2019 Update on ε_K with lattice QCD inputs

Yong-Chull Jang, Jeehun Kim, Weonjong Lee†, Sunkyu Lee, and Jaehoon Leem , Sungwoo Park
(LANL-SWME Collaboration)

Lattice Gauge Theory Research Center
Department of Physics and Astronomy
Seoul National University

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ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Master formula for ε_K in the Standard Model.

$$\varepsilon_K = \exp(i\theta)\,\sqrt{2}\,\sin(\theta)\left(C_\varepsilon\, X_{\mathsf{SD}}\,\hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\mathsf{LD}}\right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0\Gamma_2/\Gamma_1)$$

$$X_{\mathsf{SD}} = \mathrm{Im}\,\lambda_t\Big[\mathrm{Re}\,\lambda_c\,\eta_{cc}\,S_0(x_c) - \mathrm{Re}\,\lambda_t\,\eta_{tt}\,S_0(x_t) - (\mathrm{Re}\,\lambda_c - \mathrm{Re}\,\lambda_t)\,\eta_{ct}\,S_0(x_c, x_t)\Big]$$

ε_K and $\hat{B}_K$, V_{cb} II

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\,\pi^2 \Delta M_K}$$

$$\frac{\xi_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} = \text{Absorptive LD Effect} \approx -7\%$$

$$\xi_{\text{LD}} = \text{Dispersive LD Effect} \approx \pm 2\% \qquad \longrightarrow \text{systematic error}$$

- Inami-Lim functions:

$$S_0(x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} - \frac{3x_i^2 \ln x_i}{2(1-x_i)^3} \right],$$

$$S_0(x_i, x_j) = \left\{ \frac{x_i x_j}{x_i - x_j} \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \ln x_i \right.$$
$$\left. + (i \leftrightarrow j) \right\} - \frac{3 x_i x_j}{4(1-x_i)(1-x_j)}$$

ε_K and $\hat{B}_K$, V_{cb} III

$$
\begin{aligned}
S_0(x_t) &\longrightarrow +\,72.4\% \\
S_0(x_c, x_t) &\longrightarrow +\,45.4\% \\
S_0(x_c) &\longrightarrow -\,17.8\%
\end{aligned}
$$

- Dominant contribution ($\approx (72.4 + \alpha)\%$) comes with $|V_{cb}|^4$.

$$
\begin{aligned}
\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t &= -\bar{\eta}\lambda^2 |V_{cb}|^4 (1 - \bar{\rho}) \\
\mathrm{Re}\lambda_c &= -\lambda(1 - \frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5) \\
\mathrm{Re}\lambda_t &= -(1 - \frac{\lambda^2}{2}) A^2 \lambda^5 (1 - \bar{\rho}) + \mathcal{O}(\lambda^7) \\
\mathrm{Im}\lambda_t &= \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7) = -\mathrm{Im}\,\lambda_c
\end{aligned}
$$

ε_K and $\hat{B}_K$, V_{cb} IV

- Definition of $\hat{B}_K$ in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

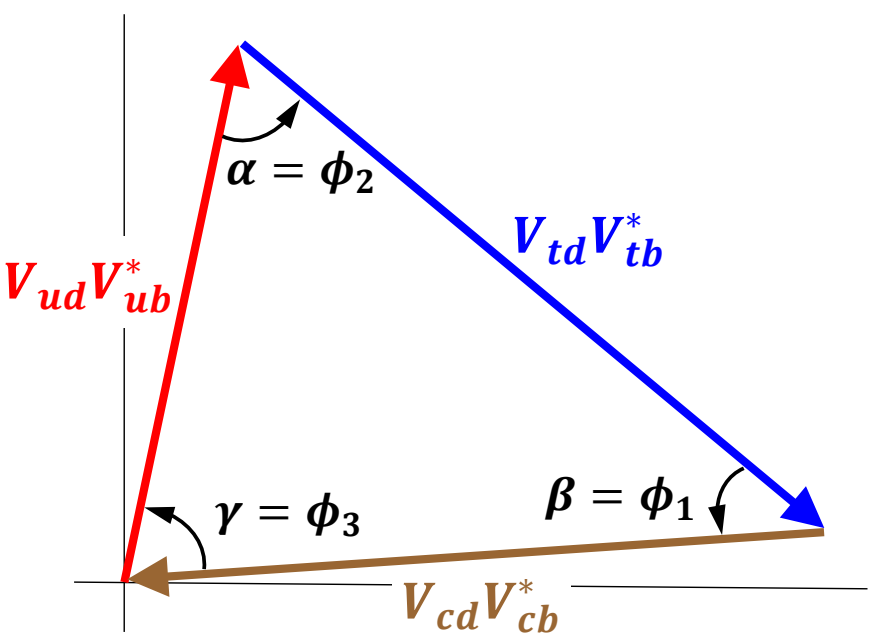
- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$

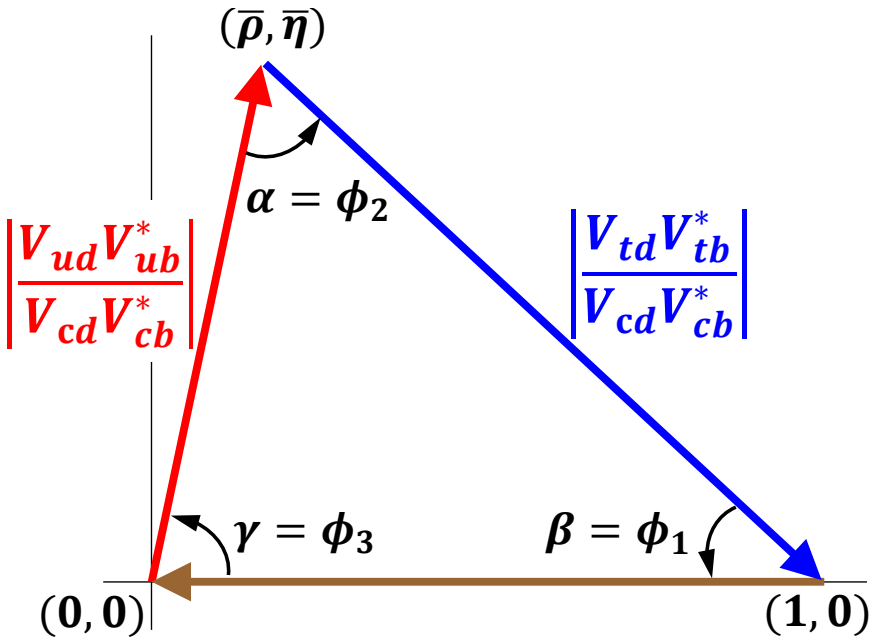
$$\phi_\varepsilon = 43.52(5)^\circ$$

ε_K with lattice QCD inputs

Unitarity Triangle $\rightarrow (\bar{\rho}, \bar{\eta})$



(a)



