The $B \to D^* \ell \nu$ semileptonic decay at non-zero recoil and its implications for $|V_{cb}|$ and $R(D^*)$

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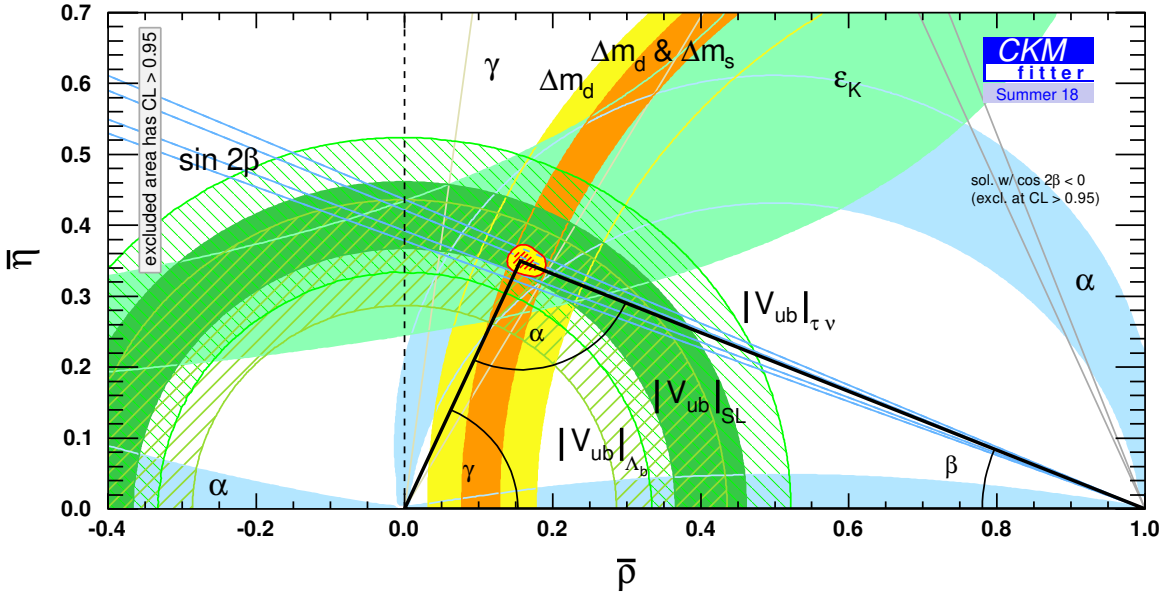
The V_{cb} matrix element: Tensions

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\lvert V_{cb}\rvert\,(\cdot 10^{-3})$	PDG 2016	PDG 2018
Exclusive	39.2 ± 0.7	41.9 ± 2.0
Inclusive	42.2 ± 0.8	42.2 ± 0.8

- Matrix must be unitary (preserve the norm)
- BUT **current tensions (2019) stand at** $\approx 2\sigma - 3\sigma$



The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}}P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}}\left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q\to\infty}\mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2\left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL) *Phys. Rev. Lett. 74 (1995) 4603-4606* *Phys.Rev. D56 (1997) 6895-6911* *Nucl.Phys. B461 (1996) 493-511*

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

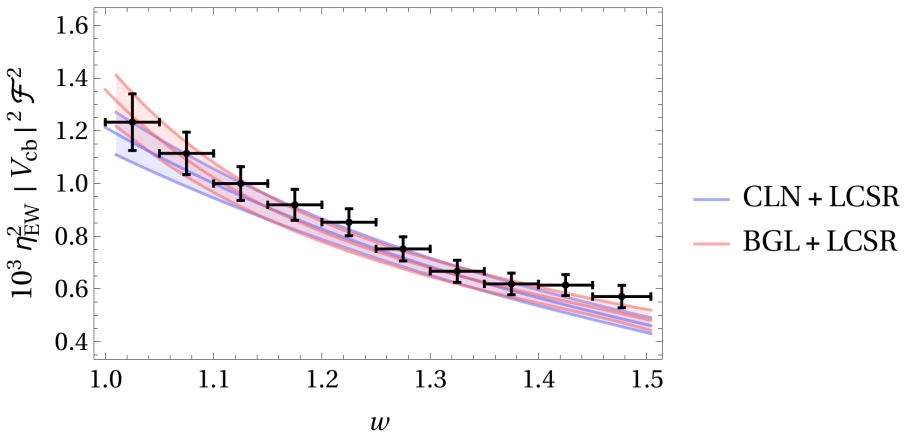
  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys. B530 (1998) 153-181*

$$\mathcal{F}(w) \propto 1 - \rho^2 z + c z^2 - d z^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

