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Non-perturbative renormalization of $O(a)$ improved tensor currents

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Introduction

- We want to use $N_\mathrm{f} = 2 + 1$ flavours of **Wilson quarks** to investigate the isovector tensor current

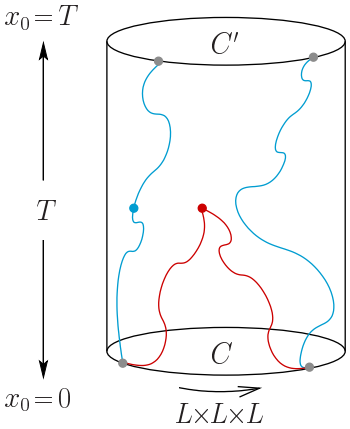
$$T^a_{\mu\nu}(x) = \mathrm{i}\bar{\psi}(x)\sigma_{\mu\nu}\frac{1}{2}\tau^a\psi(x)\,,$$

- Relevant for some interesting processes to probe the standard model
  - Rare heavy meson decays
  - Neutron beta decay observables
  - Direct dark matter detection
- The tensor current requires scale dependent renormalization and Wilson quarks additionally necessitate $\mathrm{O}(a)$ improvement of the local operator
- We want to tackle both of these challenges with **non-perturbative** methods based on the **Schrödinger functional**

The Schrödinger functional

Benefits of Schrödinger functional boundary conditions

- Provide a regularization independent renormalization scheme
- Non-perturbative couplings can be defined exactly and their scale evolution can be evaluated
- Provide an IR cutoff which allows for **mass independent** renormalization and improvement
- Gauge invariant wall sources can be used to improve the signal of fermionic observables



The Schrödinger functional

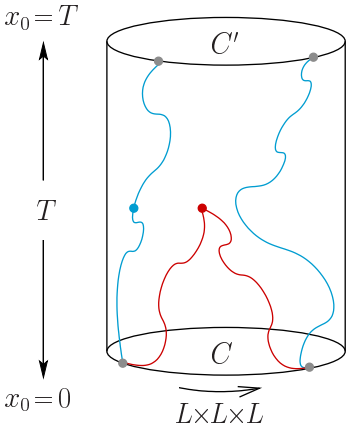
In the Schrödinger functional we use boundary sources to define boundary-to-bulk correlation functions

$$k_{\mathrm{T}}(x_0) = -\frac{a^6}{6} \sum_{\mathbf{u},\mathbf{v}} \left\langle T_{0k}(x_0)\bar{\zeta}(\mathbf{u})\gamma_k\zeta(\mathbf{v})\right\rangle$$

$$k_{\mathrm{V}}(x_0) = -\frac{a^6}{6} \sum_{\mathbf{u},\mathbf{v}} \left\langle V_{k}(x_0)\bar{\zeta}(\mathbf{u})\gamma_k\zeta(\mathbf{v})\right\rangle$$

and boundary-to-boundary correlation functions

$$k_1 = -\frac{a^{12}}{6L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \left\langle \bar{\zeta}'(\mathbf{u}')\gamma_k\zeta'(\mathbf{v}')\,\bar{\zeta}(\mathbf{u})\gamma_k\zeta(\mathbf{v})\right\rangle$$



$\mathrm{O}(a)$ improvement of the tensor current

- The Wilson fermion formulation causes not only cutoff effects of $\mathrm{O}(a)$ in the action but also in local operators. These can be canceled by appropriate counter terms to achieve $\mathrm{O}(a^2)$ scaling [Lüscher et al., Nucl. Phys. B478 (1996)]
- As we restrict ourselves to **zero momentum** only the "electric" part of the tensor current requires improvement:

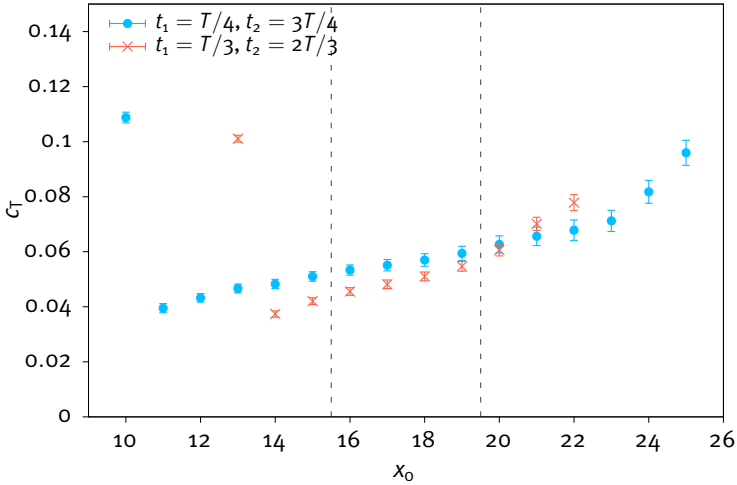
$$\left(T^a_{0k}\right)^{\mathrm{I}} = T^a_{0k} + a c_{\mathrm{T}}(g_0^2)\tilde{\partial}_0 V^a_k\,, \quad \left(T^a_{ij}\right)^{\mathrm{I}} = T^a_{ij}$$

