χSF near the electroweak scale

ALPHA Collaboration

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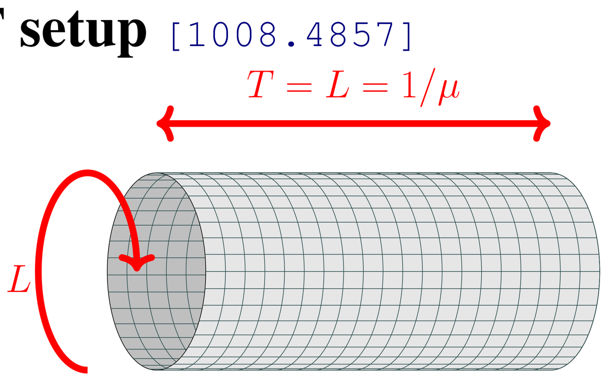
The chirally rotated Schrödinger functional (χSF) with massless Wilson fermions is a lattice regularization which endows the Schrödinger functional (SF) with the property of automatic O(a)-improvement. The χSF framework is effective in reducing lattice artefacts in correlation and step scaling functions, but especially it offers new strategies to study and simplify the pattern of renormalization. The price to pay for the automatic O(a)-improvement is the nonperturbative tuning of coefficients of new boundary counterterms. This tuning is the first phase of a long-term project, aiming at the computation of $B_{\rm K}$ low-energy contributions BSM, with Wilson fermion $N_{\rm f} = 2+1$ lattice QCD in a non-unitary (mixed-action) framework.

The χSF setup [1008.4857]



The fermion flavour doublet $\psi = \begin{pmatrix}\psi_u\\ \psi_d\end{pmatrix}$ satisfies b.c.'s

$$\tilde{Q}_\pm \equiv \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3) \quad \begin{cases}\tilde{Q}_+\psi(x)|_{x_0=0} = 0 & \tilde{Q}_-\psi(x)|_{x_0=T} = 0\\ \bar{\psi}(x)\tilde{Q}_+|_{x_0=0} = 0 & \bar{\psi}(x)\tilde{Q}_-|_{x_0=T} = 0\end{cases}$$

in time and periodic ones in space. The fermion action is

$$S_f = a^4 \sum_{x_0=0}^{T}\sum_{\mathbf{x}} \bar{\psi}(x)(\mathcal{D}_W + \delta\mathcal{D}_W + m_0)\psi(x)\,,$$

with $\mathcal{D}_W$ the standard Wilson fermion matrix and the boundary term

$$\delta\mathcal{D}_W\psi(x) = \left(\delta_{x_0,0} + \delta_{x_0,T}\right)\left[(z_f - 1) + (d_s - 1)\,a\mathbf{D}_s\right]\psi(x)\,.$$

Performing the chiral flavour rotation

$$R = \exp\left(i\frac{\alpha}{2}\gamma_5\tau^3\right)\Big|_{\alpha=\pi/2} \begin{cases}\psi & \to \psi' = R\psi\\ \bar{\psi} & \to \bar{\psi}' = \bar{\psi}R\\ O[\psi,\bar{\psi}] & \to Q[\psi,\bar{\psi}] = O[R\psi,\bar{\psi}R]\end{cases}$$

we obtain the following universality relation for SF and χSF correlation functions

$$\begin{aligned}\langle O\,\mathcal{O}\rangle^{\rm cont}_{\rm (SF)} &= \lim_{a\to 0}\left[Z_O Z_{\mathcal{O}}\,\langle O\,\mathcal{O}\rangle_{\rm (SF)} + {\rm O}(a)\right]\\ &= \lim_{a\to 0}\left[Z_Q Z_{\mathcal{Q}}\,\langle Q\,\mathcal{Q}\rangle_{(\chi{\rm SF})} + {\rm O}(a^2)\right],\end{aligned}$$

where composite operators O, Q are defined in the bulk and $\mathcal{O}, \mathcal{Q}$ are defined on a time boundary. Note that χSF incorporates automatic O(a) improvement. The price to pay is the non-perturbative tuning of the boundary counterterm coefficient z_f.