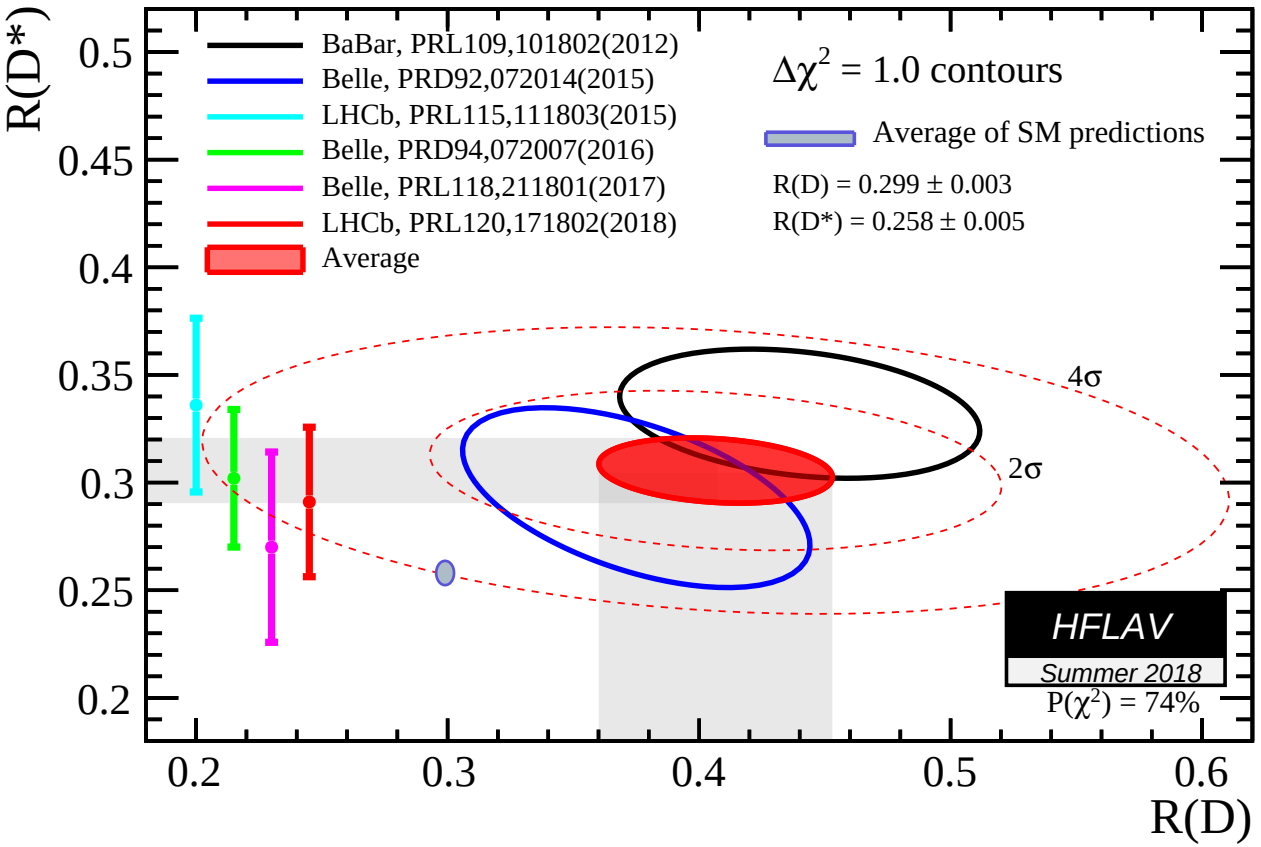
$$|V_{cb}| = 41.7 \pm 2.0(\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper arXiv:1903.10002 **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper arXiv:1809.03290v3 finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

The V_{cb} matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma - 4\sigma$ tension with the SM

Calculating V_{cb} on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\,(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\,(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w))\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating V_{cb} on the lattice: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

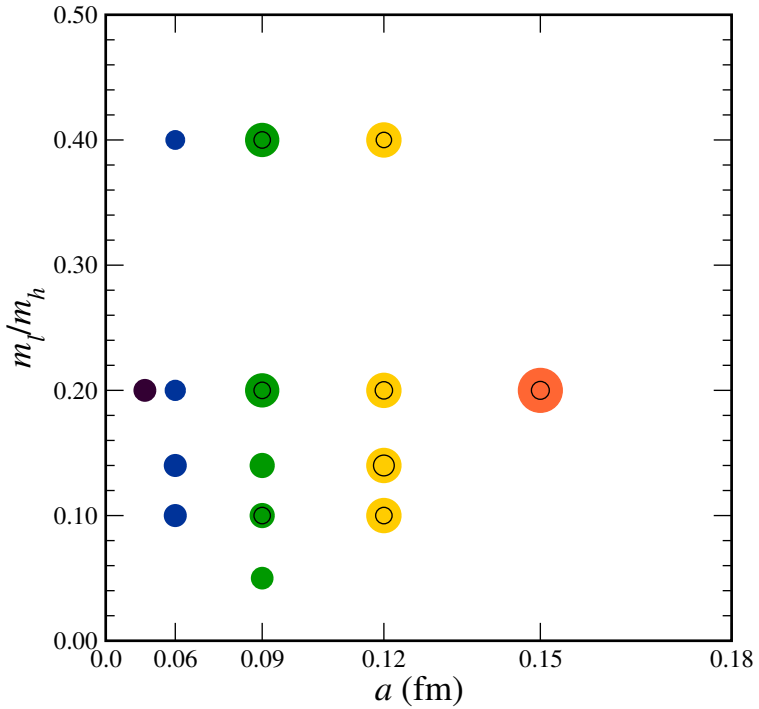
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

