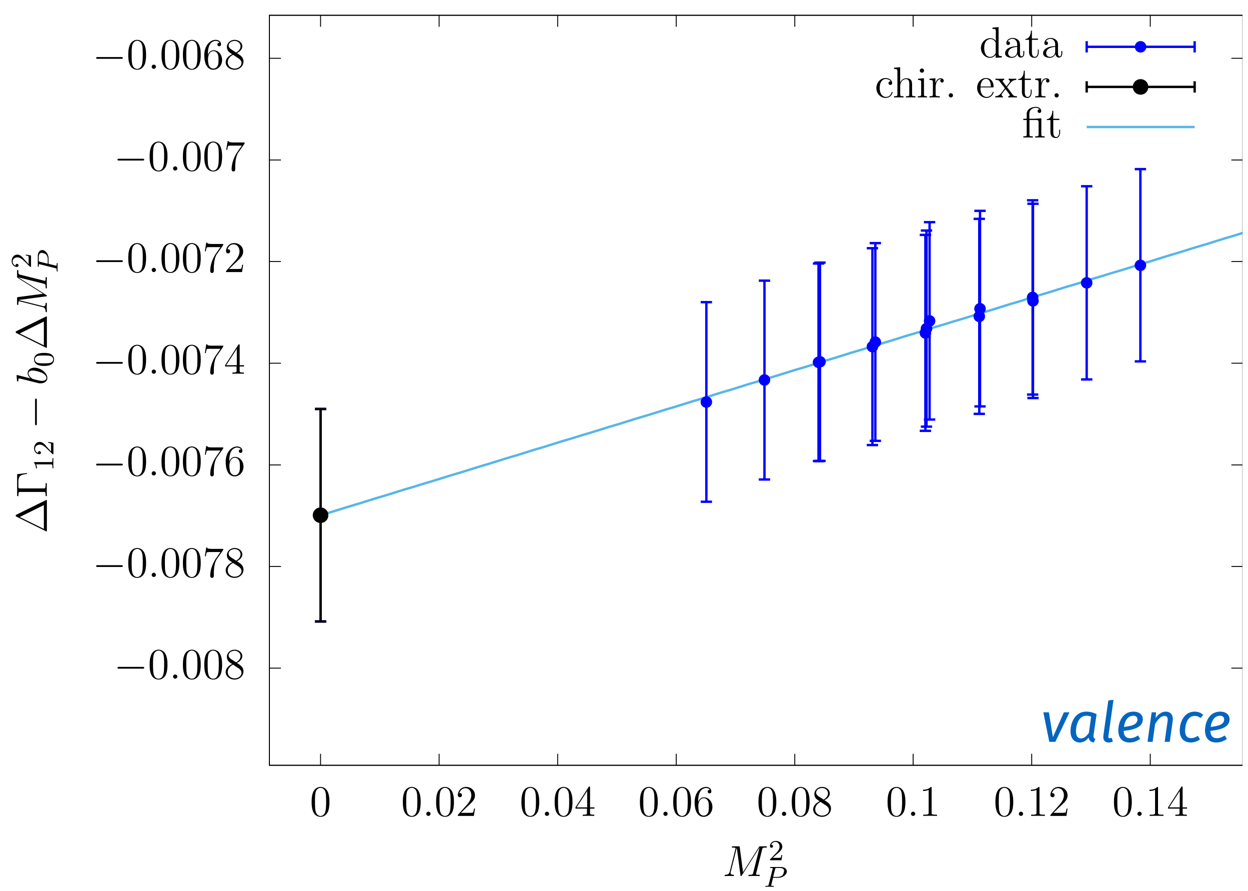


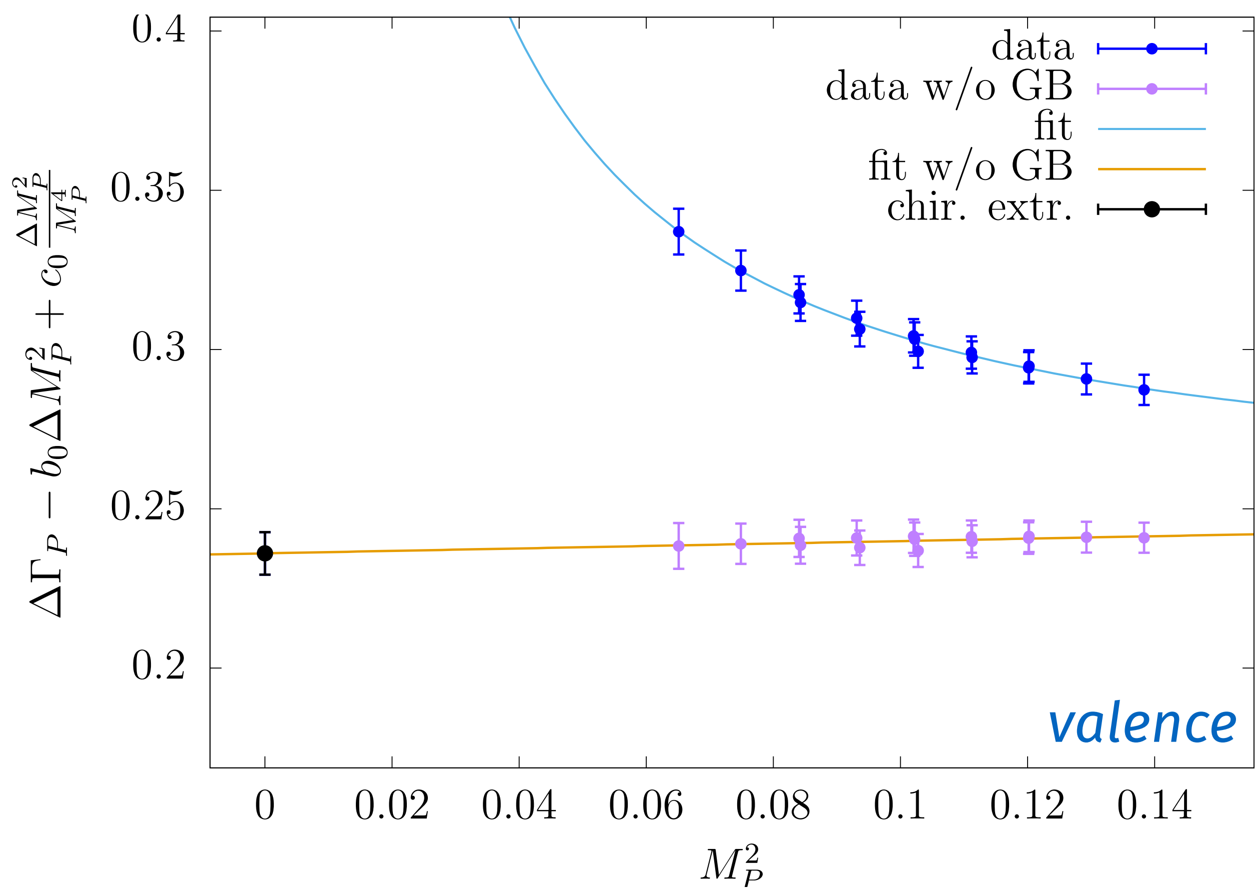
$$a\tilde{p}_\mu = \sin(ap_\mu)$$

see numerical impact e.g. on g-2 in D. Giusti's talk on Monday 17th

Chiral extrapolation [2]