Correlation functions [1808.09236]

We consider the set of fermion bilinear operators; e.g.

$$V_\mu^{f_1f_2}(x) = \bar{\psi}_{f_1}(x)\gamma_\mu\psi_{f_2}(x), \quad A_\mu^{f_1f_2}(x) = \bar{\psi}_{f_1}(x)\gamma_\mu\gamma_5\psi_{f_2}(x),$$

with flavours $f_1, f_2 \in \{u, d, u', d'\}$, and determine the χSF bulk-to-boundary correlation functions

$$g_{\rm X}^{f_1f_2}(x_0) = -\frac{1}{2}\left\langle X^{f_1f_2}(x)\,\mathcal{Q}_5^{f_2f_1}\right\rangle, \qquad X = V_0, A_0, S, P$$

$$l_{\rm Y}^{f_1f_2}(x_0) = -\frac{1}{6}\sum_{k=1}^{3}\left\langle Y_k^{f_1f_2}(x)\,\mathcal{Q}_k^{f_2f_1}\right\rangle. \qquad Y_k = V_k, A_k, T_{k0}, \widetilde{T}_{k0}$$

Up to discretization effects and boundary fields renormalization they are related to the standard SF correlation functions $f_{\rm X}$ and $k_{\rm Y}$ by universality

$$\begin{aligned}f_{\rm A}^{\rm cont} &= Z_{\rm A}g_{\rm A}^{uu'} = Z_{\rm A}g_{\rm A}^{dd'} = -iZ_{\rm V}g_{\rm V}^{ud} = iZ_{\rm V}g_{\rm V}^{du},\\ f_{\rm V}^{\rm cont} &= Z_{\rm V}g_{\rm V}^{uu'} = Z_{\rm V}g_{\rm V}^{dd'} = -iZ_{\rm A}g_{\rm A}^{ud} = iZ_{\rm A}g_{\rm A}^{du},\end{aligned}$$

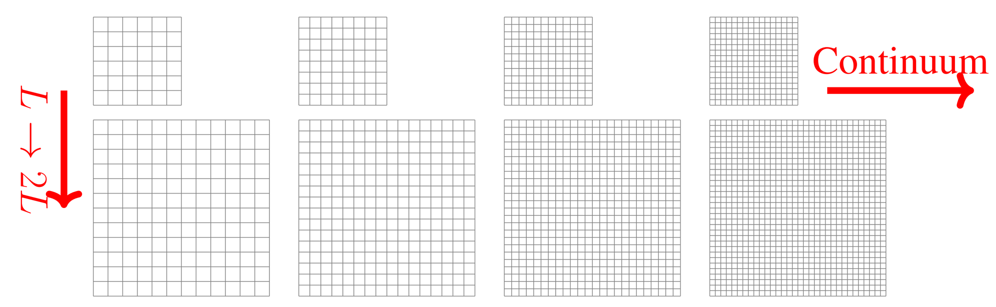
$$\begin{aligned}k_{\rm V}^{\rm cont} &= Z_{\rm V}l_{\rm V}^{uu'} = Z_{\rm V}l_{\rm V}^{dd'} = -iZ_{\rm A}l_{\rm A}^{ud} = iZ_{\rm A}l_{\rm A}^{du},\\ k_{\rm A}^{\rm cont} &= Z_{\rm A}l_{\rm A}^{uu'} = Z_{\rm A}l_{\rm A}^{dd'} = -iZ_{\rm V}l_{\rm V}^{ud} = iZ_{\rm V}l_{\rm V}^{du}.\end{aligned}$$

