Lattice QCD Impact on Determination of CKM Matrix: Status and Prospects

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Introduction I

✦ My charge was to "review very recent FLAG results on standard model parameters and renormalization"

- Chose to concentrate on CKM matrix

✦ My background:

- MILC Collaboration; Fermilab Lattice/MILC Collaborations

- Flavour Lattice Averaging Group (FLAG): B, D semileptonic working group member—*But this is not a FLAG approved talk*

✦ Primary goal is to summarize lattice QCD contribution to understanding CKM matrix

- FLAG 3: S. Aoki et al., arXiv:1607.00299, EPJC, **77**, 102 (2017)

- FLAG 4: S. Aoki et al., arXiv:1902:08191

Introduction II

✦ In addition to FLAG plots, use many from Fermilab Lattice/MILC papers

✦ Hope to have time for future prospects

- If not, there are several relevant papers:

- Belle II Physics Book, E.Kou, et al., arXiv:1808.10567, 690 pgs.

- Opportunities in Flavour Physics at HL-LHC and HE-LHC: A. Cerri at al., arXiv:1812.07638, 298 pgs.

- Opportunities for lattice QCD in quark and lepton flavor physics: C. Lehner et al. (USQCD Collaboration), arXiv:1904.09479, 36 pgs.

✦Gratefully acknowledge my collaborators in the FLAG, Fermilab Lattice, MILC Collaborations.

- Please see FLAG report and cited papers, as the total list of individuals > 50

- FLAG results come from several working groups

CKM Matrix I

✦ Some relevant processes listed under each **element**

$$
\begin{pmatrix}
\begin{array}{ccc}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
 & K \to l\nu & \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{array}
\end{pmatrix}
$$

CKM Matrix II

✦ CKM matrix is unitary.

  • Each row and column is a (complex) unit vector.

  • Each row (column) is orthogonal to the other rows (columns).

✦ Violations of unitarity are evidence of non-standard model physics.

✦ If two different processes are used to determine an element of the matrix and they do not agree, that is evidence for new physics.

✦ LQCD input for decay constants and form factors is needed to determine elements of CKM matrix

$$\mathcal{B}(D_{(s)} \to \ell\nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 m_{D_{(s)}} \left(1 - \frac{m_\ell^2}{m_{D_{(s)}}^2}\right)^2$$

First Row: Light Quarks

First Row: Light Quarks

✦ Processes involving only light quarks test first row unitarity

First Row: Light Quarks

✦ Processes involving only light quarks test first row unitarity

$$\begin{pmatrix}
\begin{array}{ccc}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
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D \to \pi l\nu & D \to K l\nu & B \to D^{(*)}l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{array}
\end{pmatrix}$$
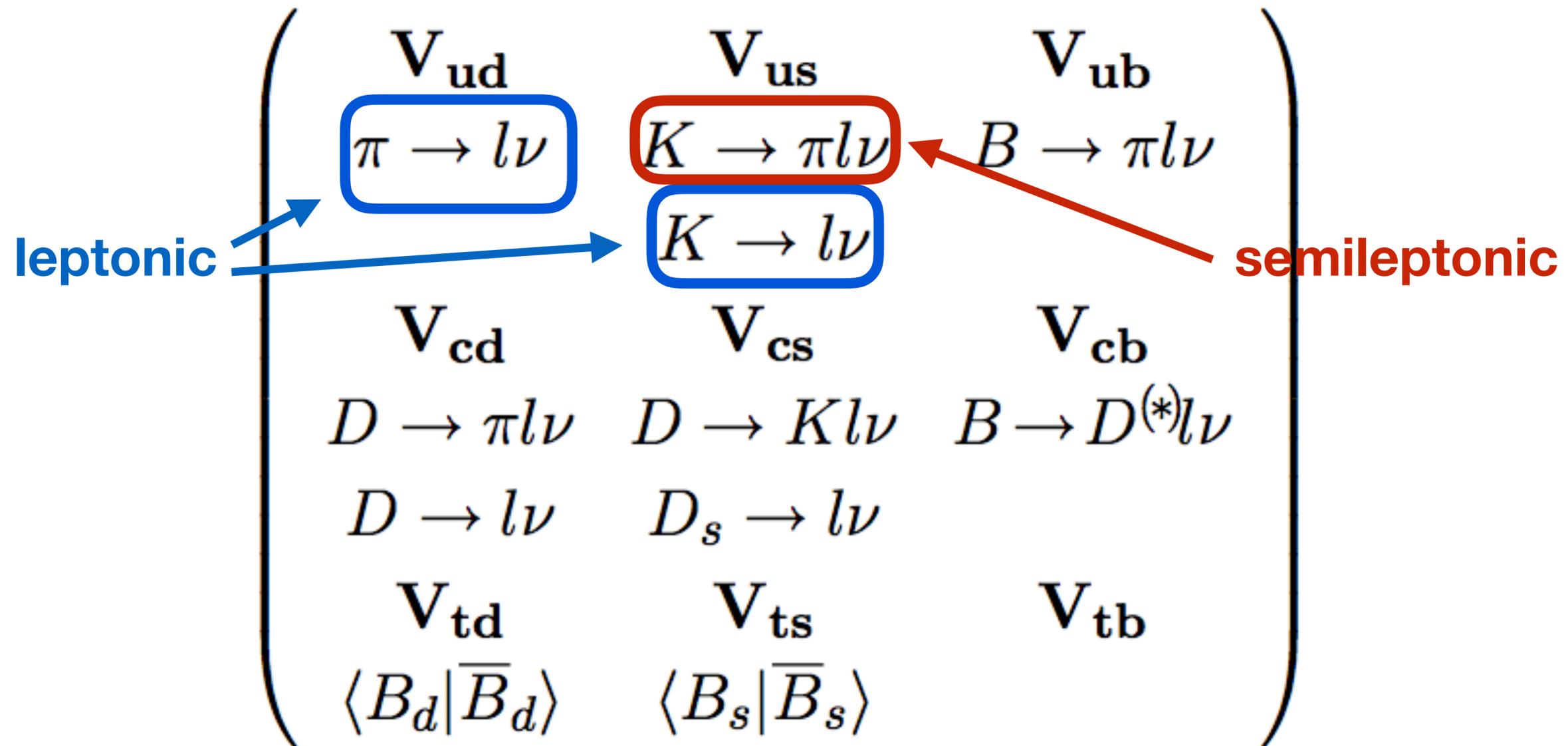
✦ Processes involving only light quarks test first row unitarity

$$
\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\boxed{\pi \to l\nu} & K \to \pi l\nu & B \to \pi l\nu \\
 & \boxed{K \to l\nu} & \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D^{(*)}l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{pmatrix}
$$

leptonic

First Row: Light Quarks

✦ Processes involving only light quarks test first row unitarity

$$\begin{pmatrix} \begin{matrix} \mathbf{V_{ud}} \\ \boxed{\pi \to l\nu} \\ \\ \mathbf{V_{cd}} \\ D \to \pi l\nu \\ D \to l\nu \\ \mathbf{V_{td}} \\ \langle B_d | \overline{B}_d \rangle \end{matrix} & \begin{matrix} \mathbf{V_{us}} \\ \boxed{K \to \pi l\nu} \\ \boxed{K \to l\nu} \\ \mathbf{V_{cs}} \\ D \to K l\nu \\ D_s \to l\nu \\ \mathbf{V_{ts}} \\ \langle B_s | \overline{B}_s \rangle \end{matrix} & \begin{matrix} \mathbf{V_{ub}} \\ B \to \pi l\nu \\ \\ \mathbf{V_{cb}} \\ B \to D^{(*)} l\nu \\ \\ \mathbf{V_{tb}} \\ \end{matrix} \end{pmatrix}$$

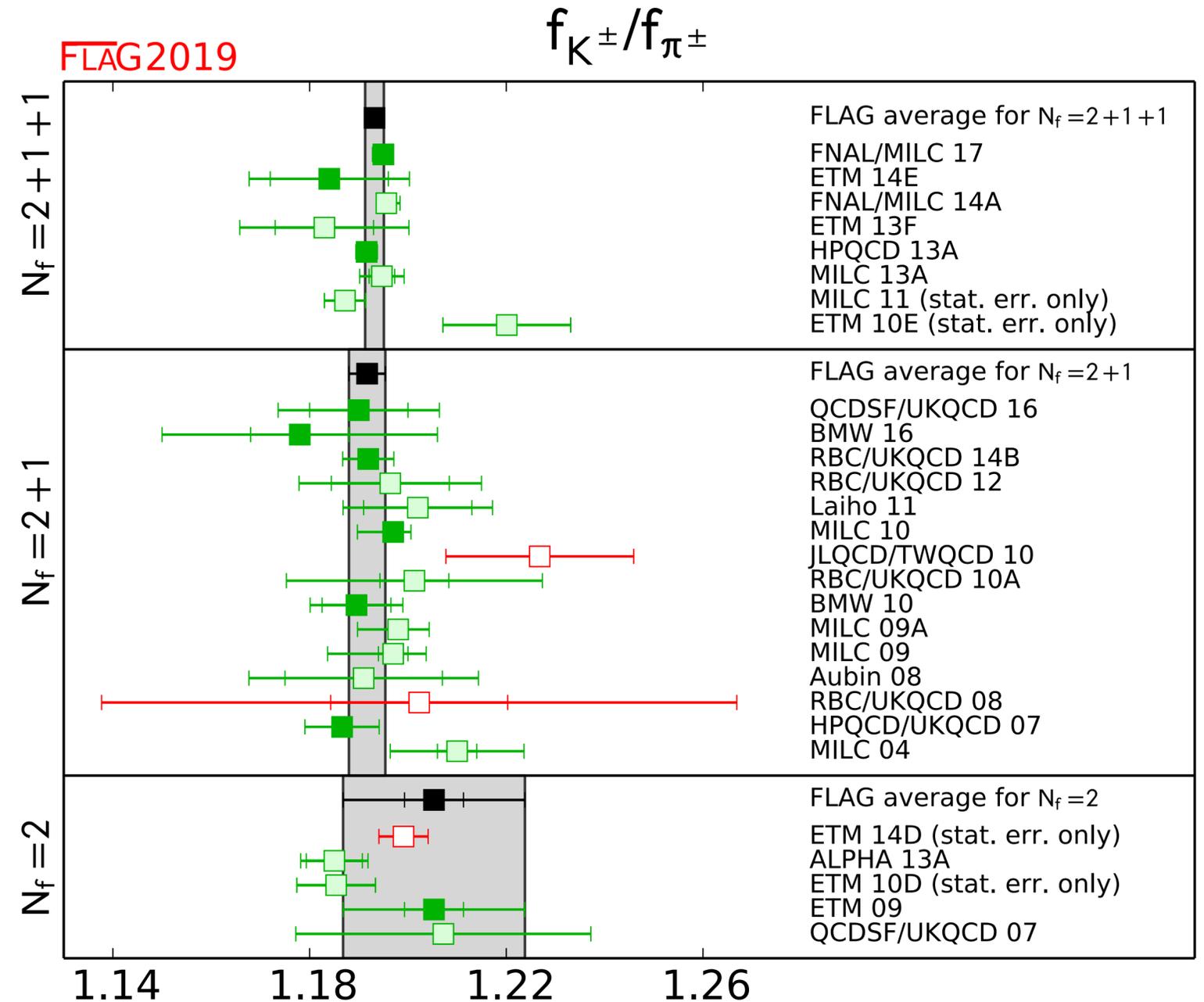**leptonic**

semileptonic

f$_\pi$ and f$_K$

- Light decay constants as summarized by FLAG
  - solid green: contributes to average
  - light green: superseded
  - red: errors not controlled
  - black points & gray bands: FLAG average
- Some calculations use f$_\pi$ to set the scale so fewer results on left
- Ratio of decay constants is easy to calculate and used to test unitarity

f_K/f_π
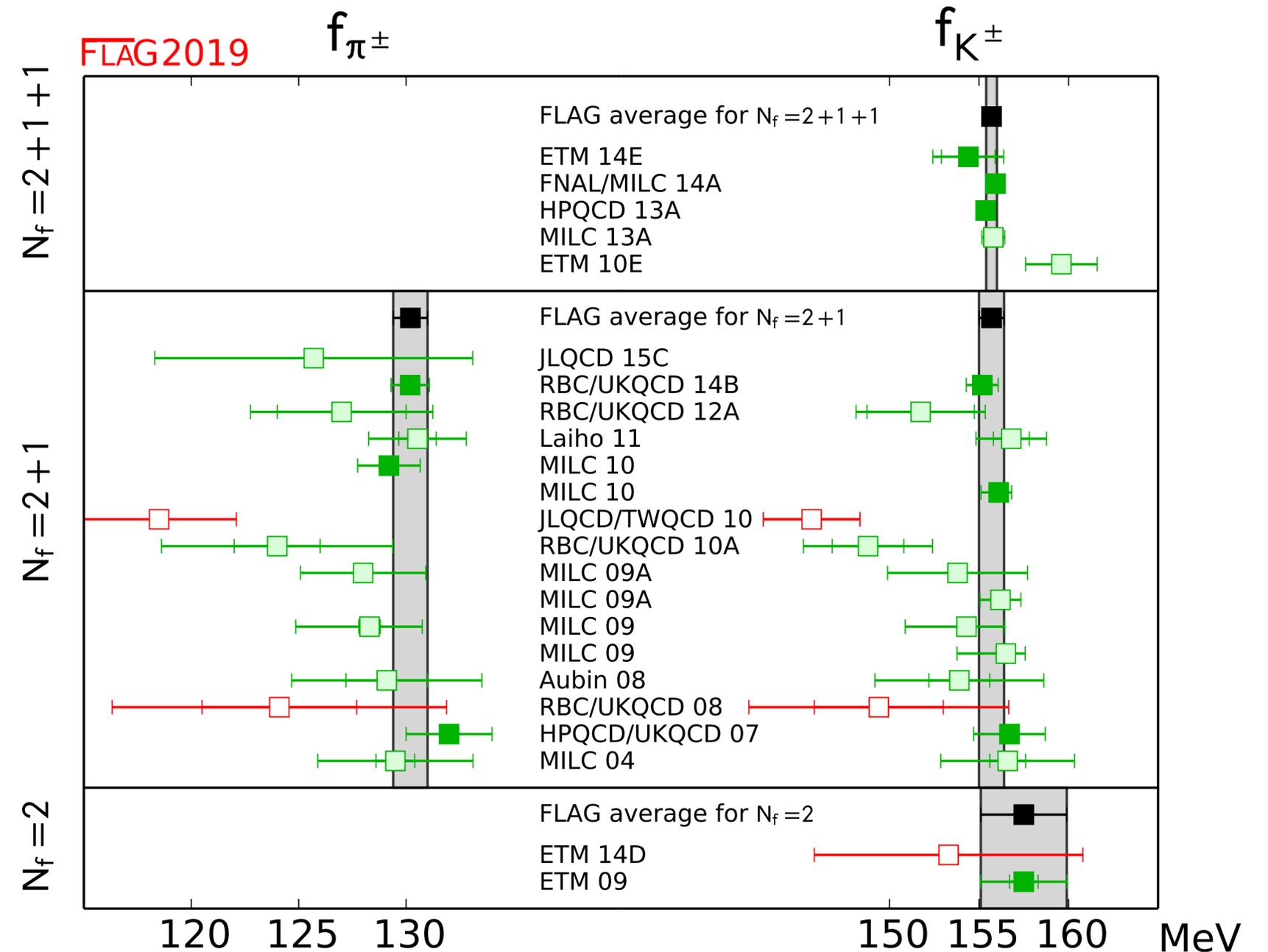
- Light decay constant ratio summarized by FLAG

- From experimental measurements summarized by the PDG and Moulson at CKM 2017:

$$\left|\frac{V_{us}}{V_{ud}}\right|\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4)$$

- error is 0.15%

$f_{K^\pm}/f_{\pi^\pm}$

FLAG 2019

$N_f = 2+1+1$

FLAG average for $N_f = 2+1+1$
FNAL/MILC 17
ETM 14E
FNAL/MILC 14A
ETM 13F
HPQCD 13A
MILC 13A
MILC 11 (stat. err. only)
ETM 10E (stat. err. only)

$N_f = 2+1$

FLAG average for $N_f = 2+1$
QCDSF/UKQCD 16
BMW 16
RBC/UKQCD 14B
RBC/UKQCD 12
Laiho 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
BMW 10
MILC 09A
MILC 09
Aubin 08
RBC/UKQCD 08
HPQCD/UKQCD 07
MILC 04

$N_f = 2$

FLAG average for $N_f = 2$
ETM 14D (stat. err. only)
ALPHA 13A
ETM 10D (stat. err. only)
ETM 09
QCDSF/UKQCD 07

1.14   1.18   1.22   1.26

f_K/f_π

- Plot from Bazavov et al., [FNAL/MILC] PRD98(2018)074512; [1712.09262]

- FLAG 2016 avg: 1.193(3)

- New result from FNAL/MILC:

$$1.1950\left(^{+15}_{-22}\right)$$

- FLAG 2019 (2+1+1): 1.1932(19)

- theory error now 0.16%

- New plot in FLAG 2019 (previous slide)

