Yang Mills short distance potential and perturbation theory

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Wuhan, Lattice 2019, June 17

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- α_s world average is dominated by lattice determinations
  FLAG 2019:

  $$\alpha^{(5)}_{\overline{\rm MS}}(M_Z) = 0.11823(81)$$

  3.5% precision on

  $$\Lambda^{(3)}_{\overline{\rm MS}} = 343(12)$$

  with phenomenology determinations (PDG 16/18)

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- perturbative potential enters predictions of e+e-
  cross-section across t-tbar `threshold' —> top quark mass at linear collider

α_s from static potential

- Advantages
  - 4-loop PT (4-loop beta-function) available, (normally only 3-loop)
  - relatively simple observable (no fermions …)

Peter 97; Schröder 99
Anzai, Kiyo, Sumino, 10
Smirnov, Sminov, Steinhauser, 10
Brambilla, Pineda, Soto, Vairo, 99
Kniehl, Penin, 99
Brambilla, Garcia i Tormo, Soto, Vairo, 07,09

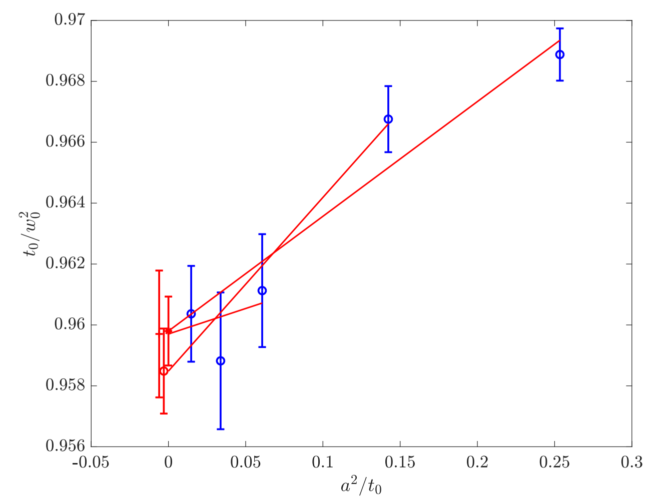
- Disadvantages
  - infrared divergences, starting at 4-loop, resummation with pNRQCD techniques $\alpha^{4+k}\,[\log(\alpha)]^k\,, \quad k = 0,1,\ldots$

    or US - scale $\alpha(1/r)\,,\ \alpha(\mu_{\rm US})\,, \quad \mu_{\rm US} \sim \dfrac{C_{\rm A}\alpha}{2r}$
  - T(loop) —> ∞ limit, noise
  - Discretisation errors at small r, window problem

Pure Yang Mills

- Precision, study of systematic effects
- Comparisons: two recent determinations of $\Lambda_{\overline{\rm MS}}$

conversion is very precise



  - FlowQCD: $w_0\,\Lambda_{\overline{\rm MS}} = 0.2154(12)$
    converted (by us): $\sqrt{8t_0}\,\Lambda_{\overline{\rm MS}} = 0.5968(33)$

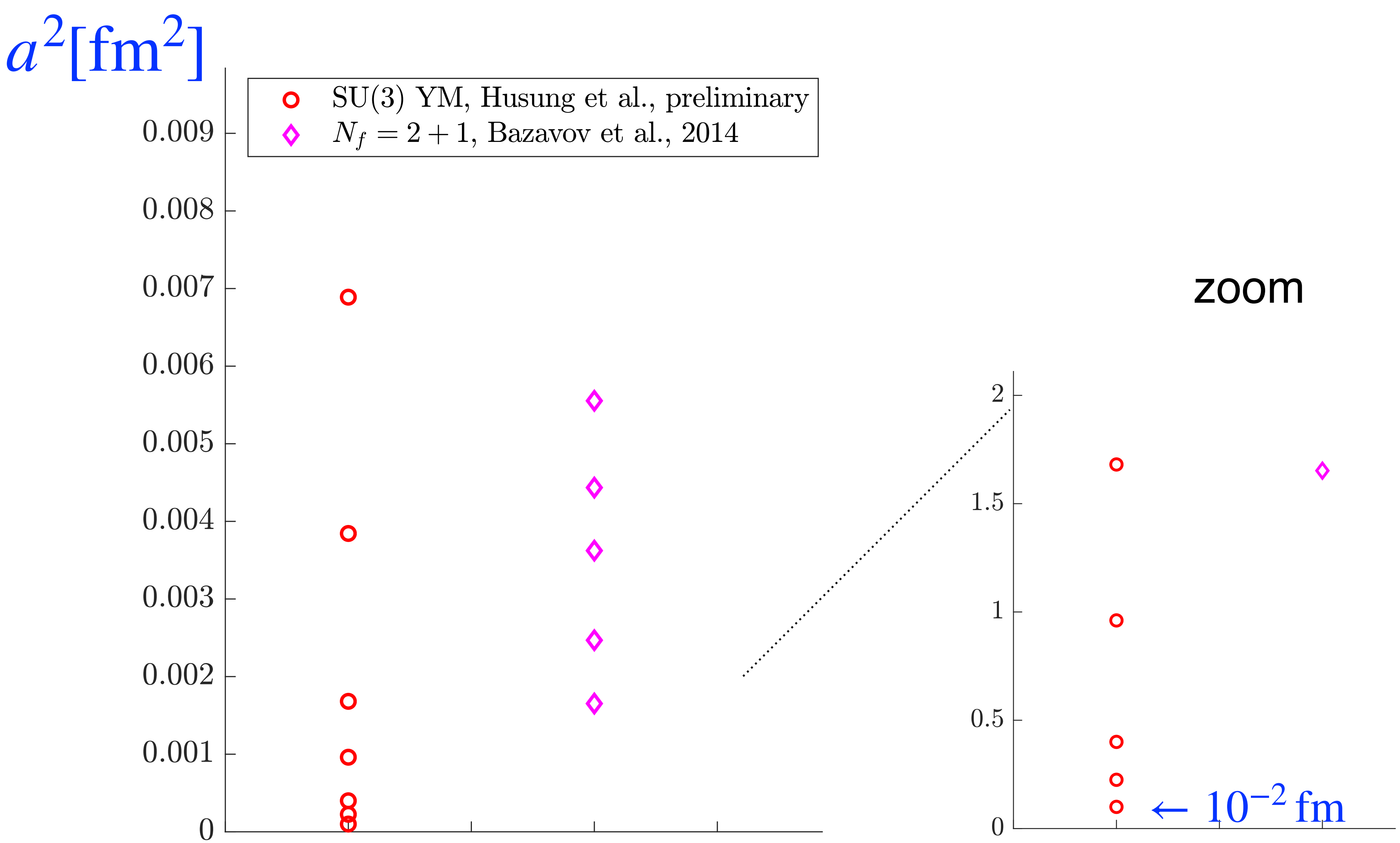
    from boosted coupling,
    contin. extrapolation $\beta \to \infty$ with a^2
    perturbative uncertainty?

  - Dalla Brida and Ramos, 2019: $\sqrt{8t_0}\,\Lambda_{\overline{\rm MS}} = 0.6227(98)$

    NP step scaling down to $\alpha_{\rm SF} < 0.1$

Scales, lattices

- L/a=32, … 192, L=2fm (big enough in YM), open BC (no topology freezing)



Strategy to get to small r (see also arXiv:1711.01860)

- basic scale from t_0 :

$$\alpha_{\mathrm{qq}}(\mu, a^2\mu^2)\,, \quad \mu = 1/r = (x\sqrt{8t_0})^{-1}$$

on ensembles with a > 0.02 fm

- Then step scaling functions

$$\Sigma(u, a/r) = \bar{g}^2_{\mathrm{qq}}(sr)\Big|_{\bar{g}^2_{\mathrm{qq}}(r)=u}$$

with $s = 3/4$ including $a = \{1.0, 1.4, 2.0\} \times 10^{-2}\,\mathrm{fm}$

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$$\alpha_{\rm qq}(\mu, a^2\mu^2)\,, \quad \mu = 1/r = (x\sqrt{8t_0})^{-1}$$

$$0.25 \leq x \leq 0.4$$

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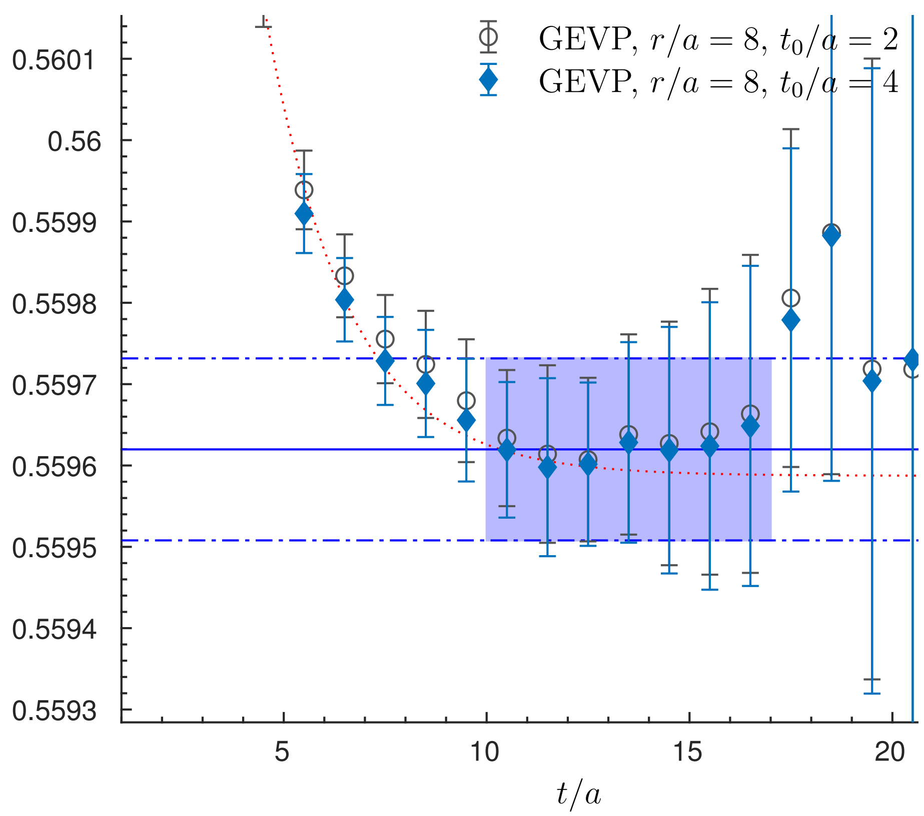
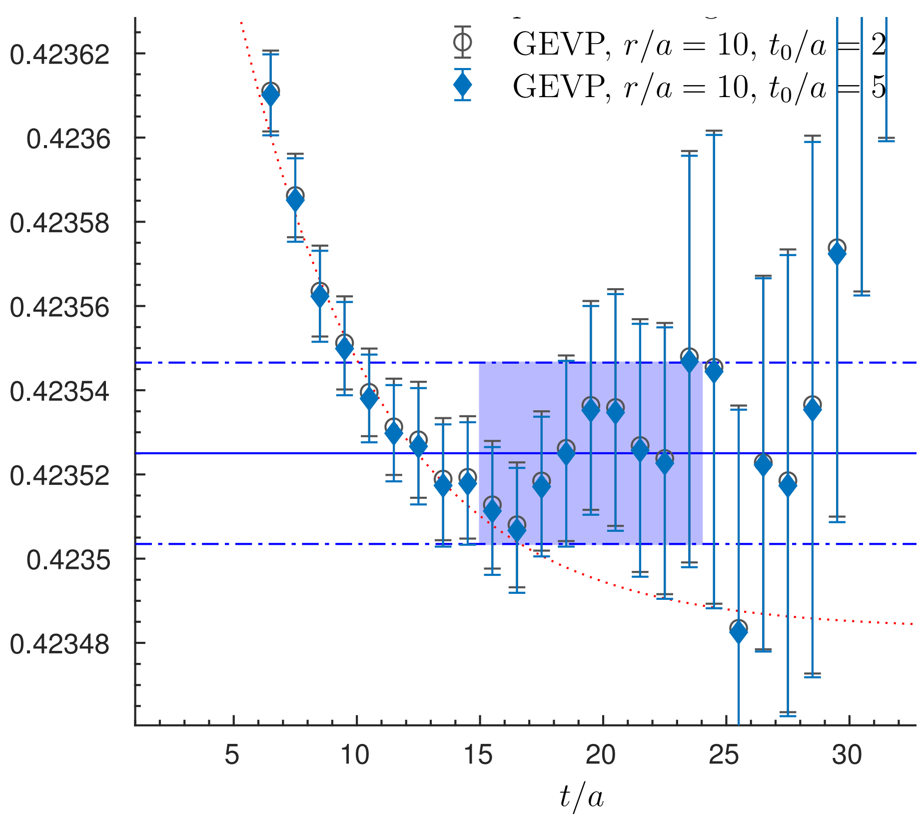
Plateaux