The SF correlation functions $f_{\rm A}, k_{\rm V}$ are non-zero, while $f_{\rm V}, k_{\rm A}$, being parity odd, are O(a). The local vector current can be replaced by the exactly conserved one $\widetilde{V}_\mu(x)$ with normalization $Z_{\widetilde{\rm V}} = 1$. Therefore $Z_{\rm A}$ may be obtained from the ratios

$$Z_{\rm A}^g = \frac{-ig_{\widetilde{\rm V}}^{ud}(x_0)}{g_{\rm A}^{uu'}(x_0)}\Bigg|_{x_0=L/2} \quad \text{or} \quad Z_{\rm A}^l = \frac{il_{\widetilde{\rm V}}^{uu'}(x_0)}{l_{\rm A}^{ud}(x_0)}\Bigg|_{x_0=L/2}. \qquad (1)$$

Computational setup and results

We obtain results for $N_{\rm f} = 3$ QCD in a non-unitary setup. Valence quark propagators are inverted with χSF boundaries on the configuration ensembles of [1802.05243], generated on lattices with standard SF boundary conditions. These configurations have been used for the RG-running of the quark mass in a range of scales $2 \lesssim \mu \lesssim 128$ GeV, in the standard framework of finite-size scaling $L \to 2L$.



Tuning

We must ensure that massless QCD with χSF boundary conditions is correctly regularized. This is achieved by tuning the bare mass parameter m_0 to its critical value, $m_{\rm cr}$, where the axial current is conserved, and by tuning the boundary counterterm coefficient z_f so that physical parity is restored. At present we choose the conditions

$$\begin{aligned} m &= \frac{\tilde{\partial}_0 f_{\rm A}^{ud}(x_0)}{2f_{\rm P}^{ud}(x_0)}\Bigg|_{x_0=L/2} = 0, & m_{\rm cr}\ \text{tuning}\,,\\ & g_{\rm A}^{ud}(x_0)\Big|_{x_0=L/2} = 0 & z_f\ \text{tuning}\,. \end{aligned} \qquad (2)$$

By requiring that eq. (2) be satisfied, z_f is tuned for each ensemble as shown in Fig. 1.

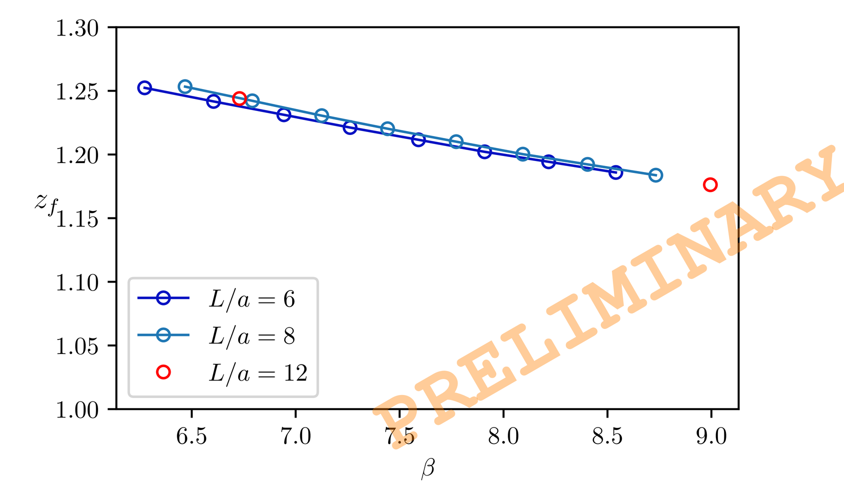


Figure 1: Results of nonperturbative tuning of z_f, according to eq. (2).

O(a)-improvement

Using eqs. (1) we obtain two estimates for $Z_{\rm A}$, which differ by discretization errors