  - Not possible to see some of the error bars as they are smaller than the plotting symbols in that plot
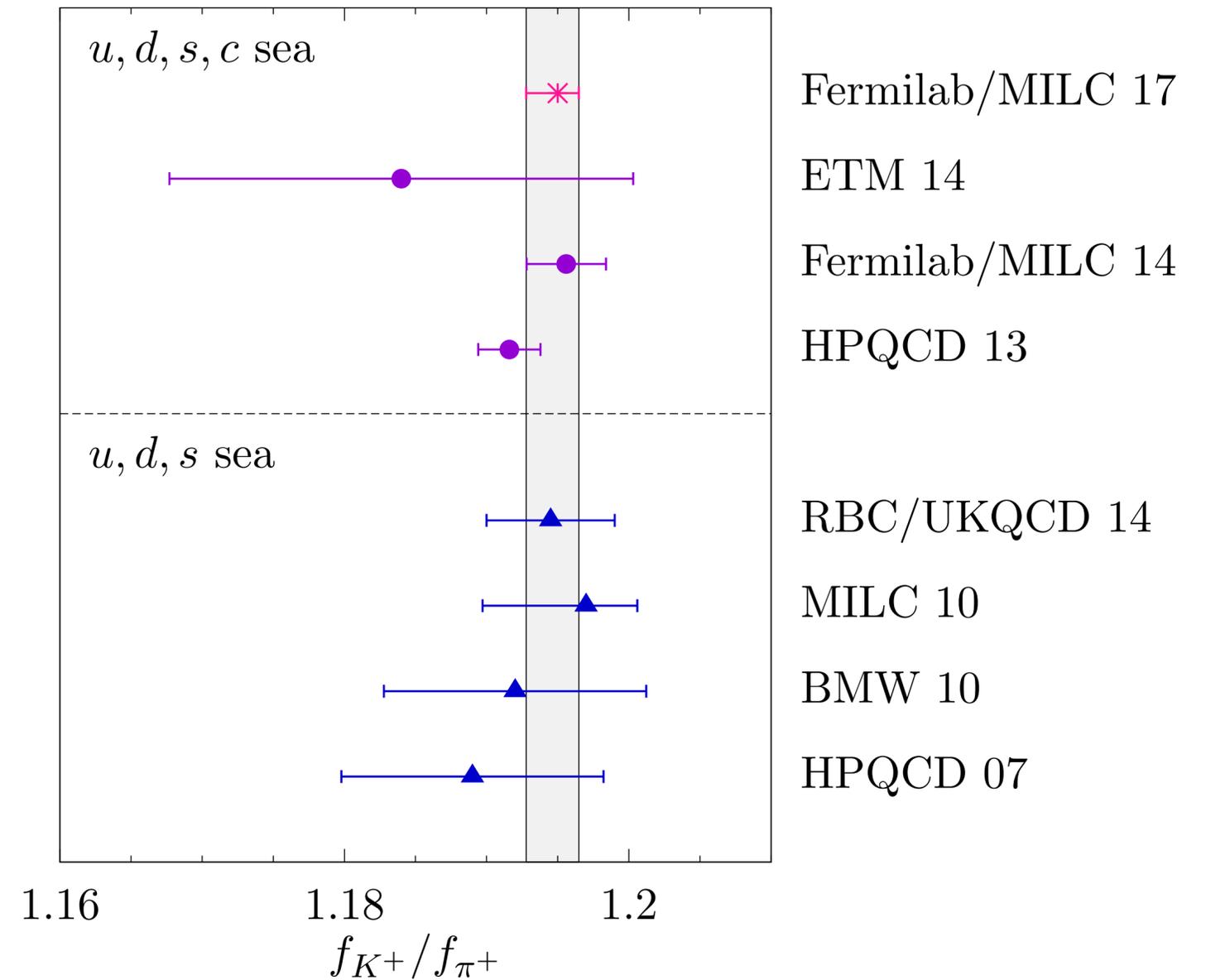
K semileptonic decay (2014)

✦ Semileptonic decays have three-body final states, so there is one kinematic variable, usually denoted q^2 , which is momentum transfer to the leptons.

$$p_K = p_\pi + q_\ell + q_\nu$$

$$q = q_\ell + q_\nu$$

✦ To extract $|V_{us}|$ we just need form factor $f_+(q^2=0)$ as experiment tells us

$$|V_{us}| f_+(0) = 0.2165(4)$$

✦ From FNAL/MILC with 2+1+1 flavors* PRL 112, 112001 (2014), arXiv:1312.1228

$$f_+(0) = 0.9704(24)(22)$$

✦ Experimental error 0.18%, theory error 0.34%

K semileptonic decay (2018)

✦ Experimental result slightly updated at CKM2016 by Moulson, PoS CKM 2016, 033 (2017); arXiv:1704.04104 [hep-ex]

$$|V_{us}|f_+(0) = 0.21654(41)$$

✦ From FNAL/MILC with 2+1+1 flavors, arXiv:1809.02827, PRD to appear.

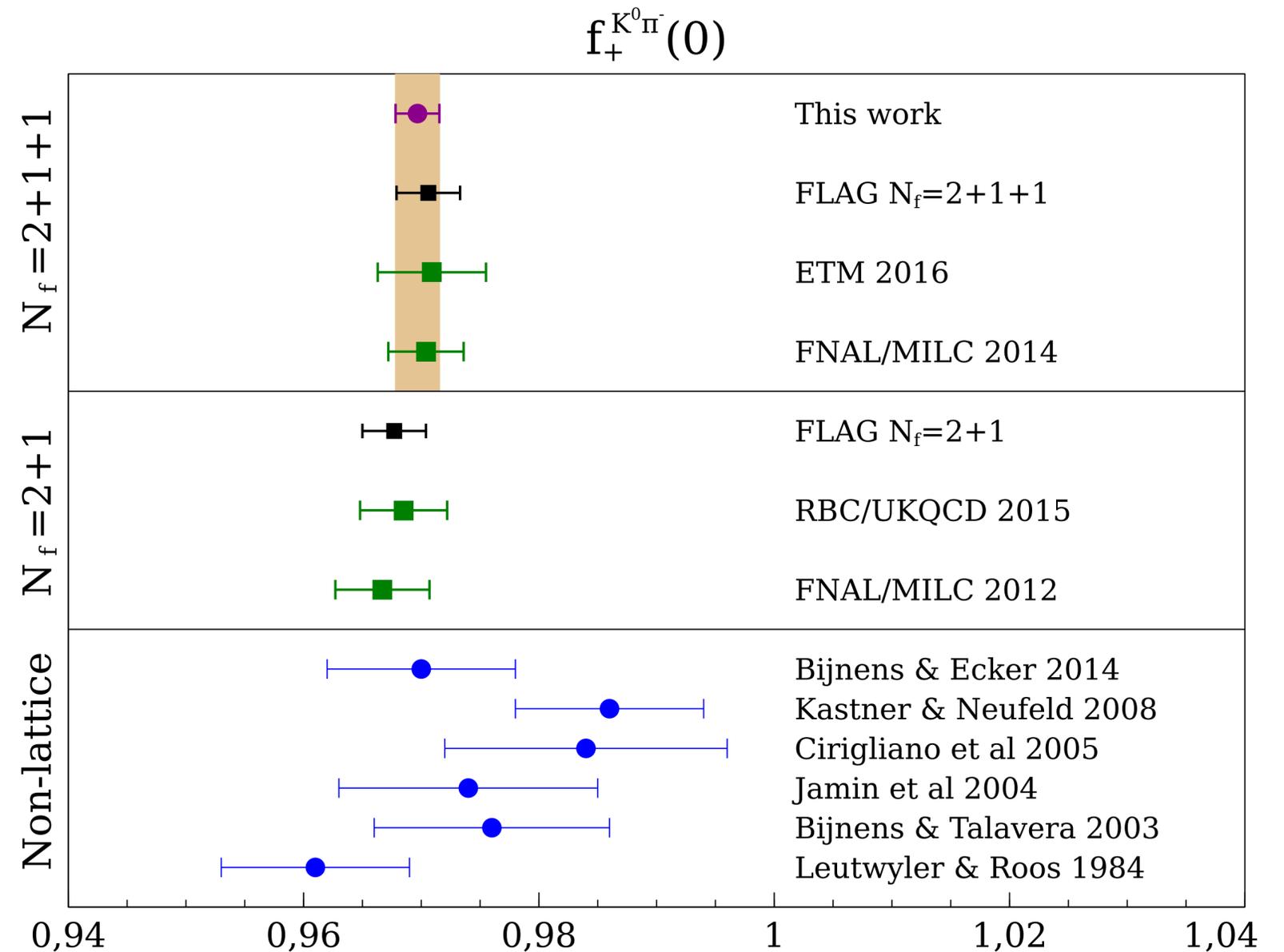$$f_+(0) = 0.9696(15)_{\mathrm{stat}}(12)_{\mathrm{sys}}$$
$$f_+(0) = 0.9696(19), \ \mathrm{i.e.}, \ 0.20\%$$

✦ Form factor error now comparable to experimental error!

$$f_+(0) = 0.9706(27), \ \mathrm{FLAG2019} \ N_f = 2+1+1$$

FNAL/MILC Comparison

- Slightly finer scale and more selective choice of earlier results

- This work is FNAL/MILC (1809:02827), PRD to appear

- Elvira Gamiz has been leading this analysis

- Use quoted experimental results and form factor or decay constant ratio for V_{us} or V_{us}/V_{ud}
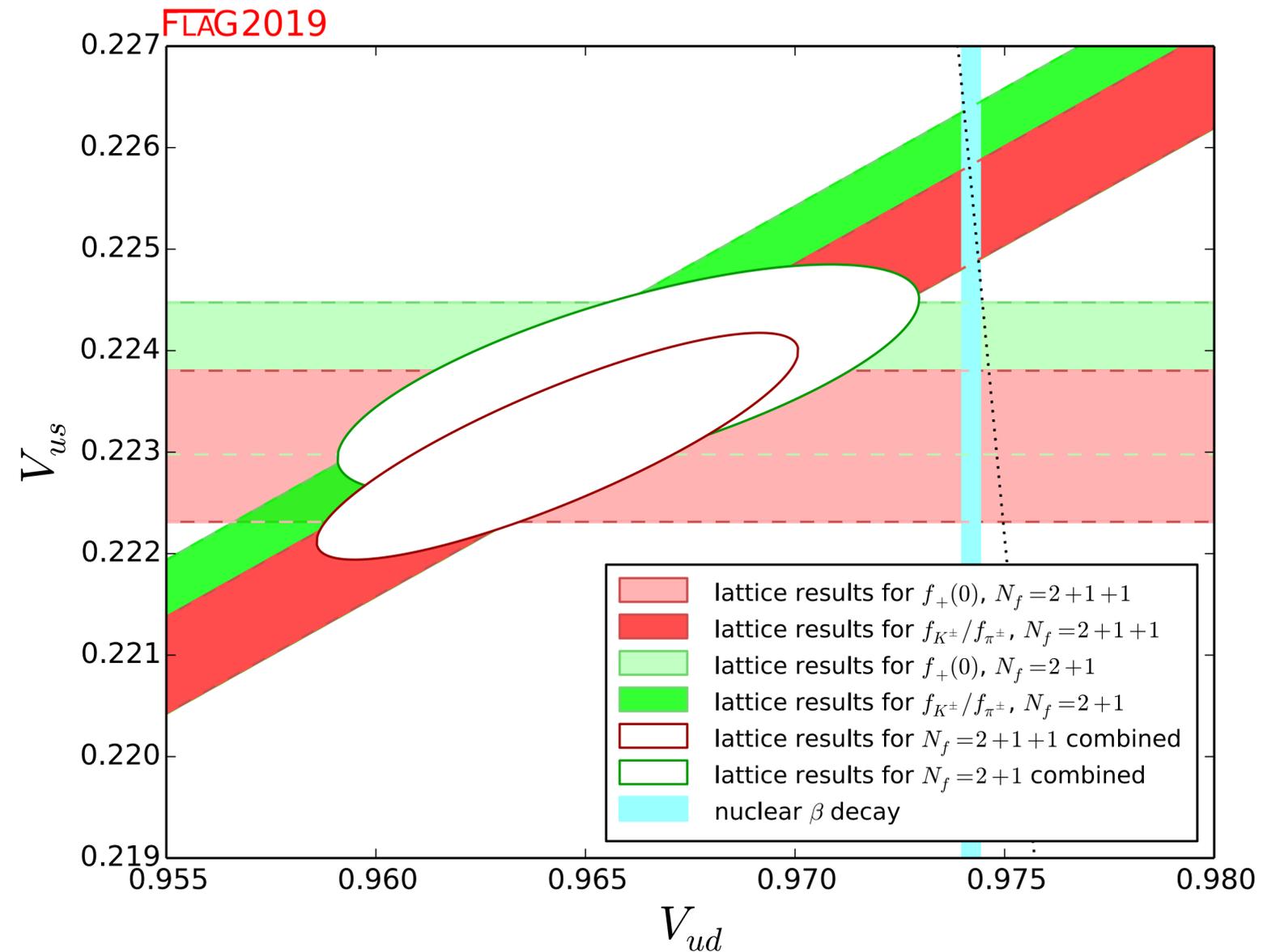
$f_+^{K^0\pi^-}(0)$

$N_f = 2+1+1$	This work
	FLAG $N_f=2+1+1$
	ETM 2016
	FNAL/MILC 2014
$N_f = 2+1$	FLAG $N_f=2+1$
	RBC/UKQCD 2015
	FNAL/MILC 2012
Non-lattice	Bijnens & Ecker 2014
	Kastner & Neufeld 2008
	Cirigliano et al 2005
	Jamin et al 2004
	Bijnens & Talavera 2003
	Leutwyler & Roos 1984

0,94   0,96   0,98   1   1,02   1,04

First Row Unitarity (FLAG)

- FLAG2019 results for 2+1 and 2+1+1 flavors

- Dotted line is unitarity

- 2+1 flavors has larger error

- Tension with unitarity has grown for both 2+1 and 2+1+1 since FLAG2016

- Horizontal red band:

  $|V_{us}| = 0.2231(7)$

  $|V_{ud}| \approx 0.965$

FLAG2019

Legend:
- lattice results for $f_+(0)$, $N_f = 2+1+1$
- lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1+1$
- lattice results for $f_+(0)$, $N_f = 2+1$
- lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1$
- lattice results for $N_f = 2+1+1$ combined
- lattice results for $N_f = 2+1$ combined
- nuclear β decay

Axis labels: V_{us} (vertical), V_{ud} (horizontal)

- K_{l3} result from latest FNAL/MILC result

- Leptonic decay (K_{l2}), nuclear β decay, and unitarity are in good agreement

- However, there is notable tension with semileptonic (K_{l3}) result

- from 1809:02827

- large blue ellipse corresponds to

$$|V_{ud}| = 0.9669\binom{+32}{-35}$$
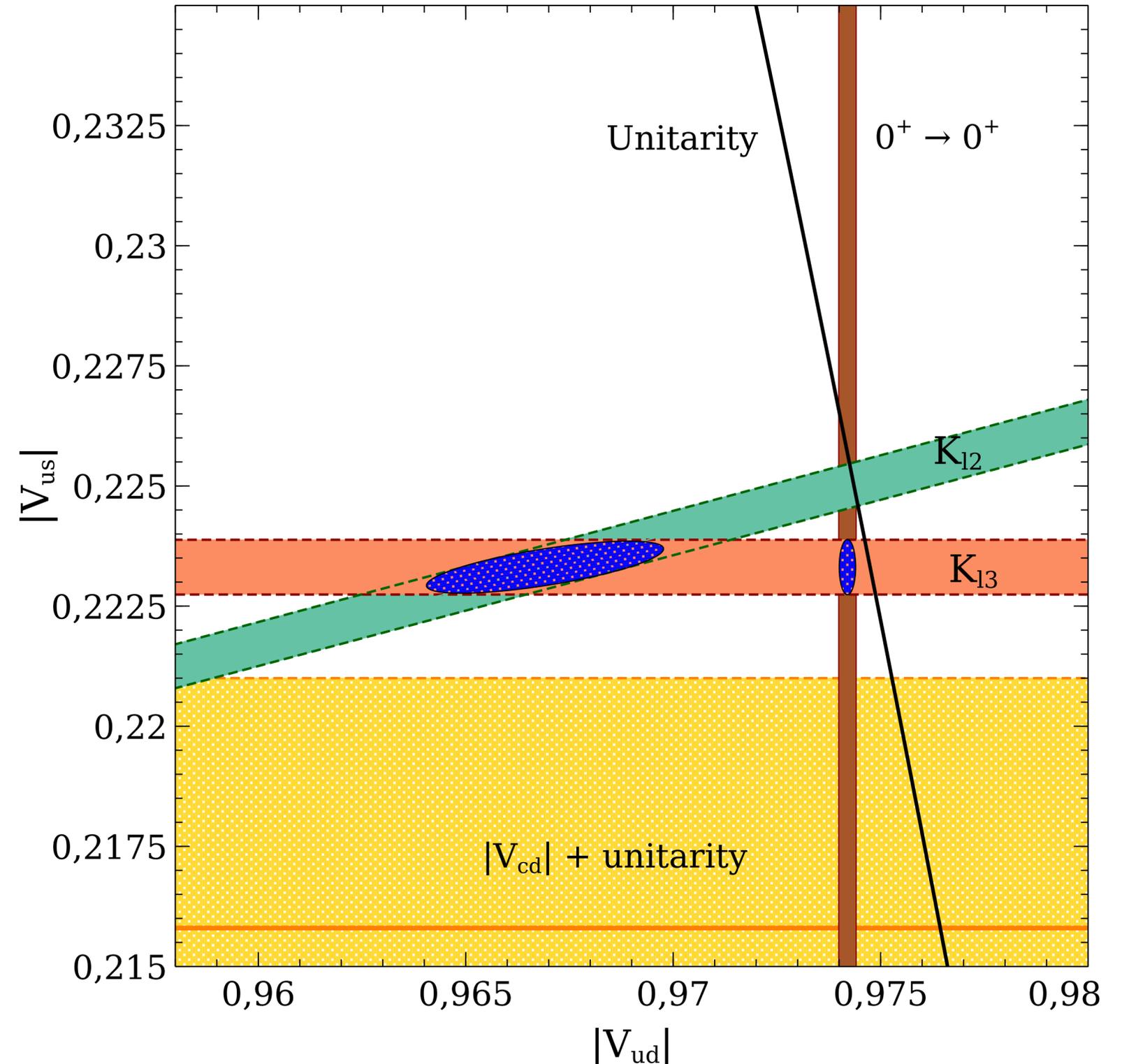
|V$_{us}$| & |V$_{ud}$| Summary

- Plot from FLAG 2019 (**assumes unitarity**)

