Non-perturbative renormalization by decoupling

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ALPHA Collaboration

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3M: A world with three heavy degenerate quarks ($M \gg \Lambda$)

Alice uses fundamental theory

$$S_{\text{fund}}[A_\mu, \psi, \bar{\psi}] = \int \mathrm{d}^4 x \left\{ -\frac{1}{2g^2} \mathrm{Tr}\,(F_{\mu\nu} F_{\mu\nu}) + \sum_{i=1}^{3} \bar{\psi}_i (\gamma_\mu D_\mu + M) \psi_i \right\}$$

Bob uses effective theory

$$S_{\text{eff}}[A_\mu] = -\frac{1}{2g_{\text{eff}}^2} \int \mathrm{d}^4 x \, \{\mathrm{Tr}\,(F_{\mu\nu} F_{\mu\nu})\} + \frac{1}{M^2} \sum_k \omega_k \int \mathrm{d}^4 x \, \mathcal{L}_k^{(6)} + \dots$$

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Decoupling

- Low energy scales $1/r_0, 1/\sqrt{8t_0}, 1/w_0, \ldots$ from effective theory

$$\mu_{\mathrm{dec}}^{\mathrm{fund}}(M) = \mu_{\mathrm{dec}}^{\mathrm{eff}} + \mathcal{O}\left(\frac{\mu_{\mathrm{dec}}^2}{M^2}\right)$$

Renormalization in 3M: the strong coupling

$$\frac{\Lambda}{\mu} = \left[b_0 \bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}x \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}.$$

$$M = \bar{m}_i(\mu)\left[2b_0\bar{g}^2(\mu)\right]^{-\frac{d_0}{2b_0}} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}.$$

$\alpha_s(\mu_{\text{high}}) \Longleftrightarrow \Lambda^{(3)}$

- Here talk about RGI: $\Lambda^{(3)}, M$ in massless scheme (Many completely equivalent ways of describing the problem)
- Scale independent
- Scheme dependence trivial
- Quarks treated as massless!!

RENORMALIZATION IN 3M: THE STRONG COUPLING

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Alice determines the strong coupling

- Determine non-perturbatively the β-function in the fundamental ($N_f = 3$) theory.
- Integral up to $\bar{g}^{(3)}(\mu_{\mathrm{dec}}(M))$ gives:

$$\frac{\Lambda^{(3)}}{\mu_{\mathrm{dec}}(M)}$$

(i.e. $\sqrt{8t_0}\,\Lambda^{(3)}, w_0\,\Lambda^{(3)}, r_0\,\Lambda^{(3)}, \ldots$).

RENORMALIZATION IN 3M: THE STRONG COUPLING

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Bob determines the strong coupling

- Determine non-perturbatively the β-function in the effective ($N_f = 0$) theory.
- Integral up to $\bar{g}^{(0)}(\mu_{\rm dec})$ gives:

$$\frac{\Lambda^{(0)}}{\mu_{\rm dec}}$$

  (i.e. $\sqrt{8t_0}\,\Lambda^{(0)}, w_0\,\Lambda^{(0)}, r_0\,\Lambda^{(0)}, \dots$).
- Match across quark threshold to convert to $\Lambda^{(3)}$

$$\frac{\Lambda^{(3)}}{\mu_{\rm dec}} = \frac{\Lambda^{(0)}}{\mu_{\rm dec}} \times \frac{1}{P(\Lambda/M)} .$$

- Matching factor $P(\Lambda/M)$ **[ALPHA 1809.03383]**:
  - Known in perturbation theory up to three-loops. Power series in $\alpha(m^\star)$
  - "Good" PT: corrections very small even at $m_{\rm c}^\star$.

Renormalization in 3M: the strong coupling

$$\frac{\Lambda}{\mu} = \left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}x \left[\frac{1}{\beta(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2 x}\right]\right\}.$$

Why does this work for Bob, and not for us? It does!