2. Chiral extrapolation, subtracting the **Goldstone boson** contamination (if present)





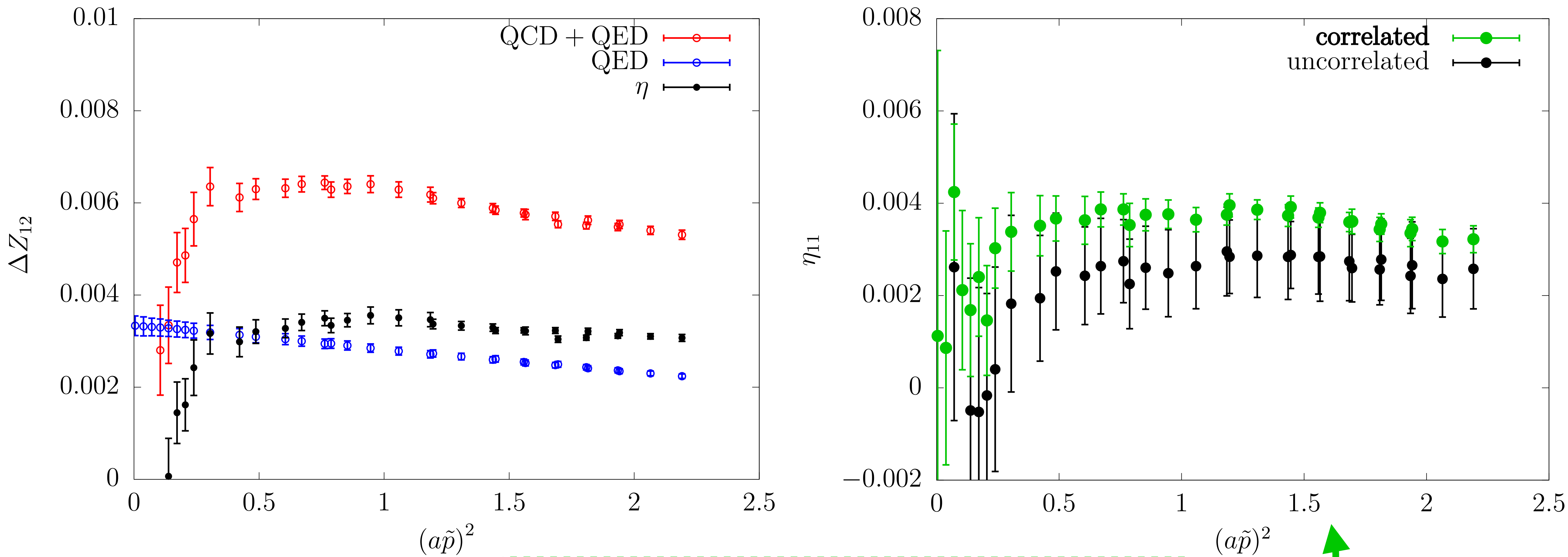
$$\Gamma_O^{\mathrm{QCD}} = a_0 + b_0\, M_P^2 + \frac{c_0}{M_P^2}$$