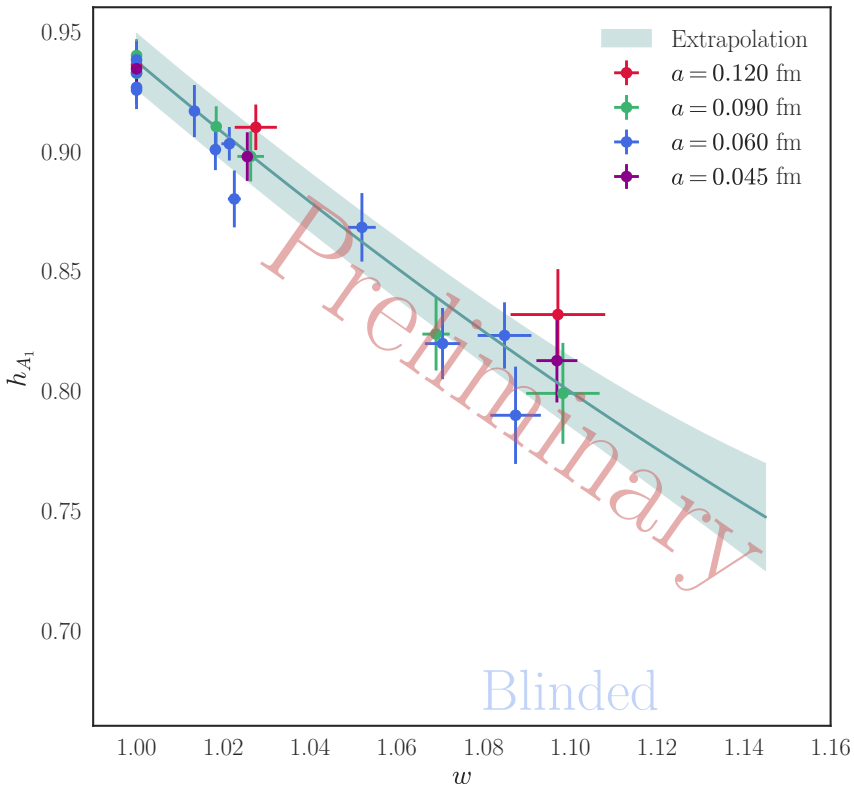
$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

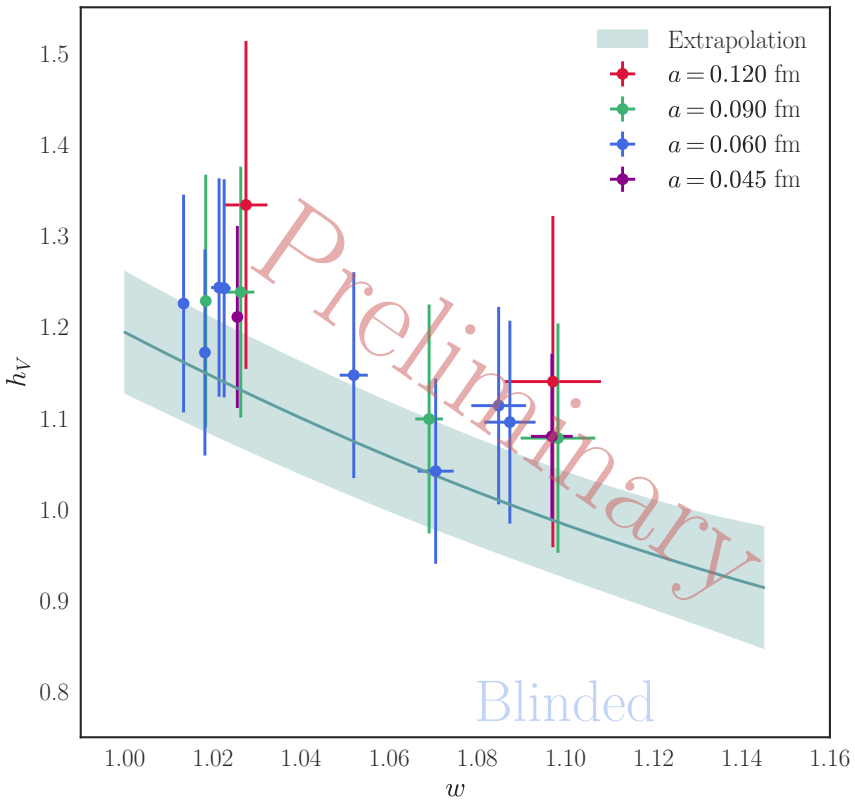
Introduction: Available data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



