Yang Mills short distance potential and perturbation theory

Nikolai Husung, Philipp Krah, Alessandro Nada
Rainer Sommer

John von Neumann Institute for Computing, DESY
&
Humboldt University, Berlin

Wuhan, Lattice 2019, June 17
Motivation

- $\alpha_s$ world average is dominated by lattice determinations
  FLAG 2019:
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11823(81)
  \]
  3.5% precision on
  \[
  \Lambda_{\text{MS}}^{(3)} = 343(12)
  \]
  with phenomenology determinations (PDG 16/18)
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11806(72), \quad \text{FLAG 19 + PDG 18}
  \]
Motivation

- $\alpha_s$ world average is dominated by lattice determinations
  
  FLAG 2019:
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11823(81)
  \]
  3.5% precision on
  \[
  \Lambda_{\text{MS}}^{(3)} = 343(12)
  \]
  with phenomenology determinations (PDG 16/18)
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11806(72), \quad \text{FLAG 19 + PDG 18}
  \]

- determination from static potential (Bazavov et al, 2016)
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11660(100)
  \]
  similar cited precision, small tension
Motivation

- $\alpha_s$ world average is dominated by lattice determinations
  FLAG 2019:
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11823(81)
  \]
  3.5% precision on
  \[
  \Lambda_{\text{MS}}^{(3)} = 343(12)
  \]
  with phenomenology determinations (PDG 16/18)
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11806(72), \quad \text{FLAG 19 + PDG 18}
  \]

- determination from static potential (Bazavov et al, 2016)
  \[
  \alpha_{\text{MS}}^{(5)}(M_Z) = 0.11660(100)
  \]
  similar cited precision, small tension

- perturbative potential enters predictions of e+e- cross-section across t-tbar `threshold’ $\rightarrow$ top quark mass at linear collider
\( \alpha_s \) from static potential

- **Advantages**
  - 4-loop PT (4-loop beta-function) available, (normally only 3-loop)
  - relatively simple observable (no fermions …)

- **Disadvantages**
  - infrared divergences, starting at 4-loop, resummation with pNRQCD techniques
  - \( \alpha^{4+k} [\log(\alpha)]^k, \; k = 0,1,… \)
  - T(loop) \( \rightarrow \infty \) limit, noise
  - Discretisation errors at small r, window problem
Pure Yang Mills

- Precision, study of systematic effects

- Comparisons: two recent determinations of $\Lambda_{\text{MS}}$

  - FlowQCD: $w_0 \Lambda_{\text{MS}} = 0.2154(12)$
    converted (by us): $\sqrt{8t_0} \Lambda_{\text{MS}} = 0.5968(33)$
    from boosted coupling, contin. extrapolation $\beta \to \infty$ with $\alpha^2$
    perturbative uncertainty?

  - Dalla Brida and Ramos, 2019: $\sqrt{8t_0} \Lambda_{\text{MS}} = 0.6227(98)$

NP step scaling down to $\alpha_{\text{SF}} < 0.1$
Scales, lattices

- $L/a=32, \ldots 192, \ L=2\text{fm}$ (big enough in YM), open BC (no topology freezing)

$\alpha^2 [\text{fm}^2]$

- $\text{SU(3) YM, Husung et al., preliminary}$
- $N_f = 2 + 1, \text{ Bazavov et al., 2014}$

Zoom

$10^{-2}$ fm
Strategy to get to small $r$ (see also arXiv:1711.01860)

- basic scale from $t_0$:

$$\alpha_{qq}(\mu, a^2\mu^2), \quad \mu = 1/r = (x\sqrt{8t_0})^{-1}$$

on ensembles with $a > 0.02$ fm

- Then step scaling functions

$$\Sigma(u, a/r) = \bar{g}_{qq}^2(sr) \bigg|_{\bar{g}_{qq}^2(r)=u}$$

with $s = 3/4$ including $a = \{1.0, 1.4, 2.0\} \times 10^{-2}$ fm
Strategy to get to small r (see also arXiv:1711.01860)

- basic scale from $t_0$:

$$\alpha_{qq}(\mu, a^2\mu^2), \quad \mu = 1/r = (x\sqrt{8t_0})^{-1}$$

0.25 ≤ x ≤ 0.4

on ensembles with $a > 0.02$ fm

- Then step scaling functions

$$\Sigma(u, a/r) = \tilde{g}_{qq}^2(sr) \bigg|_{\tilde{g}_{qq}^2(r)=u}$$

with $s = 3/4$ including $a = \{1.0, 1.4, 2.0\} \times 10^{-2}$ fm
1-link integral

GEVP with
3 trial wave-functions
\( a=0.03 \text{ fm} \quad r=0.24 \text{ fm} \)

