THE UNIVERSITY *of* EDINBURGH

Investigating Rare Kaon Decays with the All-to-All Method.

Fionn Ó hÓgáin

Lattice 2019 Wuhan

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Outline

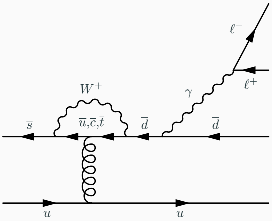
- $K \to \pi \ell^+ \ell^-$
- All-to-All method
- Exploratory 24^3 results
- Preliminary 48^3 results
- Future outlook & conclusions

$K \to \pi \ell^+ \ell^-$

Motivations

- $K \to \pi \ell^+ \ell^-$ are FCNC processes
- Forbidden at tree level

$\implies$ Sensitive to new physics



- Long distance effects dominate: use lattice QCD
- NA62 has collected $O(30k)$ $K \to \pi \mu^+ \mu^-$ samples
  - Potential for e^+e^- samples in 2021-2023 runs discussed at the Rare Kaon Decays Forum 2019 [C. Parkinson: RKF 2019]

Long-Distance Amplitude: Phenomenology

In terms of EM transition form factor $V_j(z) \quad (j = +, S)$
[G. D'Ambrosio et al. arXiv:hep-ph/9808289]

- $\mathscr{A}_\mu^j(q^2) = -iG_F \frac{V_j(z)}{(4\pi)^2} \left(q^2(k+p)_\mu - \left(M_K^2 - M_\pi^2\right)\right)$

$V_j(z) = a_j + b_j z + V_j^{\pi\pi}(z), \qquad z = q^2/M_K^2, \qquad q \equiv k - p$

Phenomenological predictions:
[V. Cirigliano et al. arXiv:1107.6001]

e	$\lvert a_S\rvert = 1.06^{+0.26}_{-0.21}$	$a_+ = -0.578 \pm 0.016$	$b_+ = -0.779 \pm 0.066$
μ	$\lvert a_S\rvert = 1.54^{+0.40}_{-0.32}$	$a_+ = -0.575 \pm 0.039$	$b_+ = -0.813 \pm 0.145$

NA62 working towards new a_+, b_+ measurements for the muon: [A. Sturgess: 2018 Thesis]

Long-Distance Amplitude: Lattice

Long-distance Minkowski amplitude:
[G. Isidori et al. arXiv:hep-lat/0506026]

- $\mathscr{A}^j_\mu(q^2) = \int \mathsf{d}^4x \, \left\langle \pi^j(\mathbf{p}) \right| \mathsf{T}\left[J_\mu(0) H_W(x)\right] \left| K^j(\mathbf{k}) \right\rangle$

$\Delta S = 1$ effective weak Hamiltonian:

- $H_W(x) = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \left(C_1(Q^u_1 - Q^c_1) + C_2(Q^u_2 - Q^c_2)\right)$

Wilson coefficients C_1 and C_2 are much larger than $C_{3,\ldots,8}$

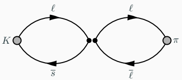
- $Q^q_1 = (\bar{s}_a \gamma^L_\mu d_a)(\bar{q}_b \gamma^{L\,\mu} q_b)$ and $Q^q_2 = (\bar{s}_a \gamma^L_\mu q_a)(\bar{q}_b \gamma^{L\,\mu} d_b)$

where $\gamma^L_\mu = \gamma_\mu(1 - \gamma_5)$.

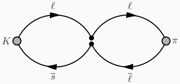
Current can be either the local or conserved lattice vector current.

Wick Contractions

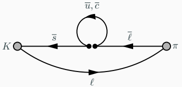
- The Wick contractions for $K \to \pi\ H_W$ 3-pt functions gives:



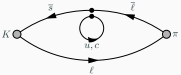
W
(Wing)



C
(Connected)

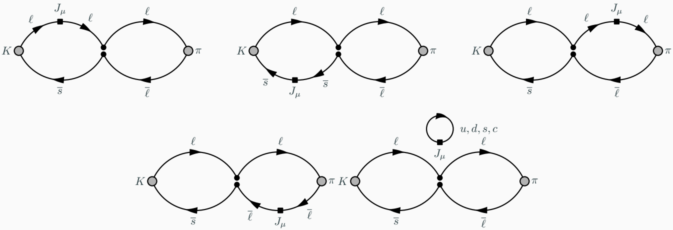


S
(Saucer)