$$\Delta\Gamma_O = a_1 + b_1\, M_P^2 + \frac{c_1}{M_P^2} + b_0\, \Delta M_P^2 - c_0\, \frac{\Delta M_P^2}{M_P^4}$$

Non-factorizable part of RC's [3]

3. Computation of the **non-factorizable** part of the RC's

$$\eta = \Delta Z_O - \Delta Z_O^{\mathrm{QED}} \qquad \mathcal{R} = (Z^{\mathrm{QED}})^{-1} Z_O (Z^{\mathrm{QCD}})^{-1} \equiv 1 + \frac{\alpha_{\mathrm{em}}}{4\pi}\eta$$

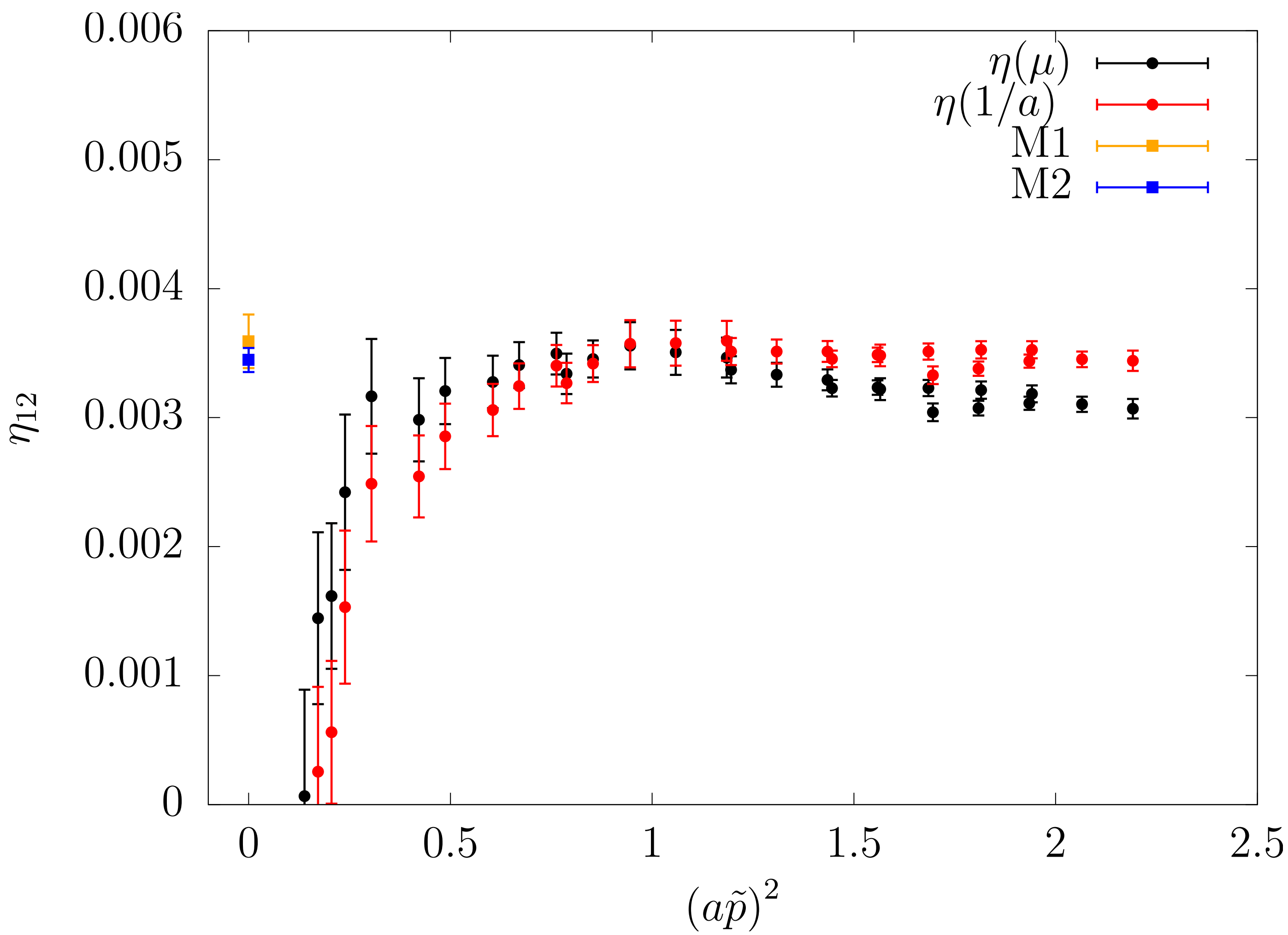


Statistical uncertainty** reduced by a factor of $\sim$**5** using **correlated stochastic photons

Evolution and extrapolation of RC's [4]

4. Evolution to a reference scale $(\mu = 1/a)$ with the residual mixed anomalous dimension of order $\mathcal{O}(\alpha_{\rm em}\alpha_{\rm s})$ computed analytically

5. Extrapolation of the RC's at zero (M1) or fixed momentum (M2)



Method M1

extrapolation to $(a\tilde{p})^2 \to 0$

fitting in the region $(a\tilde{p})^2 > 1.0$

Method M2

interpolation around

$\tilde{p}^2 = 13.0\ \mathrm{GeV}^2$