- c_{T} is only known to 1-loop PT for our setup so far [Taniguchi et al., Phys. Rev. D 58 (1998)]

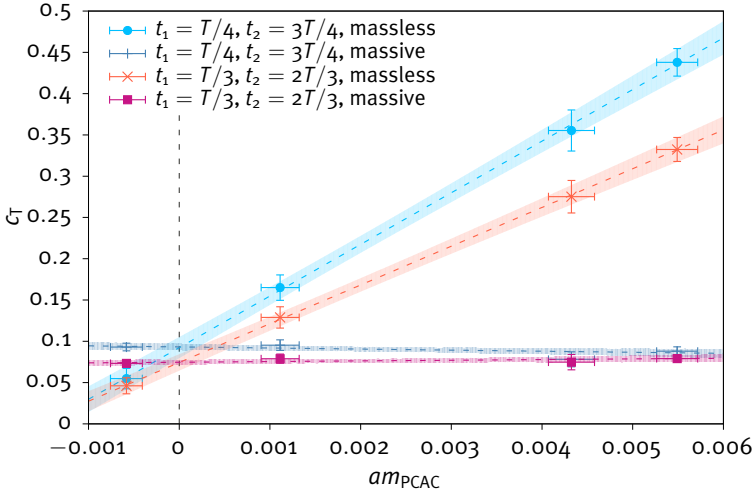
Ward identity for the determination of c_T

The **non-perturbative** determination of c_T is based on a chiral Ward identity which relates the "electric" to the "magnetic" part of the tensor current [Bhattacharya et al., Phys. Lett. B, 461 (1999)]

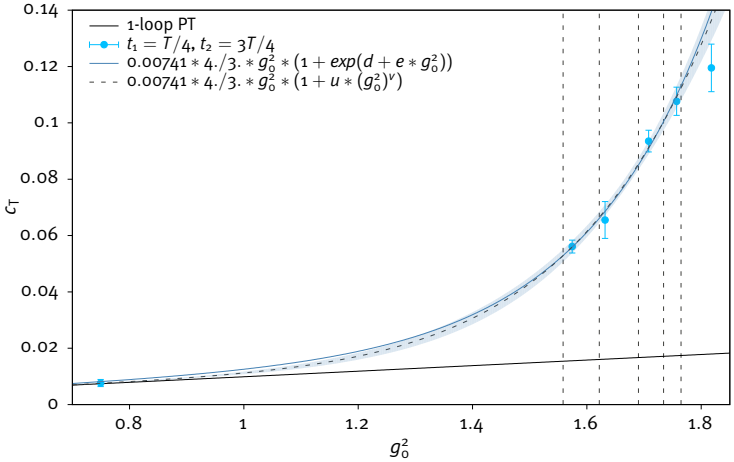
$$\epsilon_{0kij}\left(\int \mathrm{d}^3\mathbf{x}\left\langle\left[A_0^a(t_2,\mathbf{x})-A_0^a(t_1,\mathbf{x})\right]T_{ij}^b(y)\mathcal{O}_\mathrm{ext}\right\rangle\right.$$
$$\left.-2m\int \mathrm{d}^3\mathbf{x}\int_{t_1}^{t_2}\mathrm{d}x_0\left\langle P^a(x_0,\mathbf{x})T_{ij}^b(y)\mathcal{O}_\mathrm{ext}\right\rangle\right)$$
$$=2d^{abc}\left\langle T_{0k}^c(y)\mathcal{O}_\mathrm{ext}\right\rangle$$



Ward identity evaluated on a 35×24^3 lattice with $\beta = 3.81$ and $am_{\mathrm{PCAC}} = -0.00003(7)$



c_T evaluated at $\beta = 3.512$ and different quark masses. The Schrödinger functional boundary conditions allow for a controlled chiral limit.