E
(Eye)

- Including the current insertions:



All-to-All Method

All-to-All Propagators

Eigenvalues and eigenvectors of $D(x, y)$, λ_i and ϕ_i, are used to solve the low modes exactly. For a set of random noise source vectors that have the property:

- $\lim_{N_h \to \infty} \sum_{h=1}^{h=N_h} \eta_h {\eta_h}^\dagger = \mathbb{1}$

and deflating the Dirac operator

- $D_{defl}^{-1} = D^{-1} - \sum_{j=1}^{j=N_l} \frac{\phi_j \phi_j^\dagger}{\lambda_j}$

we can write

- $D_{A2A}^{-1} = \sum_{i=1}^{i=N_l} \frac{\phi_i \phi_i^\dagger}{\lambda_i} + \sum_{h=1}^{h=N_h} D_{defl}^{-1} \eta_h {\eta_h}^\dagger$

All-to-all formalism has been implemented in the C++ library, “Grid,” and the framework based on Grid, “Hadrons.”
See Antonin Portelli’s talk (Wed 09:00).

Meson Fields

The "all-to-all vectors" are defined as

$$v_i = \begin{cases} \frac{1}{\lambda_i}\phi_i \\ D_{defl}^{-1}\eta_i \end{cases} \qquad w_i = \begin{cases} \phi_i : & 1 \leq i < N_l \\ \eta_i : & N_l \leq i < N_l + N_h \end{cases}$$

such that

- $D_{A2A}^{-1}(x,y) = \sum_i v_i(x) w_i^\dagger(y)$

We can now write the meson correlator functions as

- $C(t) = \sum_{i,j} \Pi_{ji}^{(q',q)}(t_x;\Gamma_2)\Pi_{ij}^{(q,q')}(t_y;\Gamma_1)$

where $\Pi_{ij}^{(q,q')}(t_x;\Gamma) = \sum_{\vec{x}} w_i^{\dagger q}(x)\Gamma v_j^{q'}(x)$ are "Meson Fields."

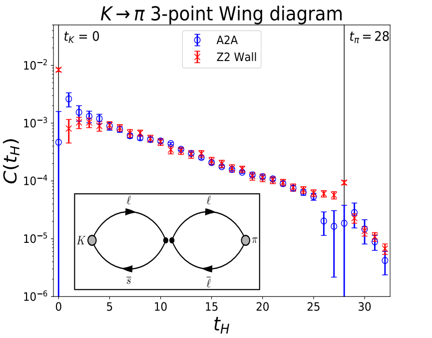
Arbitrary n-point functions can be made with appropriate MF multiplication and contraction. [J.Foley arXiv:hep-lat/0505023]

24^3 Tests

24^3 Setup & $K \to \pi$ 3-pt Function

$24^3 \times 64$ Domain Wall Fermion

- $M_\pi \approx 340MeV$
- $a^{-1} = 1.78GeV$
- $2 + 1$ flavor
- Light quarks: 600 low modes
- Spin/color/time diluted high modes

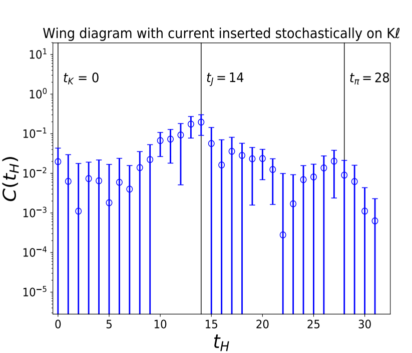


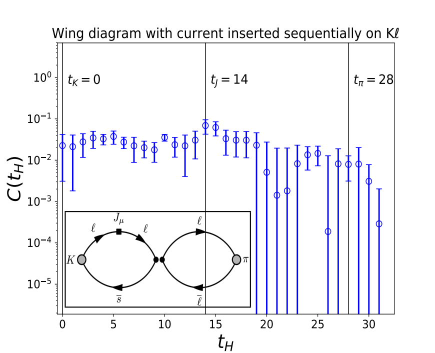
Diagrams were found by computing contractions:

- $\mathcal{P}(x_H; t_y, \Gamma) = \sum_{i,j} v_i^{(q)}(x_H)\widetilde{\Pi}_{ij}(t_y; \Gamma)v_j^{\dagger(q')}(x_H)$.

where $\widetilde{\Pi}(t_y; \Gamma)$ is some product of MFs.