- squares leptonic

- triangles semileptonic
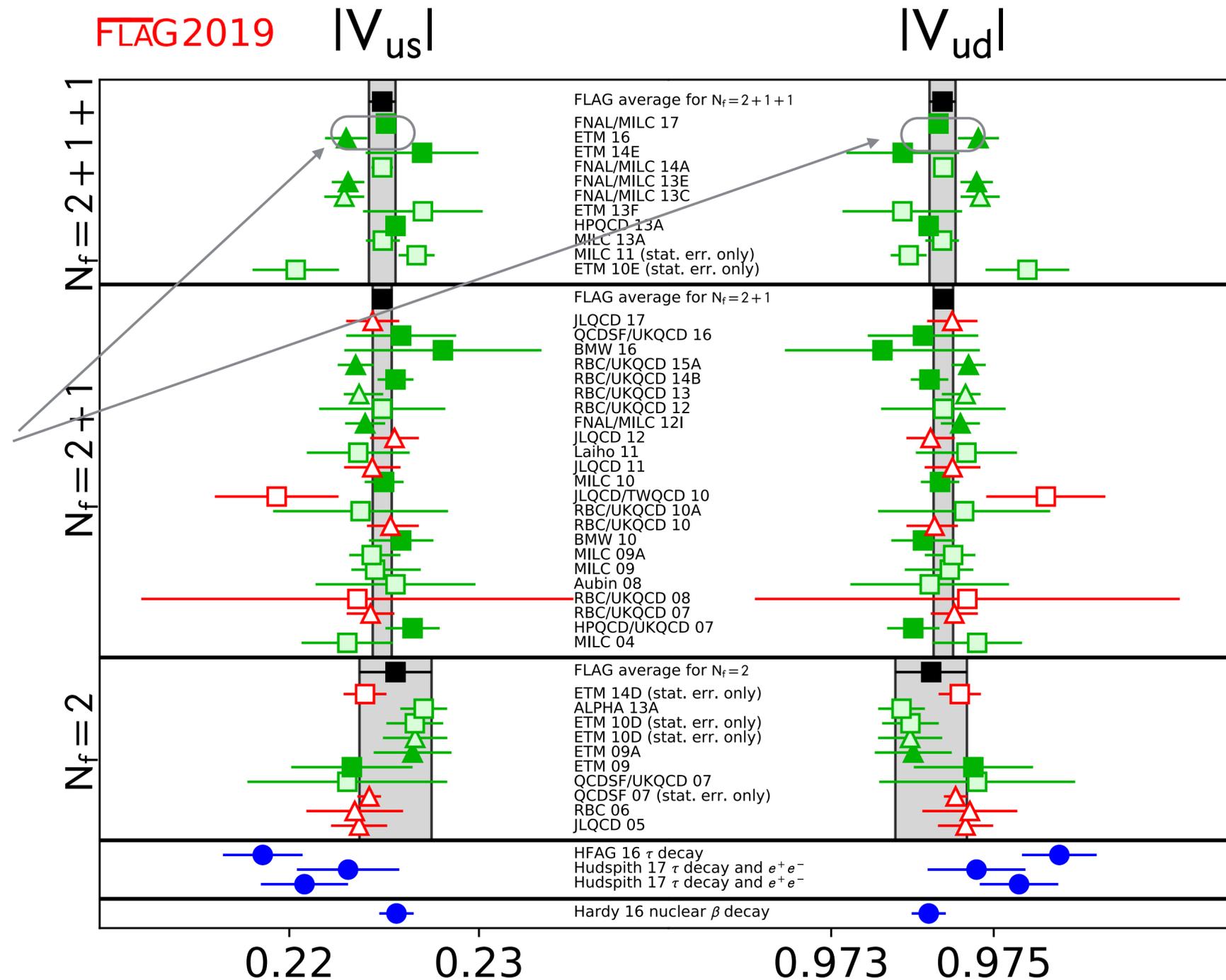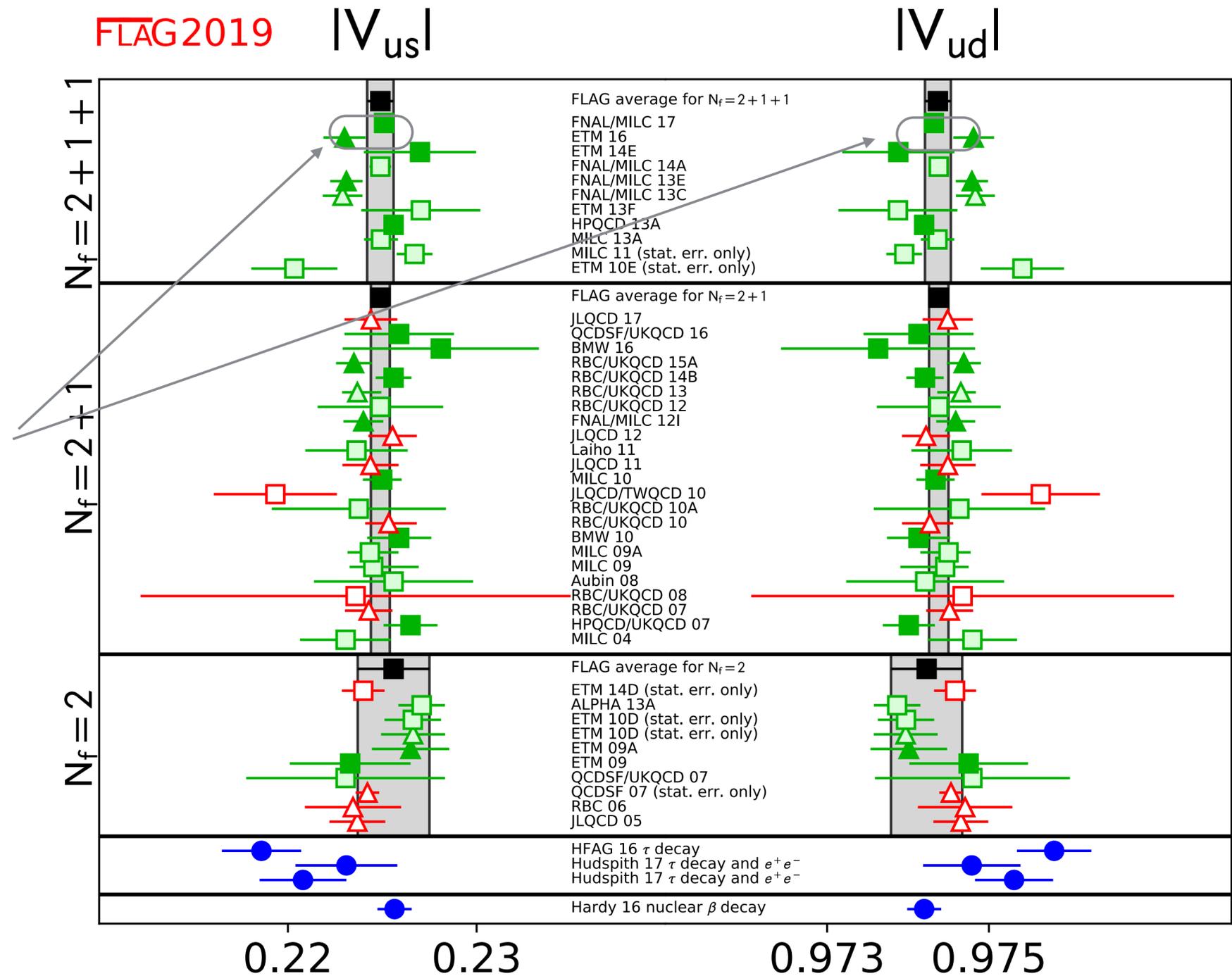
$$|V_{us}| = 0.2249(7)$$
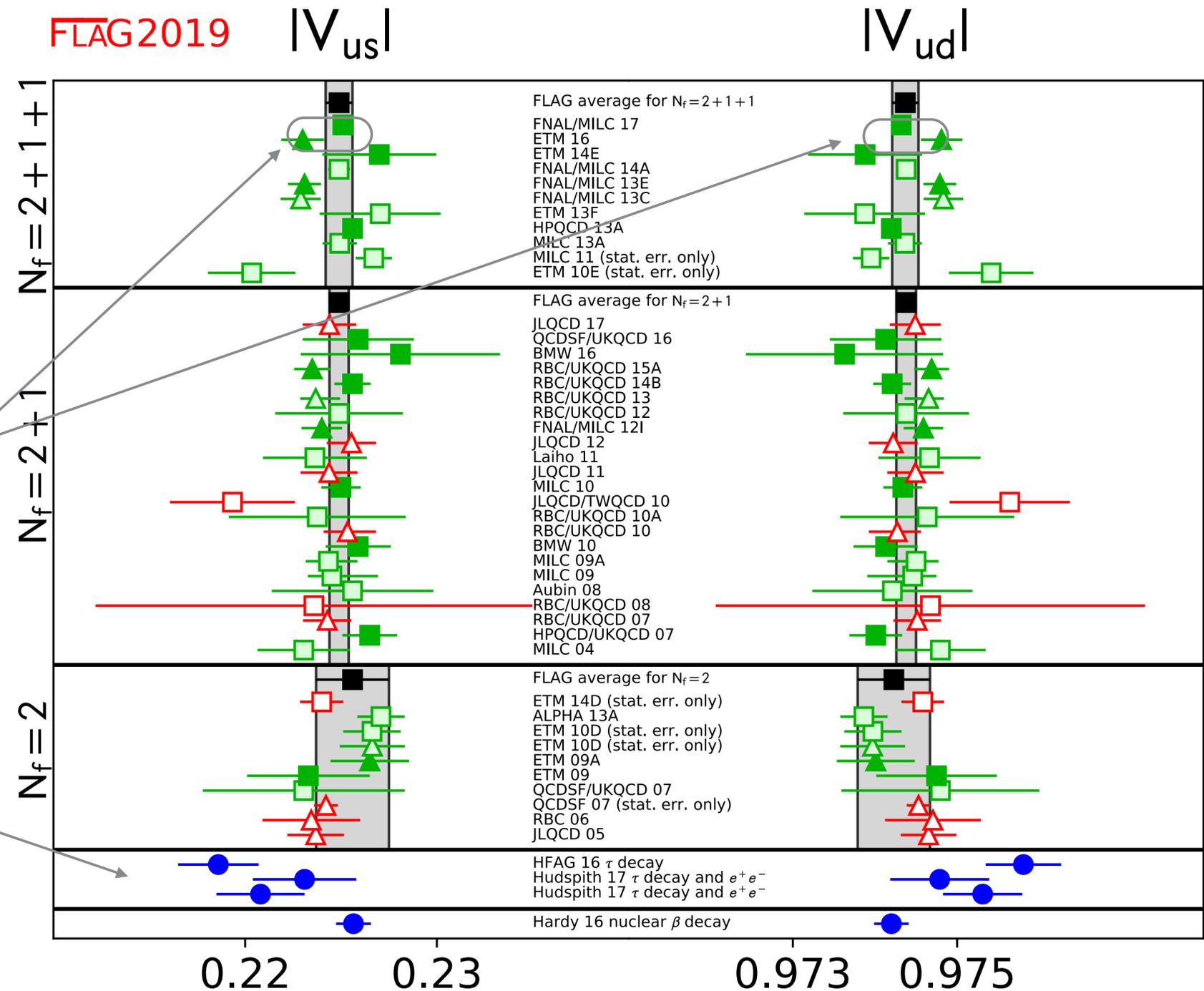$$|V_{ud}| = 0.97437(16)$$
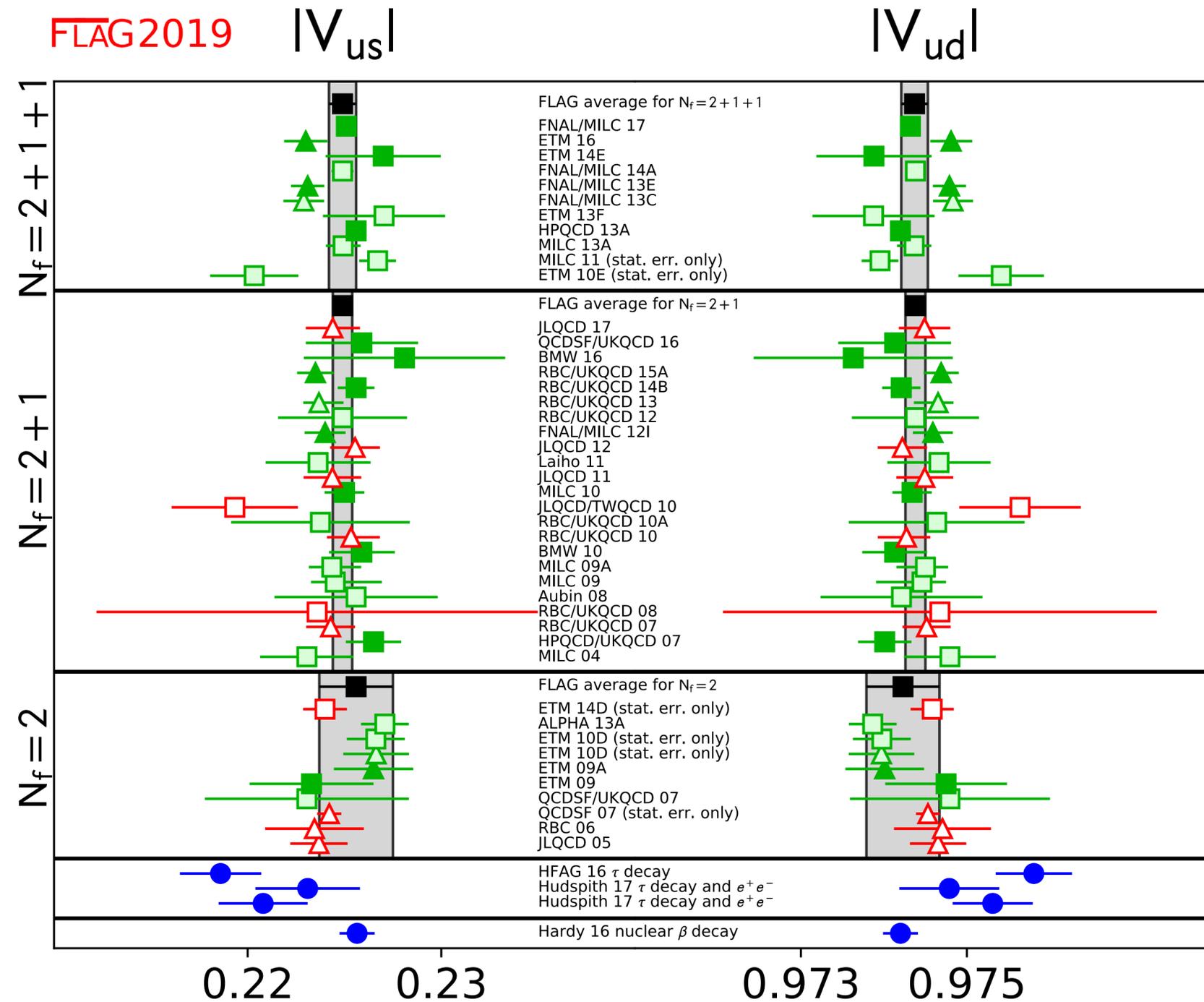
|V$_{us}$| & |V$_{ud}$| Summary

- Plot from FLAG 2019 (**assumes unitarity**)

- squares leptonic

- triangles semileptonic

- good agreement w. 2+1 flavors, some tension for 2+1+1

$$|V_{us}| = 0.2249(7)$$
$$|V_{ud}| = 0.97437(16)$$

|V$_{us}$| & |V$_{ud}$| Summary

- Plot from FLAG 2019 (**assumes unitarity**)

- squares leptonic

- triangles semileptonic

- good agreement w. 2+1 flavors, some tension for 2+1+1

- new Hudspith 17 reduce tension with lattice results

$$|V_{us}| = 0.2249(7)$$
$$|V_{ud}| = 0.97437(16)$$

|V_us| & |V_ud| Summary

- Plot from FLAG 2019 (**assumes unitarity**)

- squares leptonic

- triangles semileptonic

- good agreement w. 2+1 flavors, some tension for 2+1+1

- new Hudspith 17 reduce tension with lattice results

- For 2+1+1:
  $$|V_{us}| = 0.2249(7)$$
  $$|V_{ud}| = 0.97437(16)$$

FLAG 2019 |V_us| |V_ud|

N_f=2+1+1

FLAG average for N_f=2+1+1
FNAL/MILC 17
ETM 16
ETM 14E
FNAL/MILC 14A
FNAL/MILC 13E
FNAL/MILC 13C
ETM 13F
HPQCD 13A
MILC 13A
MILC 11 (stat. err. only)
ETM 10E (stat. err. only)

N_f=2+1

FLAG average for N_f=2+1
JLQCD 17
QCDSF/UKQCD 16
BMW 16
RBC/UKQCD 15A
RBC/UKQCD 14B
RBC/UKQCD 13
RBC/UKQCD 12
FNAL/MILC 12I
JLQCD 12
Laiho 11
JLQCD 11
MILC 10
JLQCD/TWQCD 10
RBC/UKQCD 10A
RBC/UKQCD 10
BMW 10
MILC 09A
MILC 09
Aubin 08
RBC/UKQCD 08
RBC/UKQCD 07
HPQCD/UKQCD 07
MILC 04

N_f=2

FLAG average for N_f=2
ETM 14D (stat. err. only)
ALPHA 13A
ETM 10D (stat. err. only)
ETM 10D (stat. err. only)
ETM 09A
ETM 09
QCDSF/UKQCD 07
QCDSF 07 (stat. err. only)
RBC 06
JLQCD 05

HFAG 16 τ decay
Hudspith 17 τ decay and e^+e^-
Hudspith 17 τ decay and e^+e^-

Hardy 16 nuclear β decay

0.22   0.23   0.973   0.975

Second Row: Charm Quark

✦ Processes involving charm quark test second row unitarity

$$
\begin{pmatrix}
\begin{array}{ccc}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\
 & K \to l\nu & \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\
D \to l\nu & D_s \to l\nu & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{array}
\end{pmatrix}
$$

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D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\
\boxed{D \to l\nu \quad D_s \to l\nu} & & \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle &
\end{array}
\end{pmatrix}$$

leptonic

Second Row: Charm Quark

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\end{array}
\end{pmatrix}
$$

leptonic

semileptonic

Leptonic Decays

✦ It has been about a decade since the initial tests of D and D_s meson decay constants.