- 1-link integral
- GEVP with

3 trial wave-functions
a=0.03 fm   r=0.24 fm



2 trial wave-functions
a=0.01 fm   r=0.10 fm



Force

- Tree-level improved force, r_I such that

$$F_\mathrm{TI}(r_\mathrm{I}) = \frac{1}{a}[V(r+a) - V(r)] = \frac{4}{3}\frac{\alpha}{r_\mathrm{I}^2} + \mathrm{O}(\alpha^2)$$

- Tree-level improved force with a^2 improved derivative:

$$F_\mathrm{n}(r_\mathrm{n}) = \tfrac{1}{a}[V(r+a) - V(r)] = \tfrac{1}{a}[V(r_\mathrm{n}+a/2) - V(r_\mathrm{n}-a/2)]\,, \quad r_\mathrm{n} = r + a/2$$

$$F_\mathrm{impr}(r_\mathrm{n}) = \tfrac{13}{12}F_\mathrm{n}(r_\mathrm{n}) - \tfrac{1}{24}[F_\mathrm{n}(r_\mathrm{n}+a) + F_\mathrm{n}(r_\mathrm{n}-a)]$$

$$(\tilde{r}_\mathrm{I})^{-2} = \tfrac{13}{12}(r_\mathrm{I}(r))^{-2} - \tfrac{1}{24}[(r_\mathrm{I}(r+a))^{-2} + (r_\mathrm{I}(r-a))^{-2}]$$

$$\alpha_\mathrm{qq}(1/r,a) = \alpha_\mathrm{qq}(1/r,0)\left\{1 + (a/r)^2\,[\alpha(1/a)]^{-\hat{\gamma}_0}\,A_1(r)\Big[1 + \mathrm{O}([\alpha(1/a)]^{-\Delta\hat{\gamma}})\Big]\right\} + \mathrm{O}(a^4)$$

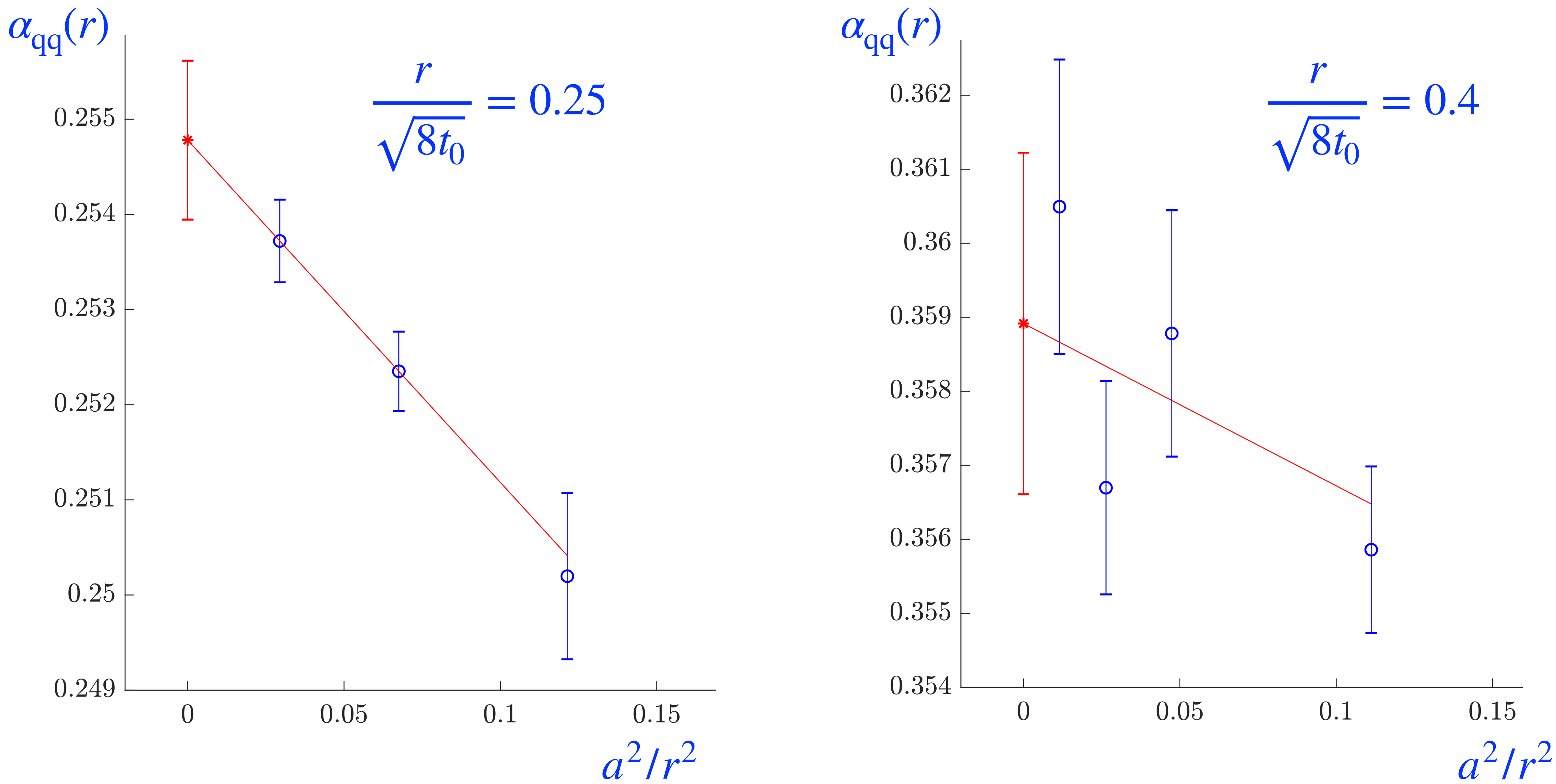
$$A_i(r) = \bar{A}_i\alpha(1/r)^{1+\hat{\gamma}_i}[1 + \mathrm{O}(\alpha(1/r)]\,.$$

$$-\hat{\gamma}_0 = 7/11 = 0.686$$

smallest eigenvalue of 1-loop anomalous dimension matrix
of Symanzik EFT d=6 operator basis [new, talk by N. Husung]

Continuum limits

- Large r region (r > 0.1fm)



- Gradient flow: log-corrections to a^2 not yet known.

Continuum limits

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- Small r region (r < 0.1fm), step scaling functions

(1) select $\alpha_i \in \{0.205, 0.215, \ldots, 0.5\}$ (qq-scheme)

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(2) fit $\Sigma(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11}\rho_i$

$\alpha(1/a)$ from $\alpha_{qq}(1/(2.5a))$ by 4-loop running

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(3) fit slopes to

$$\rho_i = \rho(\alpha_i)\,, \quad \rho(\alpha) = \alpha^{(2-7/11)} \times [\rho^{(0)} + \rho^{(1)}\alpha + \rho^{(2)}\alpha^2]$$

(4) use fitted slope function $\rho(\alpha)$ in

$$\Sigma(\alpha, s, a/r) = \sigma(\alpha, s) + (a/sr)^2[\alpha(1/a)]^{7/11}\rho(\alpha)$$

for all α in range

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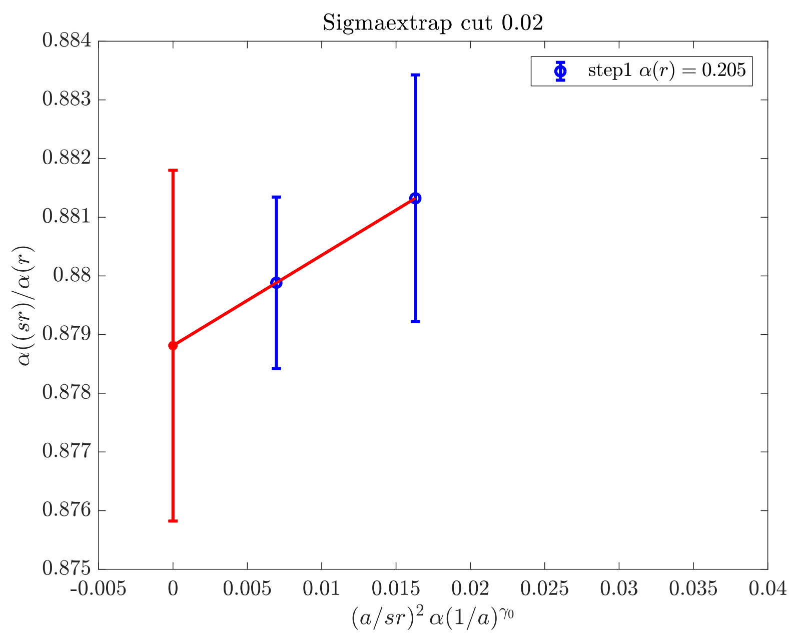