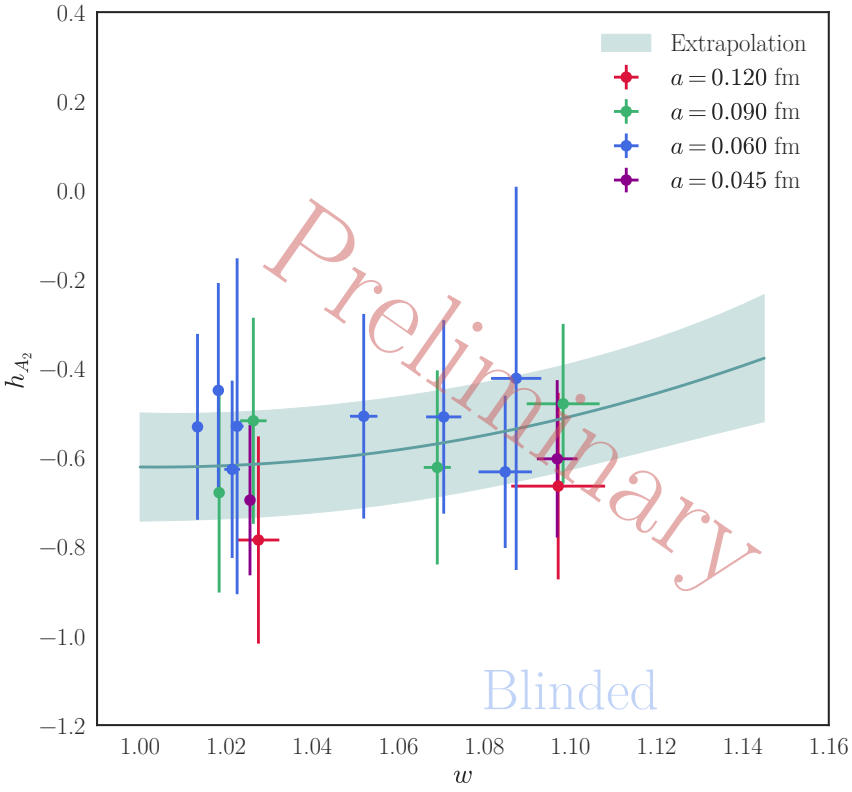
Results: Fermilab/MILC

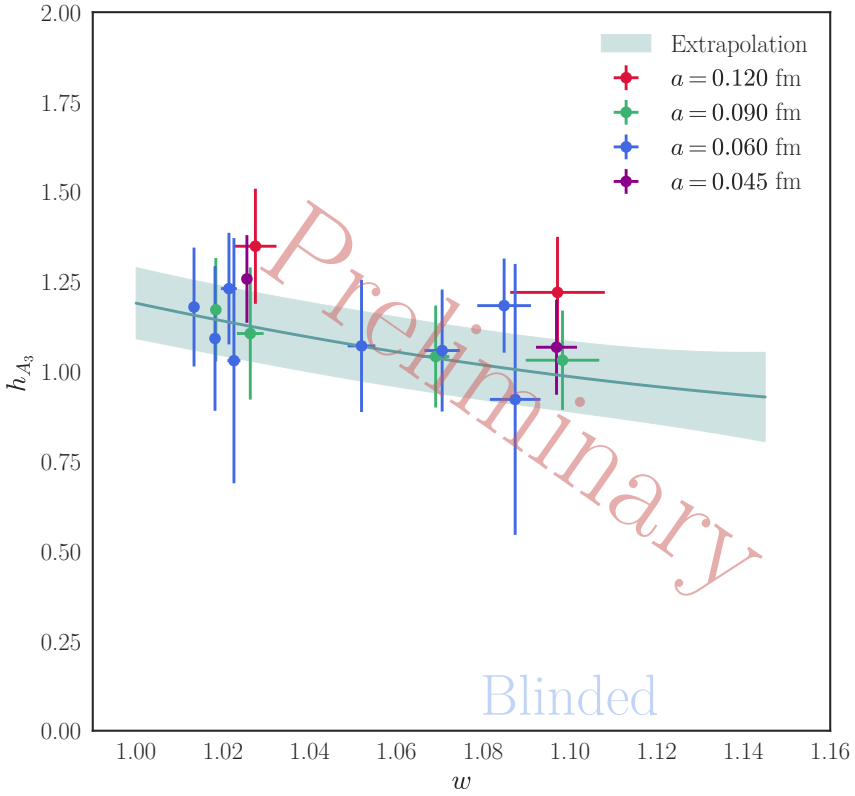




- Preliminary **blinded** results, joint fit $p - \text{value} = 0.96$
- **Excluding ensembles** with high discretization errorsin this fit, at present
- **Need to finalize study of heavy quark discretization errors** to include all ensembles

Results: Chiral-continuum fits





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Analysis: Preliminary error budget

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Statistics	1.1	0.4	4.9	1.9
Isospin effects	0.0	0.0	0.6	0.3
χPT/cont. extrapolation	**1.9**	**0.7**	**6.3**	**2.9**
Matching	*1.5*	*0.4*	*0.1*	*1.5*
Heavy quark discretization	$2^\dagger/1.4^*$	$2^\dagger/0.4^*$	$2^\dagger/5.8^*$	$2^\dagger/1.3^*$

†Preliminary value actually used during the analysis
*Estimate, currently subject of intensive study

- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- *Italic* marks errors to be reduced/removed when using HISQ for heavy quarks
  - Heavy HISQ would add errors $O(\alpha_s(am_b)^2)$ and $O((am_b)^4)$ (current errors $O(\alpha_s am_b)$)

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\rm Max}) = (1+r)\mathcal{F}_2(z = z_{\rm Max})$
- BGL (weak) unitarity constraints (all HISQ will use strong constraints)

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

Analysis: z expansion fit procedure

- Several different datasets
  - Our lattice data
  - BaBar BGL fit arXiv:1903.10002
  - Belle tagged dataset arXiv:1702.01521
  - Belle untagged dataset arXiv:1809.03290
- Several different fits
  - Lattice form factors only
  - Experimental data only (one fit per dataset)
  - Joint fit lattice + experimental data
- Each dataset is given in a different format, and requires a different amount of processing
- Different fitting strategy per dataset

