Non-perturbative renormalization of $O(a)$ improved tensor currents

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Introduction

- We want to use $N_f = 2 + 1$ flavours of Wilson quarks to investigate the isovector tensor current

\[ T_{\mu\nu}^a(x) = i\bar{\psi}(x)\sigma_{\mu\nu} \frac{1}{2} \tau^a \psi(x), \]

- Relevant for some interesting processes to probe the standard model
  - Rare heavy meson decays
  - Neutron beta decay observables
  - Direct dark matter detection

- The tensor current requires scale dependent renormalization and Wilson quarks additionally necessitate $O(a)$ improvement of the local operator

- We want to tackle both of these challenges with non-perturbative methods based on the Schrödinger functional
The Schrödinger functional

Benefits of Schrödinger functional boundary conditions

- Provide a regularization independent renormalization scheme
- Non-perturbative couplings can be defined exactly and their scale evolution can be evaluated
- Provide an IR cutoff which allows for mass independent renormalization and improvement
- Gauge invariant wall sources can be used to improve the signal of fermionic observables
The Schrödinger functional

In the Schrödinger functional we use boundary sources to define boundary-to-bulk correlation functions

\[ k_T(x_0) = - \frac{a^6}{6} \sum_{u,v} \langle T_{ok}(x_0) \bar{\zeta}(u) \gamma_k \zeta(v) \rangle \]

\[ k_V(x_0) = - \frac{a^6}{6} \sum_{u,v} \langle V_k(x_0) \bar{\zeta}(u) \gamma_k \zeta(v) \rangle \]

and boundary-to-boundary correlation functions

\[ k_1 = - \frac{a^{12}}{6L^6} \sum_{u,v,u',v'} \langle \bar{\zeta}'(u') \gamma_k \zeta'(v') \bar{\zeta}(u) \gamma_k \zeta(v) \rangle \]
O(\(a\)) improvement of the tensor current

The Wilson fermion formulation causes not only cutoff effects of O(\(a\)) in the action but also in local operators. These can be canceled by appropriate counter terms to achieve O(\(a^2\)) scaling

[\(\text{Lüscher et al., Nucl. Phys. B}478 \text{ (1996)}\)]

As we restrict ourselves to zero momentum only the "electric" part of the tensor current requires improvement:

\[
(\mathcal{T}_{ok}^a)^I = \mathcal{T}_{ok}^a + ac_T(g_0^2)\tilde{\partial}_o V_k^a, \quad (\mathcal{T}_{ij}^a)^I = \mathcal{T}_{ij}^a
\]

\(c_T\) is only known to 1-loop PT for our setup so far [\(\text{Taniguchi et al., Phys. Rev. D} 58 \text{ (1998)}\)]
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Ward identity for the determination of $c_T$

The **non-perturbative** determination of $c_T$ is based on a chiral Ward identity which relates the "electric" to the "magnetic" part of the tensor current [Bhattacharya et al., Phys. Lett. B, 461 (1999)]

\[
\begin{align*}
\epsilon_{0kij} & \left( \int d^3x \left\langle \left[ A^a_0(t_2, x) - A^a_0(t_1, x) \right] T_{ij}^b(y) \mathcal{O}_{\text{ext}} \right\rangle 

-2m \int d^3x \int_{t_1}^{t_2} dx_0 \left\langle P^a(x_0, x) T_{ij}^b(y) \mathcal{O}_{\text{ext}} \right\rangle 

= 2d^{abc} \left\langle T_{0k}^c(y) \mathcal{O}_{\text{ext}} \right\rangle
\end{align*}
\]
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Ward identity evaluated on a $35 \times 24^3$ lattice with $\beta = 3.81$ and $am_{PCAC} = -0.00003(7)$
Non-perturbative renormalization of $O(a)$ improved tensor currents

c$T$ evaluated at $\beta = 3.512$ and different quark masses. The Schrödinger functional boundary conditions allow for a controlled chiral limit.
Non-perturbative renormalization of \( O(a) \) improved tensor currents