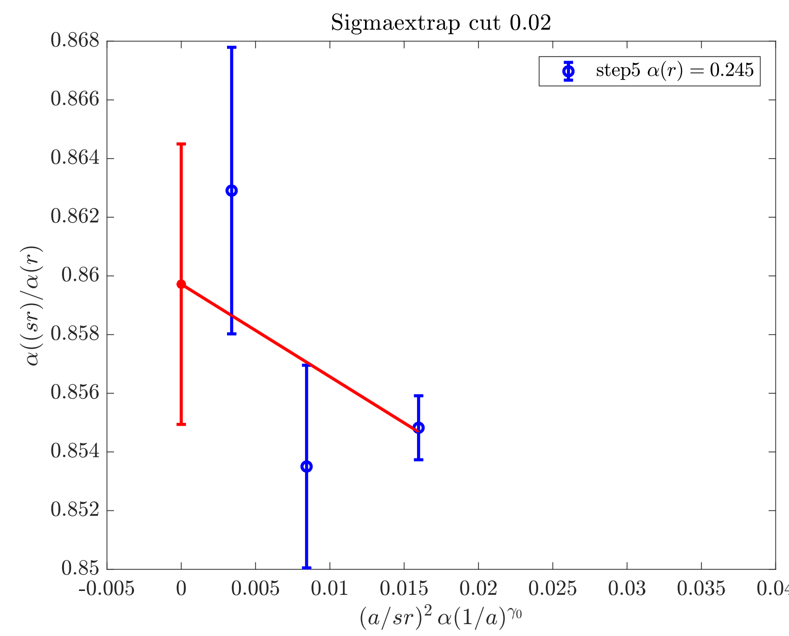

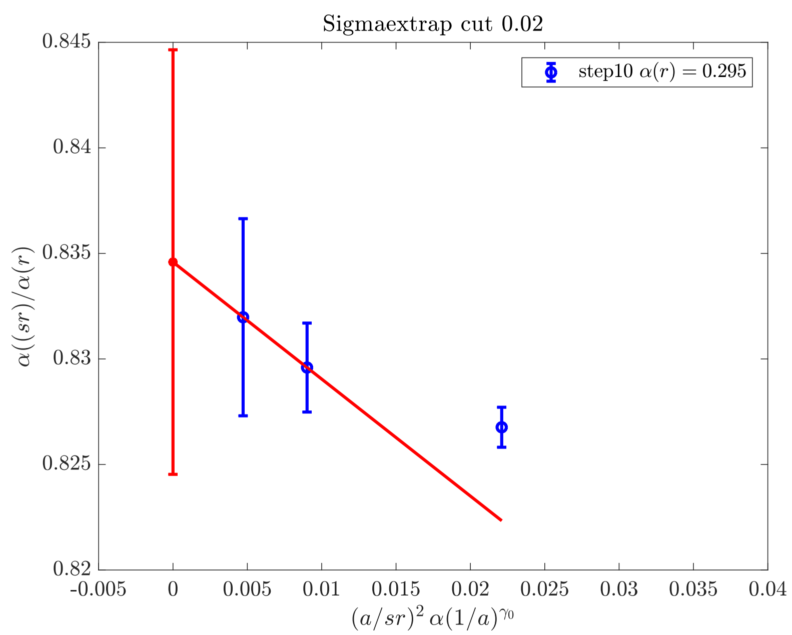

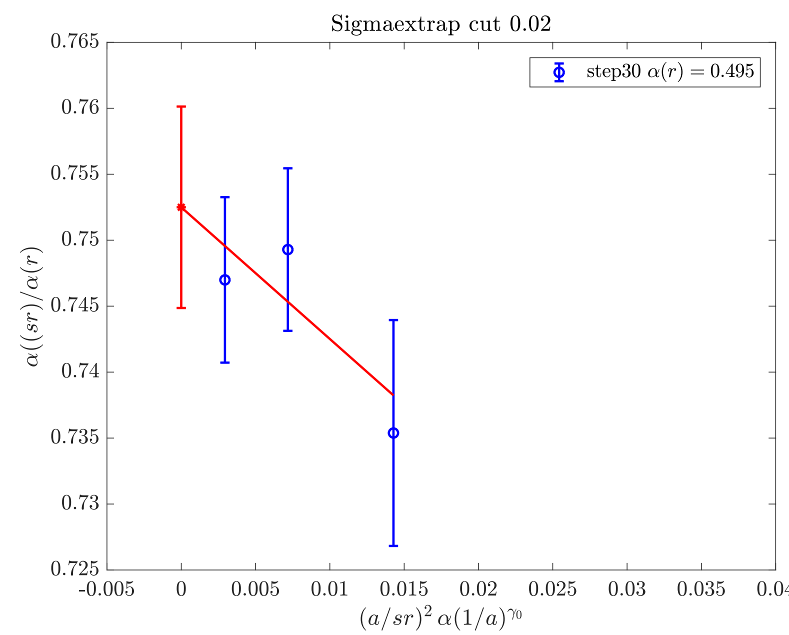

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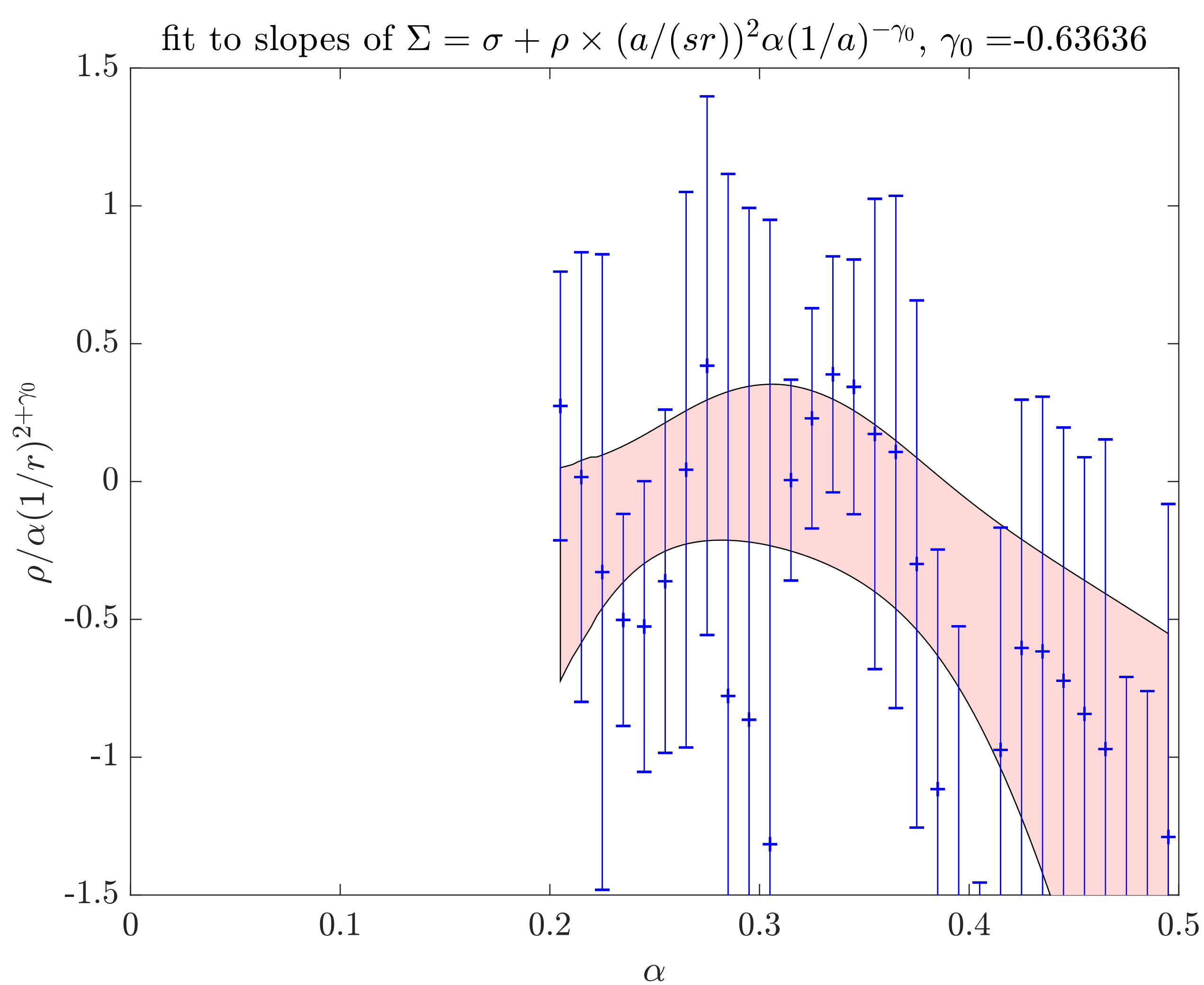

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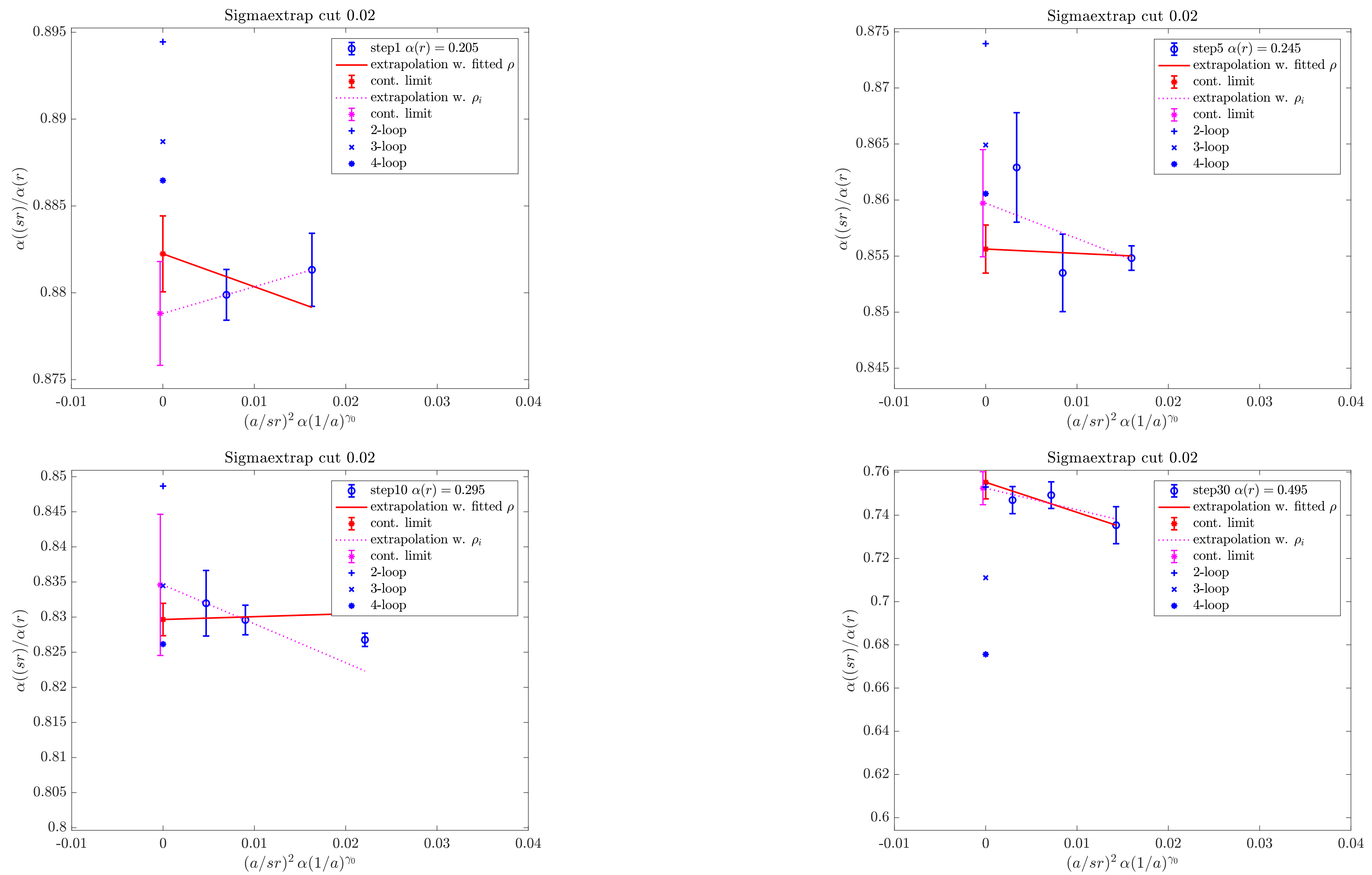

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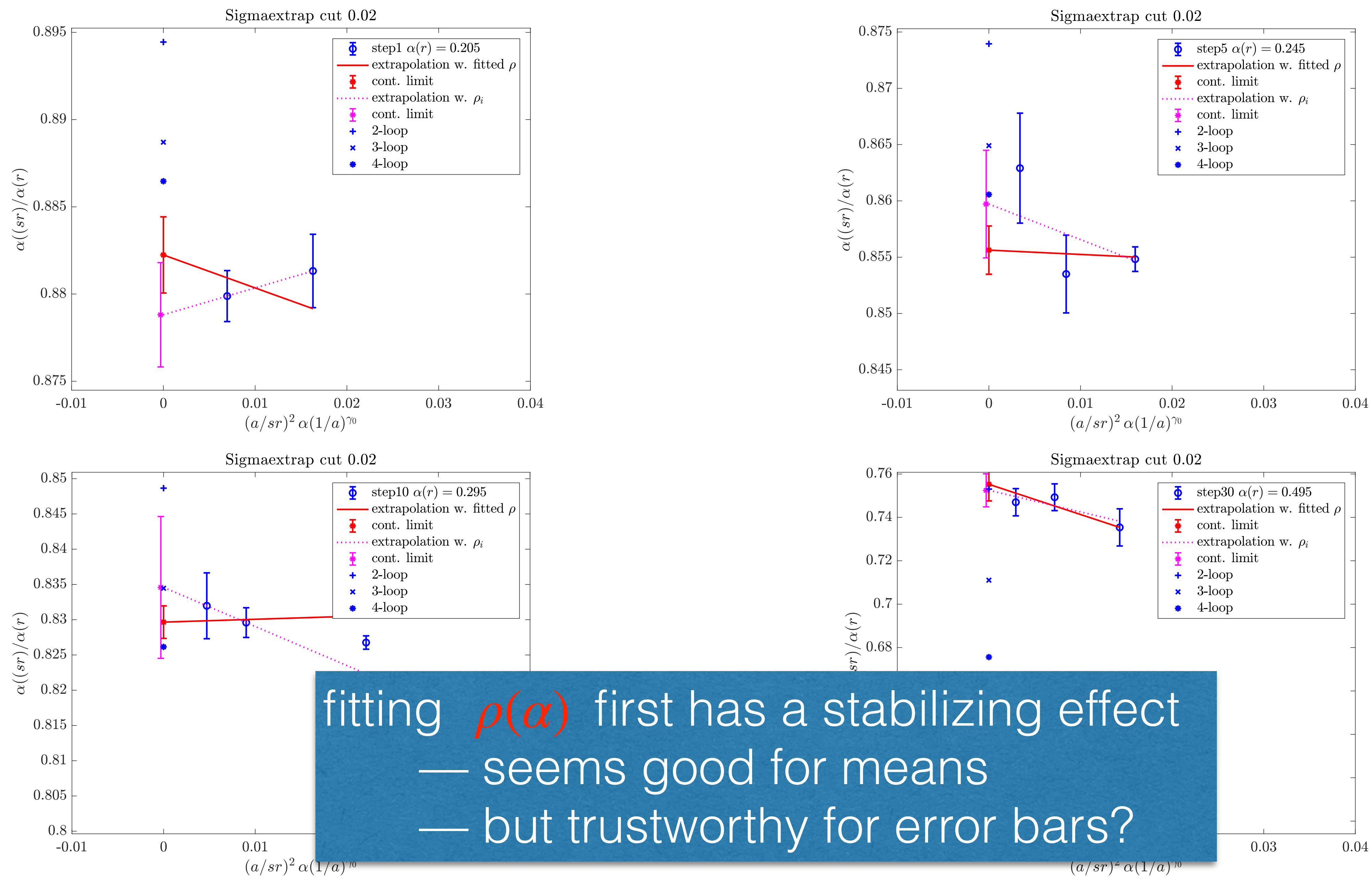