Relation between Alice and Bob results:

$$\frac{\Lambda^{(3)}}{\mu_{\mathrm{dec}}(M)} = \frac{\Lambda^{(0)}}{\mu_{\mathrm{dec}}} \times \frac{1}{P(\Lambda/M)} + \mathcal{O}(\alpha^3(m^\star)) + \mathcal{O}\left(\frac{\mu_{\mathrm{dec}}^2}{M^2}\right)$$

Three flavor $\Lambda^{(3)}$ can be computed as

$$\Lambda^{(3)} = \lim_{M\to\infty} \mu_{\mathrm{dec}}(M) \times \frac{\Lambda^{(0)}}{\mu_{\mathrm{dec}}} \times \frac{1}{P(\Lambda/M)}$$

We need

- $\Lambda^{(0)}/\mu_{\mathrm{dec}}$ (i.e. $\sqrt{8t_0}\,\Lambda$ in pure gauge).
- $\mu_{\mathrm{dec}}(M)$ (i.e. $\sqrt{8t_0}$ in fm for three degenerate heavy quarks).

Our setup: Choices optimized to be able to simulate heavy quarks

$$\Lambda^{(3)} = \mu_{\rm dec}(M) \times \frac{\Lambda^{(0)}}{\mu_{\rm dec}} \times \frac{1}{P(\Lambda/M)} + \mathcal{O}(\alpha^3(m^\star)) + \mathcal{O}\left(\frac{\mu_{\rm dec}}{M}\right) + \mathcal{O}\left(\frac{\mu_{\rm dec}^2}{M^2}\right)$$

- Work in finite volume schemes with Schrödinger Functional boundary conditions: $T \times L^3$ with Dirichlet bcs. in time. ($\mu \sim 1/L$): "Only" two scales.
- Use Gradient Flow couplings

$$\bar{g}^2(\mu) = \mathcal{N}^{-1}(c, a/L)\, t^2 \langle E(t) \rangle \Big|_{\mu^{-1}=\sqrt{8t}=cL}.$$

- Matching condition ($\{N_{\rm f} = 3, M\} \leftrightarrow \{N_{\rm f} = 0\}$) between massive scheme and effective theory

$$\bar{g}^2(\mu_{\rm dec}(M))\Big|_{N_{\rm f}=3,M,T=2L} = \bar{g}^2(\mu_{\rm dec})\Big|_{N_{\rm f}=0,T=2L}.$$

Caveats: Schrödinger Functional boundary conditions break chiral symmetry

- $\mu_{\rm dec}/M$ corrections to decoupling
- Choose $T = 2L$. Boundary effects very suppressed
- Most probably completly negligible, but keep in mind…

OUR SETUP: CHOICES OPTIMIZED TO BE ABLE TO SIMULATE HEAVY QUARKS

$$\Lambda^{(3)} = \mu_{\mathrm{dec}}(M) \times \frac{\Lambda^{(0)}}{\mu_{\mathrm{dec}}} \times \frac{1}{P(\Lambda/M)} + \mathcal{O}(\alpha^3(m^\star)) + \mathcal{O}\left(\frac{\mu_{\mathrm{dec}}}{M}\right) + \mathcal{O}\left(\frac{\mu_{\mathrm{dec}}^2}{M^2}\right)$$

We only need to fill in a table!

M [GeV]	$\mu_{\mathrm{dec}}(M)$ [GeV]	$\bar{g}^2(\mu_{\mathrm{dec}}(M))\Big\vert_{N_{\mathrm{f}}=3,M,T=2L}$	$\Lambda^{(0)}/\mu_{\mathrm{dec}}$	$1/P(\Lambda/M)$	$\Lambda^{(3)}$
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
…	…	…	…	…	…

Our setup: Most columns of the table already known

3-flavor renormalization program by ALPHA

- $\mu_{\mathrm{dec}}(M)$ [GeV]: Switch to mass-less scheme. Use ALPHA **[ALPHA 1706.03821]**

$$\bar{g}^2(\mu_{\mathrm{dec}}(M))\Big|_{N_{\mathrm{f}}=3,M,T=2L} \Longrightarrow \bar{g}^2(\mu_{\mathrm{dec}}(M))\Big|_{N_{\mathrm{f}}=3,M=0,T=L} \Longrightarrow \mu_{\mathrm{dec}}(M) \text{ in [fm]}\,.$$

- M [GeV]: NP-renormalization ALPHA **[ALPHA 1802.05243]**

$$LM = \frac{L}{a} Z_{\mathrm{m}}(\mu_{\mathrm{dec}}(M))(1 + b_{\mathrm{m}} a m_q)\, Z_{RGI}(\mu_{\mathrm{dec}}(M))\ (am_0 - am_c)$$

  - Z_{m} determined non-perturbatively ($\rightarrow$ no details here!)
  - Z_{RGI} Known non-perturbatively **[ALPHA 1802.05243]**
  - 1-loop value for $b_{\mathrm{m}}, b_{\mathrm{g}}$: Not fully $\mathcal{O}(a)$-improved.