common to all lattice spacings

(coincide in the limit $a \to 0$)

Construction of the renormalized operator [5]

6. Construction of the complete electromagnetic correction to the RC's

$$\Delta Z_O = \Delta \mathcal{Z}_O^{\mathrm{QED}} + \eta$$

*one-loop **QED** contribution*

N. Carrasco et al., PRD 91 (2015) 074506

7.

$$\vec{O}^{\mathrm{RI'}} = \left[1 + \frac{\alpha_{\mathrm{em}}}{4\pi}\left(\Delta \mathcal{Z}_O^{\mathrm{QED}} + \eta\right)\right] Z_O^{\mathrm{QCD}}\, \vec{O}^{\mathrm{bare}}$$

$$\boldsymbol{O}_{\mathbf{1}}^{\mathbf{W\text{-}reg}}(M_W) = Z^{\mathrm{W\text{-}RI'}}(aM_W, \alpha_{\mathrm{s}}(1/a), \alpha_{\mathrm{em}})\, O_1^{\mathrm{RI'}}(1/a)$$

From renormalization to decay rates

$$O_1^{\text{W-reg}}(M_W)$$

subtracting $O_1^{\text{QCD}} = (Z^{\text{QCD}}\vec{O}^{bare})_1$

$$\delta O_1^{\text{W-reg}}(M_W)$$

$$\delta A_P^{ren}$$

axial amplitude:

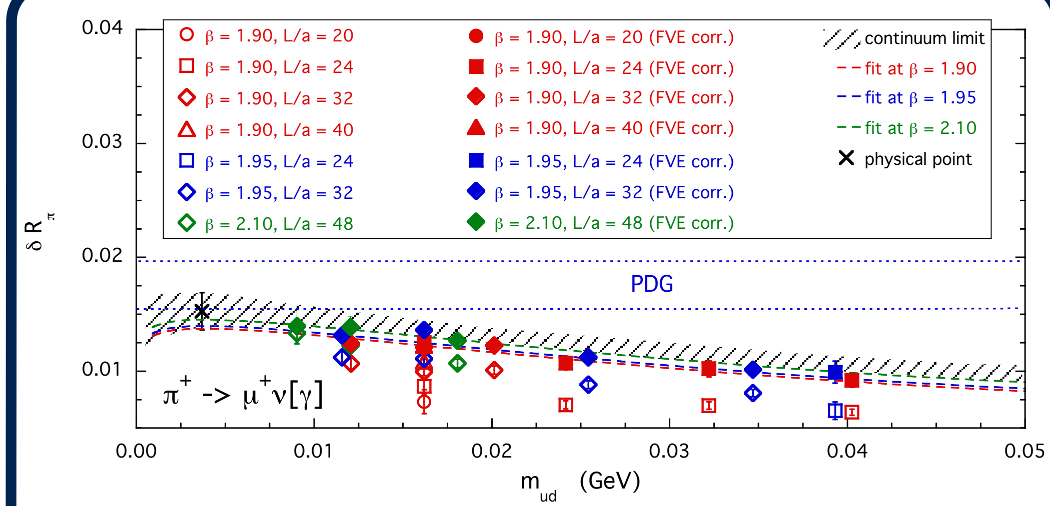
$$A_P^{(0)} = \langle 0|\bar{q}_2\gamma_0\gamma_5 q_1|P^{(0)}\rangle$$