fitting $\rho(\alpha)$ first has a stabilizing effect

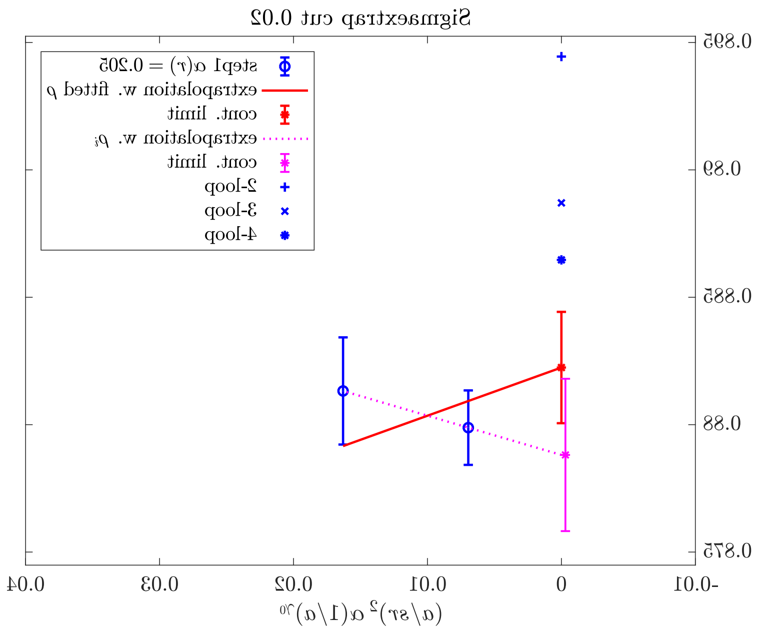
— seems good for means

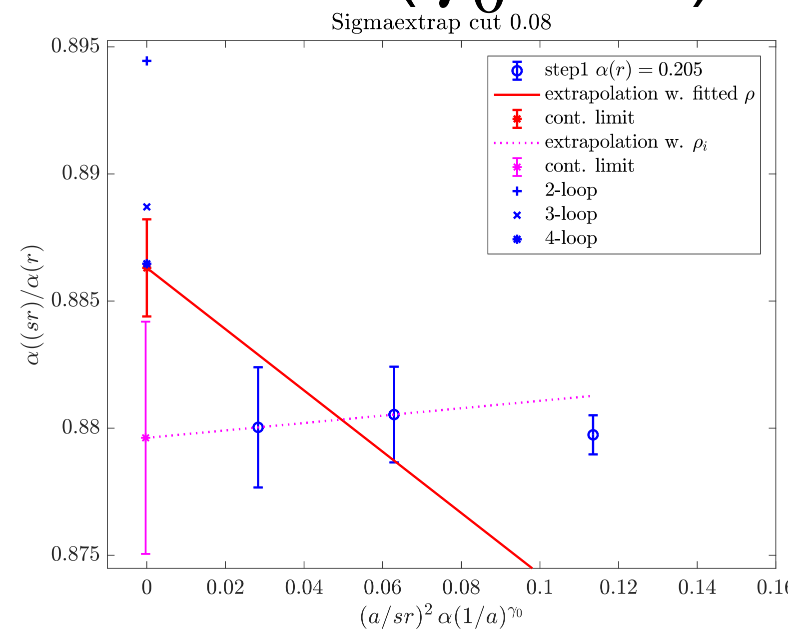
— but trustworthy for error bars?

Continuum limits: compare to standard derivative

a^2 improved derivative

standard derivative $(\gamma_0 = 0)$

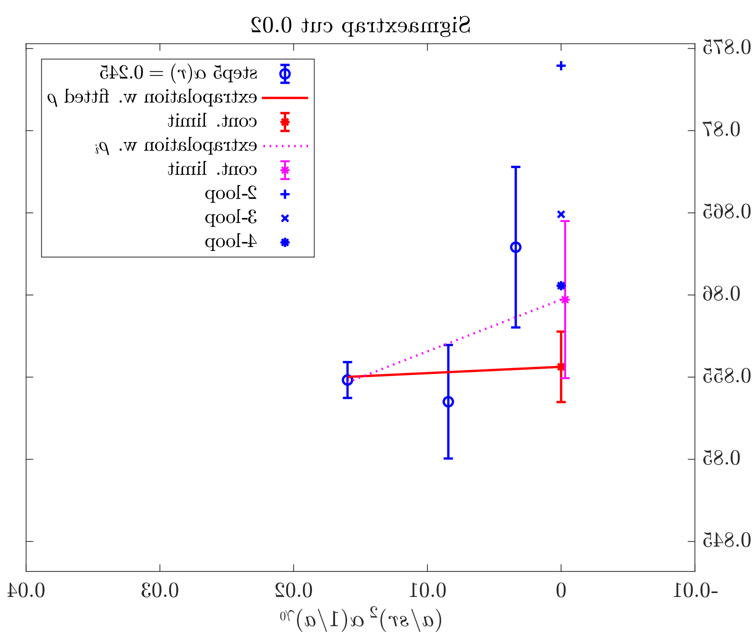


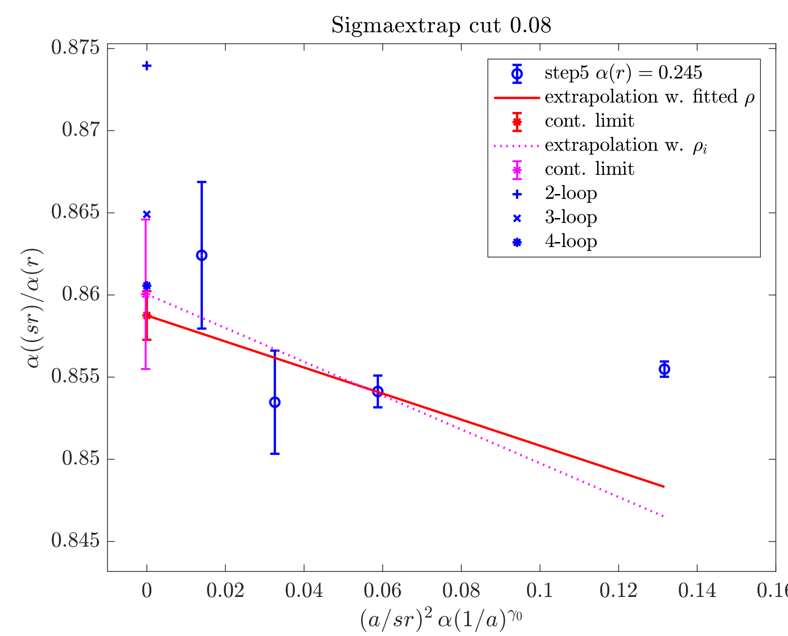


more points, but lost by cut

$$\left(\frac{a}{sr}\right)^2 \le 0.08$$

$$\left(\frac{s\,r}{a} > 3.5\right)$$





same continuum limits

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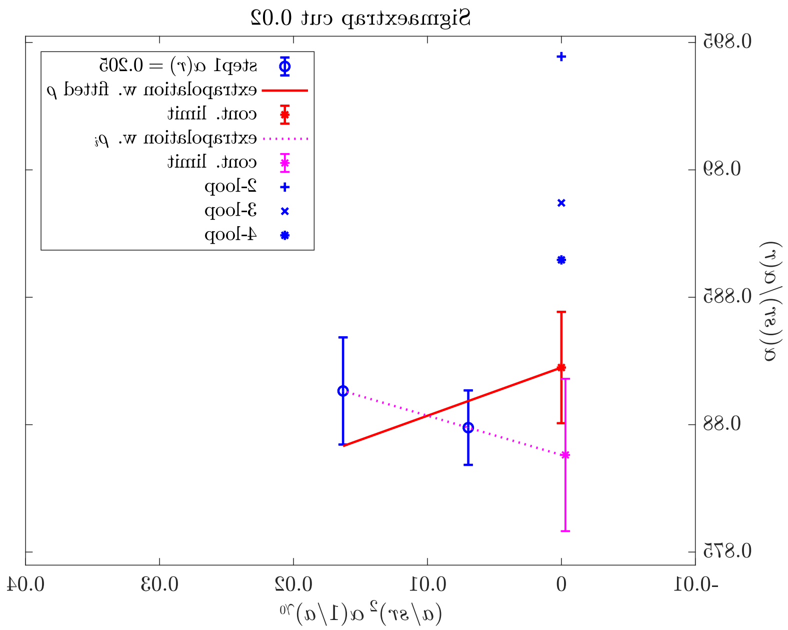
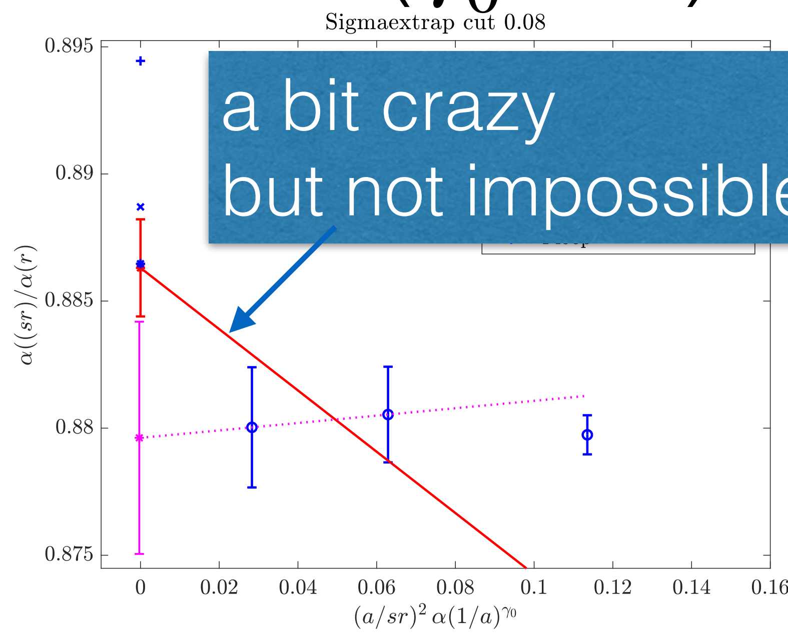
standard derivative ($\gamma_0 = 0$)