  • based on 2+1 flavor asqtad ensembles

✦ There has been a lot of progress.

✦ Until recently, FNAL/MILC results:

  • Had errors of ≈0.6%

  • We now have more results on 2+1+1 HISQ ensembles

  • Denoted "FNAL/MILC17" on next slide

CERN COURIER

Testing the lattice at CLEO-c
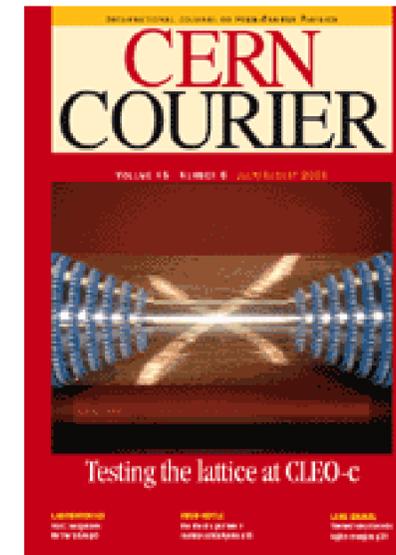
Charm Decay Constants

- FNAL/MILC PRD98, (2018) 074512; arXiv:1712.09262
- HISQ heavy quarks on HISQ sea
- errors < 0.25%

$$f_{D^+} = 212.7(0.5) \text{ MeV}$$
$$f_{D_s} = 249.9(0.4) \text{ MeV}$$

- precision of 2+1 flavor results lower
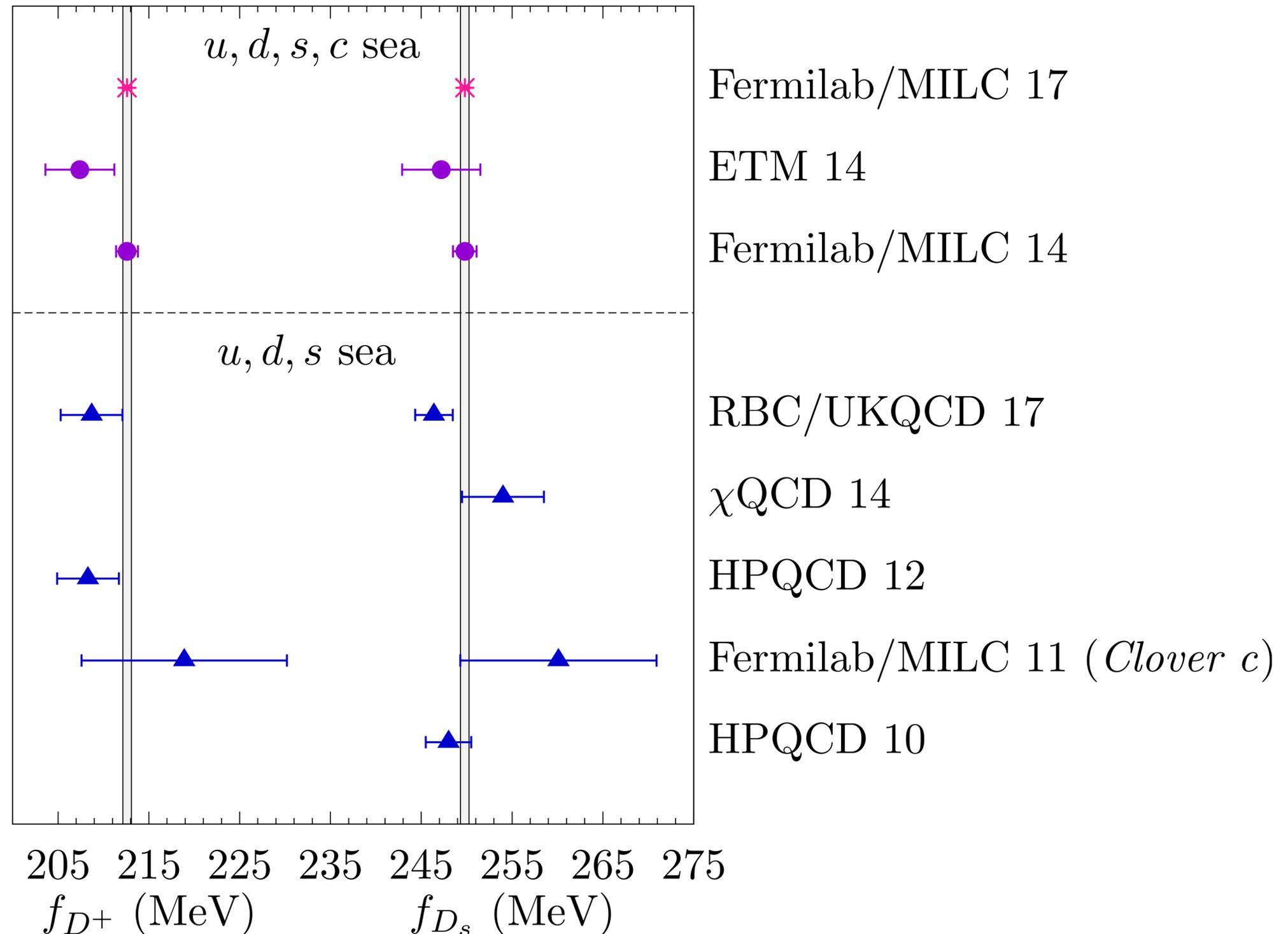
Extraction of V_{cd} & V_{cs}

✦ The experimental results for charm meson leptonic decays are summarized by the Particle Data Group (PDG):

$$f_D|V_{cd}| = 45.91(1.05)\mathrm{MeV}, \qquad f_{D_s}|V_{cs}| = 250.9(4.0)\mathrm{MeV}$$

✦ Experimental error is 1.6–2.3%.

✦ Using decay constants from LQCD, FNAL/MILC get CKM matrix elements:

$$|V_{cd}|_{\mathrm{SM}, f_{D+}} = 0.2152(5)_{f_D}(49)_{\mathrm{expt}}(6)_{\mathrm{EM}}, \qquad |V_{cs}|_{\mathrm{SM}, f_{D_s}} = 1.001(2)_{f_{D_s}}(16)_{\mathrm{expt}}(3)_{\mathrm{EM}}$$

✦ Errors are lattice, experiment, and structure dependent electromagnetic, respectively. Experimental error dominates.

✦ Values above differ slightly from FLAG2019 2+1+1 values

Extraction of V$_{cd}$ & V$_{cs}$

✦ At KEK-FF 2019, Huijing Li gave BESIII result:

$$f_{D_s}|V_{cs}| = 246.2 \pm 3.6_{\text{stat}} \pm 3.5_{\text{sys}} = 246.2(5.0)\text{MeV}$$

✦ implies

$$\text{BES III alone}: \quad |V_{cs}|_{\text{SM}, f_{D_s}} = 0.9822(2)_{f_{D_s}}(20)_{\text{expt}}(3)_{\text{EM}}$$

✦ Obviously, this will improve unitarity, as unitarity sum is reduced from 1.050 to 1.011 (see later slide)

✦ PRL 122, 071802 (2019), arXiv:1811.10890

✦ It will be good to have new experimental average from PDG or HFLAV

Charm Semileptonic Decays

✦ A single form factor describes the major contribution to the decay rate. Analysis often restricted to q²=0.

✦ Experimental input from HFLAV is:

$$f_+^{D\pi}(0)|V_{cd}| = 0.1426(19)\,, \qquad f_+^{DK}(0)|V_{cs}| = 0.7226(34)$$

✦ Errors are 1.3% and 0.5%, respectively.

✦ Next slide has FLAG 2019 summary of lattice QCD form factor results

- ETMC also has form factor for non-zero q².

- Meinel has results for Λc➙Λlν

- FNAL/MILC is completing an analysis using HISQ quarks of charm meson decays. Analysis led by Ruizi Li.

Charm semi-leptonic form factors

- For D to K form factor:

- 2+1 HPQCD
  - 0.747(19), i.e., 2.5%

- 2+1+1 ETM
  - 0.765(31), i.e., 4.2%

- FNAL/MILC (prelim)
  - 0.768(16), i.e., 2.1%

- Expect to have new form factor results as a function of q^2 in the near future.

- Theory needs to improve precision by a factor of 2-4 just to match current experimental precision.
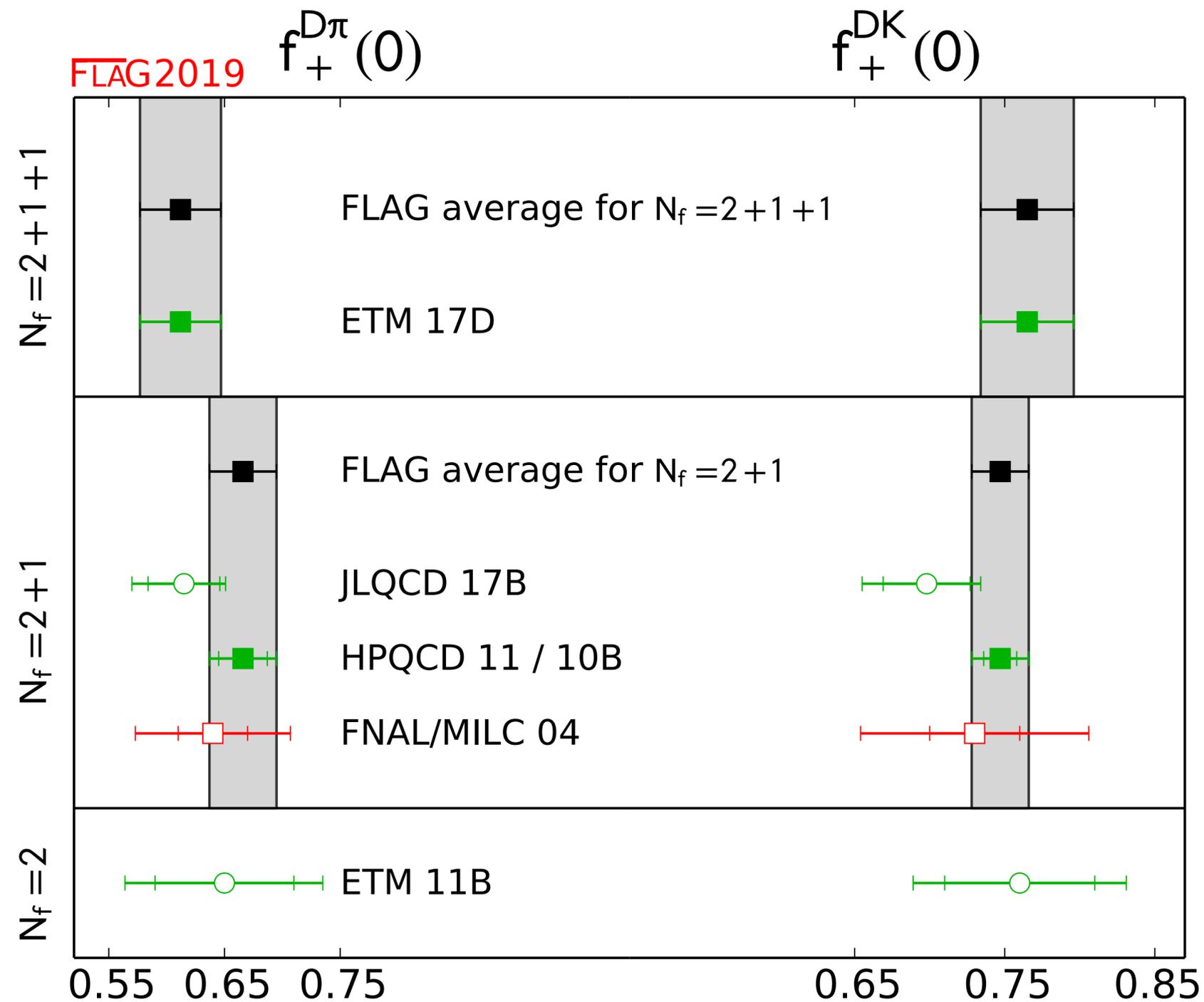
Second Row Elements

- Squares: leptonic

- Triangles: semileptonic

- ETM 17D covers full kinematic range

- Meinel 16 from baryon decay

- It will be interesting to make use of new BES III results

- Experimental errors dominate for leptonic decays, theory for semileptonic

- Next slide gives numerical values
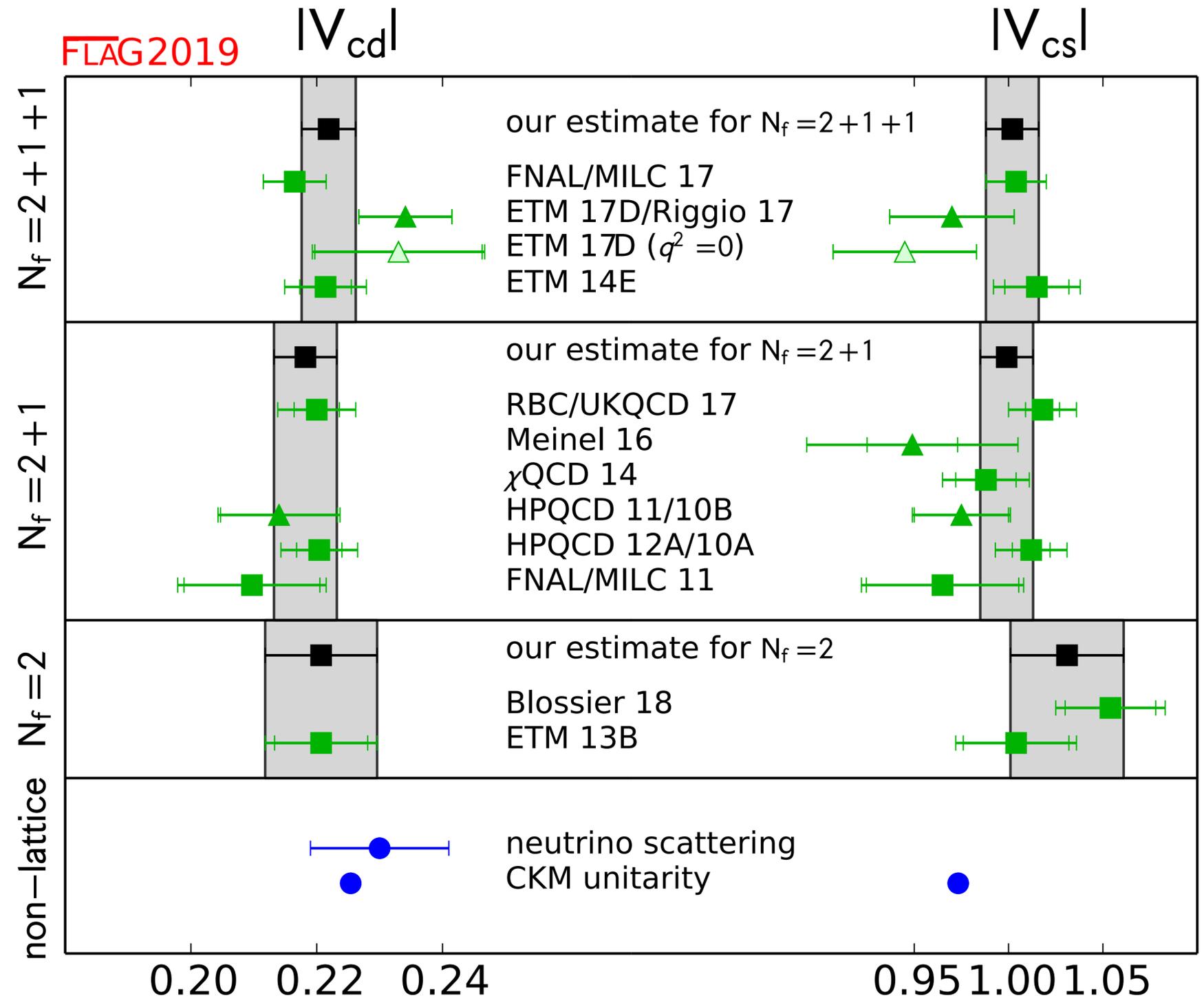
$|V_{cd}|$ & $|V_{cs}|$ Summary

leptonic decays, $N_f = 2 + 1 + 1$: $|V_{cd}| = 0.2166(7)(50)$, $|V_{cs}| = 1.004(2)(16)$,

leptonic decays, $N_f = 2 + 1$: $|V_{cd}| = 0.2197(25)(50)$, $|V_{cs}| = 1.012(7)(16)$,

semileptonic decays, $N_f = 2 + 1 + 1$: $|V_{cd}| = 0.2341(74)$, $|V_{cs}| = 0.970(33)$,

semileptonic decays, $N_f = 2 + 1$: $|V_{cd}| = 0.2141(93)(29)$, $|V_{cs}| = 0.967(25)(5)$,

semileptonic Λ_c decay, $N_f = 2 + 1$: $|V_{cs}| = 0.949(24)(51)$,

FLAG2019, $N_f = 2 + 1 + 1$: $|V_{cd}| = 0.2219(43)$, $|V_{cs}| = 1.002(14)$,

FLAG2019, $N_f = 2 + 1$: $|V_{cd}| = 0.2182(50)$, $|V_{cs}| = 0.999(14)$,

✦ Errors are 1.9-2.3% and 1.4% for the two matrix elements

✦ Our community should improve the semileptonic form factors

Second Row Unitarity

✦ Latest FNAL/MILC leptonic decay results:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.050(2)_{|V_{cd}|}(32)_{|V_{cs}|}(0)_{|V_{cb}|}$$

- 1.6 σ effect

✦ FLAG 2019 value for Nf=2+1+1 & 2+1, is 1.05(3)

- 1.67 σ, so latest result in better agreement with unitarity than previous FLAg result

✦ ETMC (arxiv:1706.03657) semileptonic results:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.996(64)$$

✦ Semileptonic decays favor a smaller value of V_{cs} hence they are in better agreement with unitarity.