Assume $V_{cb} = V_{cb}^{\mathrm{BaBar}}$ for the only Belle data fits to have a **common normalization** for the coefficients

All the experimental and theoretical **correlations are included** in all fits

Analysis: Fit procedure

Constraints

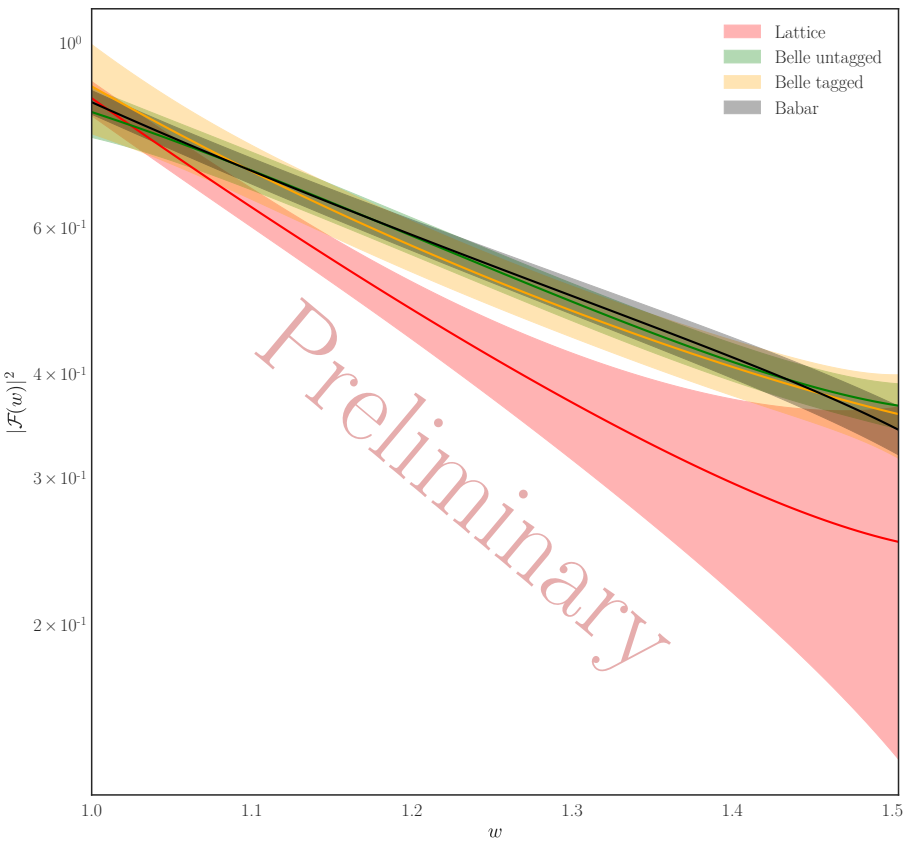
- The constraint at zero recoil is used to remove a coefficient of the BGL expansion
- The constraint at maximum recoil is imposed by adding a datapoint with small errors
- The unitarity constraints are imposed by adding hard cuts (under revision)
- In the fits shown, we do **not** impose any constraints, but they **are satisfied** automatically

How many coefficients?

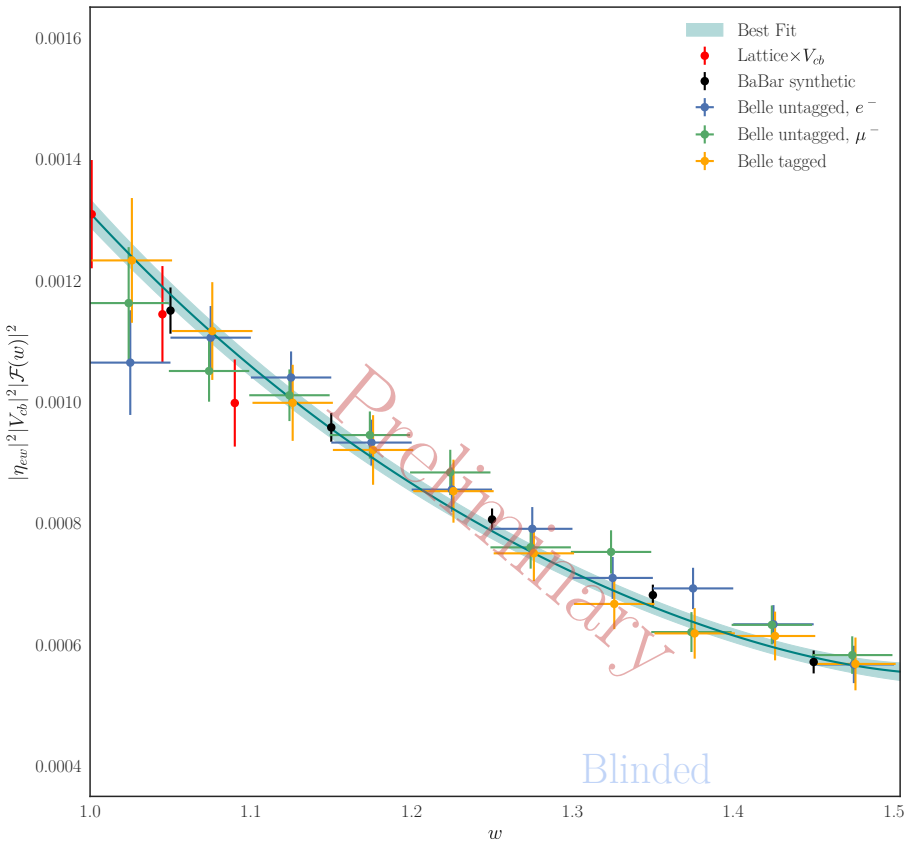
- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
- Current maximum 4 (each coefficient requires new integrals)