Cutoff effects from ambiguities in the improvement definition vanish smoothly for $a \to 0$ due to the lcp

Renormalization of the tensor current

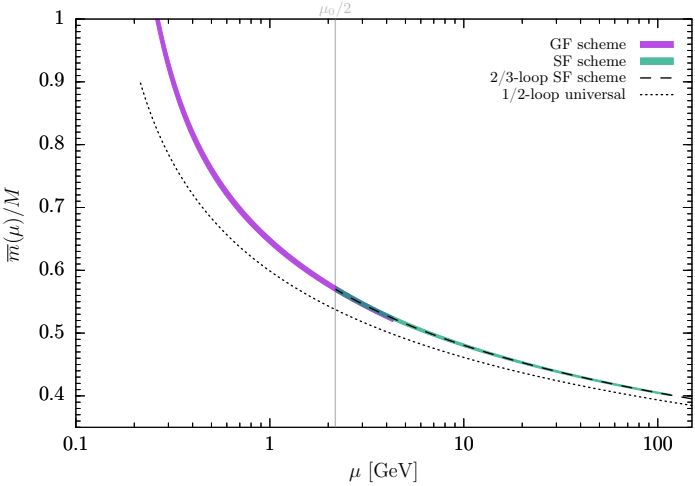
$$Z_{\mathrm{T}}^{\mathrm{RGI}}(g_0^2) = \underbrace{\frac{Z_{\mathrm{T}}^{\mathrm{RGI}}}{Z_{\mathrm{T}}(\mu_{\mathrm{pt}})}}_{\mathrm{PT}} \underbrace{\frac{Z_{\mathrm{T}}(\mu_{\mathrm{pt}})}{Z_{\mathrm{T}}(\mu_0/2)}}_{\mathrm{SF}} \underbrace{\frac{Z_{\mathrm{T}}(\mu_0/2)}{Z_{\mathrm{T}}(\mu_{\mathrm{had}})}}_{\mathrm{GF}} Z_{\mathrm{T}}(g_0^2, \mu_{\mathrm{had}})$$

PT: NLO perturbation theory in the Schrödinger functional scheme

SF: Non-perturbative running in the Schrödinger functional scheme ($\Delta_{\mathrm{stat}}\bar{g}_{\mathrm{SF}}^2 \sim \bar{g}_{\mathrm{SF}}^4$)

GF: Non-perturbative running in the Gradient flow scheme ($\Delta_{\mathrm{stat}}\bar{g}_{\mathrm{GF}}^2 \sim \bar{g}_{\mathrm{GF}}^2$)

Scheme switching scale $\mu_0/2 \approx 2$ GeV, hadronic scale $\mu_{\mathrm{had}} = 233(8)$ MeV [Campos at al., Eur. Phys. J. C (2018)]



[Campos at al. Eur. Phys. J. C (2018)]

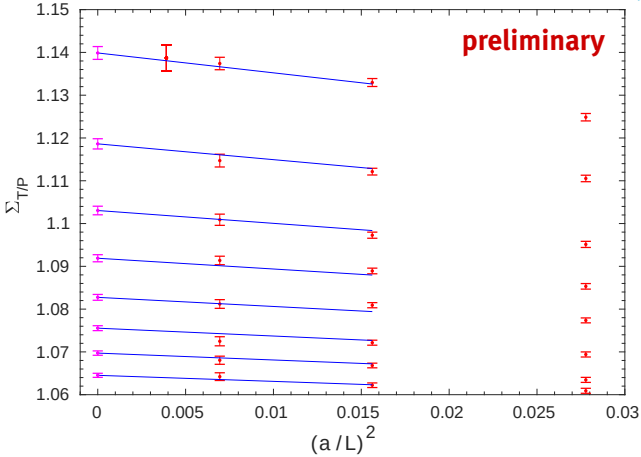
We impose a **mass independent** renormalization condition of the form

$$Z_{\mathrm{T}}(g_0^2, a/L)\frac{k_{\mathrm{T}}^{\mathrm{I}}(L/2)}{\sqrt{k_1}} = \frac{k_{\mathrm{T}}^{\mathrm{I}}(L/2)}{\sqrt{k_1}}\bigg|_{\text{tree level}}$$