(b)

Global UT Fit and Angle-Only-Fit (AOF)

Global UT Fit

- Input: $|V_{ub}|/|V_{cb}|$, Δm_d, $\Delta m_s/\Delta m_d$, ε_K, and $\sin(2\beta)$.
- Determine the UT apex ($\bar{\rho}$, $\bar{\eta}$).
- Take λ from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

  which comes from K_{l3} and $K_{\mu 2}$.
- Disadvantage: unwanted correlation between ($\bar{\rho}$, $\bar{\eta}$) and ε_K.

AOF

- Input: $\sin(2\beta)$, $\cos(2\beta)$, $\sin(\gamma)$, $\cos(\gamma)$, $\sin(2\beta+\gamma)$, $\cos(2\beta+\gamma)$, and $\sin(2\alpha)$.
- Determine the UT apex ($\bar{\rho}$, $\bar{\eta}$).
- Take λ from $|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$ which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ to determine A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

- Advantage: NO correlation between ($\bar{\rho}$, $\bar{\eta}$) and ε_K.

Input Parameters: Wolfenstein Parameters

Angle-Only-Fit (AOF)

- ε_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex ($\bar{\rho}$, $\bar{\eta}$).
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

which comes from K_{l3} and $K_{\mu 2}$.

- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	$0.22475(25)$	[1] CKMfitter 2018
	$0.22500(100)$	[2] UTfit 2018
	$0.2243(5)$	[3] $\lvert V_{us}\rvert$ (AOF)
$\bar{\rho}$	$0.1577(96)$	[1] CKMfitter 2018
	$0.148(13)$	[2] UTfit 2018
	$0.146(22)$	[4] UTfit (AOF)
$\bar{\eta}$	$0.3493(95)$	[1] CKMfitter 2018
	$0.348(10)$	[2] UTfit 2018
	$0.333(16)$	[4] UTfit (AOF)

Input Parameter: $\hat{B}_K$ (FLAG 2019)

$\hat{B}_K$ in lattice QCD with $N_f = 2 + 1$.

Collaboration	Ref.	$\hat{B}_K$
SWME 15	[5]	$0.735(5)(36)$
RBC/UKQCD 14	[6]	$0.7499(24)(150)$
Laiho 11	[7]	$0.7628(38)(205)$
BMW 11	[8]	$0.7727(81)(84)$
FLAG 19	[9]	$0.7625(97)$

Input Parameter: Exclusive $|V_{cb}|$ in units of 1.0×10^{-3}

(a) HFLAV 2017 (CLN)

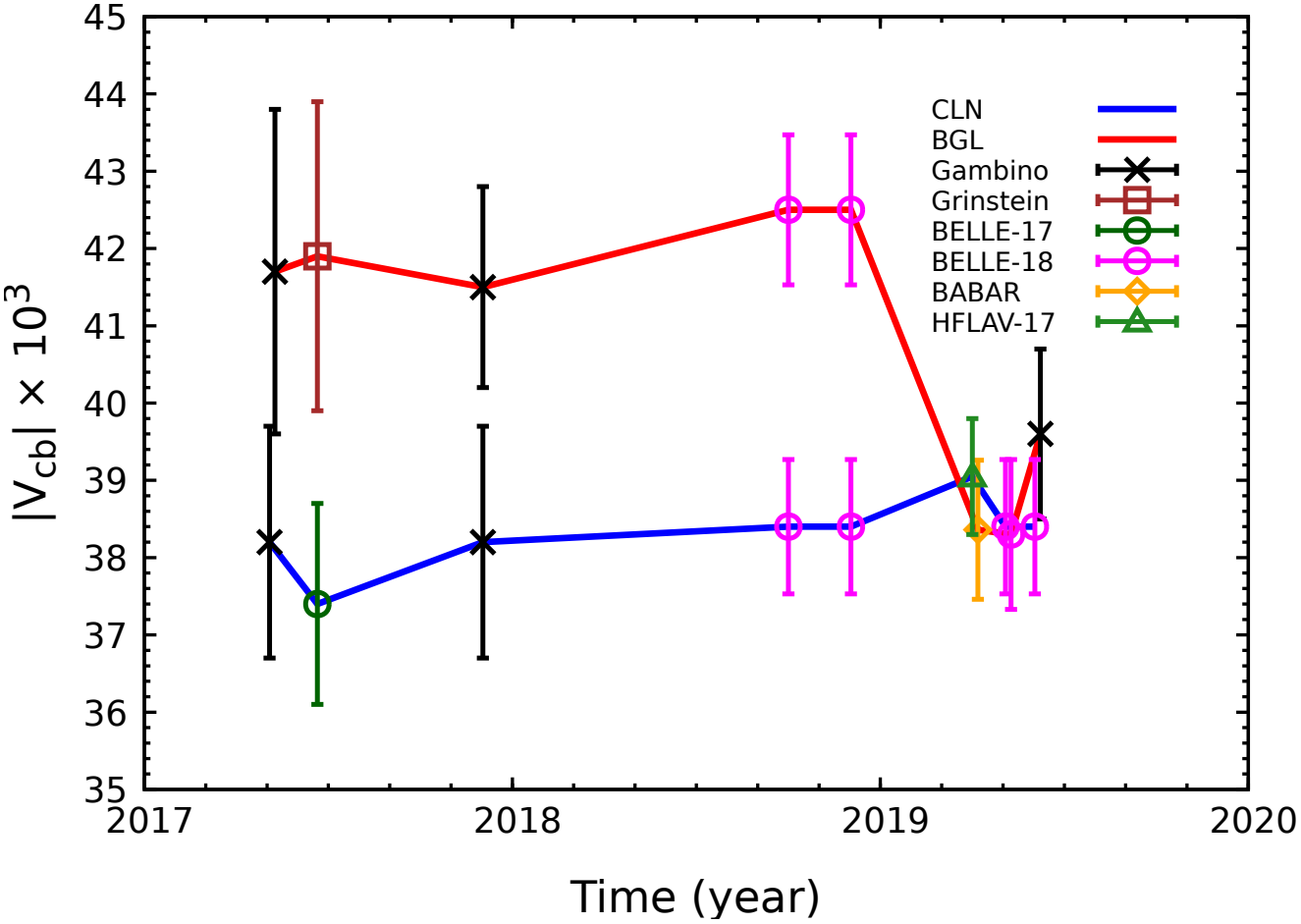
channel	value	Ref.
$B \to D^* \ell \bar{\nu}$	$39.05(47)(58)$	[10, 11]
$B \to D \ell \bar{\nu}$	$39.18(94)(36)$	[10, 12]
$\lvert V_{ub}\rvert / \lvert V_{cb}\rvert$	$0.080(4)(4)$	[10, 13]
ex-combined	$39.13(59)$	[10]