2 trial wave-functions
\( a=0.01 \text{ fm} \quad r=0.10 \text{ fm} \)
Tree-level improved force, \( r_1 \) such that
\[
F_{\text{TI}}(r_1) = \frac{1}{a} [V(r + a) - V(r)] = \frac{4}{3} \frac{\alpha}{r_1^2} + O(\alpha^2)
\]

Tree-level improved force with \( \alpha^2 \) improved derivative:
\[
F_{\text{impr}}(r_n) = \frac{13}{12} F_n(r_n) - \frac{1}{24} [F_n(r_n + a) + F_n(r_n - a)]
\]
\[
(r_{\tilde{1}})^{-2} = \frac{13}{12} (r_1(r))^{-2} - \frac{1}{24} [(r_1(r + a))^{-2} + (r_1(r - a))^{-2}]
\]
\[
\alpha_{qq}(1/r, a) = \alpha_{qq}(1/r, 0) \left\{ 1 + (a/r)^2 [\alpha(1/a)]^{-\hat{\gamma}_0} A_1(r) [1 + O([\alpha(1/a)]^{-\Delta \hat{\gamma}})] \right\} + O(a^4)
\]
\[
A_i(r) = \mathcal{A}_i \alpha(1/r)^{1+\hat{\gamma}_i} [1 + O(\alpha(1/r))].
\]

\(-\hat{\gamma}_0 = 7/11 = 0.686\)

smallest eigenvalue of 1-loop anomalous dimension matrix of Symanzik EFT d=6 operator basis [new, talk by N. Husung]
Continuum limits

- Large $r$ region ($r > 0.1 \text{fm}$)

\[ \frac{r}{\sqrt{8t_0}} = 0.25 \]

\[ \frac{r}{\sqrt{8t_0}} = 0.4 \]

- Gradient flow: log-corrections to $\alpha^2$ not yet known.
Continuum limits

- Small $r$ region ($r < 0.1\text{fm}$), step scaling functions

(1) select $\alpha_i \in \{0.205, 0.215, \ldots, 0.5\}$ (qq-scheme)
Continuum limits

- Small $r$ region ($r < 0.1$ fm), step scaling functions

1. Select $\alpha_i \in \{0.205, 0.215, \ldots, 0.5\}$ (qq-scheme)

2. Fit

$$\sum(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11}\rho_i$$

$\alpha(1/a)$ from $\alpha_{qq}(1/(2.5a))$ by 4-loop running
Continuum limits

- **Small r region** ($r < 0.1 \text{fm}$), step scaling functions

  (1) select $\alpha_i \in \{0.205, 0.215, \ldots, 0.5\}$  (qq-scheme)

  (2) fit
  \[ \Sigma(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11} \rho_i \]

  \(\alpha(1/a)\) from $\alpha_{qq}(1/(2.5a))$ by 4-loop running

  (3) fit slopes to
  \[ \rho_i = \rho(\alpha_i), \quad \rho(\alpha) = \alpha^{(2-7/11)} \times [\rho^{(0)} + \rho^{(1)}\alpha + \rho^{(2)}\alpha^2] \]

  (4) use fitted slope function $\rho(\alpha)$ in
  \[ \Sigma(\alpha, s, a/r) = \sigma(\alpha, s) + (a/sr)^2[\alpha(1/a)]^{7/11} \rho(\alpha) \]

  for all $\alpha$ in range
Continuum limits

\[(2) \text{ fit } \Sigma(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11} \rho_i \]
Continuum limits

- Small r region ($r < 0.1$ fm), step scaling functions
  - select $\alpha_i \in \{0.205, 0.215, \ldots, 0.5\}$ (qq-scheme)
  - fit $\Sigma(a_i, s, a/r) = \sigma_i + (a/sr)^2 [\alpha(1/a)]^{7/11} \rho_i$
    
    $\alpha(1/a)$ from $\alpha_{qq}(1/(2.5a))$ by 4-loop running

(3) fit slopes to

$$\rho_i = \rho(\alpha_i), \quad \rho(\alpha) = \alpha^{(2 - 7/11)} \times [\rho^{(0)} + \rho^{(1)} \alpha + \rho^{(2)} \alpha^2]$$

fit to slopes of $\Sigma = \sigma + \rho \times (a/(sr))^2 \alpha(1/a)^{-\gamma}$, $\gamma_0 = -0.63636$
Continuum limits

(4) fit \[ \Sigma(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11}\rho(\alpha_i) \]
Continuum limits

(4) fit \( \Sigma(\alpha_i, s, a/r) = \sigma_i + (a/sr)^2[\alpha(1/a)]^{7/11} \rho(\alpha_i) \)

fitting \( \rho(\alpha) \) first has a stabilizing effect — seems good for means — but trustworthy for error bars?
Continuum limits: compare to standard derivative