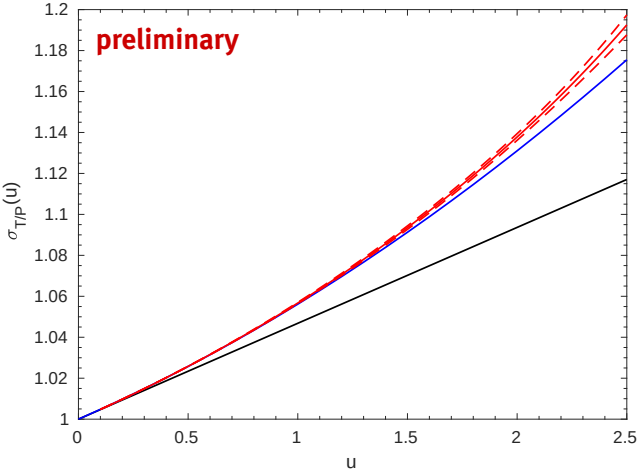
We use recursive finite size scaling and define the step scaling function as

$$\sigma_{\mathrm{T}}(u) = \lim_{a\to 0}\Sigma_{\mathrm{T}}(u, a\mu)\,, \quad \Sigma_{\mathrm{T}} = \frac{Z_{\mathrm{T}}(g_0^2, a/2L)}{Z_{\mathrm{T}}(g_0^2, a/L)}\bigg|_{\bar{g}^2(L)=u}$$

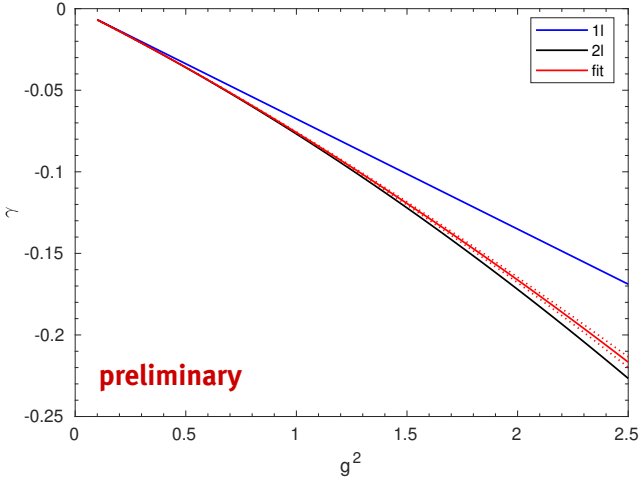
The ratio $Z_{\mathrm{T}}/Z_{\mathrm{P}}$ appears to be beneficial from a numerical perspective.



Global continuum extrapolations of $\Sigma_{T/P}$ at fixed value of the coupling in the high-energy region.



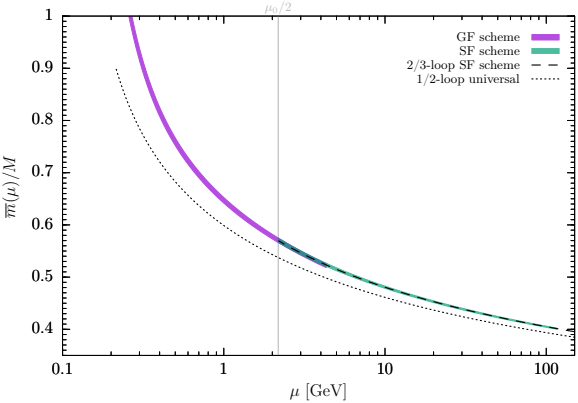
Corresponding step scaling function in the high energy region in comparison to 1-loop and 2-loop predictions.



Anomalous dimension of Z_T/Z_P.

Conclusion & Outlook

- Non-perturbative determination of c_T on a line of constant physics
- Running in the SF scheme finished
- Running in the GF scheme advanced



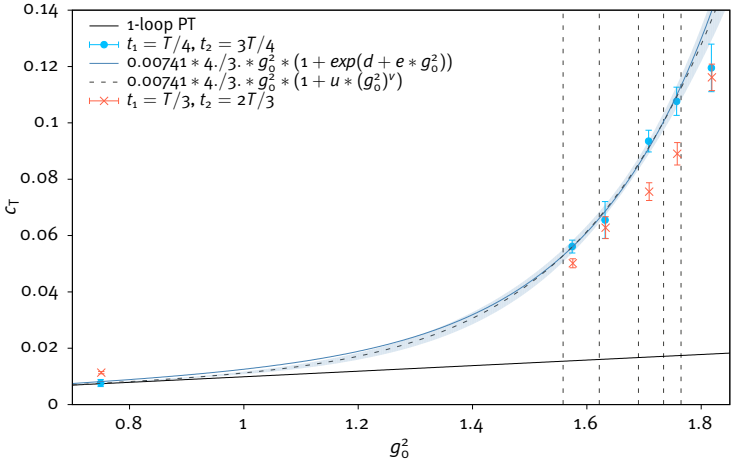
[Campos at al. Eur. Phys. J. C (2018)]

Thank you for your attention!