All-to-All Current Insertion





Stochastic solves:

- All propagators treated in A2A fashion
- Noise on current insertion unavoidable

Sequential solves:

- Removes noise at current insertion
- No t_J translation $\implies$ loss of statistics

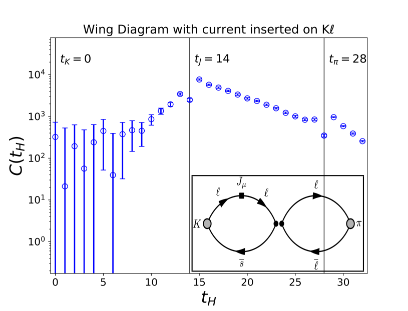
Physical Point Runs

Physical Point Setup

$48^3 \times 96$ gauge configuration [T. Blum et al. arXiv:1411.7017]

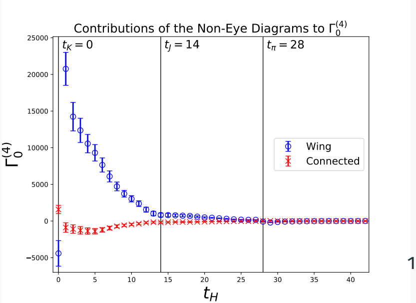
- $M_\pi \approx 140$ MeV, $M_K \approx 500$ MeV
- $a^{-1} = 1.73 GeV$
- $2+1$ flavor
- Scaled DWF action for strange quarks
- ZMöbius DWF action for light quarks
- 2000 low modes for light quarks $\implies$ deflated solves
- Spin/color/time diluted high modes
- $\mathbf{p}_K = (0,0,0)$, $\mathbf{p}_\pi = \frac{2\pi}{L}(1,0,0)$

Physical Point Preliminary Results



- Same source/sink separation as exploratory 24^3 runs
- Local current insertions
- Coulomb gauge fixed wall sources

- With Wilson coefficients and quark charges
- Analysis: remove unphysical exponential terms and integrate over t_H



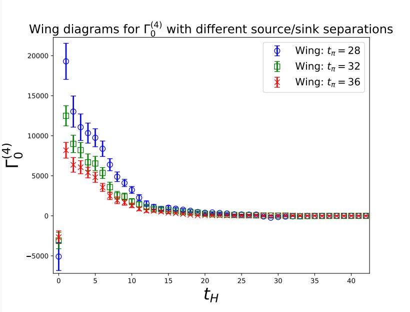
Future Outlook and Conclusions

Future Outlook

$N_t = 96$ gives more leeway for source/sink separation

- Expected range already investigated

[P.A. Boyle et al. arXiv:1504.01692]



For Eye diagrams:

- Use A2A vector for the loops
- For loop divergence we can:
  - Compute charm quark loop & employ GIM mechanism
  - Calculate in 3-flavor theory & use NPR

For disconnected diagrams:

Same approach as James Richings' QED talk (Wed 11:50)

Summary

- A2A vectors have been implemented in Grid, with modularization in Hadrons
- They are a powerful tool for approximating quark propagators
  - But they are not always appropriate to use for a full calculation
  - A2A propagators will be used to supplement the $K \to \pi\ell^+\ell^-$ calculation
- Physical point simulations have already begun
  - Eye diagrams and more kinematics to follow

Thank you.

- C. Parkinson: $K \to \pi\mu^+\mu^-$ at NA62 - The 2nd Forum on Rare Kaon Decays 2019
- G. D'Ambrosio et al.: The Decays $K \to \pi\ell^+\ell^-$ beyond Leading Order in the Chiral Expansion - arXiv:hep-ph/9808289
- V. Cirigliano et al.: Kaon Decays in the Standard Model - arXiv:1107.6001
- A. Sturgess: Tracking Optimisation and the Measurement of $K \to \pi\mu^+\mu^-$ at NA62 - Thesis (University of Birmingham)
- Gino Isidori et al,: Rare Kaon Decays on the Lattice - arXiv:hep-lat/0506026
- J. Foley et al,: Practical all-to-all propagators for lattice QCD - arXiv:hep-lat/0505023
- T. Blum et al.: Domain wall QCD with physical quark masses - arXiv:1411.7017
- P.A. Boyle et al.: The kaon semileptonic form factor in $Nf = 2 + 1$ domain wall lattice QCD with physical light quark masses - arXiv:1504.01692