(b) BABAR and BELLE 2019

channel	value	Ref.
CLN	$39.05(47)(58)$	HFLAV 17 [10]
BGL	$38.36(90)$	BABAR 19 [14]
CLN	$38.4(2)(6)(6)$	BELLE 19 [15]
BGL	$38.3(3)(7)(6)$	BELLE 19 [15]

- There is no difference between the CLN and BGL analyses.
- Refer to BABAR 2019 [14] and BELLE 2019 [15].
- Hence, the CLN method turns out to be correct and OK within our best knowledge.

CLN vs. BGL in $B \to D^* \ell \bar{\nu}$ decays



- At present, we find that there is no difference in exclusive $|V_{cb}|$ between CLN and BGL. $\Longrightarrow$ Resolved or still in question ???

Input Parameter: Inclusive $|V_{cb}|$ in units of 1.0×10^{-3}

$|V_{cb}|$ in units of 1.0×10^{-3}.

(a) Exclusive $|V_{cb}|$

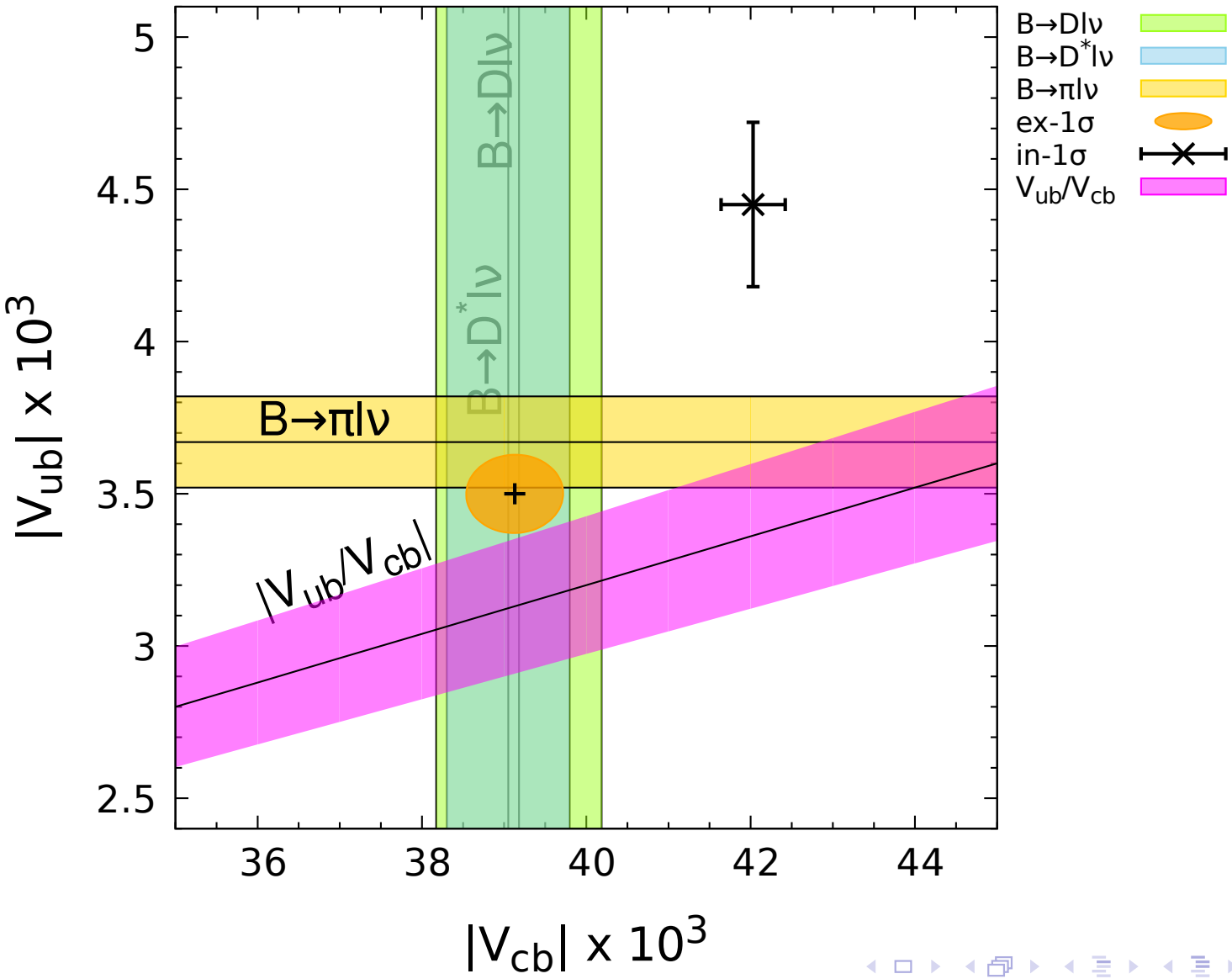
channel	value	Ref.
CLN	$39.05(47)(58)$	HFLAV 17 [10]
BGL	$38.36(90)$	BABAR 19 [14]
CLN	$38.4(2)(6)(6)$	BELLE 19 [15]
BGL	$38.3(3)(7)(6)$	BELLE 19 [15]