$L^3 \times T/a^4$	β	κ	#REP	#MDU	ID	am_{PCAC}
$12^3 \times 17$	3.3	0.13652	20	10240	A1k1	−0.00330(97)
		0.13660	10	13672	A1k2	−0.01145(84)
		0.13648	5	6876	A1k3	0.00085(170)
		0.13650	20	96640	A1k4	−0.00103(34)
$14^3 \times 21$	3.414	0.13690	32	38400	E1k1	0.00259(31)
		0.13695	48	57600	E1k2	−0.00043(28)
$16^3 \times 23$	3.512	0.13700	2	20480	B1k1	0.00584(21)
		0.13703	1	8192	B1k2	0.00443(30)
		0.13710	2	16384	B1k3	0.00112(23)
		0.13714	1	27856	B1k4	−0.00058(18)
$20^3 \times 29$	3.676	0.13680	1	8192	C1k1	0.01337(15)
		0.13700	4	15232	C1k2	0.00603(14)
		0.13719	4	15472	C1k3	−0.00111(10)
$24^3 \times 35$	3.810	0.13712	6	10272	D1k1	−0.00305(6)
		0.13701	3	5672	D1k2	0.00072(18)
		0.137033	8	44800	D1k4	−0.00003(7)

- Schrödinger functional boundary conditions
- Line of constant physics (system size $L \approx 1.2$ fm)

Table: Summary of simulation parameters of the gauge configuration ensembles used for the determination of c_{T}, as well as the number of (statistically idependent) replica per ensemble 'ID' and their total number of molecular dynamics units.



$\chi^2/\text{d.o.f} = 0.247194$

$$\mu \frac{\partial \bar{O}_{\mathrm{T}}(\mu)}{\partial \mu} = \gamma(\bar{g}(\mu))\bar{O}_{\mathrm{T}}(\mu)$$

$$\sigma_{\mathrm{T}}(u) = \exp\left\{ \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{\gamma(g')}{\beta(g')} \right\}$$

With knowledge of the scale evolution of the coupling [Dalla Brida et al., Phys. Rev. Lett. 117 (2016), Phys. Rev. D 95 (2017)] we can determine the renormalization constant ratios at different scales

$$\frac{Z_{\mathrm{T}}(2^n\mu)}{Z_{\mathrm{T}}(\mu/2)} = \prod_{k=0}^{n} \sigma_{\mathrm{T}}(u_k) \quad \text{with} \quad u_k = \bar{g}^2(2^k\mu)$$

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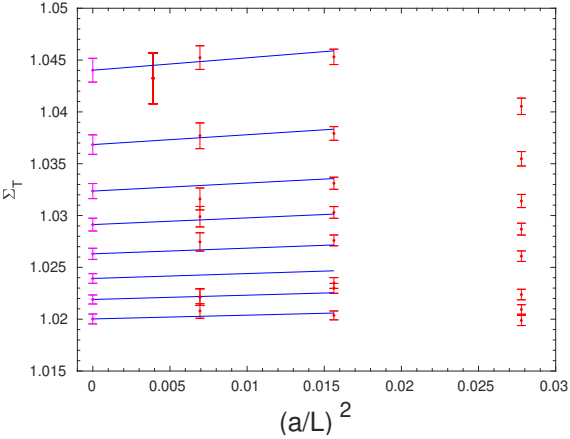
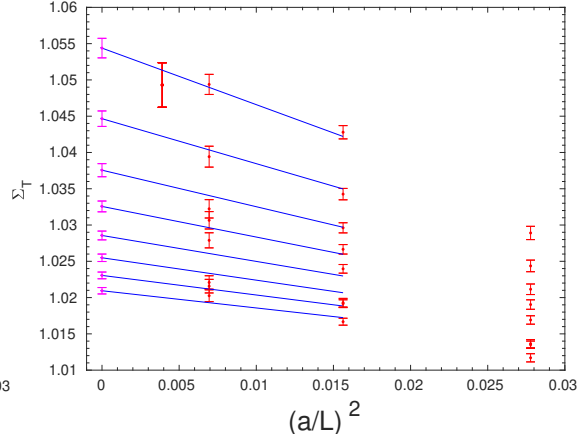


k_1 scheme



f_1 scheme