# Leptonic decays of $B_{(s)}$ and $D_{(s)}$ using the OK action

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## Motivation

- The relation between decay constant $f_X$ of a meson $X = (q_\ell \bar{q}_h)$ and the hadronic matrix element of the axial current is
$$\langle 0|A^\mu|X\rangle = ip^\mu f_X\,, \tag{1}$$
where $A^\mu = \bar{q}_h\gamma^\mu\gamma_5 q_\ell$. Here, $q_h$ ($q_\ell$) represents heavy (light) quark field in the continuum.
- Precise lattice QCD calculation on decay constants is required for the determination of CKM matrix elements: $|V_{ub}|$ (from $f_B$) or $|V_{cd}|$ (from $f_D$).
- FNAL/MILC Collaboration reported in Ref. [1] that $f_{B_s}/f_B = 1.229(26)$, $f_{D_s}/f_D = 1.188(25)$ using Fermilab action for valance bottom and charm quarks and asqtad for valance light quarks over MILC asqtad ensembles.
- Here, we use Oktay-Kronfeld (OK) action for valance bottom and charm quarks and HISQ for valance light quarks over MILC HISQ ensembles.

## MILC HISQ ensemble

Table 1: List of MILC HISQ ensembles [2]. Here, we use the labels in the first column.

| Label | $a_{r_1}$ (fm) | $N_s^3 \times N_t$ | $M_\pi$ (MeV) | $am_\ell$ | $am_s$ | $am_c$ |
|---|---|---|---|---|---|---|
| a12m310 | 0.1207(11) | $24^3 \times 64$ | 305.3(4) | 0.0102 | 0.0509 | 0.635 |
| a12m220 | 0.1184(10) | $32^3 \times 64$ | 216.9(2) | 0.00507 | 0.0507 | 0.628 |
| a09m310 | 0.0888(8) | $32^3 \times 96$ | 312.7(6) | 0.0074 | 0.037 | 0.440 |
| a09m220 | 0.0872(7) | $48^3 \times 96$ | 220.3(2) | 0.00363 | 0.0363 | 0.430 |
| a06m310 | 0.0871(6) | $48^3 \times 144$ | 319.3(5) | 0.0048 | 0.024 | 0.286 |

## The Oktay-Kronfeld action

- The OK action [3] is improved version of Fermilab action [4].
- The OK action is improved up to $\mathcal{O}(\lambda^3)$ in the HQET power counting and $\mathcal{O}(v^6)$ in the NRQCD power counting.

$$\begin{aligned} S_{\text{FNAL}} &= a^4 \sum_x \bar{\psi}_h(x)\left[\mathcal{M}_0 + \mathcal{M}_B + \mathcal{M}_E\right]\psi_h(x)\,, \\ S_{\text{OK}} &= S_{\text{FNAL}} + a^4\sum_x \bar{\psi}_h^{\text{OK}}(x)\left[\mathcal{M}_6 + \mathcal{M}_7\right]\psi_h^{\text{OK}}(x)\,, \end{aligned} \tag{2}$$

$$\begin{aligned} &\mathcal{M}_0 = m_0 + \gamma_4 D_4 + \zeta\boldsymbol{\gamma}\cdot\boldsymbol{D} - \frac{1}{2}a\Delta_4 - \frac{1}{2}r_s\zeta a\Delta^{(3)}\,, \quad \mathcal{M}_B = \frac{1}{2}c_B\zeta a i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\,, \quad \mathcal{M}_E = \frac{1}{2}c_E\zeta a\boldsymbol{\alpha}\cdot\boldsymbol{E}\,, \\ &\mathcal{M}_6 = c_1 a^2\sum_k \gamma_k D_k\Delta_k + c_2 a^2\left\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \Delta^{(3)}\right\} + c_3 a^2\left\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\right\} + c_{EE}a^2\left\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\right\}\,, \\ &\mathcal{M}_7 = c_4 a^3\sum_k \Delta_k^2 + c_5 a^3\sum_{j\neq k}\left\{i\Sigma_k B_k, \Delta_j\right\}\,. \end{aligned} \tag{3}$$