(b) Inclusive $|V_{cb}|$

channel	value	Ref.
kinetic scheme	$42.19(78)$	[10]
1S scheme	$41.98(45)$	[10]

- There is $3 \sim 4\sigma$ difference in $|V_{cb}|$ between the exclusive and inclusive decay channels.
- This issue remains unresolved yet.

Current Status of $|V_{cb}|$ in 2018



Input Parameter: ξ_0

Indirect Method

$$\xi_0 = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}, \quad \xi_2 = \frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2}.$$

ξ_0	$-1.63(19) \times 10^{-4}$	RBC-UK-2015 [16]

- RBC-UKQCD calculated $\mathrm{Im}A_2$. $\mathrm{Im}A_2 \to \xi_2 \to \varepsilon'_K/\varepsilon_K \to \xi_0$

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = \frac{1}{\sqrt{2}|\varepsilon_K|}\omega\left(\xi_2 - \xi_0\right).$$

  Other inputs ω, ε_K and $\varepsilon'_K/\varepsilon_K$ are taken from the experimental values.

- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$
- Isopspin breaking effect: (at most 15% of ξ_0) $\to$ (1% in ε_K) $\to$ neglected!

Input Parameter: ξ_0

Direct Method

- RBC-UKQCD calculated $\mathrm{Im}A_0$. $\mathrm{Im}A_0 \to \xi_0$.

$$\xi_0 = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} = -0.57(49) \times 10^{-4}$$

  Other input $\mathrm{Re}\, A_0$ is taken from the experimental value.

- RBC-UKQCD also calculated δ_0

$$\delta_0 = 23.8(49)(12)^\circ[2015] \to 23.8(49)(112)^\circ[2018]$$

  This value is within 2σ from the experimental value: $\delta_0 = 39.1(6)^\circ$.

- It appears to me that this puzzle might be resolved by multi-state fitting: RBC-UKQCD, Wang Tianle [Lattice 2018].
- Here, we use the indirect method to determine ξ_0.

Input Parameter: ξ_{LD}

$$\xi_{\mathsf{LD}} = \frac{m'_{\mathsf{LD}}}{\sqrt{2}\,\Delta M_K}$$

$$m'_{\mathsf{LD}} = -\mathrm{Im}\left[\mathcal{P}\sum_C \frac{\langle \overline{K}^0|H_{\mathsf{w}}|C\rangle\langle C|H_{\mathsf{w}}|K^0\rangle}{m_{K^0} - E_C}\right]$$

- RBC-UKQCD rough estimate [PRD 88, 014508] gives

$$\xi_{\mathsf{LD}} = (0 \pm 1.6)\%$$

- BGI estimate [PLB 68, 309, 2010] gives

$$\xi_{\mathsf{LD}} = -0.4(3) \times \frac{\xi_0}{\sqrt{2}}$$

- Precision measurement of lattice QCD is not available yet.

Input Parameter: Charm Quark Mass $m_c(m_c)$

$m_c(m_c)$ in lattice QCD with $N_f = 2+1$ and $N_f = 2+1+1$.

Collaboration	N_f	$m_c(m_c)$	Ref.
FLAG 2019	$2+1$	$1.275(5)$	[9]
FLAG 2019	$2+1+1$	$1.280(13)$	[9]

- The results for $m_c(m_c)$ with $N_f = 2+1+1$ are inconsistent with each other.
- Hence, we use the results for $m_c(m_c)$ with $N_f = 2+1$.

Input Parameter: top quark mass $m_t(m_t)$

$m_t(m_t)$ in the $\overline{\text{MS}}$ scheme in units of GeV.

Collaboration	M_t	$m_t(m_t)$	Ref.
PDG 2016	173.5 ± 1.1	$163.65 \pm 1.05 \pm 0.17$	[17]
PDG 2018	173.0 ± 0.4	$163.17 \pm 0.38 \pm 0.17$	[3]

- M_t is the pole mass of top quarks.
- CMS and ATLAS have done a great job in reducing the error.
- Here, we use the results for $m_t(m_t)$ obtained from PDG 2018.