a bit crazy
but not impossible

more points,
but lost
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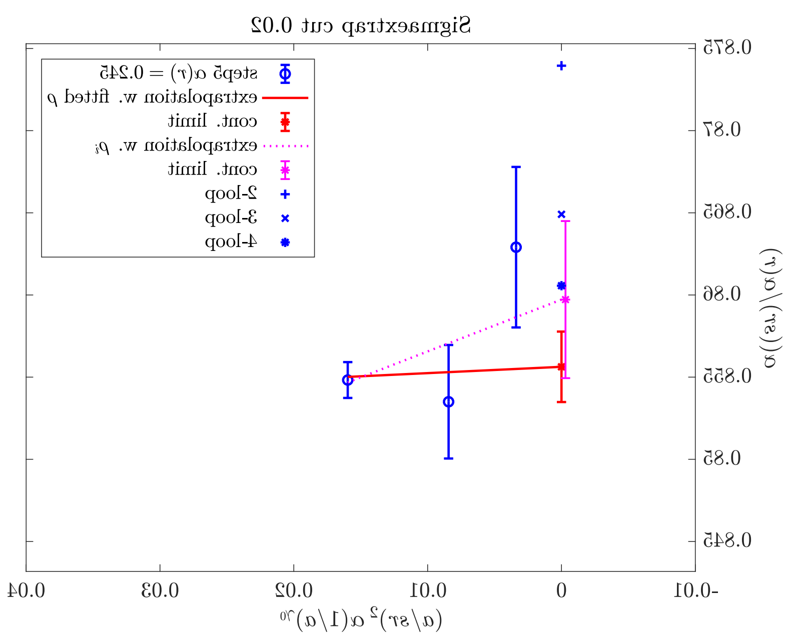
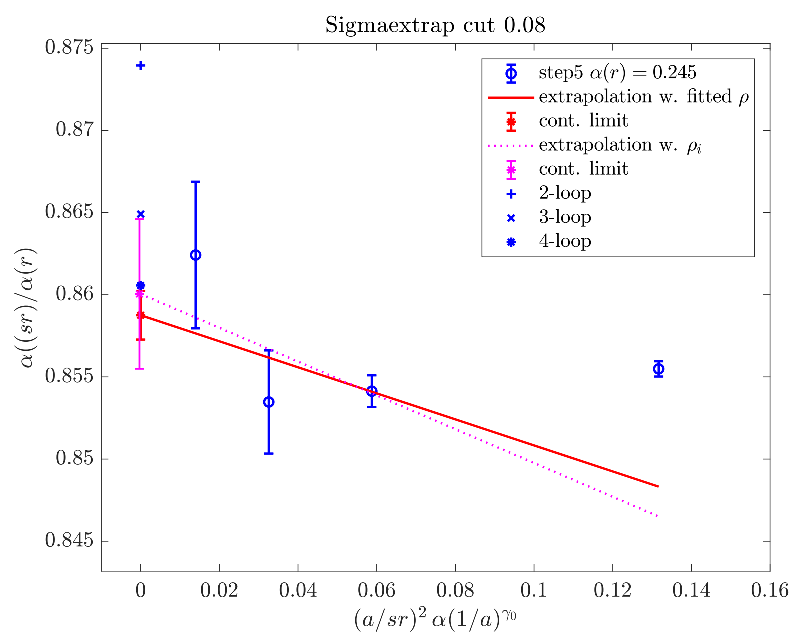
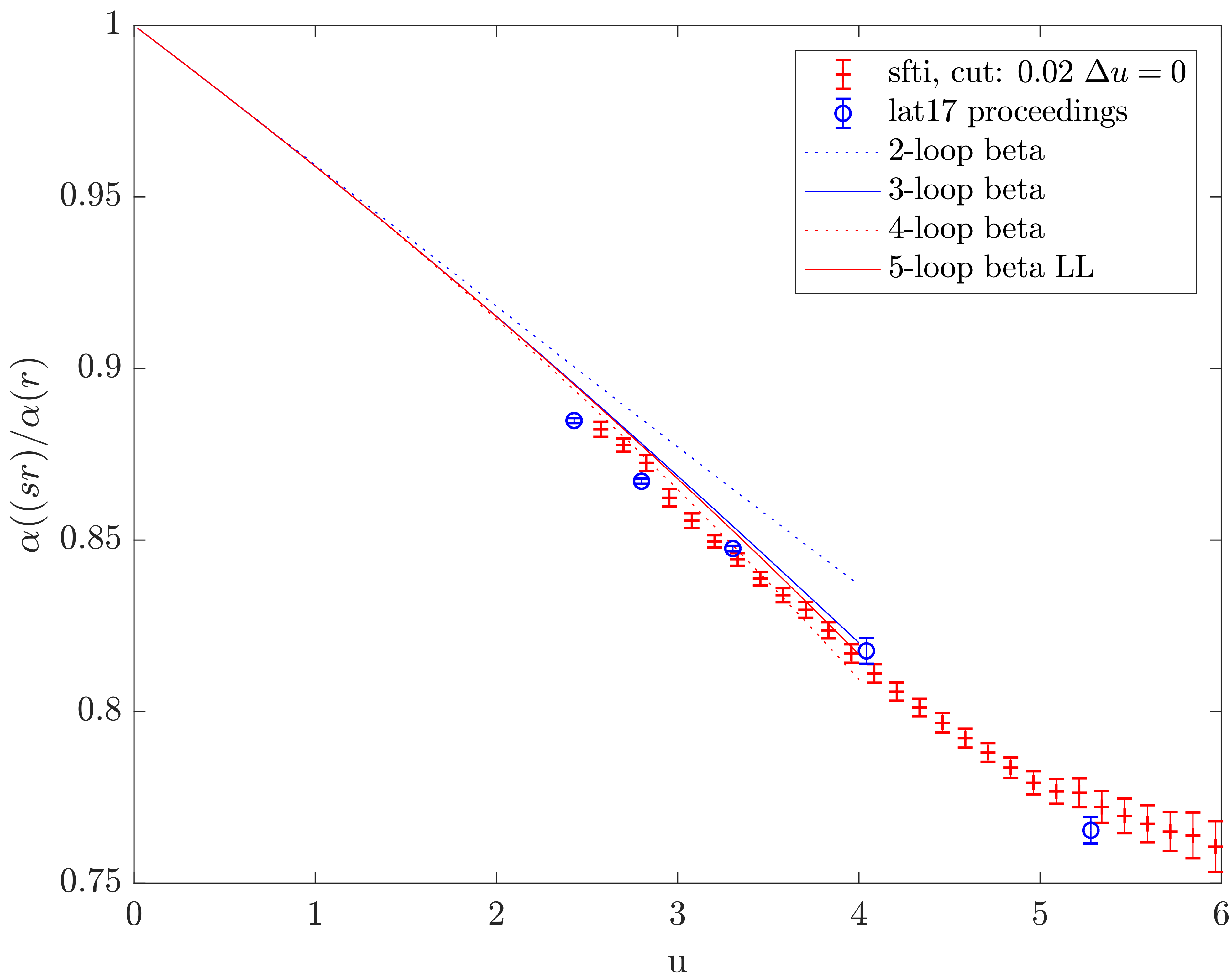




same
continuum
limits

Results (from 2-stage continuum limit, improved derivative)



Λ - parameter

- $\Lambda_{\overline{\rm MS}}$ from $\Lambda_{\rm qq}$ locally ($\alpha_{\rm qq}$ by $\alpha_{\rm qq}$)

$$\Lambda = \mu \left(b_0\overline{g}(\mu)^2\right)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\overline{g}(\mu)^2)} \exp\left\{-\int_0^{\overline{g}(\mu)} \mathrm{d}x \left[\frac{1}{\beta_{\rm n-loop}(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

$$\times \left[1 + \mathrm{O}(\overline{g}(\mu)^{2(n-1)})\right]$$

$$\beta_{3-\rm loop}(g) = -\,g^3\,[\,b_0 + b_1g^2 + b_2g^4]$$

4-loop: $+b_3g^6 + b_{3\rm L}g^6\log(\alpha)$

4-loop LL: $+b_{4\rm L}g^8\log(\alpha)+b_{4\rm LL}g^8[\log(\alpha)]^2$

computed from

Peter 97; Schröder 99
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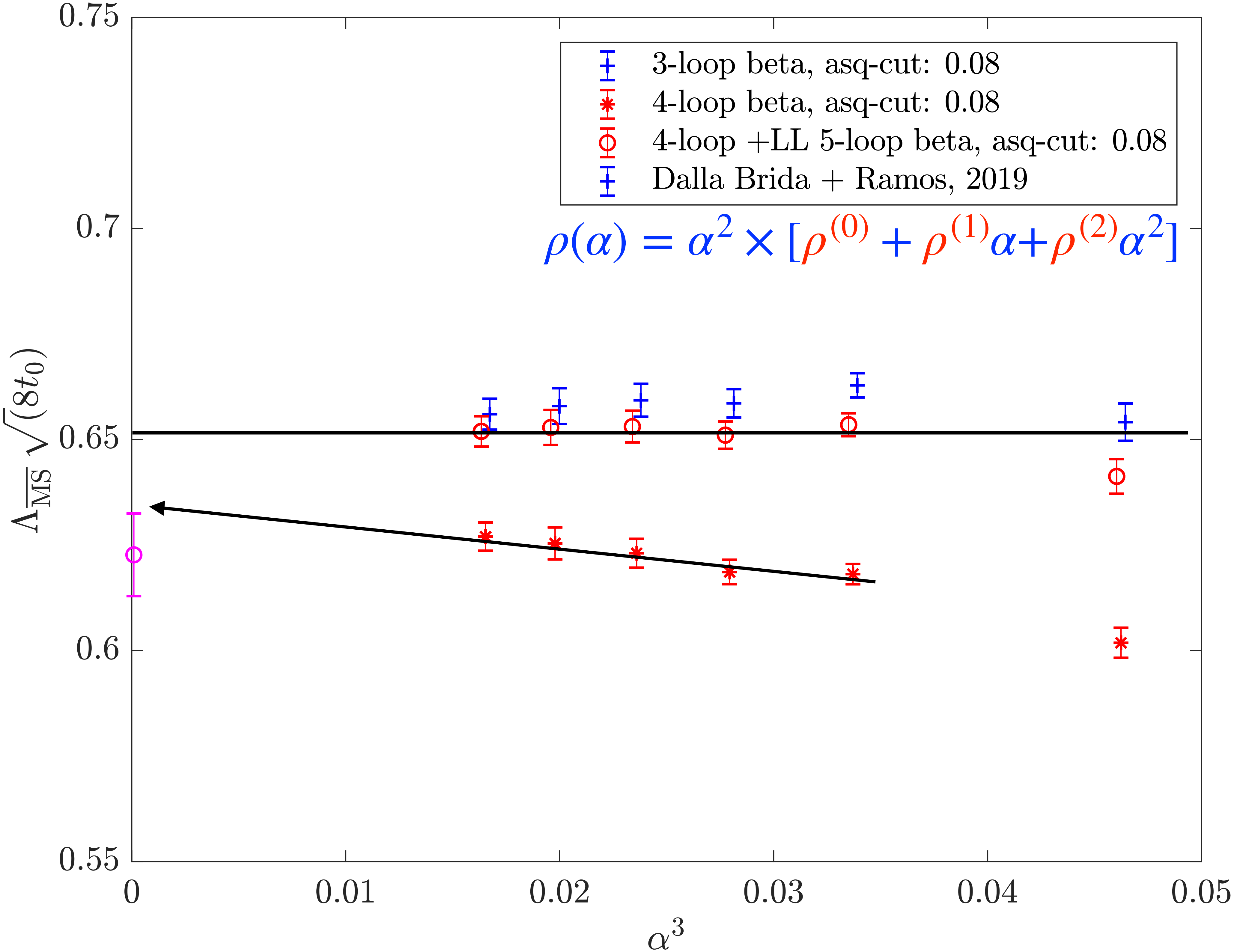

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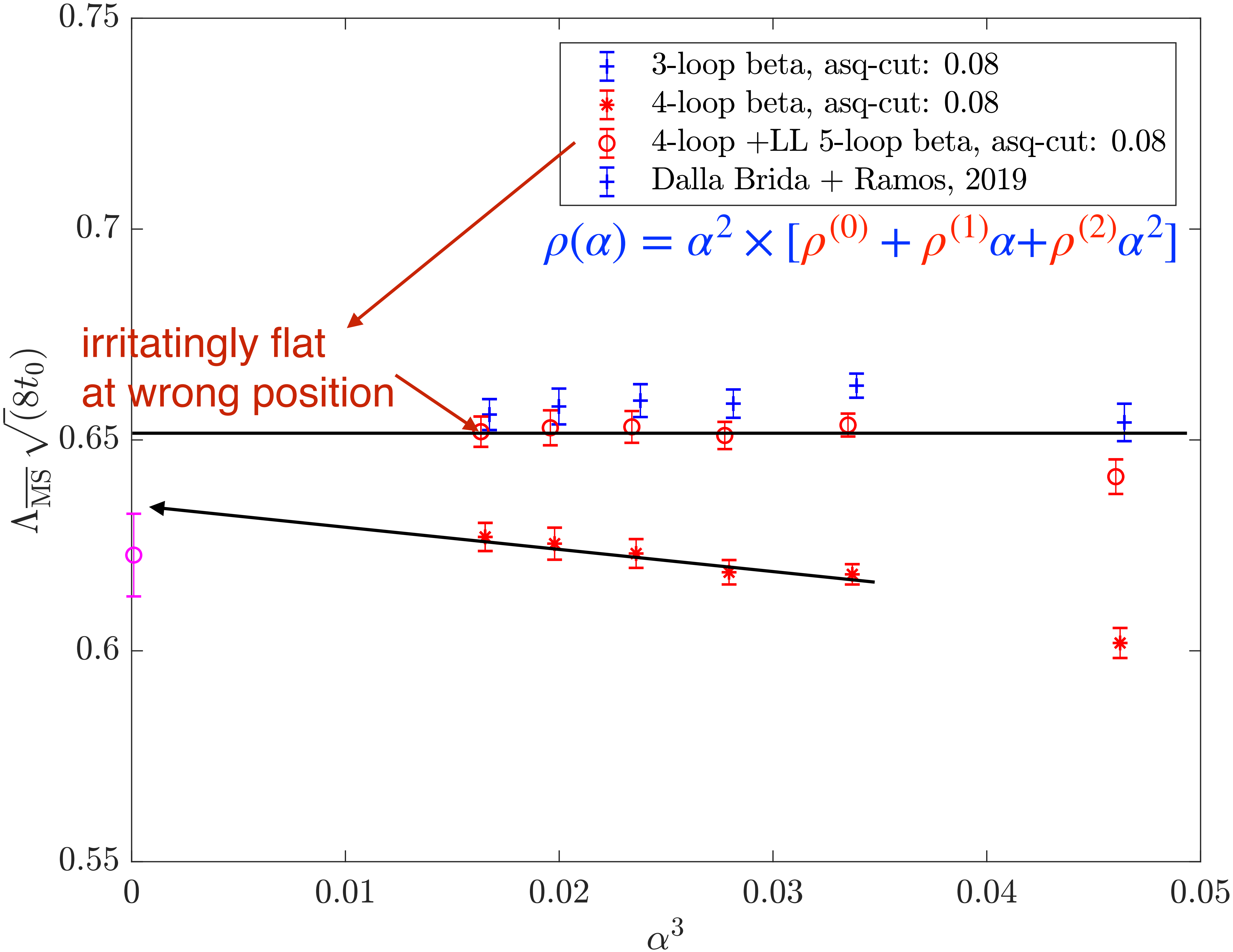