$$Z_{\rm A}^g(\beta) = Z_{\rm A}^l(\beta) + {\rm O}(a^2).$$

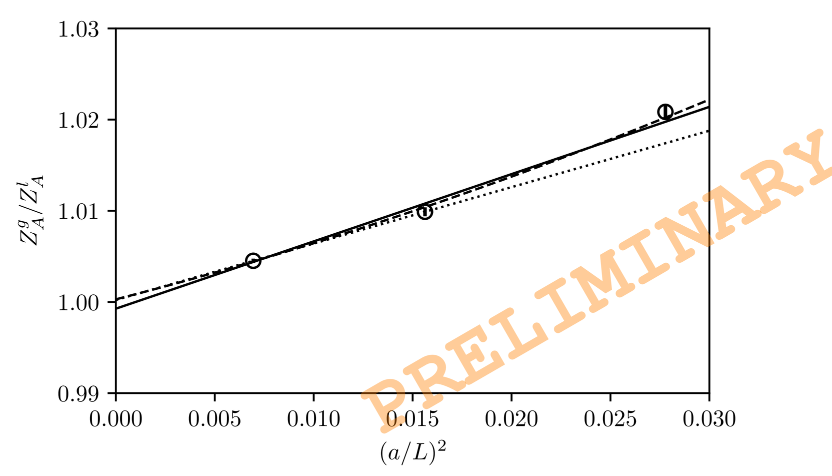


Figure 2: Ratio of two different definitions of Z_A (see eq. (1)) calculated on our ensembles with $1/L = 4$ GeV. The numerator and denominator differ only by lattice artifacts and here we confirm that, after tuning z_f, the ratio scales as a^2 and goes to 1 in the continuum.

Finally we study $Z_A(g_0^2)$ over the full range of ensembles.

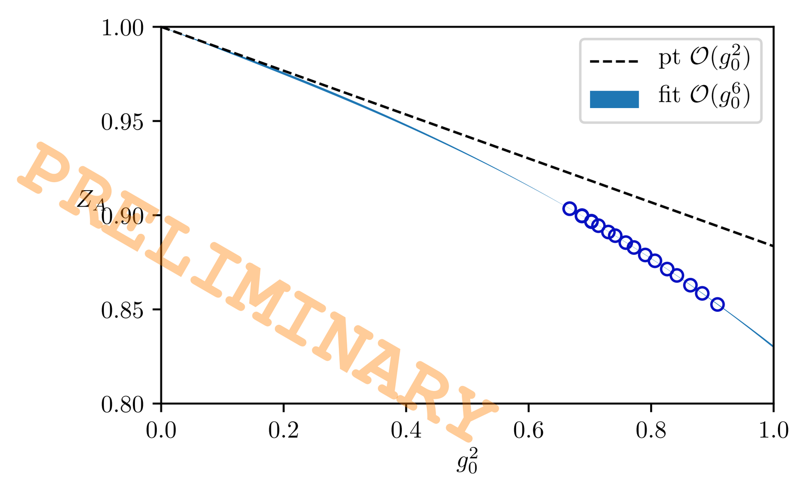


Figure 3: $Z_A(g_0^2)$ calculated across the full range of ensembles available. A fit to the data matches onto the asympotic perturbative result in the limit $g_0^2 \to 0$.

Outlook for 4 fermions [1605.09053]

Renormalization

Automatic O(a) improvement is especially valuable in simplifying the renormalization of four fermion operators. They enter the most general expression of the effective Hamiltonian which describes flavour physics processes at low energy in the Standard Model (SM) and its extensions (BSM). Here we focus on $\Delta F = 2$ transitions. The 4-quark operators with four distinct flavours

$$O_{XY}^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1\Gamma_X\psi_2)(\bar{\psi}_3\Gamma_Y\psi_4) \pm (2 \leftrightarrow 4)\right]$$

can be classified as parity even and parity odd:

$$\begin{aligned} O_k^{e,\pm} &\in \left\{O^\pm_{VV+AA}, O^\pm_{VV-AA}, O^\pm_{SS-PP}, O^\pm_{SS+PP}, O^\pm_{TT}\right\},\\ O_k^{o,\pm} &\in \left\{O^\pm_{VA+AV}, O^\pm_{VA-AV}, O^\pm_{SP-PS}, O^\pm_{SP+PS}, O^\pm_{T\tilde{T}}\right\}. \end{aligned}$$