Third Column: Bottom Quark

semileptonic

✦ Some processes involving bottom quark:

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to \pi l\nu & \boxed{B \to \pi l\nu} \\ & K \to l\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & \boxed{B \to D^{(*)} l\nu} \\ D \to l\nu & D_s \to l\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

semileptonic

B Hadron Decays

✦ Leptonic and semileptonic decays studied in LQCD

- Mesons are extensively studied

- In the past couple of years, nice results for baryon form factors from Meinel and collaborators

✦ Rare decays involving flavor changing neutral currents (FCNC) also studied

- FCNC vanish at tree level in Standard Model, so a good place to look for new physics

- Some tension between recent SM predictions from LQCD and LHCb measurements

- Alternative to B meson mixing for determining $|V_{td}|$ and $|V_{ts}|$

✦ First let's look at leptonic decay constants…

B Meson Leptonic Decays

- Latest results are from FNAL/MILC (arXiv:1712.09262; PRD98 074512, 2018)

- errors are <1.3 MeV or <0.7%

- Good agreement with other calculations whose errors range down to 5—7 MeV

- BaBar and Belle measurements don't agree very well; Belle II will provide higher precision results

- Also see FLAG 4 for more history
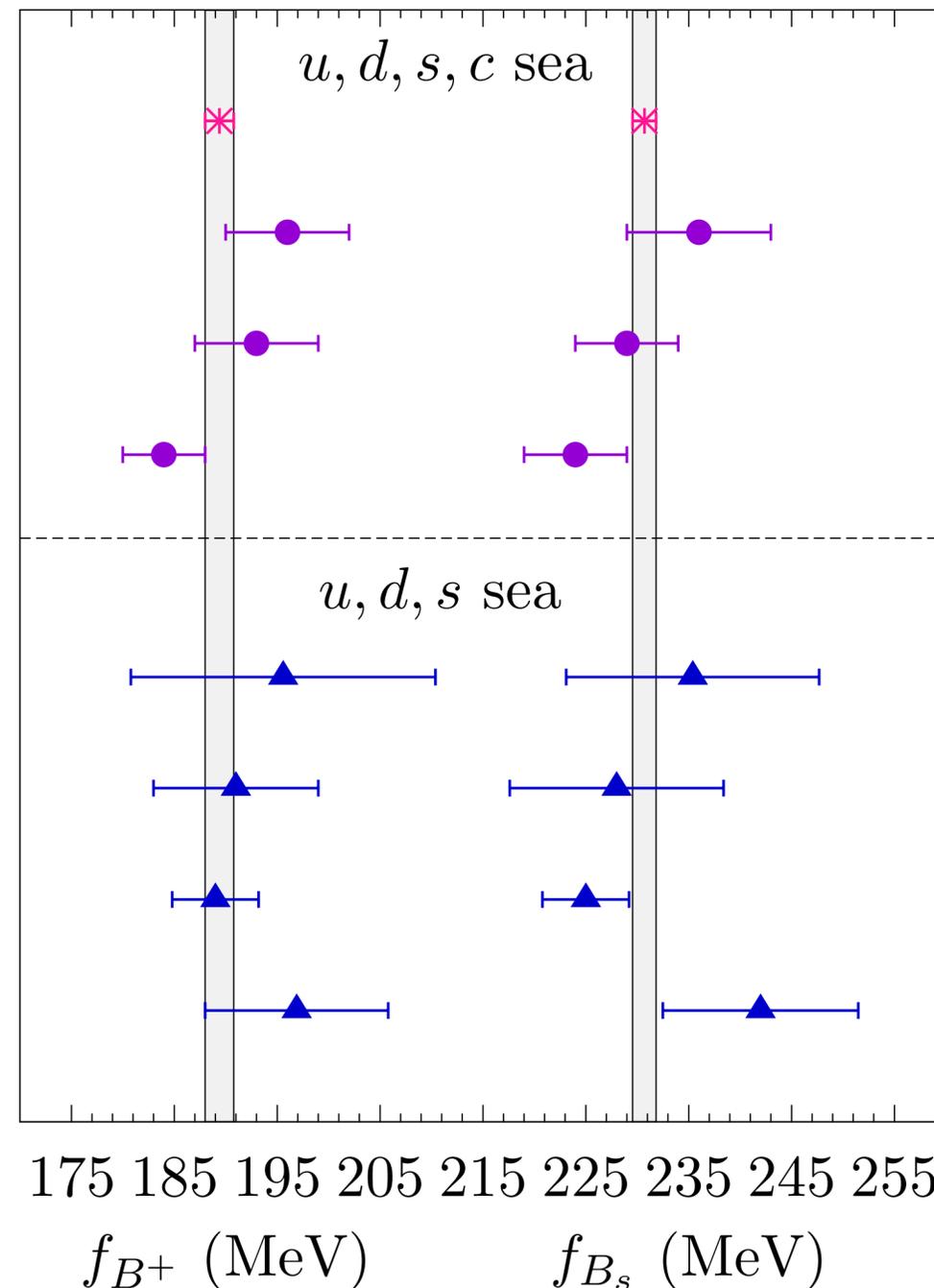
B Semileptonic & Rare Decays

✦ Could probably talk for an entire hour about this:

- For |Vub| there are many possible decays

$$B \to \pi\ell\nu,\ B_s \to K\ell\nu,\ B_s \to K^*\ell\nu,\ \Lambda_b \to p\ell\nu$$

- For |Vcb|

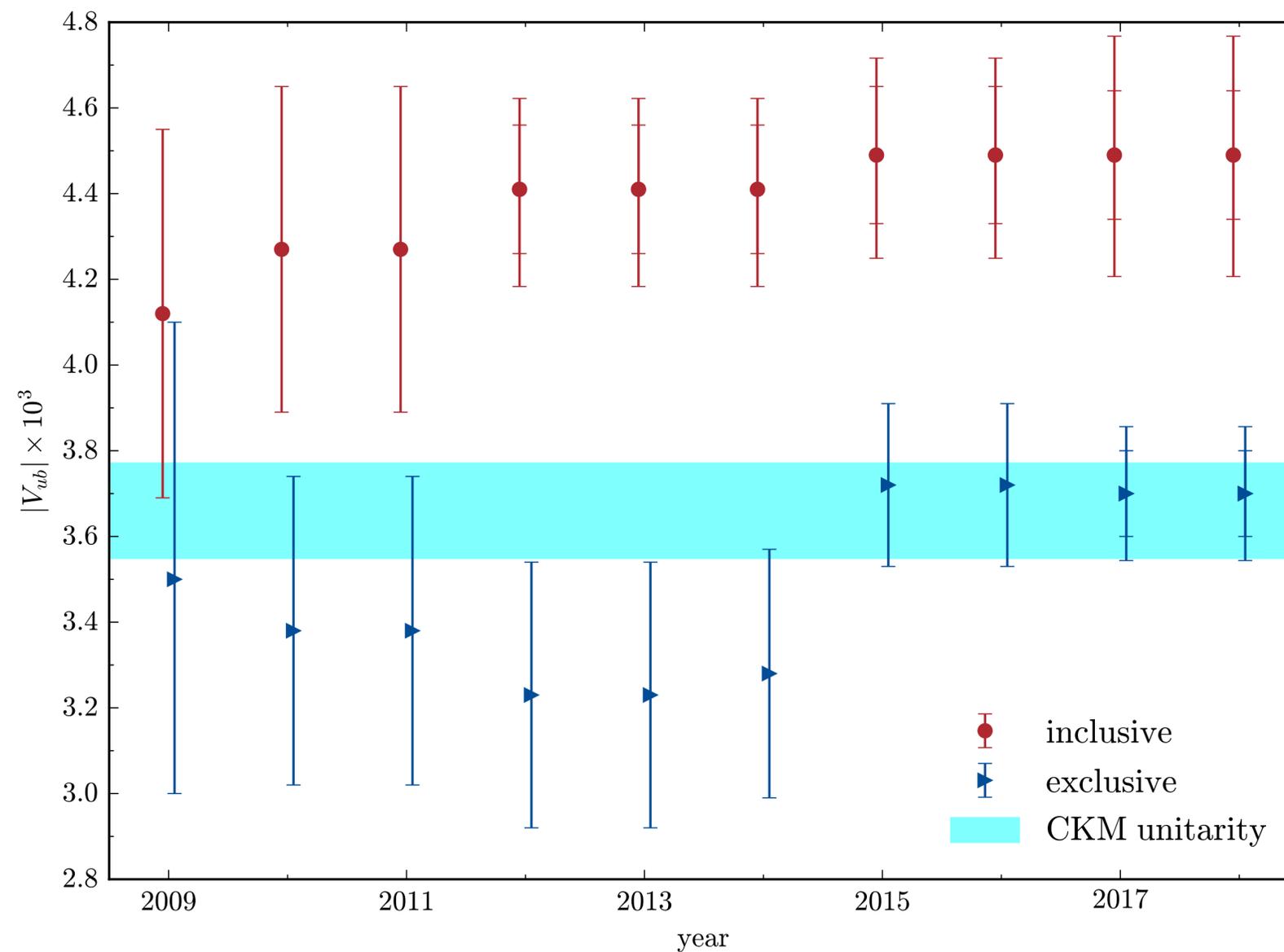$$B \to D\ell\nu,\ B \to D^*\ell\nu,\ B_s \to D_s^{(*)}\ell\nu,\ \Lambda_b \to \Lambda_c\ell\nu$$

- Lepton universality:
$$\ell = e,\ \mu,\ \text{or}\ \tau$$
- Rare decays, *e.g.*, flavor changing neutral currents

$$B^0 \to \mu^+\mu^-,\ B_s \to \mu^+\mu^-,\ B \to K\ell^+\ell^-$$

✦ First look at long-standing tension between CKM matrix element as determined in inclusive vs. exclusive decays

Inclusive vs. Exclusive Decays

✦ Long standing difference between |Vub| determined from inclusive and exclusive decays

Exclusive B Decay History

✦ FNAL/MILC updated form factors for semileptonic decays PRD 92, 014024 (2015),arXiv:1503.07839

✦ FLAG 2013:

$$|V_{ub}| = 3.37(21) \times 10^{-3}, \quad N_f = 2 + 1; \text{BaBar}$$

$$|V_{ub}| = 3.47(22) \times 10^{-3}, \quad N_f = 2 + 1; \text{Belle}$$
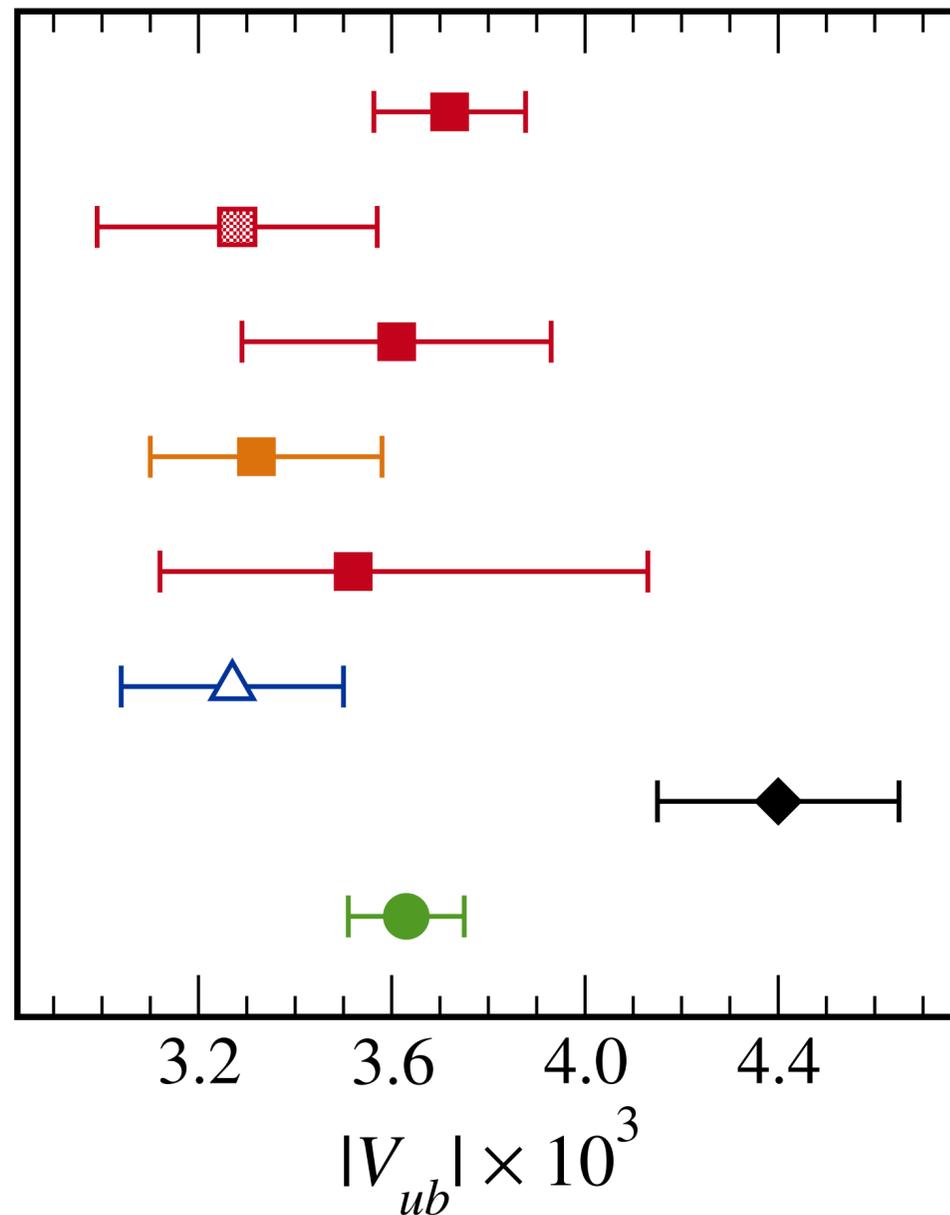
✦ 2015 FNAL/MILC result

$$|V_{ub}| = 3.72(16) \times 10^{-3}, \quad N_f = 2 + 1; \text{BaBar\&Belle}$$

✦ This result decreased, but did not eliminate tension between exclusive and inclusive results.

✦ Next slide also includes Lambda baryon decay result

Updated Semileptonic |V_ub|

Lattice error now comparable to experimental error.
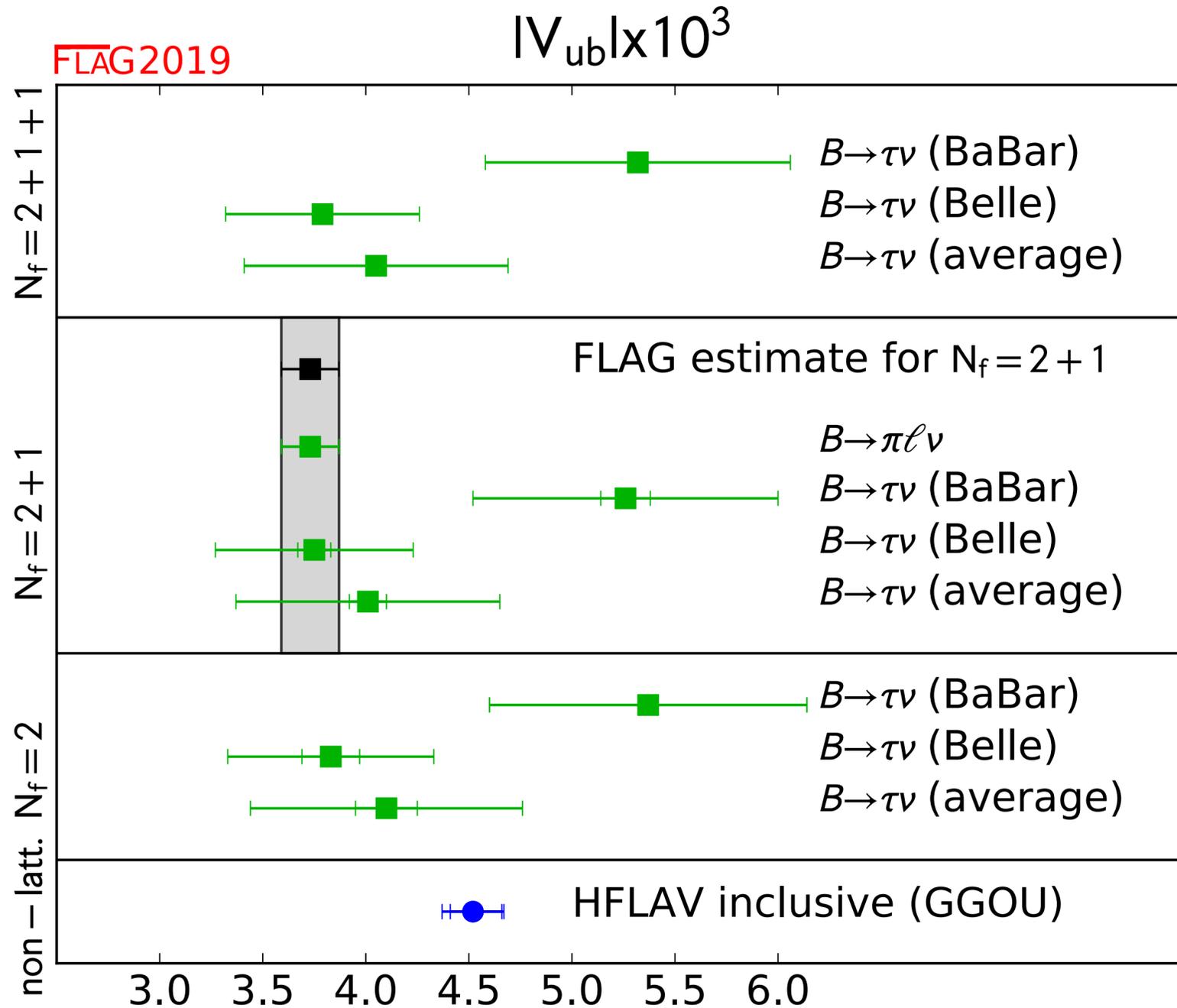
Form Factors for B→πlν: I
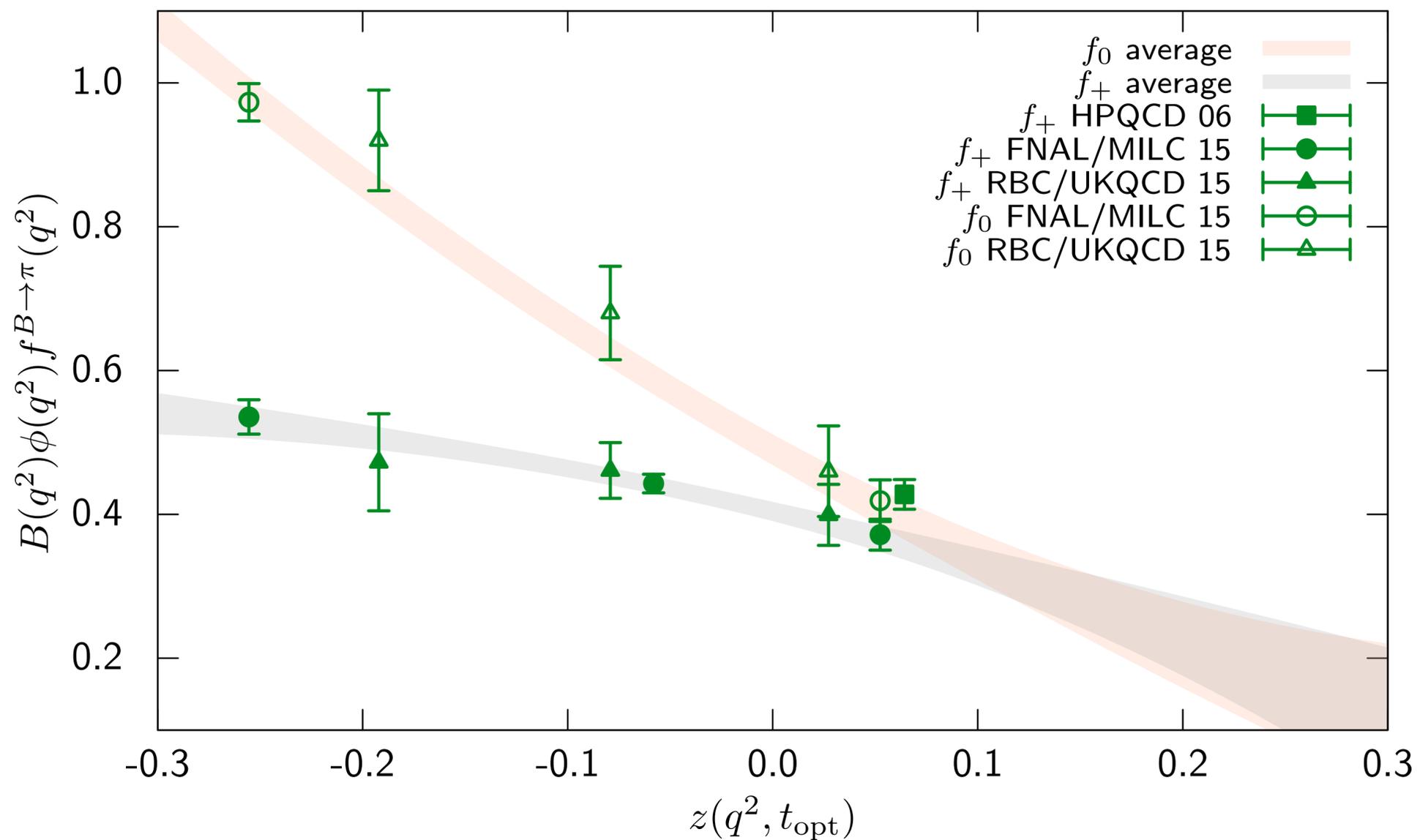
- FLAG 2019