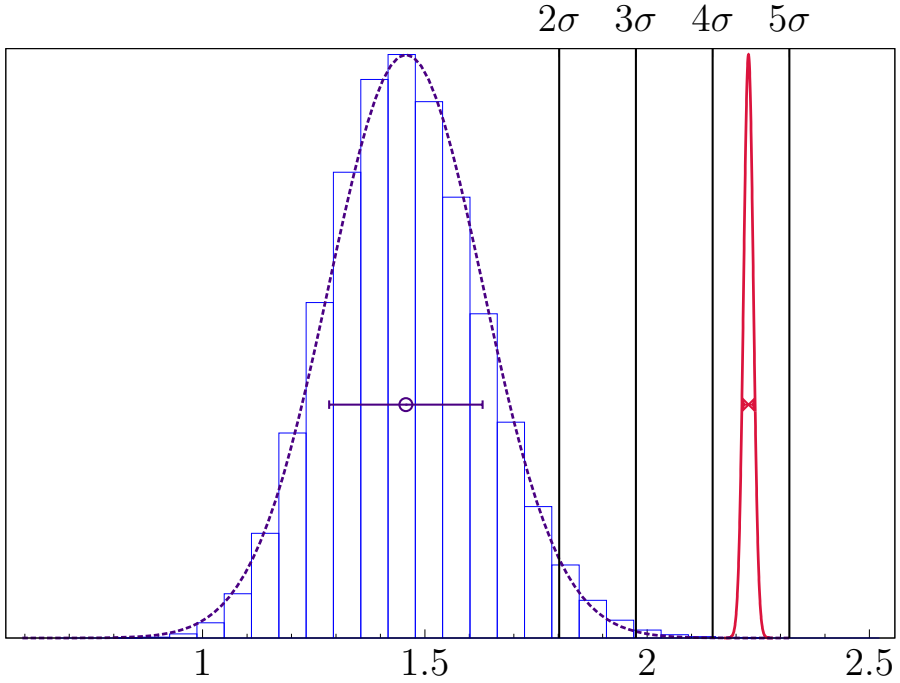
Other Input Parameters

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$	PDG 18 [3]
M_W	$80.379(12)\,\mathrm{GeV}$	PDG 18 [3]
θ	$43.52(5)^\circ$	PDG 18 [3]
m_{K^0}	$497.611(13)\,\mathrm{MeV}$	PDG 18 [3]
ΔM_K	$3.484(6) \times 10^{-12}\,\mathrm{MeV}$	PDG 18 [3]
F_K	$155.7(3)\,\mathrm{MeV}$	FLAG 19 [9]

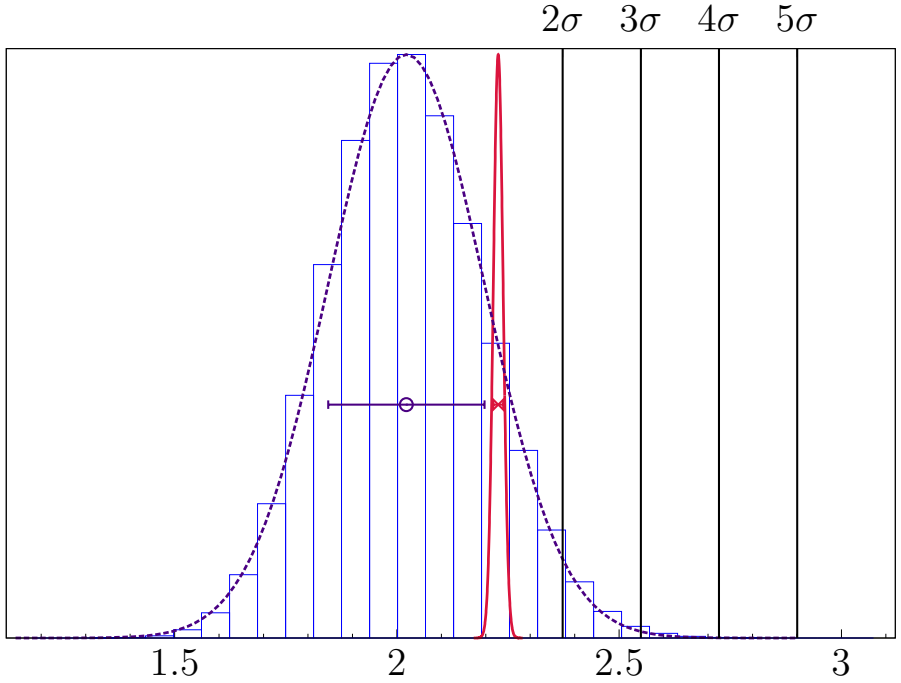
Input	Value	Ref.
η_{cc}	$1.72(27)$	[18]
η_{tt}	$0.5765(65)$	[19]
η_{ct}	$0.496(47)$	[20]

ε_K from AOF, Exclusive $|V_{cb}|$ (BELLE 19, CLN)

RBC-UKQCD estimate for ξ_{LD}



Exclusive $|V_{cb}|$, BELLE 19, CLN



Inclusive $|V_{cb}|$ (1S)

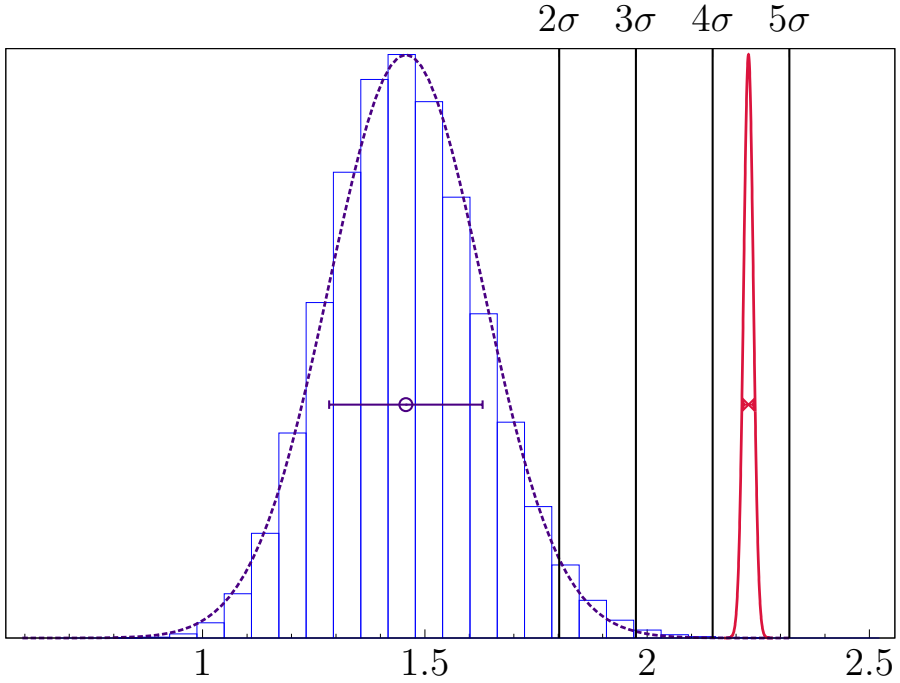
- With exclusive $|V_{cb}|$ (BELLE 19, CLN), it has 4.5σ tension.