$$\delta R_P^{ren} = 2\frac{\delta A_P^{ren}}{A_P^{(0)}}$$

complete correction:

$$\delta R_P = \delta R_P^{ren} + \delta R_P^{ampl}$$

Final results

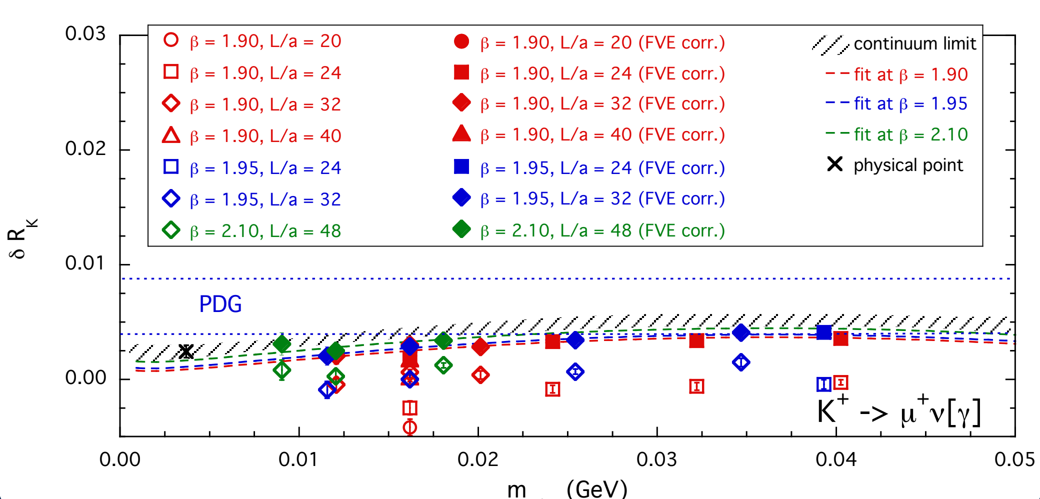


$$\delta R_{\pi^\pm} = 0.0153\,(16)_{stat}\,(10)_{syst} = 0.0153\,(19)$$

MDC et al., arXiv:1904.08731 [hep-lat]

NEW

$$\delta R_{K^\pm} = 0.0024\,(6)_{stat}\,(8)_{syst} = 0.0024\,(10)$$



$$\chi\text{PT/PDG}: \quad \delta R_{\pi^\pm} = 0.0176\,(21) \quad \delta R_{K^\pm} = 0.0064\,(24)$$

V. Cirigliano and H. Neufeld, PLB 700 (2011) 7

$$\eta = 0: \quad \delta R_{\pi^\pm} = 0.0149\,(16)_{stat}\,(9)_{syst}\,(?)_\eta \quad 3\%$$
$$\delta R_{K^\pm} = 0.0022\,(5)_{stat}\,(7)_{syst}\,(?)_\eta \quad 8\%$$

D. Giusti, arXiv:1807.11681 [hep-lat]

$$\delta R_{K\pi} = -0.0126\,(14)$$

$$\delta R_{K\pi} = -0.0122\,(16)$$

D. Giusti et al., PRL 120 (2018) 072001

~~$|V_{ud}|$~~ ***not predictable*** 0.20%

$$|V_{us}| = 0.22538\,(46)$$

PDG: 0.31%

$$|V_{us}| = 0.2253\,(7)$$

0.5‰

$$|V_u|^2 = 0.99988\,(46)$$

☑ ***1st row unitarity***

Conclusions and future perspectives

- We presented a **strategy** to compute renormalization **corrections** on the lattice, they reduce significantly **systematic uncertainties** on the decay rate

- The introduction of the **mixed** anomalous dimension reduces the **error** in the **matching** from $\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}(1/a))$ to $\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}(M_W))$. Greater precision can be achieved with a **three-loop** calculation of diagrams of order $\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}^2)$

- The renormalization corrections are related to the **operator** and not to the process. Therefore, they are valid also for **semileptonic** decays
  [Talk by C. Sachrajda on Wednesday 19th]

- The **NP procedure** can be easily extended to other renormalization schemes such as **RI-MOM** and **RI-SMOM**

- The calculation of **disconnected diagrams** would reduce the systematical error related to **quenched-QED** approximation

- A **non-perturbative** calculation of the **real emission** amplitude is ongoing
  [Talk by G. Martinelli on Monday 17th]

