The $B \to D^* \ell\nu$ semileptonic decay at non-zero recoil and its implications for $|V_{cb}|$ and $R(D^*)$

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The $V_{cb}$ matrix element: Tensions

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

| $|V_{cb}| \cdot 10^{-3}$ | PDG 2016 | PDG 2018 |
|-------------------------|----------|----------|
| Exclusive               | 39.2 ± 0.7 | 41.9 ± 2.0 |
| Inclusive               | 42.2 ± 0.8 | 42.2 ± 0.8 |

- Matrix must be unitary (preserve the norm)
- **BUT current tensions (2019) stand at**
  - $\approx 2\sigma - 3\sigma$
The $V_{cb}$ matrix element: Measurement from exclusive processes

\[
\frac{d\Gamma}{dw}(\bar{B} \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^2} (w^2 - 1)^{1/2} P(w) |\eta_{ew}|^2 |\mathcal{F}(w)|^2 |V_{cb}|^2
\]

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative

- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q \to \infty} \mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$

- Reduction in the phase space $(w^2 - 1)^{1/2}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $|V_{cb}|^2 |\eta_{ew} \mathcal{F}(w)|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations
The $V_{cb}$ matrix element: The parametrization issue

All the parametrizations perform an expansion in the $z$ parameter

$$z = \frac{\sqrt{w + 1} - \sqrt{2N}}{\sqrt{w + 1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL)
  
  $$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - $B_{f_X}$ Blaschke factors, includes contributions from the poles
  - $\phi_{f_X}$ is called outer function and must be computed for each form factor
  - Weak unitarity constraints $\sum_{n} |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN)

  $$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho), \, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$
The $V_{cb}$ matrix element: The parametrization issue

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

$$|V_{cb}| = 41.7 \pm 2.0(\times 10^{-3})$$

Latest Belle dataset and Babar analysis seem to contradict this picture

- From Babar’s paper arXiv:1903.10002 BGL is compatible with CLN and far from the inclusive value
- Belle’s paper arXiv:1809.03290v3 finds similar results in its last revision

The discrepancy inclusive-exclusive is not well understood

- Data at $w \gtrsim 1$ is urgently needed to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$
The $V_{cb}$ matrix element: Tensions in lepton universality

\[ R\left( D^{(*)}\right) = \frac{\mathcal{B}\left( B \rightarrow D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left( B \rightarrow D^{(*)}\ell\nu_\ell\right)} \]

- Current $\approx 3\sigma - 4\sigma$ tension with the SM
Calculating $V_{cb}$ on the lattice: Formalism

- Form factors

\[
\frac{\langle D^* (p_{D^*}, \epsilon^\nu) | V^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} = \frac{1}{2} \epsilon^{\nu*} \varepsilon^{\mu \nu} v^\rho_B v^\sigma_{D^*} h_V(w)
\]

\[
\frac{\langle D^* (p_{D^*}, \epsilon^\nu) | A^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} = \frac{i}{2} \epsilon^{\nu*} [g^{\mu \nu} (1 + w) h_{A_1}(w) - v^\nu_B (v^\mu_B h_{A_2}(w) + v^\mu_{D^*} h_{A_3}(w))]
\]

- $V$ and $A$ are the vector/axial currents in the continuum
- The $h_X$ enter in the definition of $F$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice
Calculating $V_{cb}$ on the lattice: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B m_{D^*}} (w + 1) \left( h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}} h_V(w) \right)$$

$$H_0 = \sqrt{m_B m_{D^*}} (w+1) m_B \left[(w-r) h_{A_1}(w) - (w-1) (r h_{A_2}(w) + h_{A_3}(w)) \right] / \sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}} \left[ (1+w) h_{A_1}(w) + (wr-1) h_{A_2}(w) + (r-w) h_{A_3}(w) \right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w) |\mathcal{F}|^2 = \frac{1 - 2wr + r^2}{12 m_B m_{D^*} (1-r)^2} \left( H_0^2(w) + H_+^2(w) + H_-^2(w) \right)$$
Using $15 \, N_f = 2 + 1$ MILC ensembles of sea asqtad quarks

The heavy quarks are treated using the Fermilab action
Preliminary blinded results, joint fit $p - value = 0.96$

- Excluding ensembles with high discretization errors in this fit, at present
- Need to finalize study of heavy quark discretization errors to include all ensembles
• Preliminary blinded results, joint fit $p$-value = 0.96
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## Analysis: Preliminary error budget

<table>
<thead>
<tr>
<th>Source</th>
<th>$h_V$ (%)</th>
<th>$h_{A_1}$ (%)</th>
<th>$h_{A_2}$ (%)</th>
<th>$h_{A_3}$ (%)</th>
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<tbody>
<tr>
<td>Statistics</td>
<td>1.1</td>
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<td>Isospin effects</td>
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<td>$\chi$PT/cont. extrapolation</td>
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<td>0.7</td>
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<tr>
<td>Matching</td>
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<td>0.4</td>
<td>0.1</td>
<td>1.5</td>
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<tr>
<td>Heavy quark discretization</td>
<td>$2^\dagger/1.4^*$</td>
<td>$2^\dagger/0.4^*$</td>
<td>$2^\dagger/5.8^*$</td>
<td>$2^\dagger/1.3^*$</td>
</tr>
</tbody>
</table>