$$|\varepsilon_K|^{\text{Exp}} = (2.228 \pm 0.011) \times 10^{-3}$$

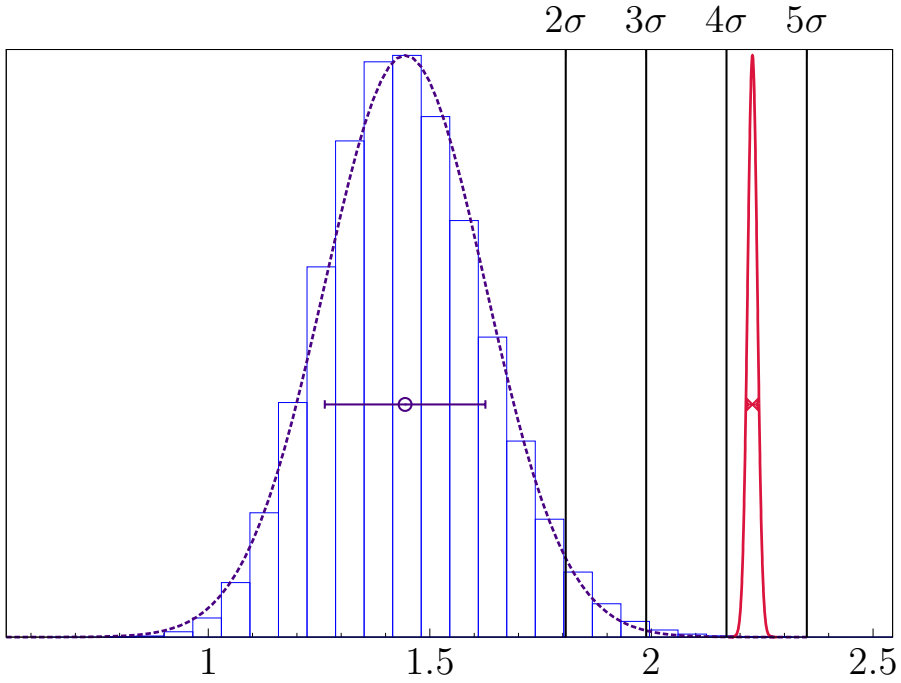
$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = (1.457 \pm 0.173) \times 10^{-3}$$

ε_K from AOF, Exclusive $|V_{cb}|$ (BELLE 19, CNL vs. BGL)

RBC-UKQCD estimate for ξ_{LD}



Exclusive $|V_{cb}|$ (BELLE 19, CNL)



Exclusive $|V_{cb}|$ (BELLE 19, BGL)

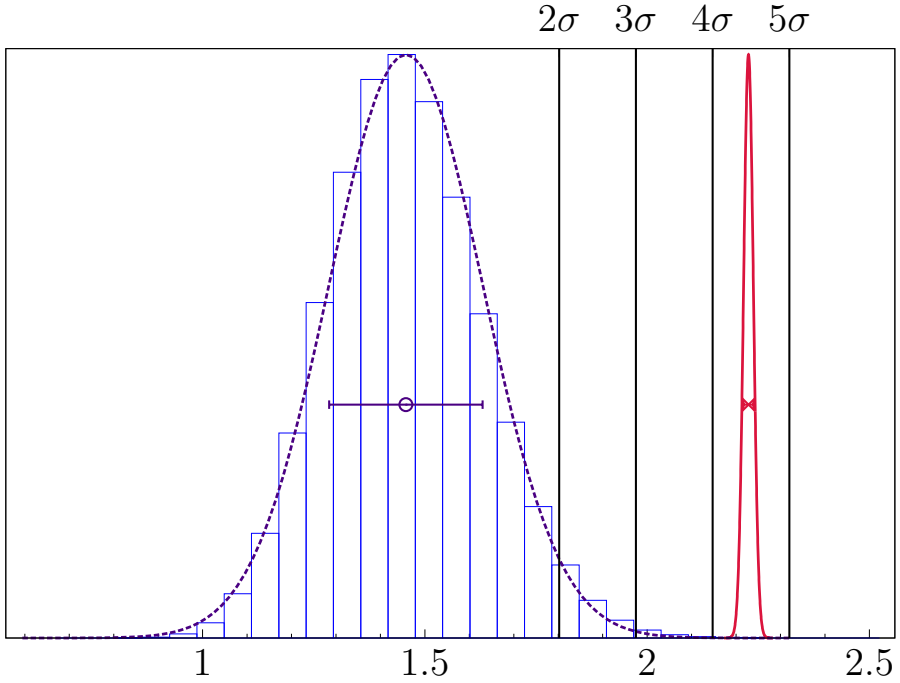
- CLN has 4.5σ tension, and BGL has 4.3σ tension.

$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = (1.457 \pm 0.173) \times 10^{-3} \qquad \text{(CLN)}$$

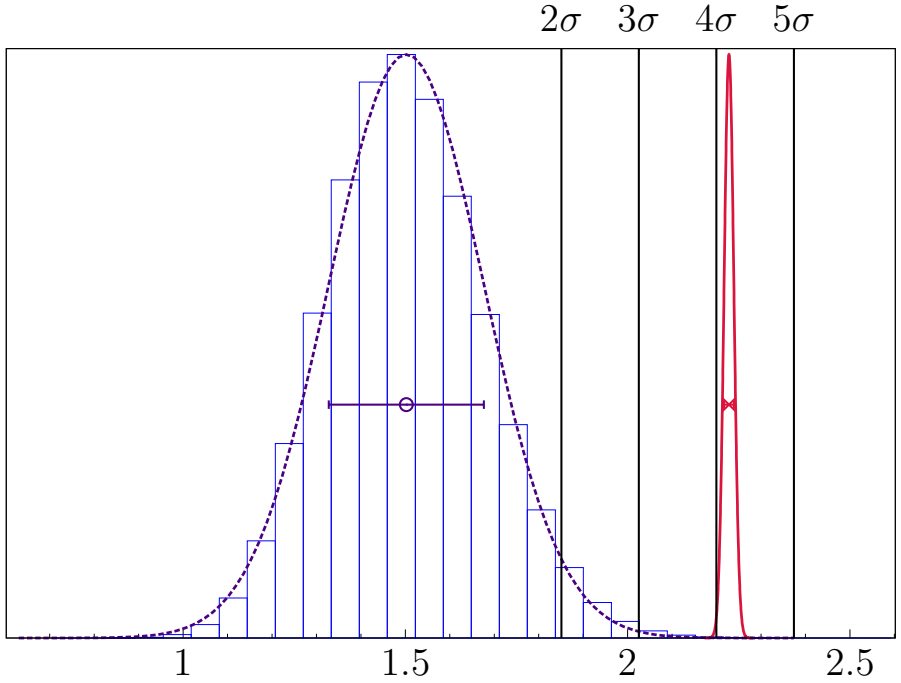
$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = (1.444 \pm 0.181) \times 10^{-3} \qquad \text{(BGL)}$$