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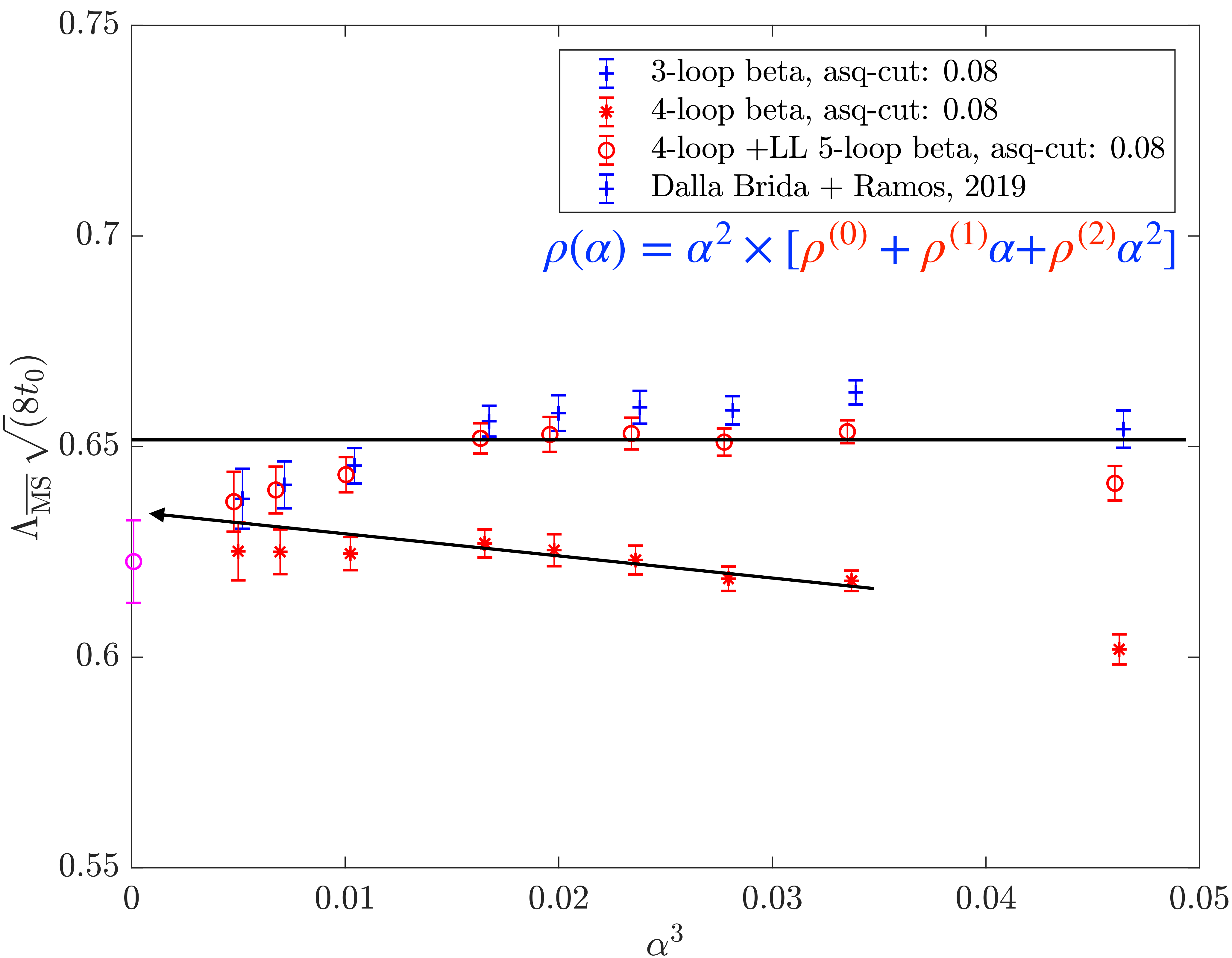

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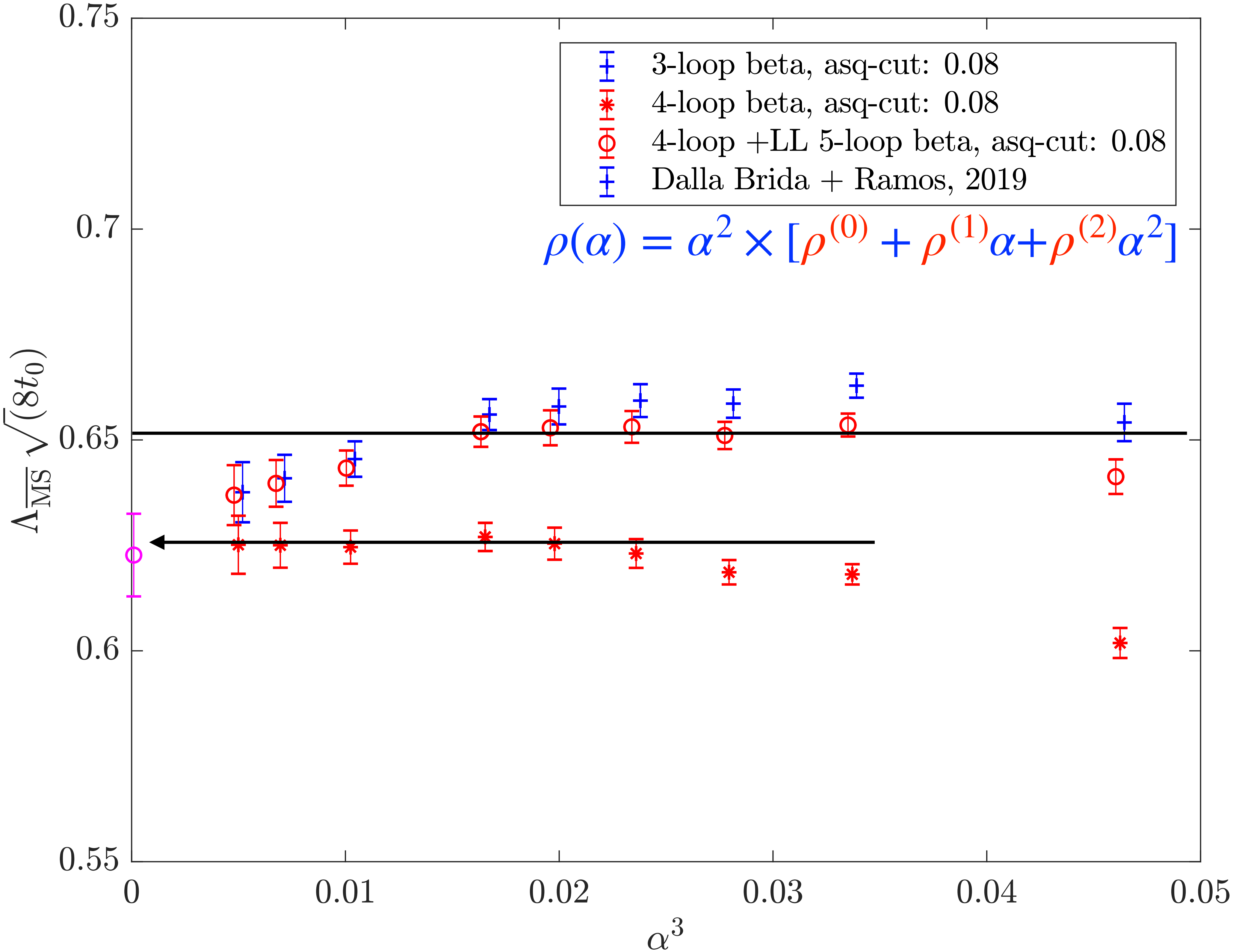

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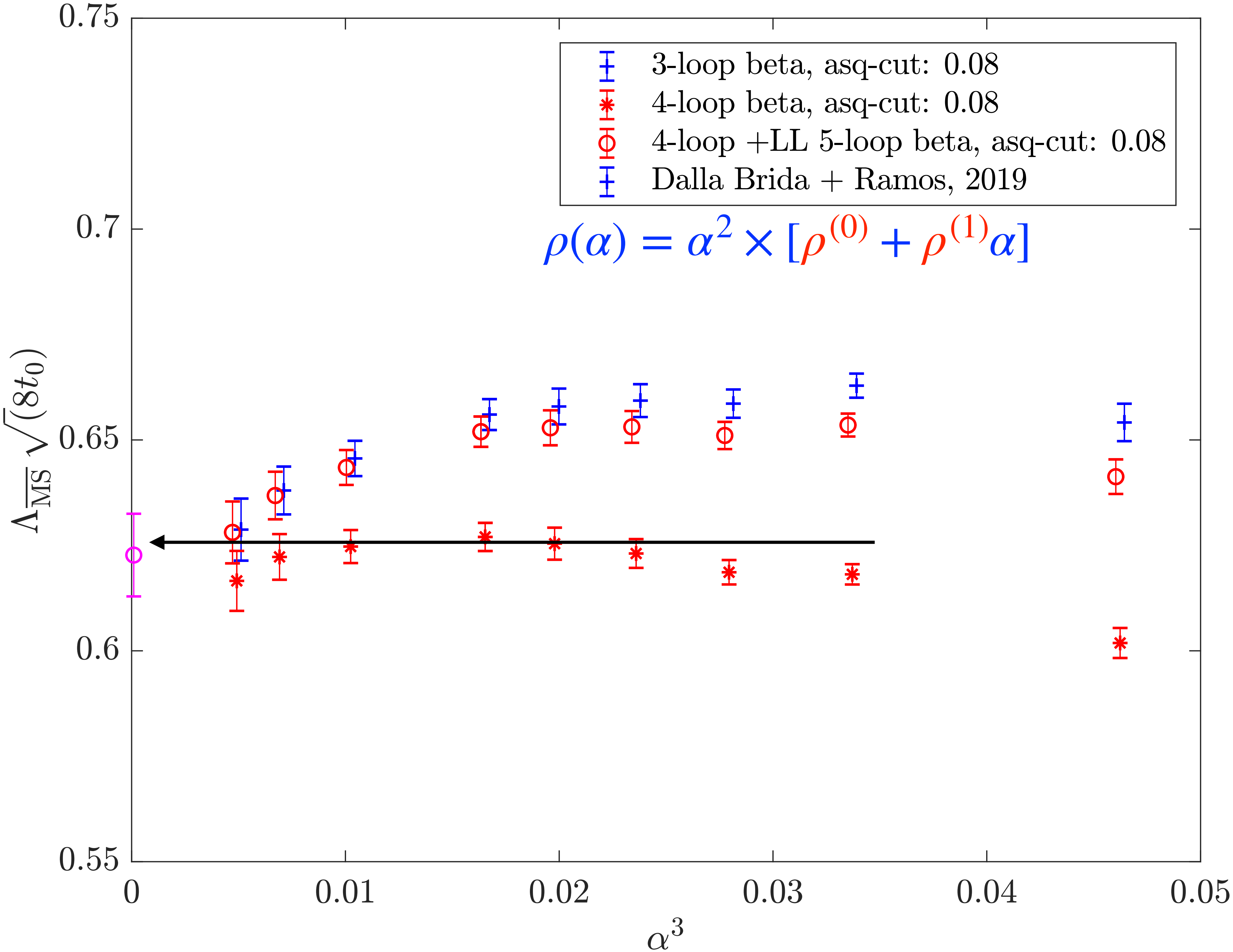