Results: Pure-lattice prediction and joint fit

Separate fits

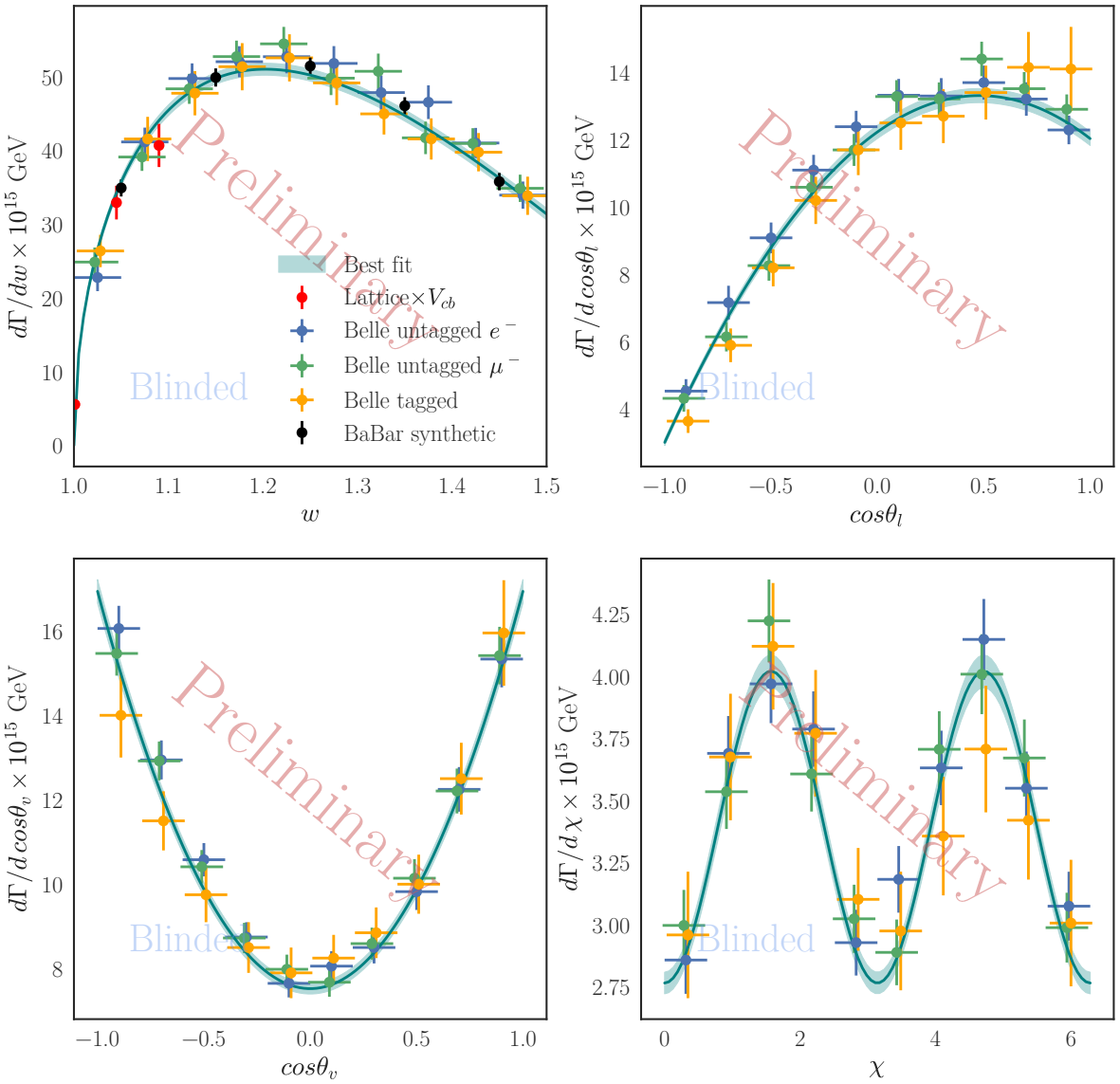


Joint fit



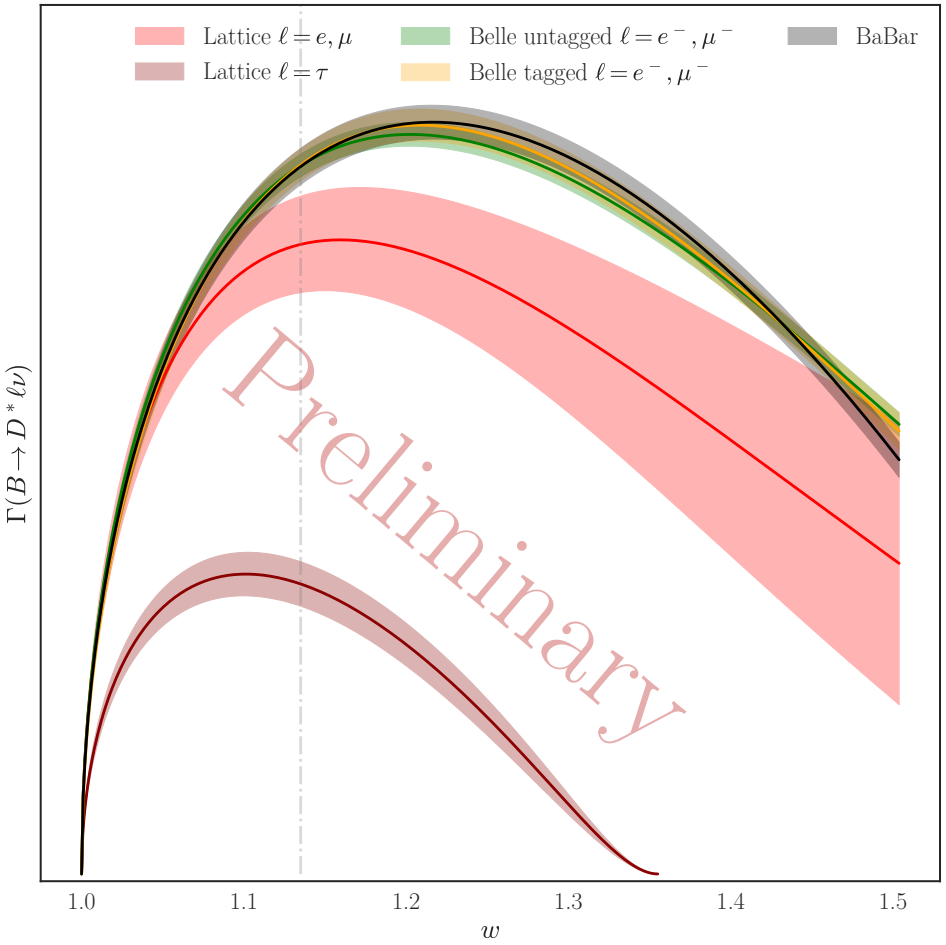
- Lattice + Belle + BaBar BGL $p-\text{value} = 0.17$
- Lattice only BGL $p-\text{value} = 0.05$
- Belle untagged BGL $p-\text{value} = 0.18$, tagged $p-\text{value} \approx 1$