- $\Lambda^{(0)}/\mu_{\mathrm{dec}}$: Known very precisely **[M. Dalla Brida, A. Ramos 1905.05147]** ($\rightarrow$ no details here!)

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- M [GeV]: NP-renormalization ALPHA **[ALPHA 1802.05243]**

$$LM = \frac{L}{a} Z_\mathrm{m}(\mu_{\mathrm{dec}}(M))(1 + b_\mathrm{m} a m_q)\, Z_{RGI}(\mu_{\mathrm{dec}}(M))\ (am_0 - am_c)$$

  - Z_m determined non-perturbatively ($\rightarrow$ no details here!)

Missing piece: massive $\leftrightarrow$ massless: LCP accureately known at $M = 0$

L/a	β	$\bar{g}^2(\mu_{\mathrm{dec}}(M))\Big\vert_{N_\mathrm{f}=3,M=0,T=L}$	$\mu_{\mathrm{dec}}(M)$ [GeV]
12	4.3020	3.9533(59)	0.789(15)
16	4.4662	3.9496(77)	0.789(15)
20	4.5997	3.9648(97)	0.789(15)
24	4.7141	3.959(50)	0.789(15)
32	4.90	3.949(11)	0.789(15)

DETERMINE MASSIVE COUPLING FOR MATCHING (PRELIMINARY)

Example: $L/a = 20$

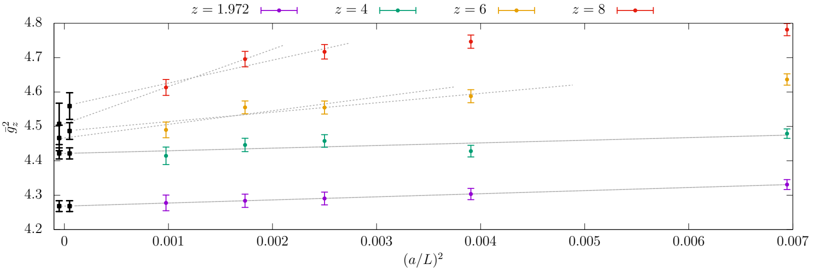
β	κ	$z = M/\mu_{\mathrm{dec}}(M)$	M [GeV]	$\bar{g}^2(\mu_{\mathrm{dec}}(M))\big\vert_{N_\mathrm{f}=3,M,T=2L}$
4.5997	0.1352889	0	0	3.9648(97)
4.6083	0.133831710060	1.972(18)	1.6	4.290(15)
4.6172	0.132345249425	4.000(37)	3.2	4.458(14)
4.6266	0.130827894135	6.000(58)	4.7	4.555(14)
4.6364	0.129273827559	8.000(85)	6.3	4.717(14)

Extrapolate results to the continuum: Example $z = 6$

We determine $\bar{g}^2(\mu_{\mathrm{dec}}(M))\big\vert_{N_\mathrm{f}=3,M,T=2L}$ change cuts $aM < 0.35, 0.5$ and flow

L/a	β	aM	ZFL	WFL
12	4.3499	0.50	4.636(17)	5.477(23)
16	4.5008	0.37	4.588(19)	5.023(22)
20	4.6266	0.30	4.555(19)	4.823(21)
24	4.7359	0.25	4.555(19)	4.738(20)
32	4.9159	0.18	4.490(23)	4.590(24)
∞	$aM < 0.50$		4.487(24)	4.470(26)
∞	$aM < 0.35$		4.466(37)	4.458(39)

Continuum extrapolations with two cuts: $aM < 0.50, 0.35$ (PRELIMINARY)