Backup Slides

Detailed Lattice setup

ETMC configurations with $N_f = 4$ in **isosymmetric QCD**

	$a\mu^{\text{sea}}$	$am^{\text{sea}}_{\text{PCAC}}$	am^{sea}_0	θ^{sea}	$a\mu^{\text{val}}$	$am^{\text{val}}_{\text{PCAC}}$
$\beta = 1.90$ ($L = 24$, $T = 48$)						
A4m	0.0080	-0.0390(01)	0.0285(01)	-1.286(01)	{0.0060, 0.0080, 0.0120,	-0.0142(02)
A4p		0.0398(01)	0.0290(01)	+1.291(01)	0.0170, 0.0210 ,0.0260}	+0.0147(02)
A1m	0.0080	-0.0273(02)	0.0207(01)	-1.174(03)	{0.0060, 0.0080, 0.0120,	-0.0163(02)
A1p		+0.0275(04)	0.0209(01)	+1.177(05)	0.0170, 0.021 0,0.0260}	+0.0159(02)
$\beta = 1.95$ ($L = 24$, $T = 48$)						
B1m	0.0085	-0.0413(02)	0.0329(01)	-1.309(01)	{0.0085, 0.0150, 0.0203,	-0.0216(02)
B1p		+0.0425(02)	0.0338(01)	+1.317(01)	0.0252, 0.02 98}	+0.0195(02)
B7m	0.0085	-0.0353(01)	0.0285(01)	-1.268(01)	{0.0085, 0.0150, 0.0203,	-0.0180(02)
B7p		+0.0361(01)	0.0285(01)	+1.268(01)	0.0252, 0.02 98}	+0.0181(01)
B8m	0.0020	-0.0363(01)	0.0280(01)	-1.499(01)	{0.0085, 0.0150, 0.0203,	-0.0194(01)
B8p		+0.0363(01)	0.0274(01)	+1.498(01)	0.0252, 0.02 98}	+0.0183(02)
B3m	0.0180	-0.0160(02)	0.0218(01)	-0.601(06)	{0.0060,0.0085,0.0120,0.0150,	-0.0160(02)
B3p		+0.0163(02)	0.0219(01)	+0.610(06)	0.0180,0.0203, 0.0252,0.0298}	+0.0162(02)
B2m	0.0085	-0.0209(02)	0.0182(01)	-1.085(03)	{0.0085, 0.0150, 0.0203,	-0.0213(02)
B2p		+0.0191(02)	0.0170(02)	+1.046(06)	0.0252, 0.02 98}	+0.0191(02)
B4m	0.0085	-0.0146(02)	0.0141(01)	-0.923(04)	{0.0060,0.0085,0.0120,0.0150,	-0.0146(02)
B4p		+0.0151(02)	0.0144(01)	+0.940(07)	0.0180,0.0203, 0.0252,0.0298}	+0.0151(02)
$\beta = 2.10$ ($L = 32$, $T = 64$)						
C5m	0.0078	-0.00821(11)	0.0102(01)	-0.700(07)	{0.0048,0.0078,0.0119,	-0.0082(01)
C5p		+0.00823(08)	0.0102(01)	+0.701(05)	0.0190,0.0242 ,0.0293}	+0.0082(01)
C4m	0.0064	-0.00682(13)	0.0084(01)	-0.706(09)	{0.0039,0.0078,0.0119,	-0.0068(01)
C4p		+0.00685(12)	0.0084(01)	+0.708(09)	0.0190,0.0242 ,0.0293}	+0.0069(01)
C3m	0.0046	-0.00585(08)	0.0066(01)	-0.794(07)	{0.0025,0.0046,0.0090,0.0152,	-0.0059(01)
C3p		+0.00559(14)	0.0064(01)	+0.771(13)	0.0201,0.0249 ,0.0297}	+0.0056(01)
C2m	0.0030	-0.00403(14)	0.0044(01)	-0.821(17)	{0.0013,0.0030,0.0080,0.0143,	-0.0040(01)
C2p		+0.00421(13)	0.0045(01)	+0.843(15)	0.0195,0.0247 ,0.0298}	+0.0042(01)

N. Carrasco et al. [ETMC], NP B887 (2014) 19

- **Out of maximal twist:** pairs of ensembles have opposite twisting angle. $\mathcal{O}(a^2)$ improvement achieved averaging on the angles.
- **Lattice spacings:** $\{0.0885(36),\, 0.0815(30),\, 0.0619(18)\}\,\text{fm}$
- **Boundary conditions:** antiperiodic (no zero modes)
- 150 **gauge ensembles**

Two simulations:

QCD

action: Iwasaki
gauge: Landau
massive quarks

FREE THEORY

QCD links = off
gauge: Landau
massless quarks

Using a different V_{ud}

We have used:

$$V_{ud} = 0.97420\,(21)$$

from **superallowed β-decays**

J. Hardy & I. S. Towner, PoS CKM 2016 (2016) 028

***New* proposed value:**

$$V_{ud} = 0.97370\,(14)$$

from **dispersion relations** and **neutrino scattering data**

C. Y. Seng et al., PRL 121 (2018) 241804
C. Y. Seng et al., arXiv:1812.03352 [nucl-th]

- Impact on V_{us} :

$$V_{us} = 0.22538\,(46) \rightarrow V_{us} = 0.22526\,(46)$$

compatible within the uncertainty

- Impact on 1st row unitarity :

$$|V_u|^2 = 0.99988\,(46) \rightarrow |V_u|^2 = 0.99885\,(34)$$

! $\sim \mathbf{3.5\,\sigma}$ *tension with unitarity*

Error budget

$$\delta R_\pi = 0.0153\,(16)_{stat+fit}\,(4)_{input}\,(3)_{chir}\,(6)_{FVE}\,(2)_{discr}\,(6)_{qQED}$$

$$\delta R_K = 0.0024\,(6)_{stat+fit}\,(3)_{input}\,(1)_{chir}\,(3)_{FVE}\,(2)_{discr}\,(6)_{qQED}$$

$stat + fit$: induced by both the **statistical errors** and the **fitting procedure**

$input$: from the uncertainties of the **input parameters** of the iso-QCD analysis

$chir$: from the inclusion or exclusion of the **chiral logaritm** in the fit Ansatz

FVE: from the **subtraction** of $\mathcal{O}(1/L)$ **"universal" FVE** only or also the $\mathcal{O}(1/L^2)$ **"point-like" FVE**

$discr$: from the inclusion or exclusion of a **discretization term** $\mathcal{O}(a^2)$ in the fit Ansatz

$qQED$: from using in the fit Ansatz the **chiral log coefficient** evaluated in **QED** or in **qQED**

Details on the extraction of decay rates

$$O_1^{\text{W-reg}}(M_W) = Z^{\text{W-RI'}}\left(\frac{M_W}{\mu}, \alpha_s(\mu), \alpha_{\text{em}}\right)\left[\hat{Z}_O(a\mu)\vec{O}^{bare}(a)\right]_1\Big|_{\mu=1/a}$$

$$\begin{aligned}\delta O_1^{\text{W-reg}}(M_W) &= \frac{\alpha_{\text{em}}}{4\pi}\left[\mathcal{F}^{\text{W-RI'}} + \Delta Z_{11}^{\text{QED}}(1/a) + \eta_{11}(\alpha_s(1/a))\right] O_1^{\chi}(a) \\ &+ \frac{\alpha_{\text{em}}}{4\pi}\left[\Delta Z_{12}^{\text{QED}} + \eta_{12}(\alpha_s(1/a))\right] O_2^{\chi}(a)\end{aligned}$$

$$O_i^{\chi} \equiv (Z^{\text{QCD}}\vec{O}^{\text{bare}})_i$$

perturbation theory

$$\mathcal{F}^{\text{W-RI'}} = 2\left(1 - \frac{\alpha_s(1/a)}{4\pi}\right)\log\left(a^2 M_W^2\right) - 5.7825 + 1.2373\,\xi$$

$$\Delta Z_{11}^{\text{QED}}(1/a) = -9.7565 - 1.2373\,\xi\ , \quad \Delta Z_{12}^{\text{QED}} = -0.5357$$