Results: Joint fit, angular bins

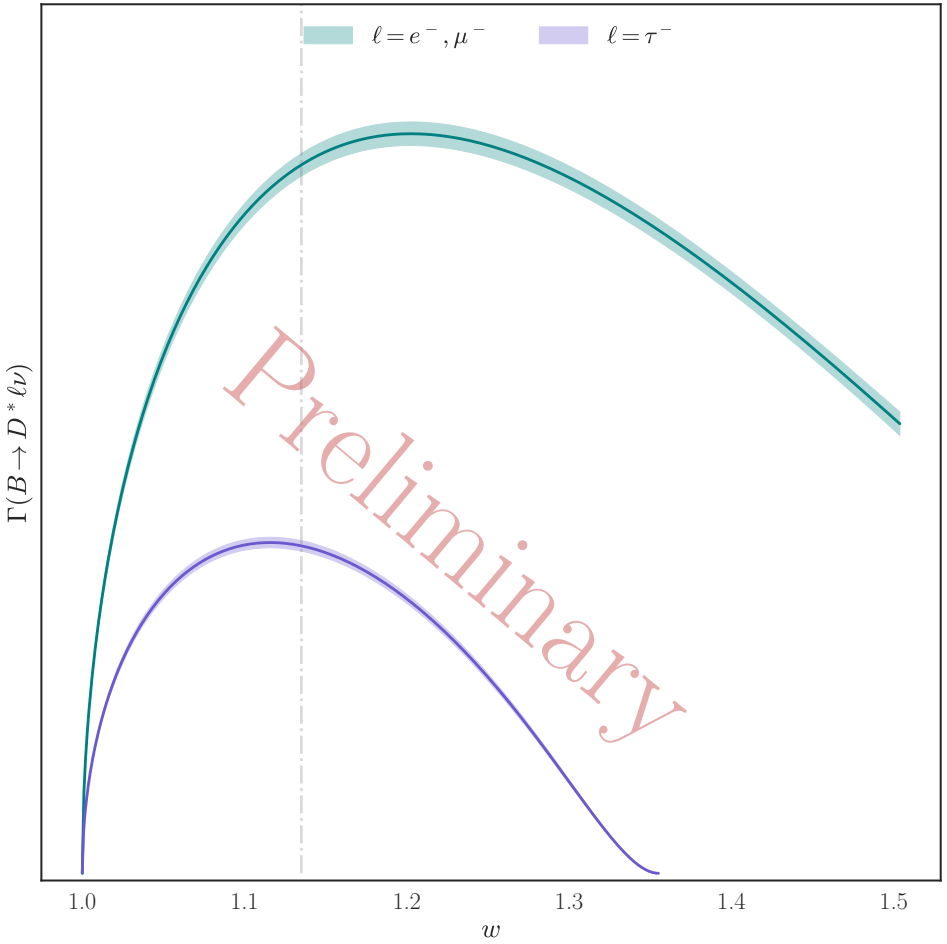


Results: $R(D^*)$

Separate fits



Joint fit



Conclusions

What to expect

- Error on V_{cb} from this analysis might not be improved compared to the previous determinations that used exp data + CLN + lattice $\mathcal{F}(1)$
  - Errors might not be improved compared to previous lattice estimations
- The **main new information of this analysis** won't come from the zero-recoil value, but from the **slope at small recoil**
- Main sources of errors of our form factors are
  - χPT-continuum extrapolation
  - HQ discretization
  - Matching
- The analysis of heavy quark discretization errors is **in progress**
- **We need to understand better the current lattice and experimental data**

Conclusions

The future

- Well established roadmap to reduce errors in our calculation with newer lattice ensembles
- Our next analysis will be joint $B \to D$ and $B \to D^*$ to benefit from strong unitarity constraints