Cutoff effects from ambiguities in the improvement definition vanish smoothly for \( a \to 0 \) due to the lcp
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**Renormalization of the tensor current**

\[
Z^\text{RGI}_T(g_0^2) = \frac{Z^\text{RGI}_T(\mu_{pt})}{Z_T(\mu_{pt})} \frac{Z_T(\mu_{o}/2)}{Z_T(\mu_{o}/2)} \frac{Z_T(\mu_{had})}{Z_T(\mu_{had})} Z_T(g_0^2, \mu_{had})
\]

**PT:** NLO perturbation theory in the Schrödinger functional scheme

**SF:** Non-perturbative running in the Schrödinger functional scheme ($\Delta_{\text{stat}}g_{SF}^2 \sim \bar{g}_{SF}^4$)

**GF:** Non-perturbative running in the Gradient flow scheme ($\Delta_{\text{stat}}g_{GF}^2 \sim \bar{g}_{GF}^2$)

Scheme switching scale $\mu_{o}/2 \approx 2$ GeV, hadronic scale $\mu_{had} = 233(8)$ MeV [Campos at al., Eur. Phys. J. C (2018)]
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\[ \frac{\pi_\mu(\mu)}{M} \]

GF scheme
SF scheme
2/3-loop SF scheme
1/2-loop universal

We impose a **mass independent** renormalization condition of the form

\[
Z_T(g_0^2, a/L) \frac{k^l_T(L/2)}{\sqrt{k_1}} = \frac{k^l_T(L/2)}{\sqrt{k_1}} \bigg|_{\text{tree level}}
\]

We use recursive finite size scaling and define the step scaling function as

\[
\sigma_T(u) = \lim_{a \to 0} \Sigma_T(u, a \mu), \quad \Sigma_T = \left. \frac{Z_T(g_0^2, a/2L)}{Z_T(g_0^2, a/L)} \right|_{\bar{g}^2(L)=u}
\]

The ratio \(Z_T/Z_P\) appears to be beneficial from a numerical perspective.
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Global continuum extrapolations of $\Sigma_{T/P}$ at fixed value of the coupling in the high-energy region.
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Corresponding step scaling function in the high energy region in comparison to 1-loop and 2-loop predictions.

preliminary
Non-perturbative renormalization of $O(a)$ improved tensor currents

Anomalous dimension of $Z_T/Z_P$.

preliminary
Conclusion & Outlook

- Non-perturbative determination of $c_T$ on a line of constant physics
- Running in the SF scheme finished
- Running in the GF scheme advanced

Thank you for your attention!
Non-perturbative renormalization of $O(a)$ improved tensor currents

- Schrödinger functional boundary conditions
- Line of constant physics (system size $L \approx 1.2$ fm)

<table>
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<th>$\beta$</th>
<th>$\kappa$</th>
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Table: Summary of simulation parameters of the gauge configuration ensembles used for the determination of $c_T$, as well as the number of (statistically independent) replica per ensemble ‘ID’ and their total number of molecular dynamics units.
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\[ \chi^2 / \text{d.o.f} = 0.247194 \]
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\[ \mu \frac{\partial \bar{O}_T(\mu)}{\partial \mu} = \gamma(\bar{g}(\mu)) \bar{O}_T(\mu) \]

\[ \sigma_T(u) = \exp \left\{ \int \frac{\sqrt{\sigma(u)}}{\sqrt{u}} \frac{\gamma(g')}{\beta(g')} \right\} \]

With knowledge of the scale evolution of the coupling [Dalla Brida et al., Phys. Rev. Lett. 117 (2016), Phys. Rev. D 95 (2017)] we can determine the renormalization constant ratios at different scales

\[ \frac{Z_T(2^n \mu)}{Z_T(\mu/2)} = \prod_{k=0}^{n} \sigma_T(u_k) \quad \text{with} \quad u_k = \bar{g}^2(2^k \mu) \]
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$k_1$ scheme

$f_1$ scheme