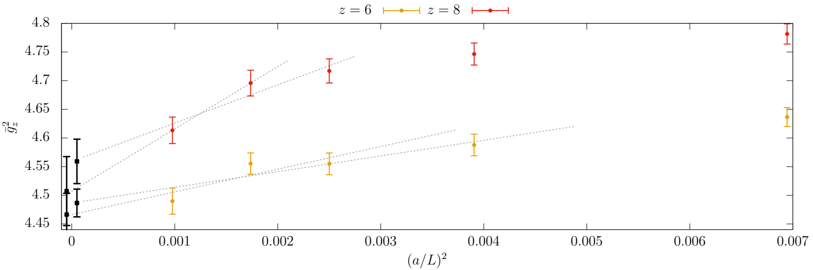
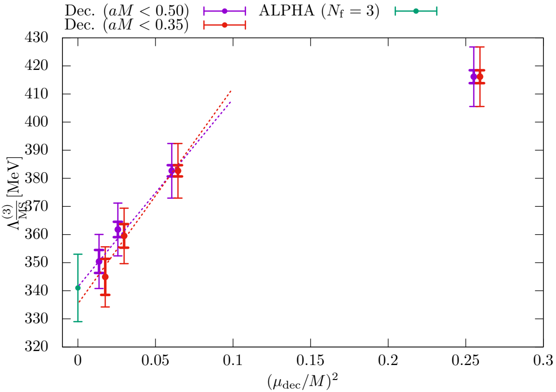


Table can be filled (PRELIMINARY)

M [GeV]	$\mu_{\mathrm{dec}}(M)$ [GeV]	$\bar{g}^2(\mu_{\mathrm{low}}(M))\Big\vert_{N_{\mathrm{f}}=3,M,T=2L}$	$\Lambda^{(0)}/\mu_{\mathrm{low}}$	$\frac{1}{P(\Lambda/M)}$	$\Lambda^{(3)}$ [MeV]
1.6	0.789(15)	4.559(39)	0.689(11)	0.7662(44)	416(11)
3.2	0.789(15)	4.421(16)	0.725(11)	0.6693(37)	382.7(96)
4.7	0.789(15)	4.466(37)	0.741(12)	0.6198(34)	362.0(92)
6.3	0.789(15)	4.507(60)	0.757(13)	0.5871(32)	350.3(92)



CONCLUSIONS: NON-PERTURBATIVE RENORMALIZATION BY DECOUPLING

- Lattice QCD can determine scales at un-physical values of the parameters

$$\Lambda^{(N_\mathrm{f})} = \lim_{M\to\infty} \mu_\mathrm{dec}(M) \times \frac{\Lambda^{(0)}}{\mu_\mathrm{dec}} \times \frac{1}{P(\Lambda/M)}$$

  with
  - $\mu_\mathrm{dec}(M)$: Scale with N_f heavy quarks ($M \gg \Lambda$)
  - $\Lambda^{(0)}/\mu_\mathrm{dec}$: Computed non-perturbatively in pure gauge
  - $P(\Lambda/M)$: Perturbative relation between fundamental and effective theories
- Method is generic
  - Similar expressions for other RGI invariants: $M, \hat{B}_K, \hat{B}_B, \ldots$
  - Valid in finite or infinite volume renormalization schemes
  - If you can, just compute $\sqrt{8t_0}, w_0$ with 3-4 quarks as heavy as possible!
- Still working (larger L/a) but It works!!
  - Finite volume setup: Small PT corrections $\mathcal{O}(\alpha^3(m^\star))$, window problem ameliorated
  - $\mu_\mathrm{dec}(M) = 789(15)$ MeV. Applied with $M = 1.6, \ldots, 6.3$ GeV
  - Non-perturbative running in pure gauge from $\mu = 789$ MeV to $\mu = \infty$
  - $\Lambda^{(3)}$ in agreement with current knowledge
- Probably best approach to reduce error in α_s substantially
  - Running done in pure gauge!
  - Better precision in pure gauge: Small lattice spacing, efficient algorithms.
  - Switch massive $\leftrightarrow$ massless schemes little effect in total error.
  - $\lim_{M\to\infty}$ can be controlled