- This roadmap is to be followed in other processes involving other CKM matrix elements

BACKUP

Analysis: z expansion fit procedure

Lattice data points

- Generate synthetic data from the chiral-continuum extrapolation
- Fitted directly to the z-expansion of the corresponding form factor

Belle untagged dataset (folded)

- e^- and μ^- datasets fitted separately
- Given the fit function, we compute the count prediction on each bin using a similar procedure as in the Belle collaboration

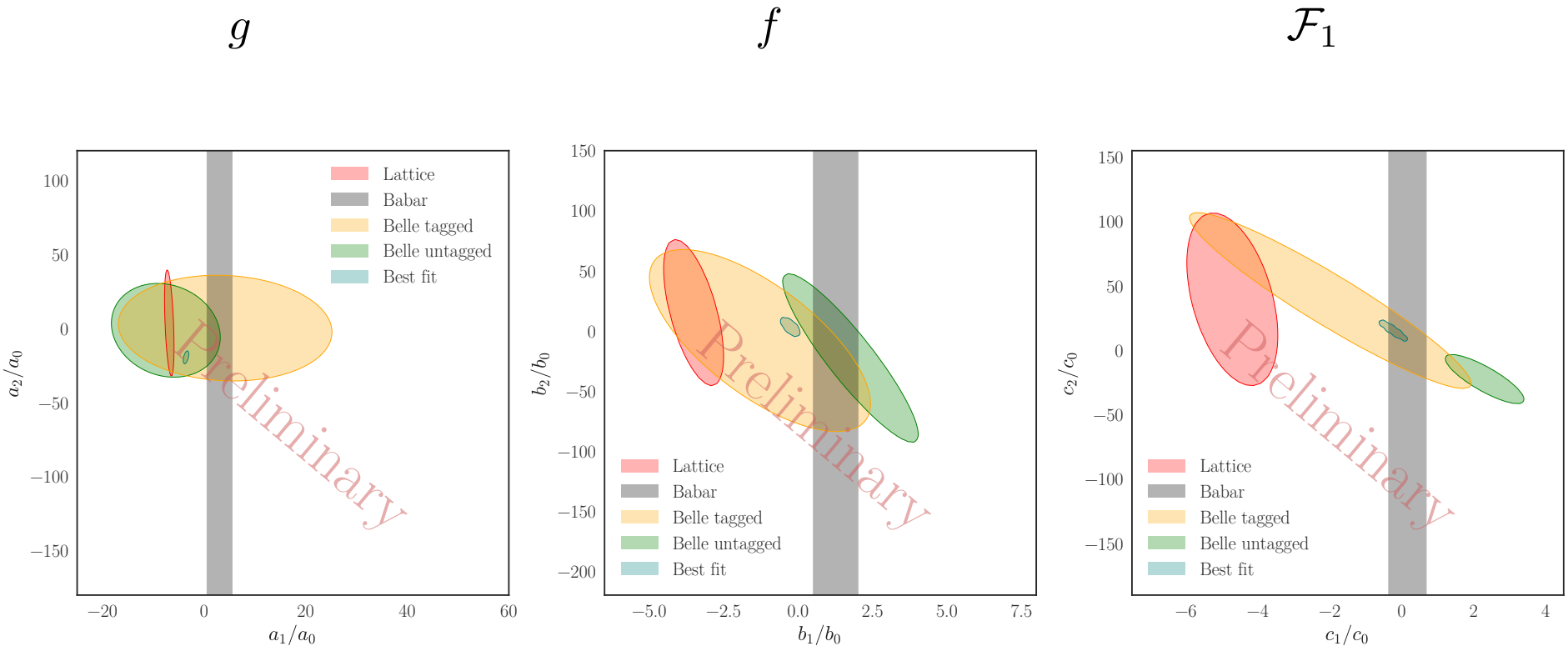
Belle tagged dataset (unfolded, binned)

- Integrate the three non-relevant variables in the fit function and fit the result to the data

BaBar fit results

- 5 independent parameters $+ V_{cb}$
- Generate 5 synthetic datapoints on the w bins and fit them normally

Results: Tensions in the BGL coefficients



- The b_j represent the small recoil behavior $\sim h_{A_1}$
- The c_j represent the large recoil behavior $\sim H_0$