- Shows both form factors f_0, f_+ using z rather than q^2

- FNAL/MILC & HPQCD results on 2+1 flavor asqtad configurations

- RBC/UKQCD on domain wall

Form Factors for B→πlν: II
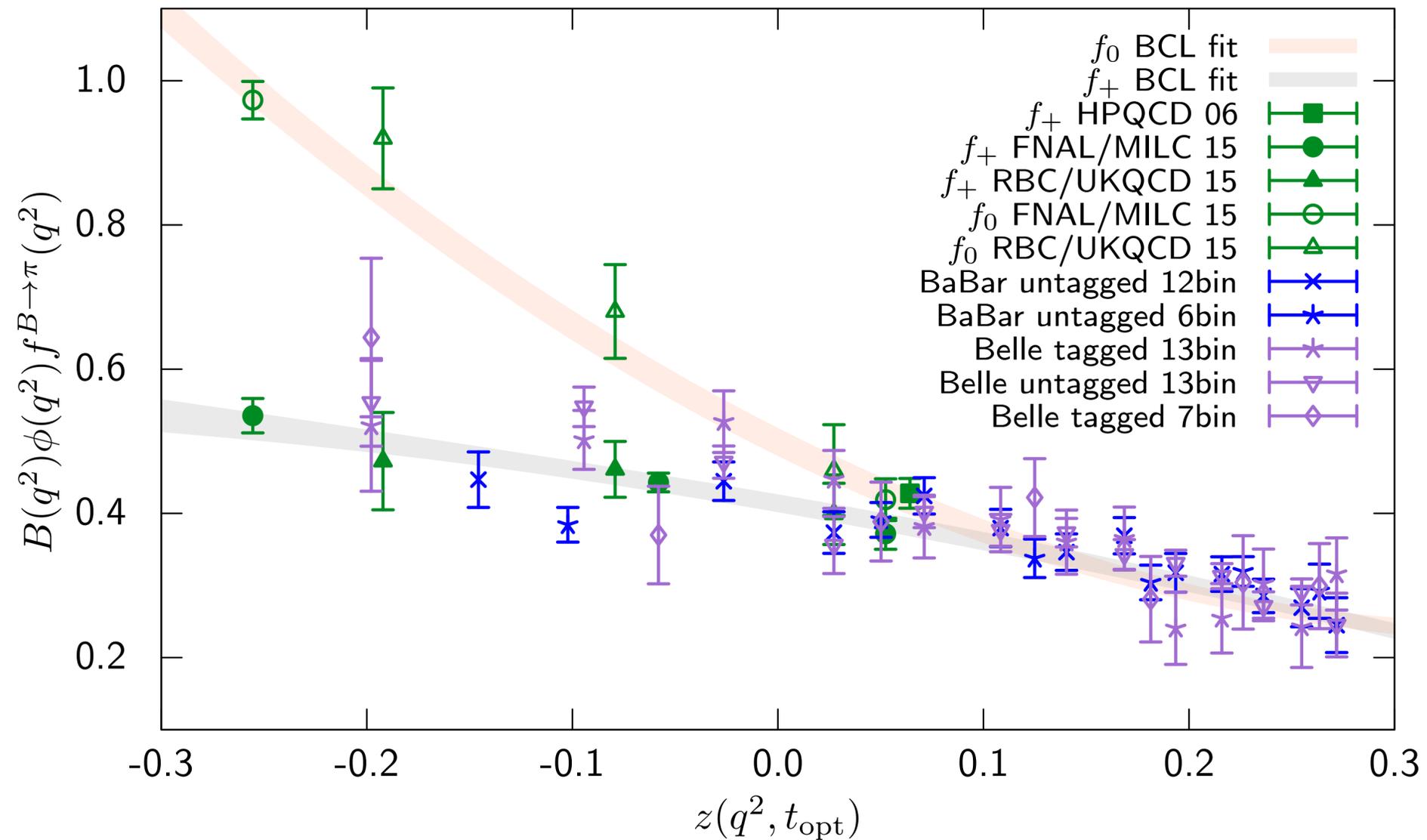
- FLAG 2019

- Joint fit to data and lattice QCD form factors

- Note how experiment at positive z, i.e., small q^2 constrains form factors in that region

- Theory dominates at negative z

- Joint fit determines V_{ub}

Plot legend:
- f_0 BCL fit
- f_+ BCL fit
- f_+ HPQCD 06
- f_+ FNAL/MILC 15
- f_+ RBC/UKQCD 15
- f_0 FNAL/MILC 15
- f_0 RBC/UKQCD 15
- BaBar untagged 12bin
- BaBar untagged 6bin
- Belle tagged 13bin
- Belle untagged 13bin
- Belle tagged 7bin

Axis labels: $B(q^2)\phi(q^2)f^{B\to\pi}(q^2)$ vs $z(q^2, t_{\mathrm{opt}})$

$|V_{ub}|$ from FLAG

IV$_{ub}$Ix10^3

FLAG2019

$N_f = 2+1+1$
- $B \to \tau\nu$ (BaBar)
- $B \to \tau\nu$ (Belle)
- $B \to \tau\nu$ (average)

$N_f = 2+1$
- FLAG estimate for $N_f = 2+1$
- $B \to \pi\ell\nu$
- $B \to \tau\nu$ (BaBar)
- $B \to \tau\nu$ (Belle)
- $B \to \tau\nu$ (average)

non−latt. $N_f = 2$
- $B \to \tau\nu$ (BaBar)
- $B \to \tau\nu$ (Belle)
- $B \to \tau\nu$ (average)
- HFLAV inclusive (GGOU)

- BaBar and Belle leptonic decays results don't agree very well.

- Leptonic error totally dominated by experiment.

- Semileptonic result is more precise.

- Tension between inclusive and exclusive determinations.

- Critical role for Belle II for both leptonic and semileptonic results

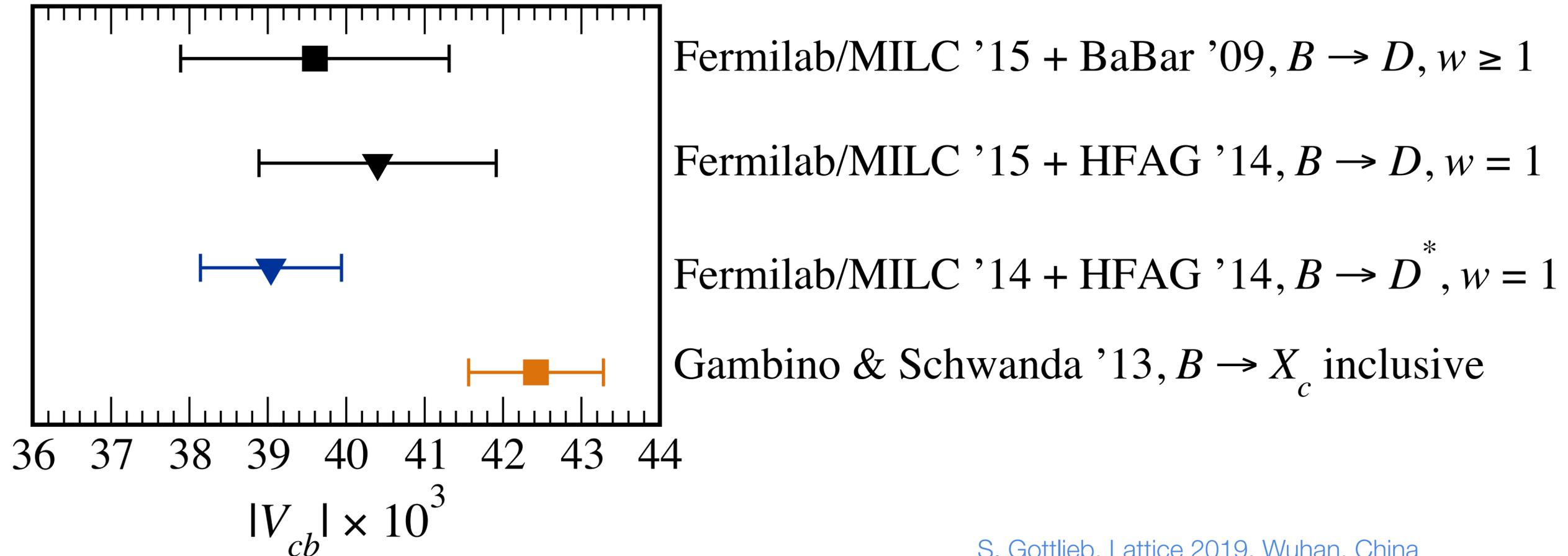- Lattice semileptonic will improve in the next couple of years as N$_f$=2+1+1 results become available.

$$V_{ub} = 3.73(0.14) \times 10^{-3}, \; i.e., 3.8\%$$

$|V_{cb}|$ circa 2015

✦ Exclusive calculations of B→D* l ν and B→D l ν yield V_{cb}

✦ Experimental error dominant for B→D l ν (3.9% vs 1.4%)

✦ Again, tension between exclusive and inclusive results

PRD 92, 034506 (2015), arXiv:1503.07237 [hep−lat]

✦ But two exclusive decay modes consistent

Fermilab/MILC '15 + BaBar '09, $B \to D$, $w \geq 1$

Fermilab/MILC '15 + HFAG '14, $B \to D$, $w = 1$

Fermilab/MILC '14 + HFAG '14, $B \to D^*$, $w = 1$

Gambino & Schwanda '13, $B \to X_c$ inclusive

$|V_{cb}| \times 10^3$

|V_{cb}| Update

✦ Inclusive-exclusive tension may be explained—At least I thought that last year! (See Vaquero talk this afternoon and Lytle plenary tomorrow.)

✦ Uncertainty from parameterization of form factors

- Bigi, Gambino, & Schacht PLB 769 441 (2017)

- Grinstein & Kobach PLB 771, 359 (2017)

- Find 10% shift in experimental input switching from CNL to BGL for $B \to D^* \ell \nu$ Belle 2017 data arXiv:1702.01521

$$CNL: (38.2 \pm 1.5) \times 10^{-3} \qquad BGL: (41.7 \pm 2.0) \times 10^{-3}, \qquad \text{for } |V_{cb}|$$

- for $B \to D \ell \nu$ Bigi & Gambino PRD 97, 094008 (2016) used FNAL/MILC & HPQCD lattice input with updated Belle data

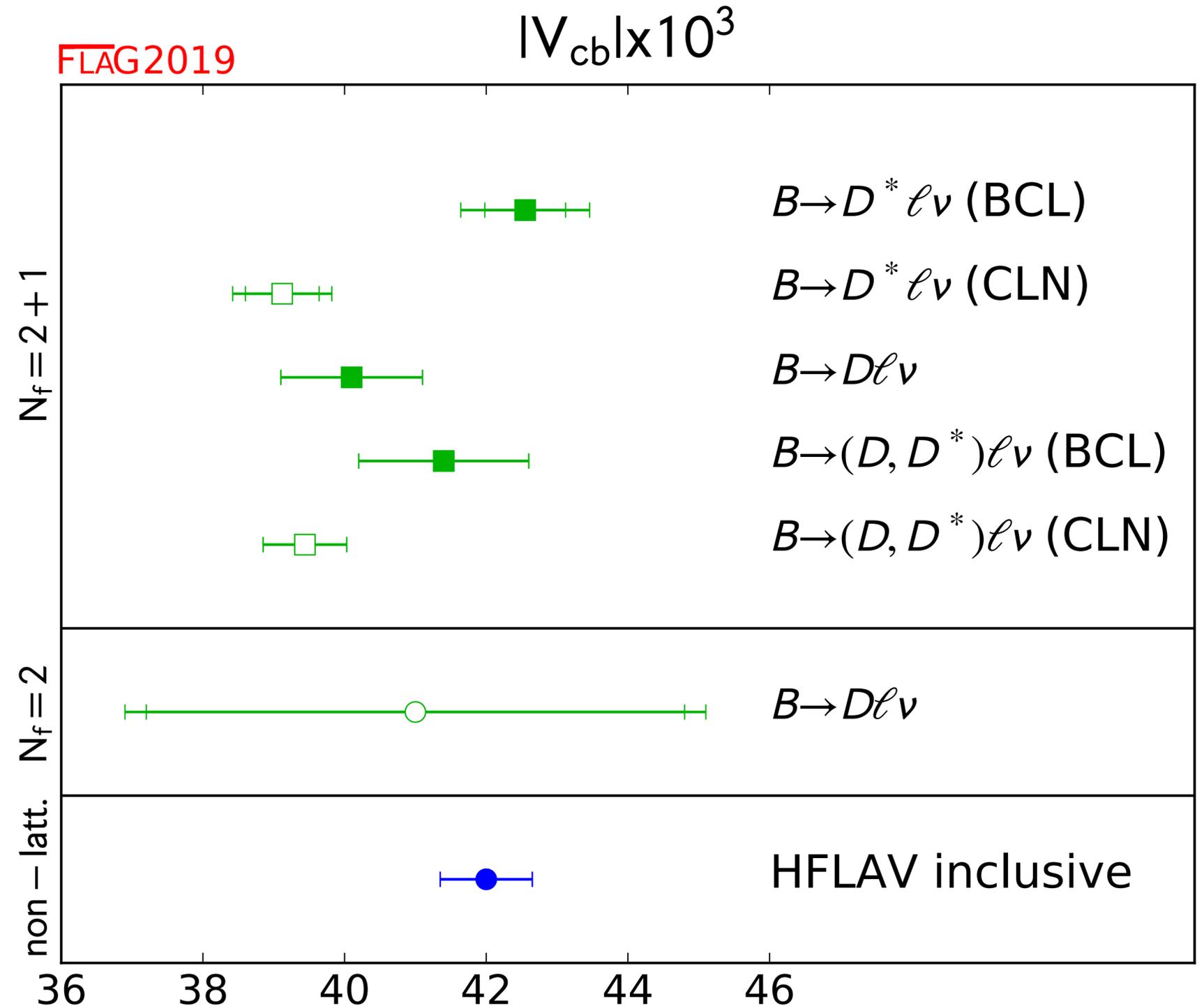$$BGL, N = 4: (40.49 \pm 0.97) \times 10^{-3}, \qquad \text{for } |V_{cb}|$$