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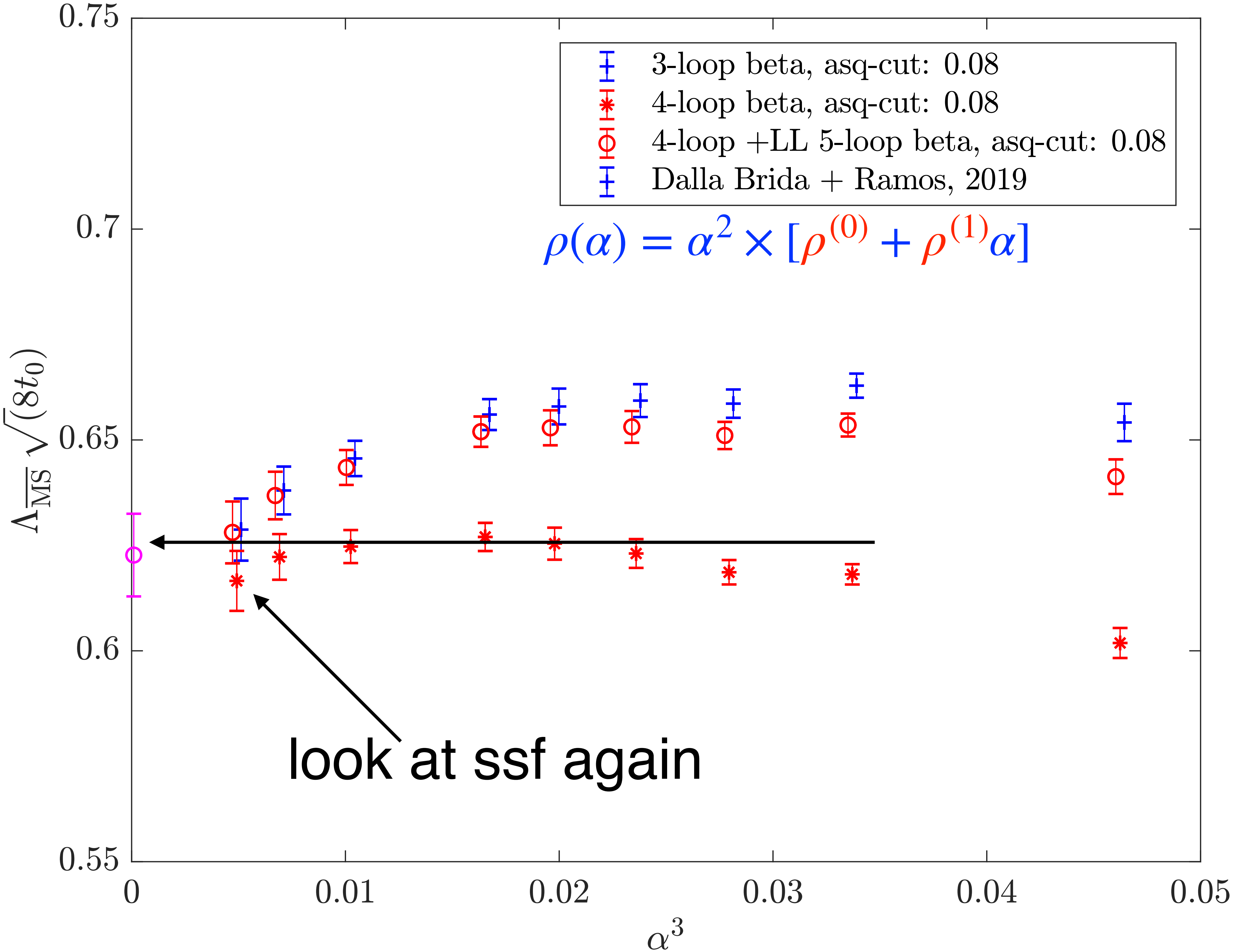
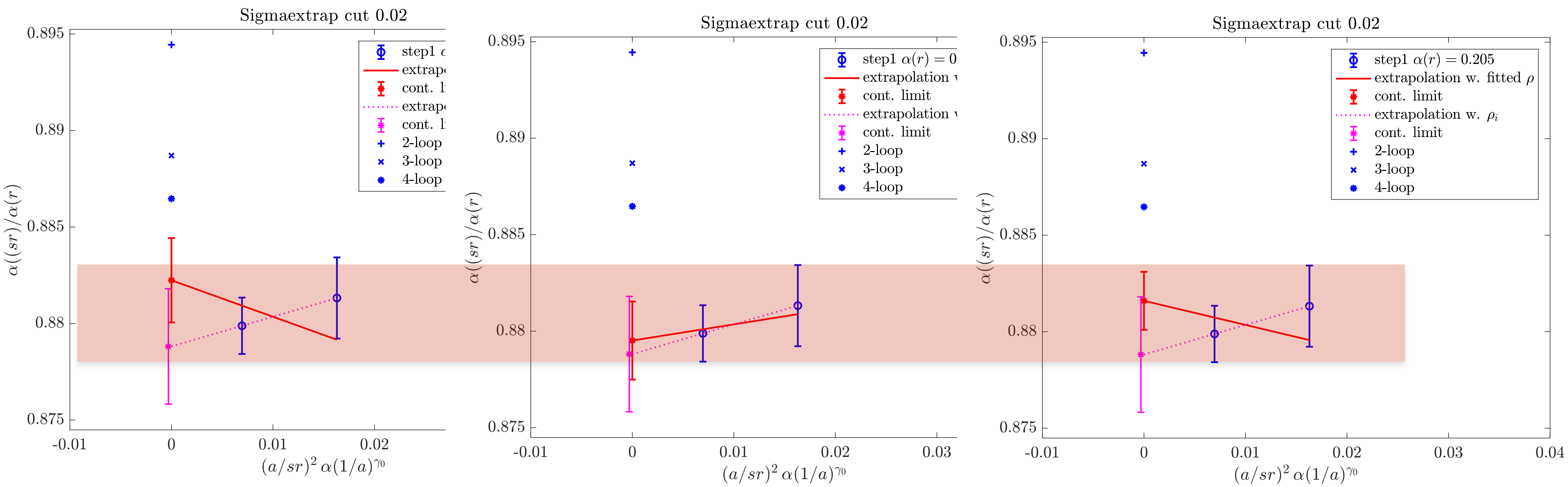


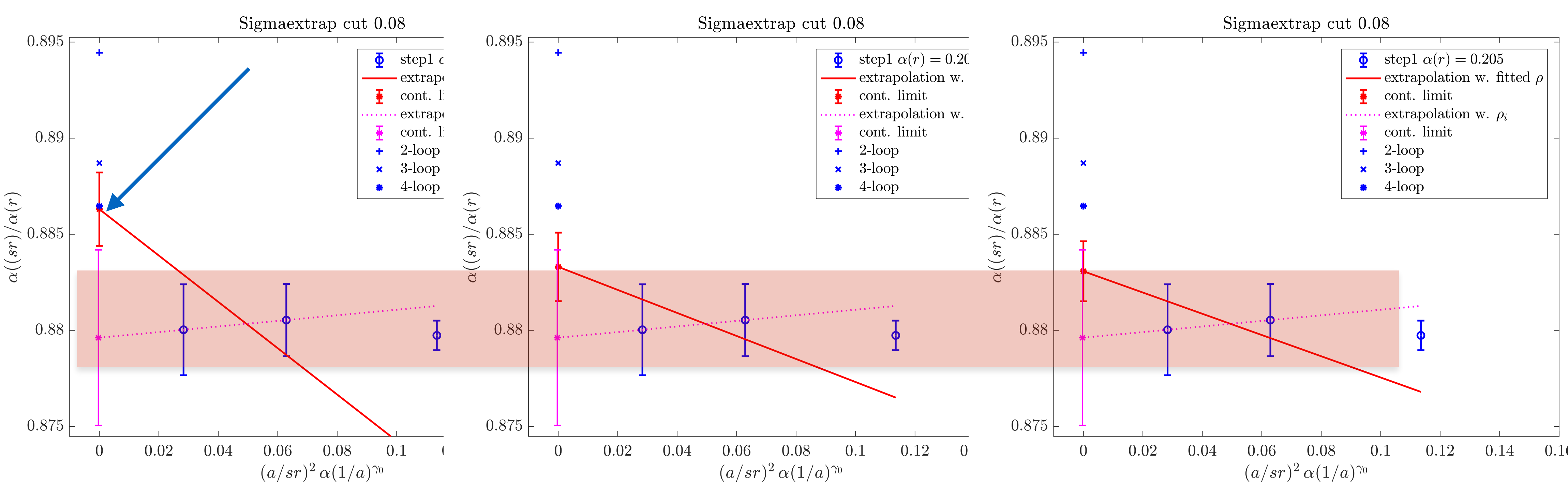
Continuum limits: compare to standard derivative

a^2 improved derivative



$\gamma_0 = 7/11$

standard derivative



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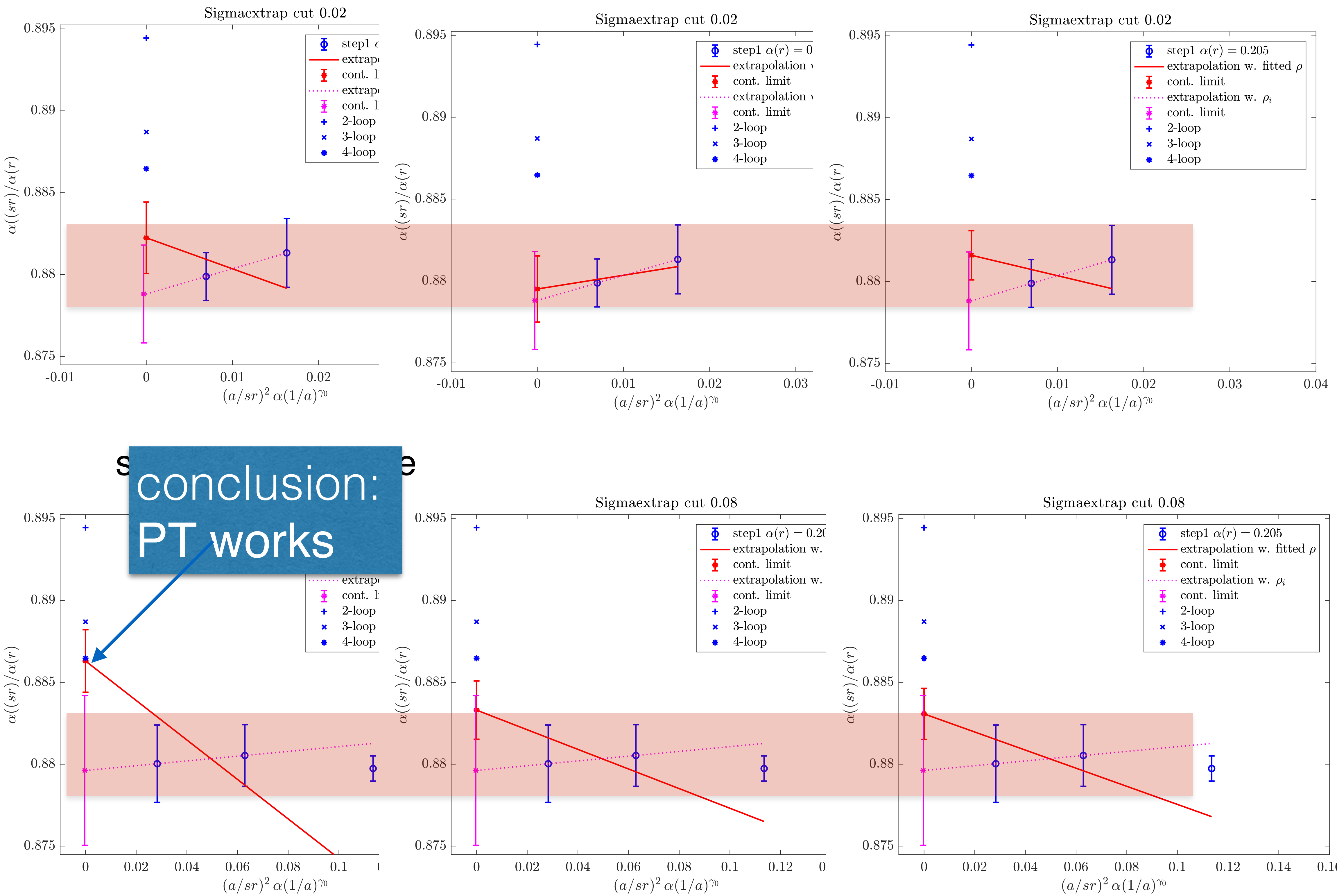

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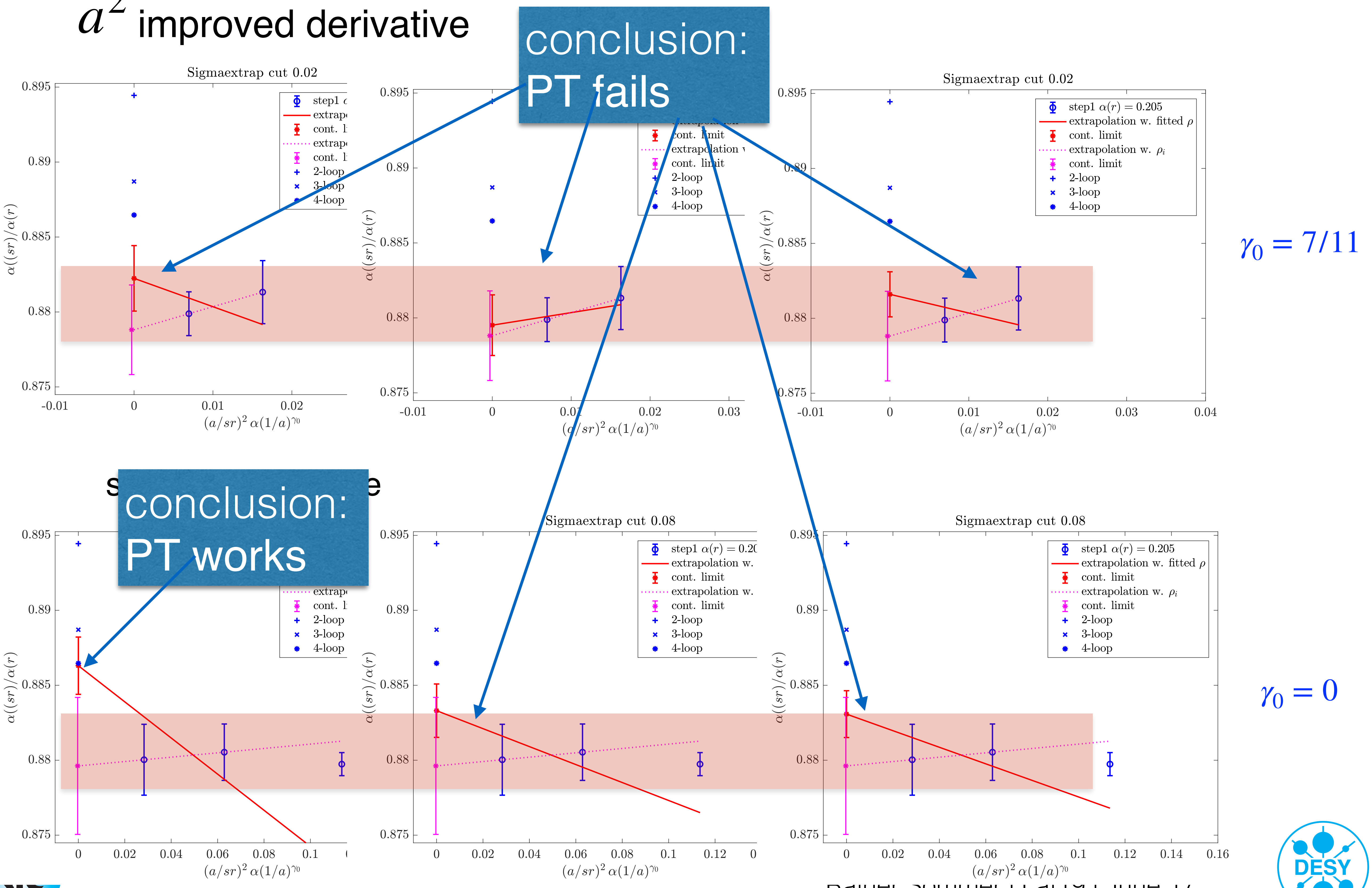