$a^2$ improved derivative          standard derivative $(\gamma_0 = 0)$

more points, but lost by cut

\[
\left(\frac{a}{sr}\right)^2 \leq 0.08 \\
\left(\frac{sr}{a}\right) > 3.5
\]

same continuum limits
Continuum limits: compare to standard derivative

\( a^2 \) improved derivative

\[
(\frac{a}{sr})^2 \leq 0.08 \quad (sr > 3.5)
\]
more points, but lost by cut

\( \gamma_0 = 0 \)

standard derivative

a bit crazy but not impossible

same continuum limits
Results (from 2-stage continuum limit, improved derivative)
**Λ - parameter**

- \( \Lambda_{\overline{\text{MS}}} \) from \( \Lambda_{\text{qq}} \) locally (\( \alpha_{\text{qq}} \) by \( \alpha_{\text{qq}} \))

\[
\Lambda = \mu \left( b_0 \bar{g}(\mu)^2 \right)^{-b_1/(2b_0^2)} e^{-1/(2b_0 \bar{g}(\mu)^2)} \exp \left\{ - \int_0^{\bar{g}(\mu)} dx \left[ \frac{1}{\beta_{n_{\text{loop}}}(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\}
\]

\[
\times \left[ 1 + O(\bar{g}(\mu)^{2(n-1)}) \right]
\]

\[
\beta_{3_{\text{loop}}}(g) = - g^3 \left[ b_0 + b_1 g^2 + b_2 g^4 \right]
\]

- 4-loop: \( + b_3 g^6 + b_{3L} g^6 \log(\alpha) \)
- 4-loop LL: \( + b_{4L} g^8 \log(\alpha) + b_{4LL} g^8 [\log(\alpha)]^2 \)

computed from

Peter 97; Schröder 99
Anzai, Kiyo, Sumino, 10
Smirnov, Sminov, Steinhauser, 10
Brambilla, Pineda, Soto, Vairo, 99
Kniehl, Penin, 99
Brambilla, Garcia i Tormo, Soto, Vairo, 07,09
Results (from 2-stage continuum limit, standard derivative)

\[ \rho(\alpha) = \alpha^2 \times [\rho^{(0)} + \rho^{(1)} \alpha + \rho^{(2)} \alpha^2] \]
Results (from 2-stage continuum limit, standard derivative)

\[ \rho(\alpha) = \alpha^2 \times \left[ \rho^{(0)} + \rho^{(1)} \alpha + \rho^{(2)} \alpha^2 \right] \]

irritatingly flat at wrong position

\[ \Delta \text{MS} \sqrt{\langle 8\eta_0 \rangle} \]

\[ \alpha^3 \]
Results (from 2-stage continuum limit, standard derivative)

\[ \rho(\alpha) = \alpha^2 \times [\rho^{(0)} + \rho^{(1)}\alpha + \rho^{(2)}\alpha^2] \]
Results (from 2-stage continuum limit, standard derivative)

\[ \rho(\alpha) = \alpha^2 \times [\rho^{(0)} + \rho^{(1)}\alpha + \rho^{(2)}\alpha^2] \]
Results (from 2-stage continuum limit, standard derivative)

\[ \rho(\alpha) = \alpha^2 \times [\rho^{(0)} + \rho^{(1)}\alpha] \]
Results (from 2-stage continuum limit, standard derivative)

$$\rho(\alpha) = \alpha^2 \times [\rho^{(0)} + \rho^{(1)}\alpha]$$

look at ssf again
Continuum limits: compare to standard derivative

\( a^2 \) improved derivative

\[
\gamma_0 = \frac{7}{11}
\]

\[
\gamma_0 = 0
\]

standard derivative
Continuum limits: compare to standard derivative

\[ a^2 \text{ improved derivative} \]

\[ \gamma_0 = \frac{7}{11} \]

\[ \gamma_0 = 0 \]

**conclusion:**
PT works
Continuum limits: compare to standard derivative

\[ a^2 \] improved derivative

**Conclusion:**
- PT fails
- PT works

\[ \gamma_0 = \frac{7}{11} \]

\[ \gamma_0 = 0 \]
Continuum limits: compare to standard derivative

$a^2$ improved derivative

standard derivative

cut of 0.12
Conclusions

- Continuum limit with known leading first log-correction to $a^2$ scaling
  \[ a^2 [\alpha(1/a)]^{7/11} \]

- Semiquantitative agreement with perturbation theory is convincing for distances below 0.1 fm

- Precision test of PT is very difficult
  
  - do known US contributions apply / help for accessible $\alpha$ ?
  
  - can continuum limit be controlled sufficiently well?
  
  - uncertainty in $\Lambda$ very difficult to assess even with a=0.01 fm lattice
Conclusions

- uncertainty in Lambda very difficult to assess even with $a=0.01$ fm lattice

7% difference from combination of $\alpha \to 0$ and $a \to 0$?
Conclusions

- uncertainty in Lambda very difficult to assess even with $a=0.01$ fm lattice

7% difference from combination of $\alpha \rightarrow 0$ and $a \rightarrow 0$?
Thank you