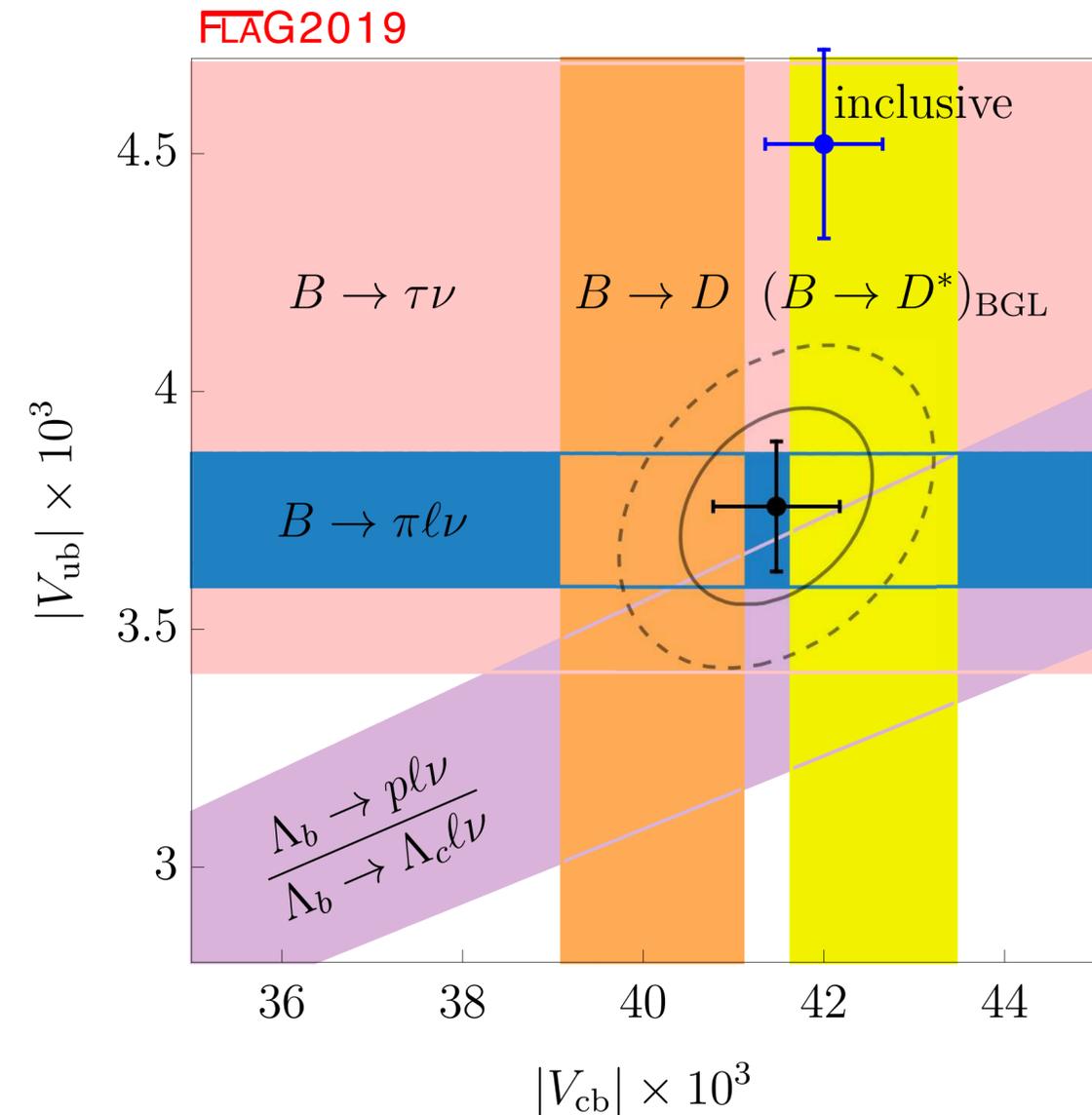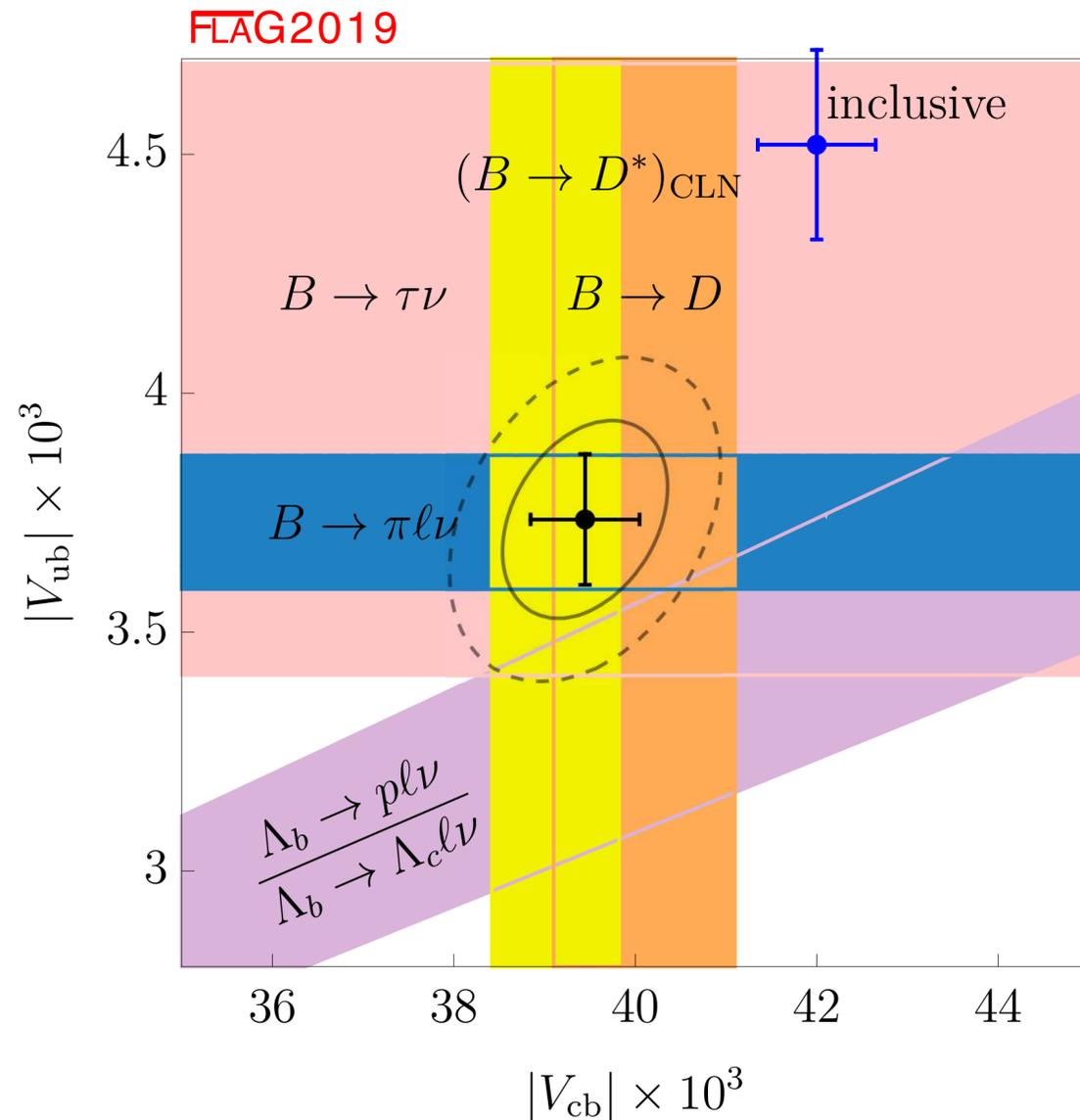- PDG quotes (42.2±0.8)×10⁻³ for inclusive determination

- Future LQCD and Belle II will play an important role here

|V$_{cb}$| from FLAG

- Good agreement between the two decay channels for 2+1 flavor lattice form factors.

- Tension between inclusive and exclusive is not that bad.

$|V_{cb}| \times 10^3$

V$_{ub}$ and V$_{cb}$

Using BGL rather than CNL for B→D* eliminates tension between inclusive and exclusive determinations of Vcb.

Tension for B→π remains!

Lambda decay not in fit to exclusive decays.

Third Row: B Meson Mixing

Third Row: B Meson Mixing
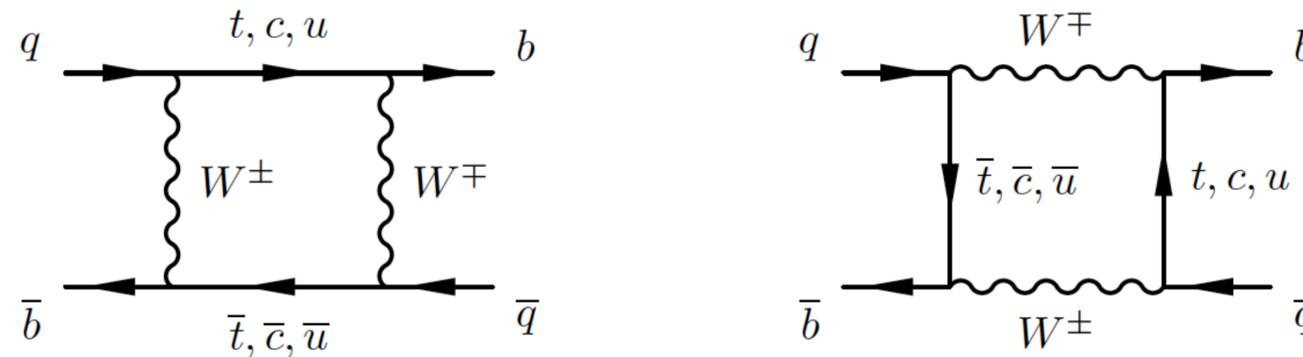
✦ Neutral B meson mixing helps determine third row

$$
\begin{pmatrix}
\begin{array}{c} \mathbf{V_{ud}} \\ \pi \to l\nu \end{array} & \begin{array}{c} \mathbf{V_{us}} \\ K \to \pi l\nu \\ K \to l\nu \end{array} & \begin{array}{c} \mathbf{V_{ub}} \\ B \to \pi l\nu \end{array} \\
\begin{array}{c} \mathbf{V_{cd}} \\ D \to \pi l\nu \\ D \to l\nu \end{array} & \begin{array}{c} \mathbf{V_{cs}} \\ D \to K l\nu \\ D_s \to l\nu \end{array} & \begin{array}{c} \mathbf{V_{cb}} \\ B \to D^{(*)} l\nu \end{array} \\
\begin{array}{c} \mathbf{V_{td}} \\ \langle B_d | \overline{B}_d \rangle \end{array} & \begin{array}{c} \mathbf{V_{ts}} \\ \langle B_s | \overline{B}_s \rangle \end{array} & \mathbf{V_{tb}}
\end{array}
\end{pmatrix}
$$

B meson mixing

✦ Neutral B meson mixing is a loop level process:

✦ q can be a d or s quark

✦ Experiments can measure mass difference and lifetime difference for the two resulting eigenstates

✦ They also measure a CP violating phase

✦ Short distance expansion of the loops results in effective weak Hamiltonian involving 4-quark operators

✦ GIM and loop suppression, so good place to look for BSM

Mixing Calculation

✦ First we define the effective Hamiltonian in terms of eight operators. However, in the SM we only need the first one.

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} C_i \mathcal{O}_i^q + \sum_{i=1}^{3} \tilde{C}_i \tilde{\mathcal{O}}_i^q$$

✦ Matrix elements of this four-quark operator are calculated using lattice QCD

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma_\mu L q^\alpha \, \bar{b}^\beta \gamma_\mu L q^\beta$$

$$\langle \mathcal{O}_1^q \rangle (\mu) = \langle \bar{B}_q^0 | \mathcal{O}_1^q | B_q^0 \rangle (\mu) \equiv \frac{2}{3} f_{B_q}^2 M_{B_q}^2 B_{B_q}^{(1)}(\mu)$$

✦ The last factor on previous line is called the bag parameter. In vacuum saturation, it would be 1.
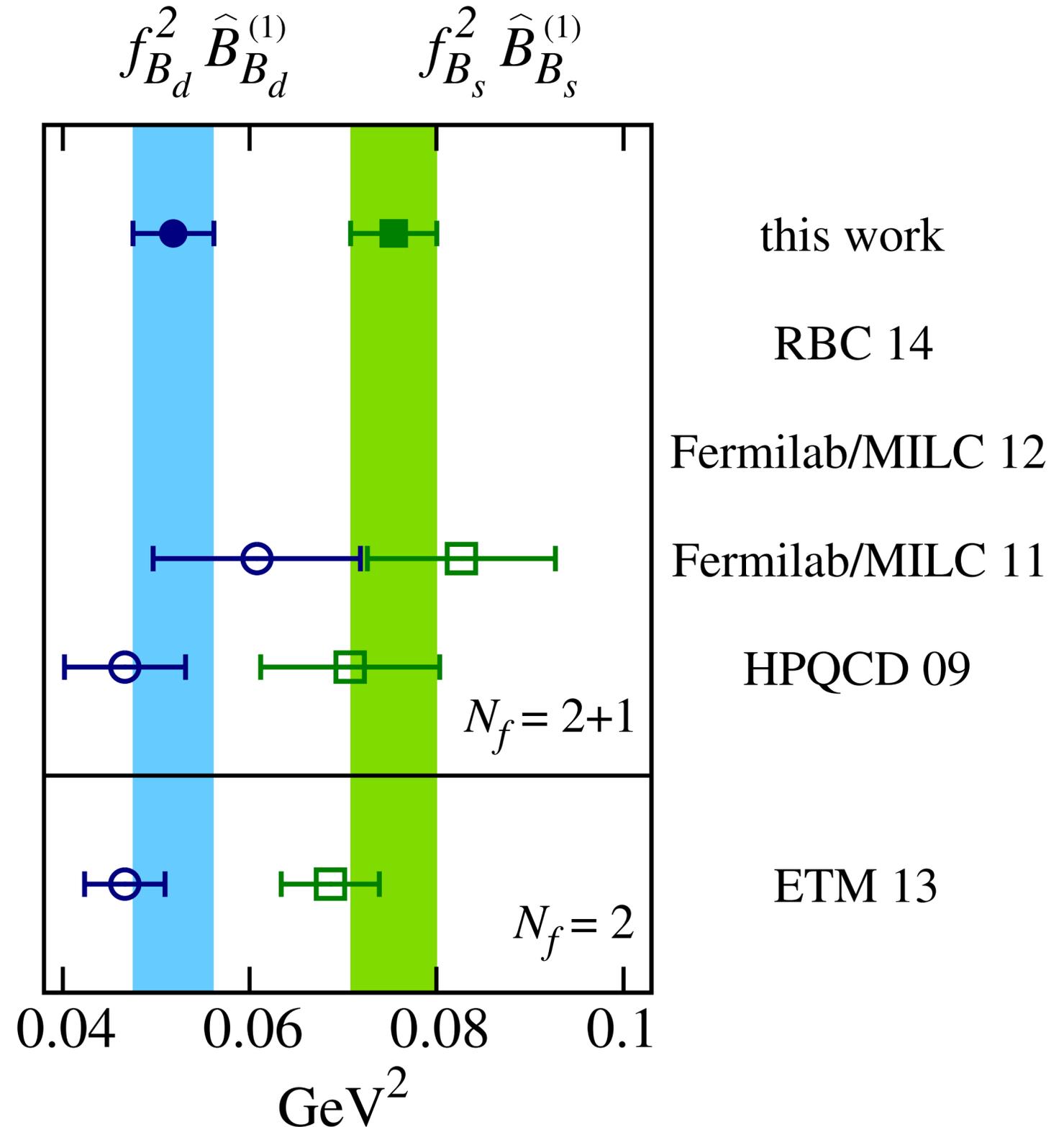
Mixing Calculation II

✦ It is also convenient to define a renormalization group invariant bag parameter and use it to define physical quantities

$$\hat{B}_{B_q}^{(1)} = \alpha_s(\mu)^{-\gamma_0/(2\beta_0)} \left[1 + \frac{\alpha_s(\mu)}{4\pi} \left(\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right) \right] B_{B_q}^{(1)}(\mu)$$
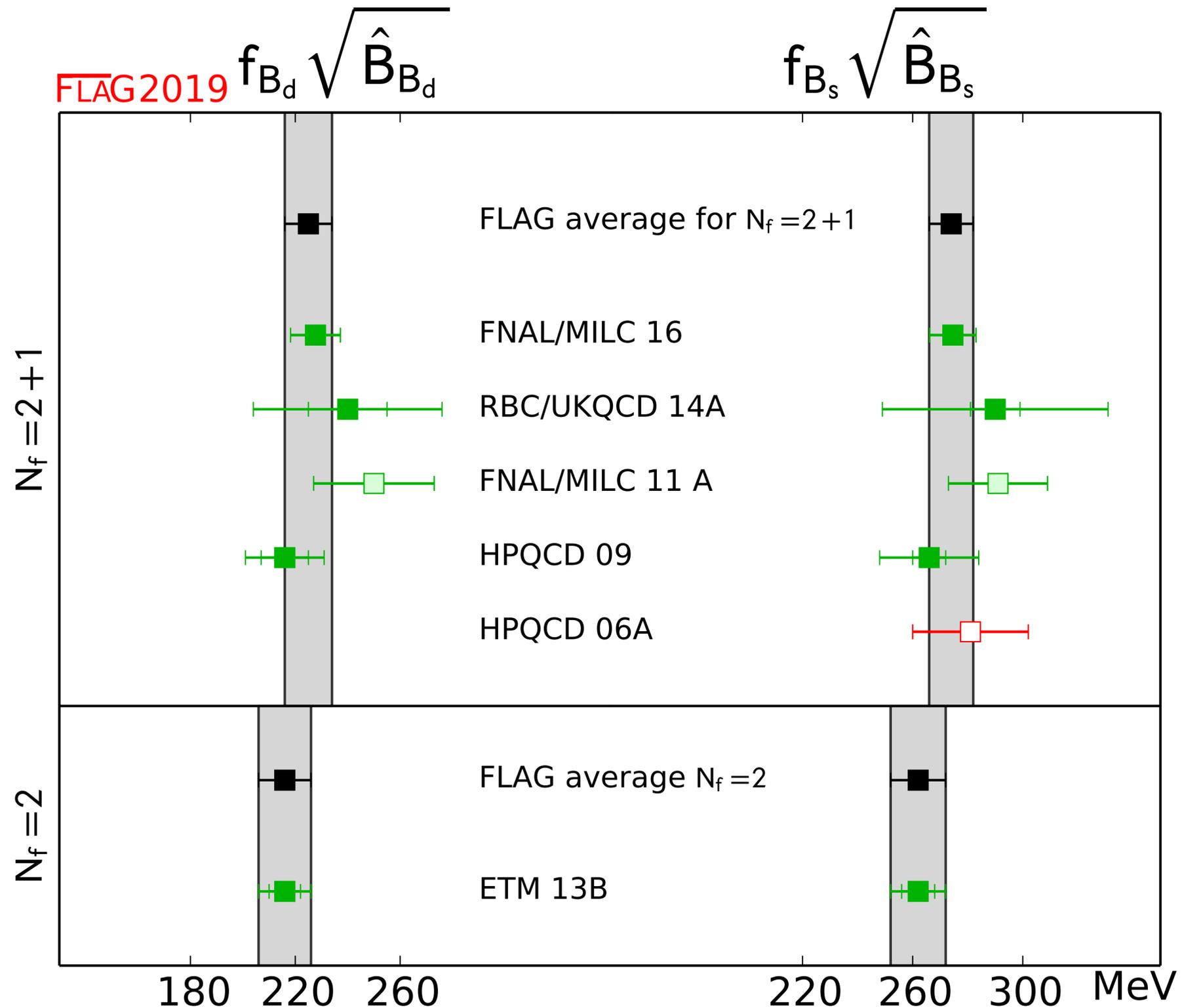
$$\Delta M_q = \frac{G_F^2 m_W^2 M_{B_q}}{6\pi^2} S_0(x_t) \, \eta_{2B} \, |V_{tq}^* V_{tb}|^2 \, f_{B_q}^2 \, \hat{B}_{B_q}^{(1)}$$

FNAL/MILC Mixing Results

- PRD 93 (2016) 113016, arXiv: 1602.03560

- RBC 14, FNAL/MILC 12 contain ratio, which is not plotted here, but is in the publication.