ε_K from AOF, Exclusive $|V_{cb}|$ (BELLE 19, CLN)

RBC-UKQCD vs. BGI estimate for ξ_{LD}



RBC-UKQCD estimate for ξ_{LD}



BGI estimate for ξ_{LD}

- RBC-UK estimate $\rightarrow$ 4.5σ tension, and BGI estimate $\rightarrow$ 4.2σ tension.

$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = (1.457 \pm 0.173) \times 10^{-3} \qquad \text{(RBC-UKQCD estimate for } \xi_{\text{LD}})$$

$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = (1.502 \pm 0.174) \times 10^{-3} \qquad \text{(BGI estimate for } \xi_{\text{LD}})$$

Current Status of ε_K

- FLAG 2019 + PDG 2018: (in units of 1.0×10^{-3}, AOF)

$$|\varepsilon_K|^{\text{SM}}_{\text{excl}} = 1.457 \pm 0.173 \quad \text{for Exclusive } V_{cb} \text{ (Lattice QCD + CLN)}$$
$$|\varepsilon_K|^{\text{SM}}_{\text{incl}} = 2.021 \pm 0.176 \quad \text{for Inclusive } V_{cb} \text{ (Heavy Quark Expansion)}$$

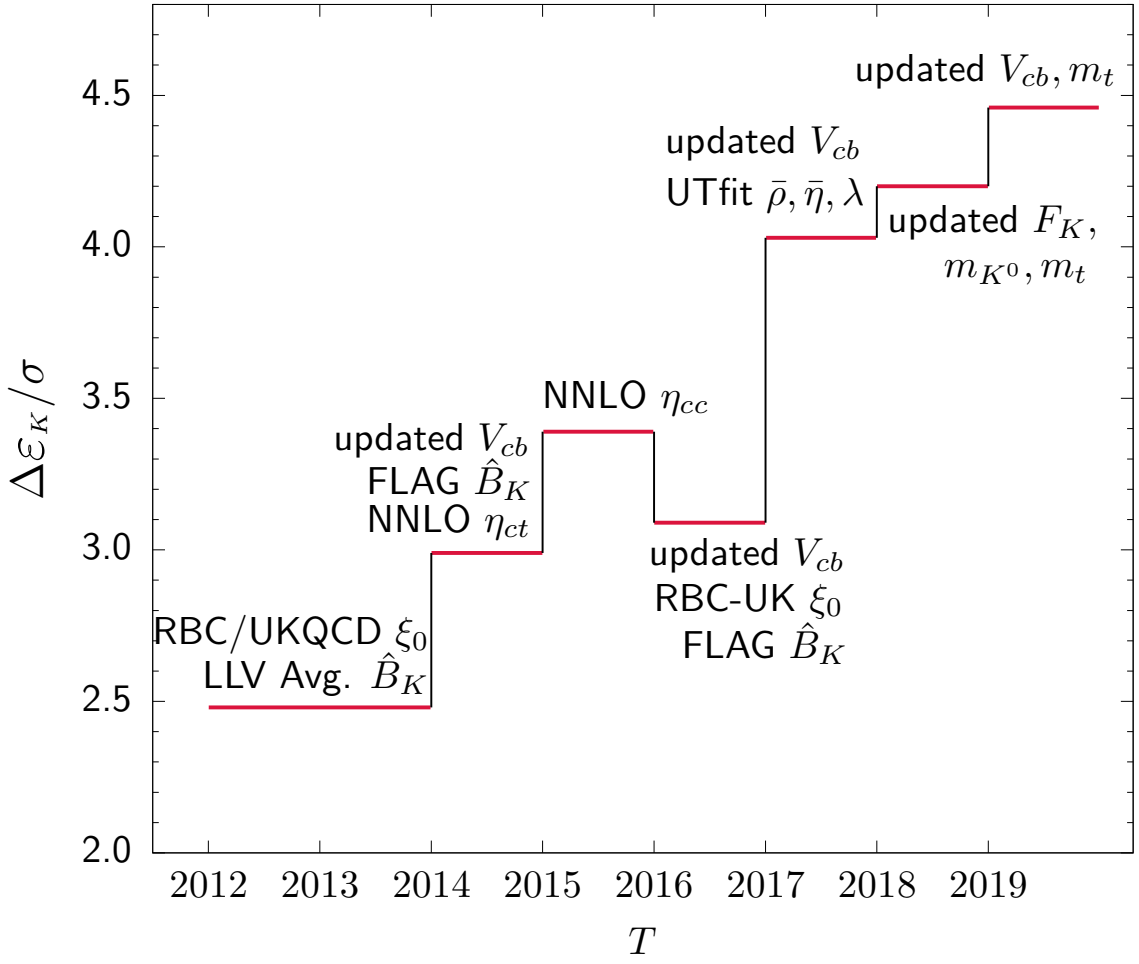
- Experiments:

$$|\varepsilon_K|^{\text{Exp}} = 2.228 \pm 0.011$$

- Hence, we observe $4.5\sigma \sim 4.2\sigma$ difference between the SM theory (Lattice QCD) and experiments.