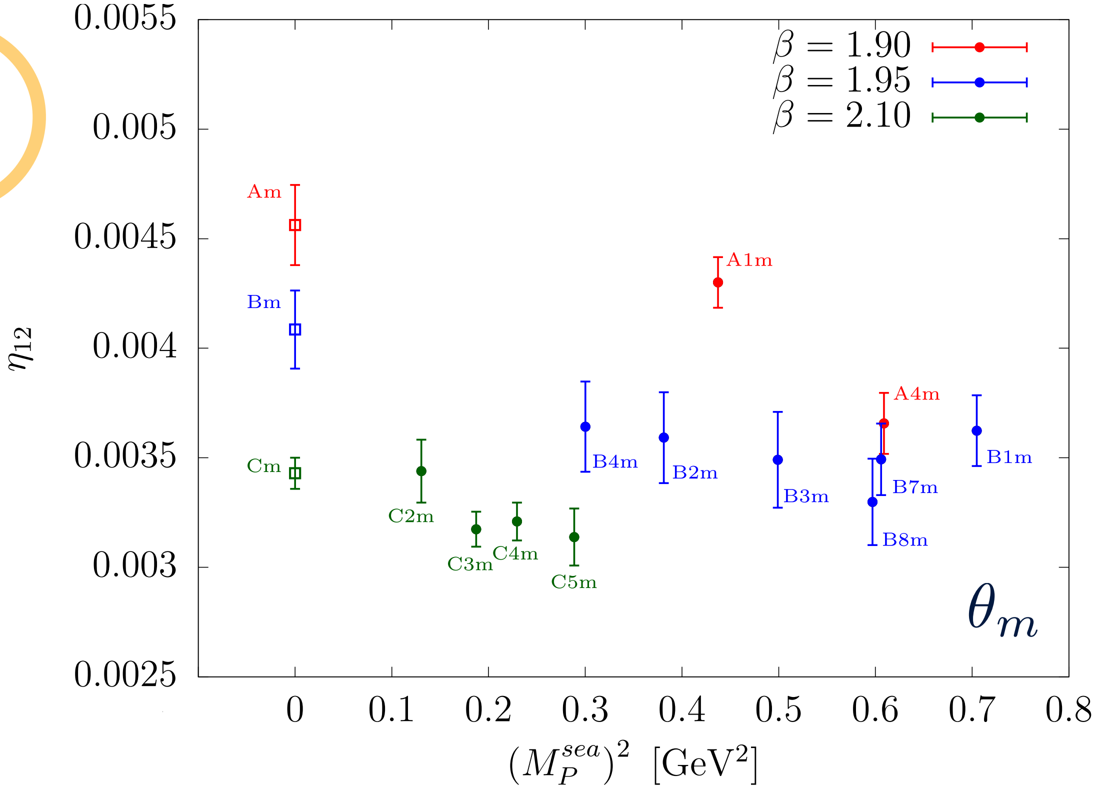
$$\delta A_P^{ren} = Z^{\text{W-reg}} A_P^{(0)} = \frac{\langle 0|\,\text{Tr}\left[\delta O_1^{\text{W-reg}}(M_W)\,\bar{\ell}\gamma_0(1-\gamma_5)\nu\right]\,|P^{(0)}\rangle}{\langle 0|\,\text{Tr}\left[O_1^{\chi}(a)\,\bar{\ell}\gamma_0(1-\gamma_5)\nu\right]\,|P^{(0)}\rangle} A_P^{(0)}$$

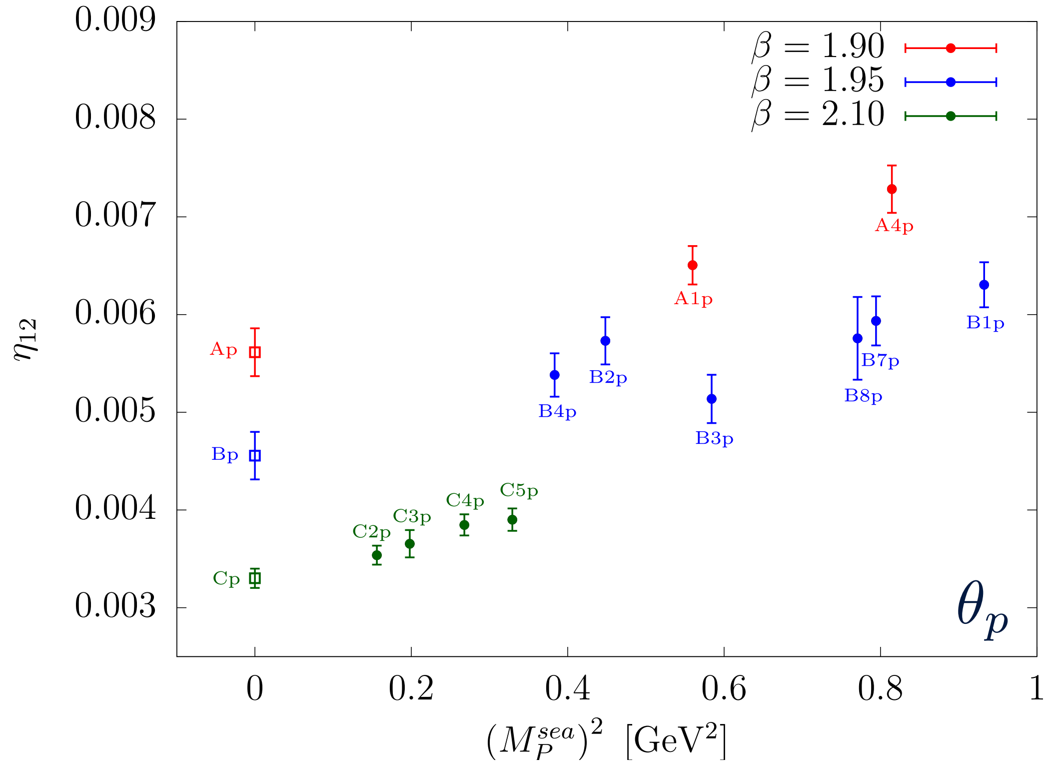
and therefore we can compute the **correction to the decay rate**

$$\boldsymbol{\Gamma\left(P^{\pm} \to \ell^{\pm}\nu_{\ell}[\gamma]\right) = \Gamma_P^{(0)}\left[1 + \delta R_P\right]}$$

Combined sea quark extrapolation

M1





M2