- Next slide has FLAG comparison

- FNAL/MILC paper also contains operators needed to bound BSM contributions
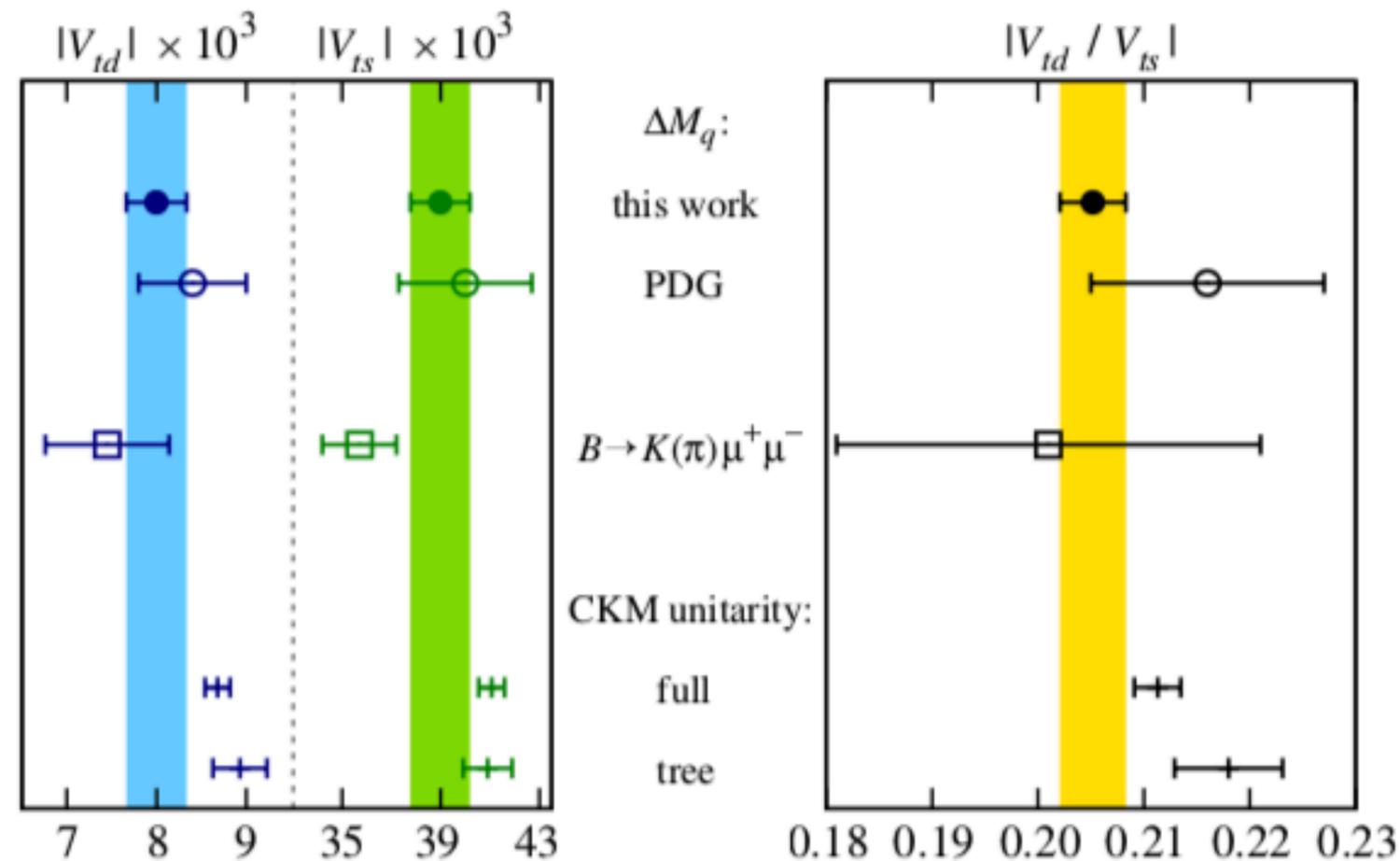

$$f^2_{B_d} \widehat{B}^{(1)}_{B_d} \qquad f^2_{B_s} \widehat{B}^{(1)}_{B_s}$$

this work

RBC 14

Fermilab/MILC 12

Fermilab/MILC 11

HPQCD 09

$N_f = 2+1$

ETM 13

$N_f = 2$

0.04   0.06   0.08   0.1

GeV^2

FLAG Summary for B Mixing

$f_{B_d} \sqrt{\hat{B}_{B_d}}$ $f_{B_s} \sqrt{\hat{B}_{B_s}}$

FLAG2019

- Note, this is square root of what is shown on previous slide.
- Two separate sets of x-axes allow display of RBC/UKQCD result.
- FLAG does not calculate impact of these calculations of CKM matrix, but FNAL/MILC do.
- Next slide shows result.

FLAG average for $N_f = 2+1$

FNAL/MILC 16

RBC/UKQCD 14A

FNAL/MILC 11 A

HPQCD 09

HPQCD 06A

$N_f = 2+1$

FLAG average $N_f = 2$

ETM 13B

$N_f = 2$

180   220   260   220   260   300   MeV

CKM Third Row Elements

✦ Using experimental results on B$_{(S)}$ mixing, FNAL/MILC find:

FIG. 16. (*left*) Recent determinations $|V_{td}|$ and $|V_{ts}|$, and (*right*) their ratio. The filled circles and vertical bands show our new results in Eqs. (9.17)–(9.19), while the open circles show the previous values from B_q-mixing [102]. The squares show the determinations from semileptonic $B \to \pi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ decays [183], while the plus symbols show the values inferred from CKM unitarity [158]. The error bars on our results do not include the estimated charm-sea uncertainties, which are too small to be visible.

$$|V_{td}| = 8.00(33)_{\mathrm{lat}}(2)_{\Delta M_q}(3)(8)_{\mathrm{no-c}} \times 10^{-3}$$

$$|V_{ts}| = 39.0(1.2)_{\mathrm{lat}}(0.0)_{\Delta M_q}(0.2)(0.4)_{\mathrm{no-c}} \times 10^{-3}$$

$$|V_{td}/V_{ts}| = 0.2052(31)_{\mathrm{lat}}(4)_{\Delta M_q}(0)(10)_{\mathrm{no-c}}$$

✦ Errors are lattice, experimental mass difference, other parameters, lack of charm quark in sea.

✦ Lattice errors clearly dominant since we can use ensembles with c quarks

✦ errors 4.3%, 3.2%, 1.6%

Summary

Quantity	value	percentage error	Comment
$|V_{ud}|$	0.9737(16)	0.16	FLAG (with unitarity)
$|V_{ud}|$	0.9669(34)	0.35	FNAL/MILC (K_{l2} & K_{l3})
$|V_{us}|$	0.2249(7)	0.31	FLAG (with unitarity)
$|V_{us}|$	0.22333(61)	0.27	FNAL/MILC (K_{l2} & K_{l3})
$|V_{cd}|$	0.2219(43)	1.9	FLAG (2+1+1)
$|V_{cs}|$	1.002(14)	1.4	FLAG (2+1+1)
$|V_{ub}| \times 10^3$	3.76(14)	3.7	FLAG (BGL, combined)
$|V_{cb}| \times 10^3$	41.47(70)	1.7	FLAG (BGL, combined)
$|V_{td}| \times 10^3$	8.00(34)	4.3	FNAL/MILC
$|V_{ts}| \times 10^3$	39.0(1.3)	3.2	FNAL/MILC

Prospects

- ✦ "It is tough to make predictions, especially about the future" — Yogi Berra
- ✦ In some quantities lattice error is the limiting factor
- ✦ Even where it is comparable, BES III, Belle II, and LHCb will reduce experimental error
- ✦ Belle II theory report assumes a factor of 5 improvement in 10 years
- ✦ USQCD white paper mainly emphases the opportunities
- ✦ LHC report assumes a factor of 3 improvement
- ✦ FLAG does not make projections, so I will just show a result from Belle II
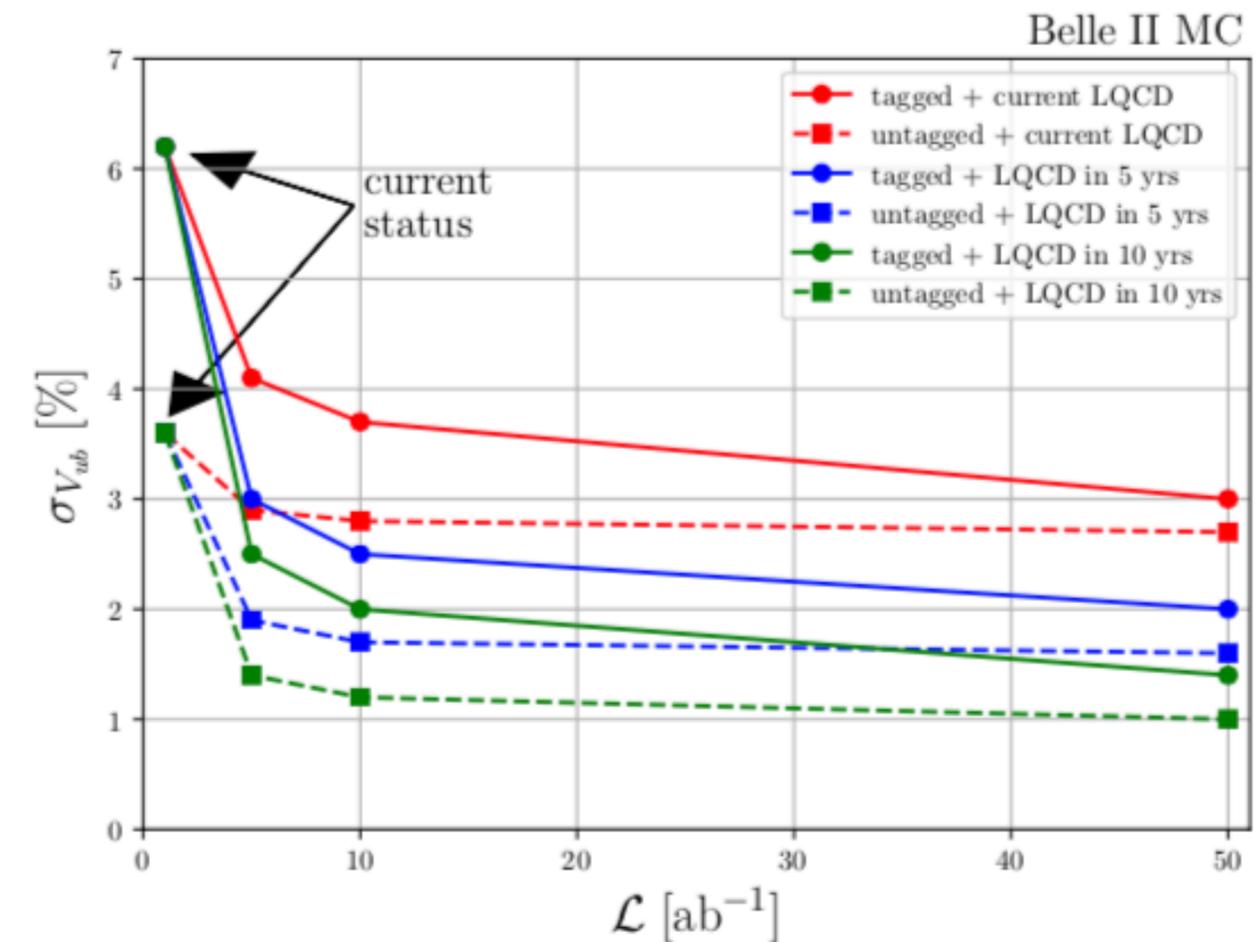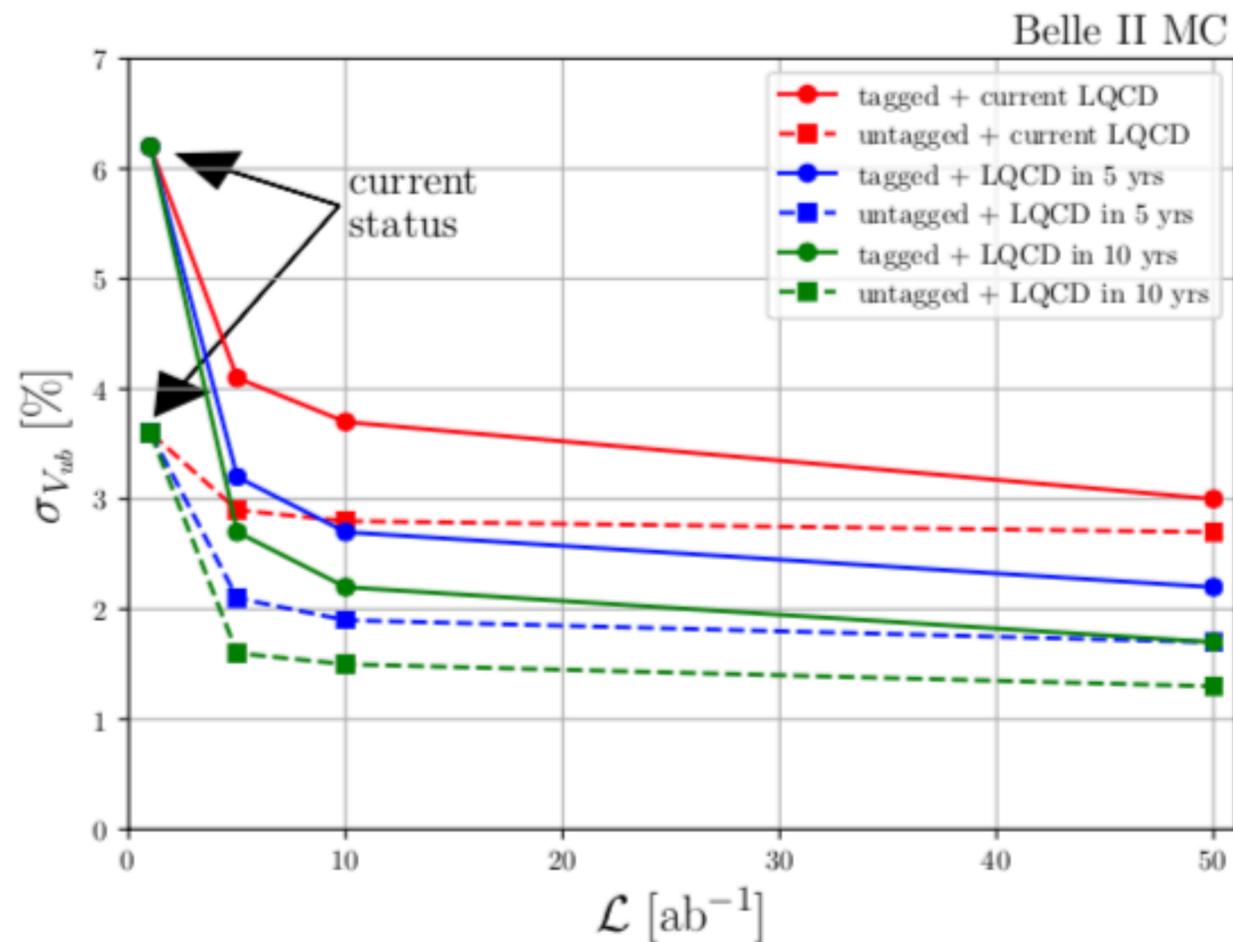- ✦ Many more details in the three reports

Fig. 85: Projections of V_{ub} error to various luminosity values and lattice-QCD error forecasts for $B \to \pi \ell \nu$ tagged and untagged modes. The figure on the left is obtained by using lattice forecasts with EM corrections and the figure on the right by forecasts without these corrections.

Conclusions

✦ There has been very significant progress using lattice QCD to calculate quark masses and matrix elements needed to determine the CKM matrix.

- • A number of quantities are available to sub-percent accuracy.

- • Expect to increase LQCD precision by factor of 3-5 over the next 5-10 years

✦ Interplay between theory and experiment will provide more and more stringent tests of the Standard Model

✦ Getting to the point where electromagnetic corrections important to lattice QCD calculations

- • Rome-Southampton group has been leading on that issue

- • Session 11:10 to 12:30 on Wednesday

✦ BESIII, Belle II, and LHCb have a large role to play in the future of flavor physics—can hardly wait for new results!