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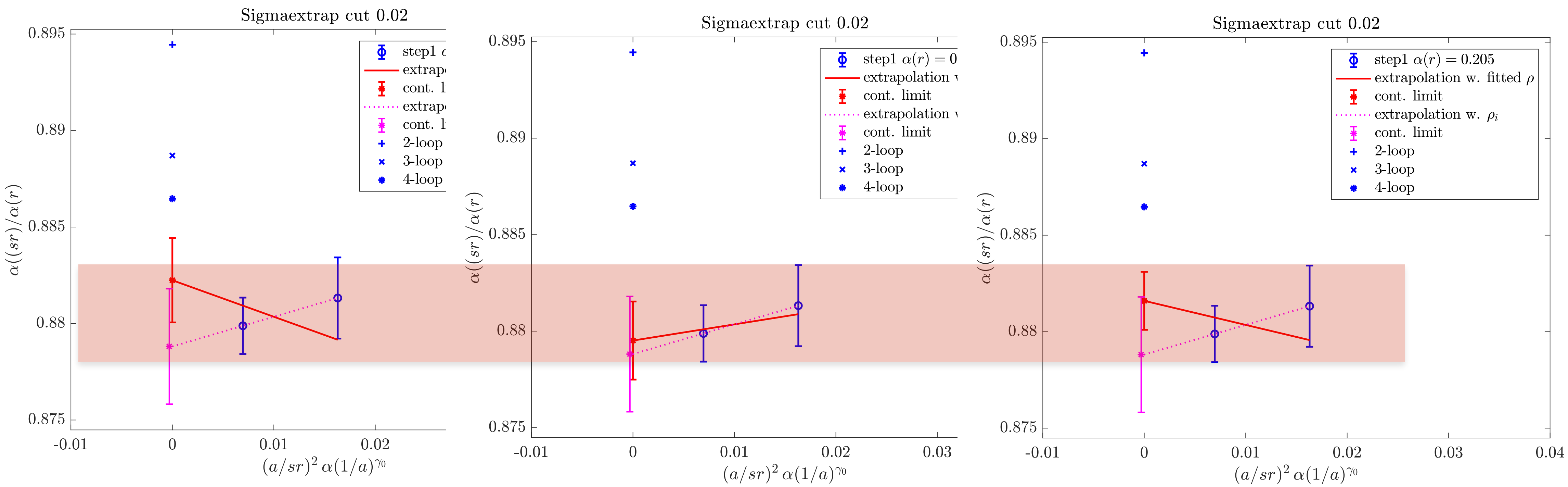

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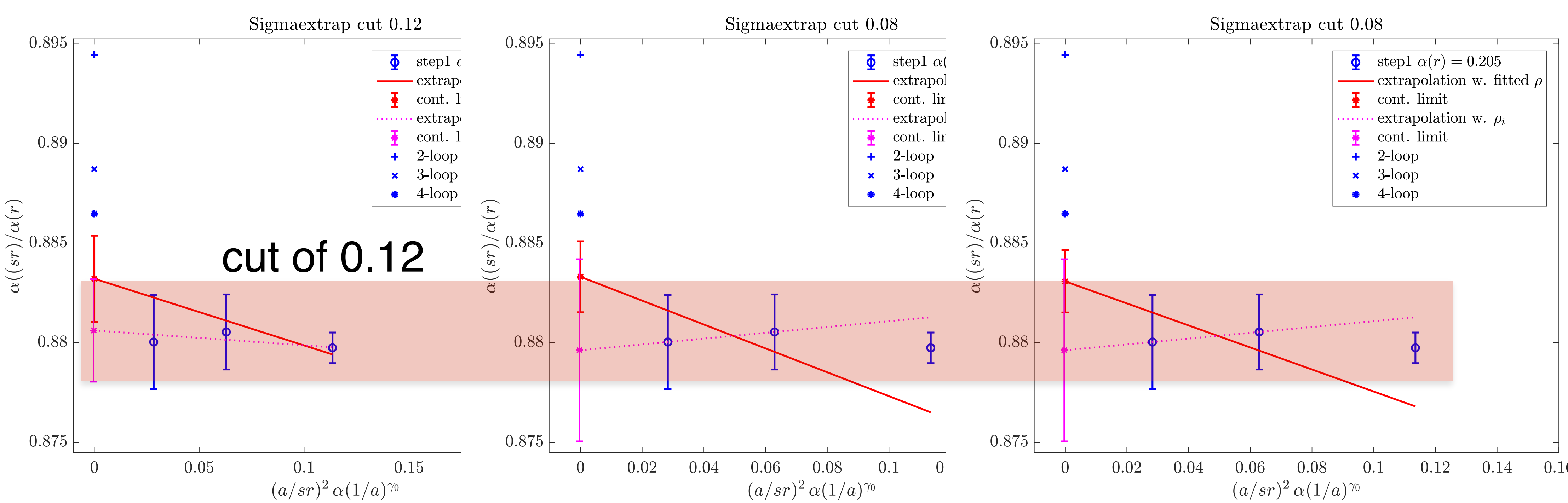


Conclusions

- Continuum limit with known leading first log-correction to a^2 scaling

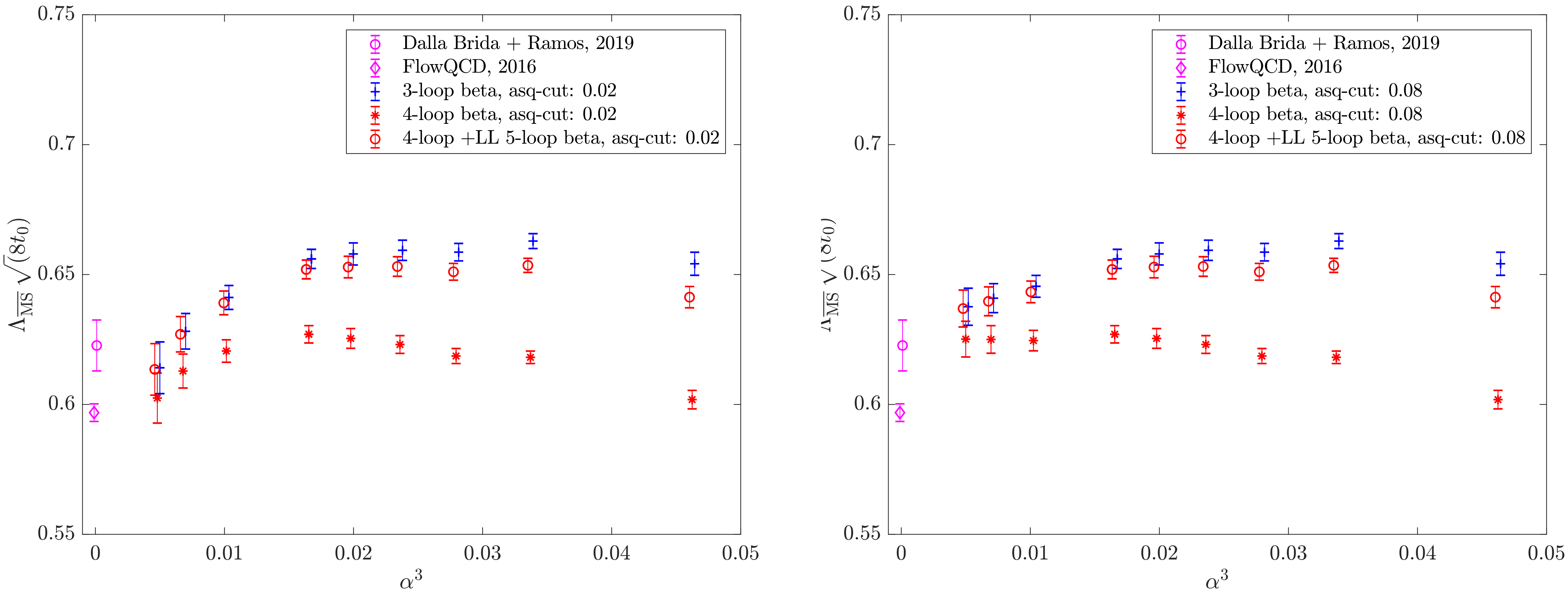
$$a^2\,[\alpha(1/a)]^{7/11}$$

- Semiquantitative agreement with perturbation theory is convincing for distances below 0.1 fm

- Precision test of PT is very difficult
  - do known US contributions apply / help for accessible α ?
  - can continuum limit be controlled sufficiently well?
  - uncertainty in Λ very difficult to assess even with a=0.01 fm lattice

Conclusions

- uncertainty in Lambda very difficult to assess even with a=0.01 fm lattice



7% difference from combination of $\alpha \to 0$ and $a \to 0$?

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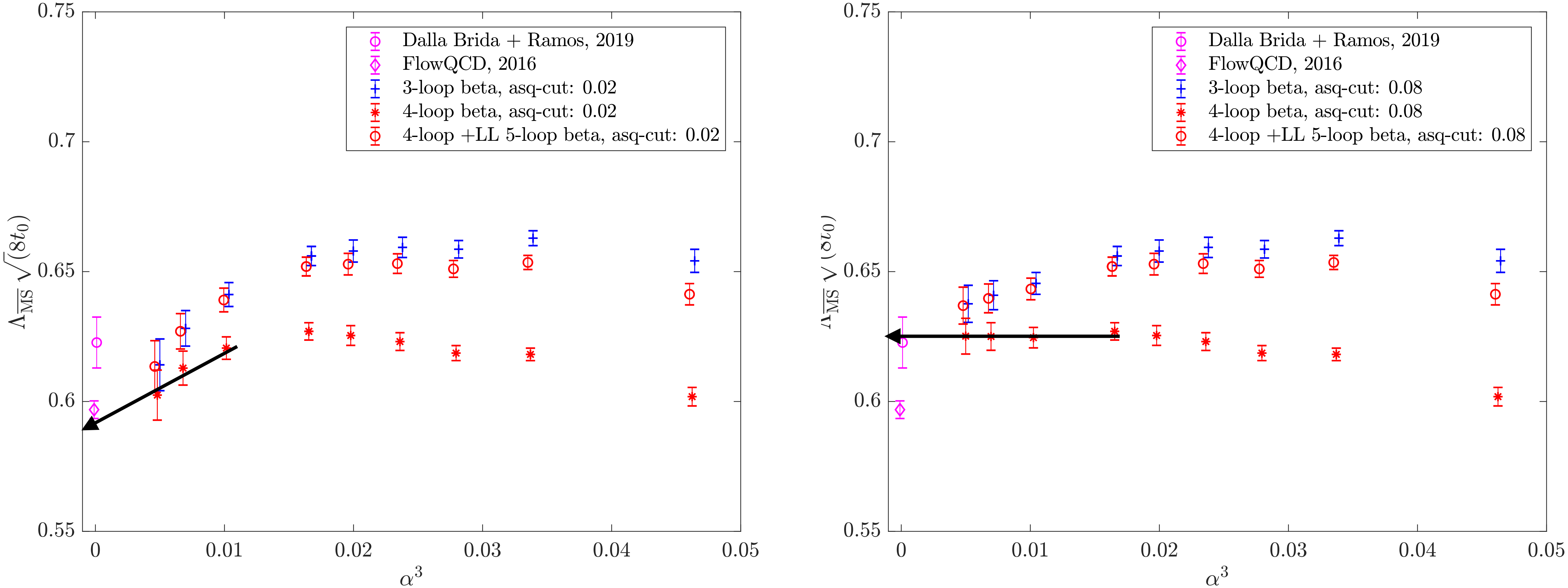

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Thank you