Due to the explicit breaking of chiral symmetry of the Wilson regularisation, the operators in general mix as follows:

$$\begin{aligned} O_i^{e,\pm} &= \sum_j Z_{ij}^{e,\pm}\left(\delta_{jm} + \Delta^{e,\pm}_{jm}\right)O_m^{e,\pm}\,,\\ O_i^{o,\pm} &= \sum_j Z_{ij}^{o,\pm}\left(\delta_{jm} + \Delta^{o,\pm}_{jm}\right)O_m^{o,\pm}\,. \end{aligned}$$

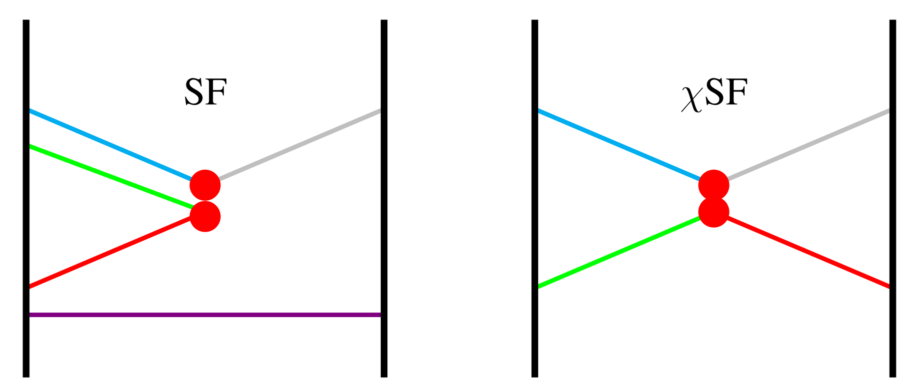
The parity-odd sector has a simpler, continuum-like mixing pattern ($\Delta^{o,\pm}_{jm} = 0$). Renormalization conditions are imposed on parity odd operators by setting suitable renormalized correlation functions equal to their tree level values at the scale $\mu = L^{-1}$. In a standard SF setup parity conservation imposes the use of 4 point correlation functions; e.g.

$$T_i(x_0) = \langle \mathcal{O}_5'^{45} O_i^{o,1234}(x_0)\mathcal{O}_5^{21}\mathcal{O}_5^{53}\rangle.$$

These are statistically noisy and suffer from O(a) discretisation errors. The most convenient renormalization scheme is χSF, imposed on 3 point correlation functions; e.g

$$G_i(x_0) = \langle \mathcal{Q}_5'^{21} Q_i^{o,1234}(x_0)\mathcal{Q}_5^{43}\rangle.$$

They are statistically less noisy and automatically $O(a)$ improved.



Performing suitable chiral rotations, the χSF flavours are rotated into the physical flavours. Thus we can map the renormalized $[G_i(x_0)]_{\rm R}$ into the continuum correlation function

$$[G_i(x_0)]_{\rm R} \to [F_i(x_0)]_{\rm R} = \langle \mathcal{O}_5'^{21} O_i^{e,1234}(x_0)\mathcal{O}_5^{43}\rangle^{\rm cont}.$$

Physical determinations

The renormalization program will be employed in the lattice computation of the physical $B_{\rm K}$-parameter which controls the $\bar{K}^0 - K^0$ meson oscillations, towards a better understanding of the physics of CP violation. To accomodate physical meson states the configurations of $N_{\rm f} = 2+1$ CLS ensembles are now characterised by *large* physical volumes with open boundary conditions and by *non-zero* quark masses [1411.3982],[1608.08900]. The sea quarks are Wilson/Clover. Valence fermions are fully twisted [1812.01474], with three flavours tuned at twisted angle $\alpha = \pi/2$ and the fourth one at $\alpha = -\pi/2$. Unitarity is lost at finite lattice spacing, but it is recovered in the continuum limit. Performing distinct *Osterwalder-Seiler* chiral rotations for each flavour, correlation functions with parity-odd operators (renormalized in the χSF scheme as described above) are mapped onto the 3-point correlation functions of parity even operators with pseudoscalar sources, from which the B-parameters are readily extracted:

$$B_{{\rm K}i}(\mu) \propto \langle \bar{K}^0|\,[O_i^e(\mu)]_R\,|K^0\rangle\,.$$