$^\dagger$Preliminary value actually used during the analysis  
*Estimate, currently subject of intensive study

- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- **Italic** marks errors to be reduced/removed when using HISQ for heavy quarks

- Heavy HISQ would add errors $O(\alpha_s (am_b)^2)$ and $O((am_b)^4)$ (current errors $O(\alpha_s am_b)$)
The BGL expansion is performed on different (more convenient) form factors

\[ g = \frac{h_V(w)}{\sqrt{m_B m_{D^*}}} \]

\[ f = \sqrt{m_B m_{D^*}(1 + w)} h_{A_1}(w) \]

\[ \mathcal{F}_1 = \sqrt{q^2} H_0 \]

\[ \mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*} \sqrt{w^2 - 1}} H_S \]

Constraint \( \mathcal{F}_1(z = 0) = (m_B - m_{D^*}) f(z = 0) \)

Constraint \( (1 + w)m_B^2(1 - r)\mathcal{F}_1(z = z_{Max}) = (1 + r)\mathcal{F}_2(z = z_{Max}) \)

BGL (weak) unitarity constraints (all HISQ will use strong constraints)

\[ \sum_j a_j^2 \leq 1, \quad \sum_j b_j^2 + c_j^2 \leq 1, \quad \sum_j d_j^2 \leq 1 \]
Analysis: $\sim$ expansion fit procedure

- Several different datasets
  - Our lattice data
  - BaBar BGL fit
  - Belle tagged dataset
  - Belle untagged dataset

- Several different fits
  - Lattice form factors only
  - Experimental data only (one fit per dataset)
  - Joint fit lattice + experimental data

- Each dataset is given in a different format, and requires a different amount of processing

- Different fitting strategy per dataset

Assume $V_{cb} = V_{cb}^{BaBar}$ for the only Belle data fits to have a common normalization for the coefficients

All the experimental and theoretical correlations are included in all fits
Constraints

- The constraint at zero recoil is used to remove a coefficient of the BGL expansion.
- The constraint at maximum recoil is imposed by adding a datapoint with small errors.
- The unitarity constraints are imposed by adding hard cuts (under revision).
- In the fits shown, we do not impose any constraints, but they are satisfied automatically.

How many coefficients?

- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
- Current maximum 4 (each coefficient requires new integrals)
Results: Pure-lattice prediction and joint fit

Separate fits

- Lattice + Belle + BaBar BGL $p$-value = 0.17
- Lattice only BGL $p$-value = 0.05
- Belle untagged BGL $p$-value = 0.18, tagged $p$-value $\approx 1$

Joint fit

- Best Fit
- Lattice $\times V_{cb}$
- BaBar synthetic
- Belle untagged, $e^-$
- Belle untagged, $\mu^-$
- Belle tagged

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Results: Joint fit, angular bins

\[ \frac{d\Gamma}{dw} \times 10^{15} \text{ GeV} \]

\[ \frac{d\Gamma}{d\cos{\theta}_l} \times 10^{15} \text{ GeV} \]

\[ \frac{d\Gamma}{d\cos{\theta}_v} \times 10^{15} \text{ GeV} \]

\[ \frac{d\Gamma}{d\chi} \times 10^{15} \text{ GeV} \]
Results: $R(D^*)$

**Separate fits**

- Lattice $\ell = e, \mu$
- Belle untagged $\ell = e^-, \mu^-$
- BaBar
- Lattice $\ell = \tau$
- Belle tagged $\ell = e^-, \mu^-$

**Joint fit**

- $\ell = e^-, \mu^-$
- $\ell = \tau^-$

$\Gamma(B \to D^*\ell\bar{\nu})$ vs $w$

Preliminary
Conclusions

What to expect

- Error on $V_{cb}$ from this analysis might not be improved compared to the previous determinations that used exp data + CLN + lattice $F(1)$
  - Errors might not be improved compared to previous lattice estimations
- The main new information of this analysis won’t come from the zero-recoil value, but from the slope at small recoil
- Main sources of errors of our form factors are
  - $\chi$PT-continuum extrapolation
  - HQ discretization
  - Matching
- The analysis of heavy quark discretization errors is in progress
- We need to understand better the current lattice and experimental data
The future

- Well established roadmap to reduce errors in our calculation with newer lattice ensembles
- Our next analysis will be joint $B \rightarrow D$ and $B \rightarrow D^*$ to benefit from strong unitarity constraints
- This roadmap is to be followed in other processes involving other CKM matrix elements
BACKUP
Analysis: $z$ expansion fit procedure

### Lattice data points
- Generate synthetic data from the chiral-continuum extrapolation
- Fitted directly to the $z$-expansion of the corresponding form factor

### Belle untagged dataset (folded)
- $e^-$ and $\mu^-$ datasets fitted separately
- Given the fit function, we compute the count prediction on each bin using a similar procedure as in the Belle collaboration

### Belle tagged dataset (unfolded, binned)
- Integrate the three non-relevant variables in the fit function and fit the result to the data

### BaBar fit results
- 5 independent parameters + $V_{cb}$
- Generate 5 synthetic datapoints on the $w$ bins and fit them normally
The $b_j$ represent the small recoil behavior $\sim h_{A_1}$

The $c_j$ represent the large recoil behavior $\sim H_0$