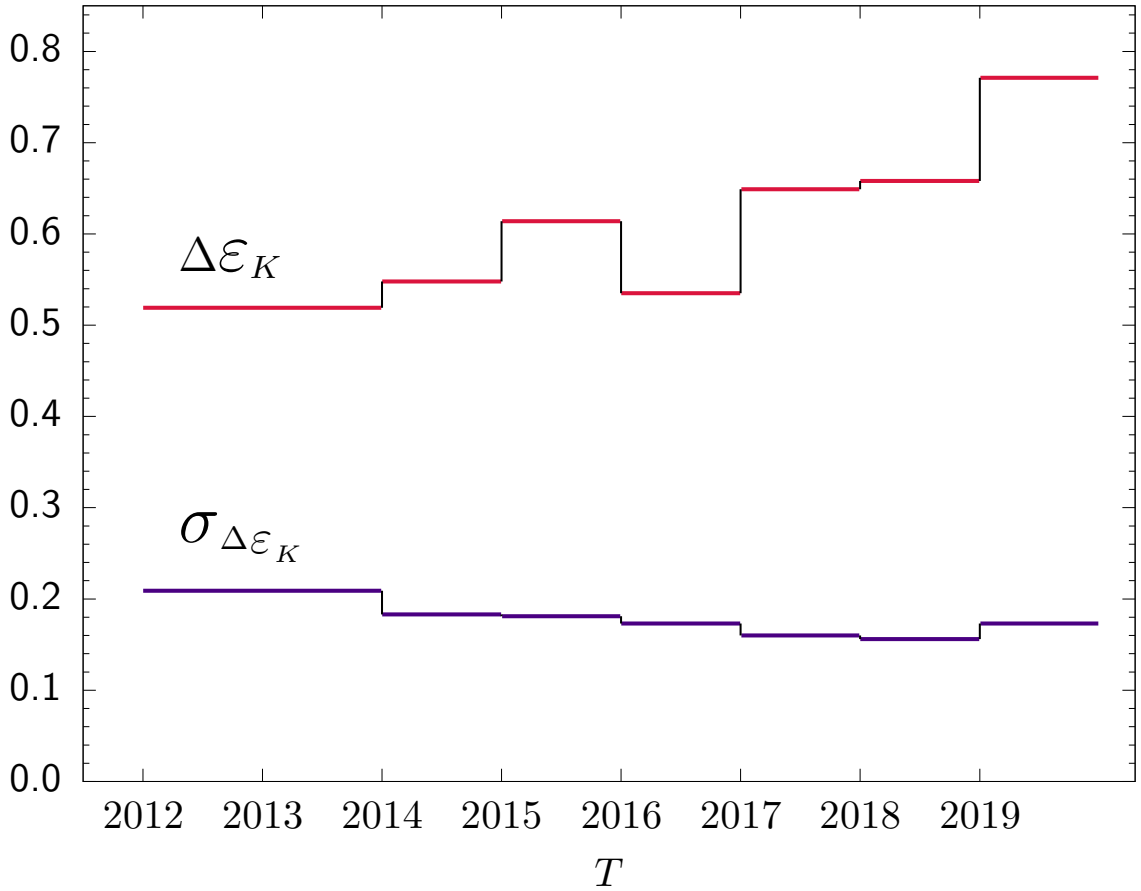
- What does this mean? $\longrightarrow$ Breakdown of SM ?

Time Evolution of $\Delta\varepsilon_K$ on the Lattice



- $\Delta\varepsilon_K \equiv |\varepsilon_K|^{\text{Exp}} - |\varepsilon_K|^{\text{SM}}_{\text{excl}} \Longleftarrow |V_{cb}|$ (CLN), and ξ_{LD} (RBC-UK)

Time Evolution of Average and Error for $\Delta\varepsilon_K$



- The average $\Delta\varepsilon_K$ has increased by 49% with some fluctuations.
- The error $\sigma_{\Delta\varepsilon_K}$ has decreased by 17% with some fluctuation: HFLAV 2017 $\rightarrow$ BELLE 2019.

Error Budget of $\Delta\varepsilon_K$: $|V_{cb}|$ (CLN), ξ_{LD} (RBC-UK)

source	error (%)	memo
$\|V_{cb}\|$	50.2	Exclusive (CLN)
$\bar{\eta}$	19.1	AOF
η_{ct}	16.3	$c-t$ Box
η_{cc}	6.9	$c-c$ Box
$\bar{\rho}$	2.8	AOF
ξ_{LD}	1.7	Long-distance
$\hat{B}_K$	1.3	FLAG
ξ_0	0.58	Indirect
η_{tt}	0.54	$t-t$ Box
λ	0.16	$\|V_{us}\|$ (PDG)
$\vdots$	$\vdots$	$\vdots$

- The error from $|V_{cb}|$ is dominant.

To Do List

- It would be highly desirable if the HFLAV group may perform a comprehensive reanalysis over the entire sets of the experimental data including both BABAR and BELLE for $\bar{B} \to D^* \ell \bar{\nu}$ using the BGL method and compare the results with those of CLN.
- It would be nice to reduce overall errors on $|V_{cb}|$: $1.4\% \to 0.8\%$. [OK action project: LANL-SWME: posters in Lattice 2019]
- It would be nice to reduce overall errors on $\bar{\eta}$. [BELLE2]
- It would be nice to reduce overall errors on ξ_0 and ξ_2. [RBC-UKQCD]

Summary and Conclusion

Summary

1. We find that

$$\Delta\varepsilon_K^{\text{excl}} = 4.5(3)\sigma \qquad \text{(Lattice QCD)} \tag{1}$$

$$\Delta\varepsilon_K^{\text{incl}} = 1.2\sigma \qquad \text{(HQE, QCD Sum Rules)} \tag{2}$$

2. It is too early to conclude that there might be something wrong with the SM yet.
3. Let us wait for the next round reanalysis of the HFLAV group on the entire data sets of the $\bar{B} \to D^* \ell \bar{\nu}$ decays, using both CLN and BGL.
4. Meanwhile, it would be very helpful to reduce the errors for $|V_{cb}|$, $\bar{\eta}$, ξ_0, ξ_2, and ξ_{LD} in lattice QCD.
5. Please stay tuned for the update.

Thank God for your help !!!

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