## Current improvement and correlator

For calculation of decay constants, the lattice axial current is
$$A^4(t,\mathbf{x}) = Z_A\bar{\Psi}_h(t,\mathbf{x})\gamma^4\gamma_5\psi_\ell(t,\mathbf{x})\,, \tag{4}$$
where $Z_A = e^{m_1 a/2}$ is the tree-level renormalization factor for the current where $m_1 a = \log(1+m_0 a)$. Current improvement is designed to remove discretization error in the lattice current up to the $\lambda^3$ order. The rotated heavy-quark field is sufficient to achieve this goal at the tree level:
$$\begin{aligned} \Psi_h(t,\mathbf{x}) = \Big[&1 + d_1 a\boldsymbol{\gamma}\cdot\boldsymbol{D} + d_2 a^2\Delta^{(3)} + d_B a^2 i\boldsymbol{\Sigma}\cdot\boldsymbol{B} - d_E a^2\boldsymbol{\alpha}\cdot\boldsymbol{E} + d_{rE}a^3\left\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\right\} \\ &- d_3 a^3\sum_i \gamma_i D_i\Delta_i - d_4 a^3\left\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \Delta^{(3)}\right\} - d_5 a^3\left\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\right\} \\ &+ d_{EE}a^3\left\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\right\} - d_6 a^3\left[\gamma_4 D_4, \Delta^{(3)}\right] - d_7 a^3\left[\gamma_4 D_4, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\right]\Big]\psi_h^{\text{OK}}(t,\mathbf{x})\,. \end{aligned}$$
The meson-meson (MM) and meson-current (MC) correlator are
$$C_{\text{MM}}(t) = \sum_{\mathbf{x}}\left\langle \mathcal{O}^\dagger(t,\mathbf{x})\mathcal{O}(0)\right\rangle\,, \quad C_{\text{MC}}(t) = \sum_{\mathbf{x}}\left\langle A^{4\dagger}(t,\mathbf{x})\mathcal{O}(0)\right\rangle \tag{5}$$
where
$$\mathcal{O}(t,\mathbf{x}) = \bar{\psi}_h^{\text{OK}}(t,\mathbf{x})\gamma_5\psi_\ell(t,\mathbf{x})\,. \tag{6}$$

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## Correlator fit

The fitting function is
$$C_Y(t) = g_Y(t) + g_Y(T-t)\,, \quad g_Y(t) = A_{Y,0}e^{-E_0 t}\sum_{i=0}^{n}\left[R_{Y,i}e^{-\Delta E_i t} - (-1)^t R^p_{Y,i}e^{-\Delta E^p_i t}\right]$$
where Y=MC or MM, and we determine the ground state amplitude $A_{Y,0}$ and the ground state energy $E_0$. Here, $R^{(p)}_{Y,i} = A^{(p)}_{Y,i}/A_{Y,0}$ and $\Delta E^{(p)}_i = E^{(p)}_i - E_0$. By definition, $R_0 = 1$, $\Delta E_0 = 0$. Here, we use 3+2 multistate fit ($n = 2$ and $R^p_{Y,2} = 0$).

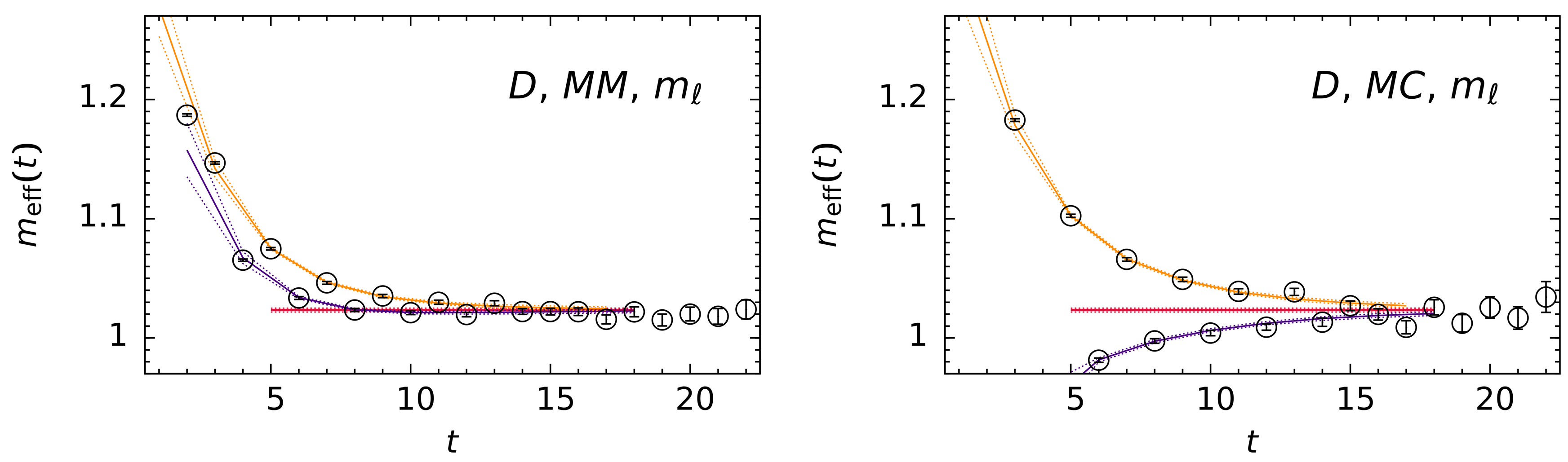


Figure 1: Effective mass plots of meson-meson (MM) and meson-current (MC) correlator for 3+2 multistate fit on a12m220 ensemble where $m_\ell = 0.1 m_s$. Here, the orange (purple) curves connect fitting results on the odd (even) time slices. Here, the horizontal red line represents the fit result on the ground state energy whose length represents the fit range. Here, $m_{\text{eff}}(t) = \frac{1}{2}\log\left(\frac{C_Y(t)}{C_Y(t+2)}\right)$.

We determine the fit ranges in the multistate fit so that we can obtain $A_{\text{MM},0}$ and $E_0$. If not, we use Bayesian priors for $R^{(p)}_{\text{MM},i}$ and $\Delta E^{(p)}_i$, and try again. Next, we do the fit for the MC correlator to determine $A_{\text{MC},0}$ around the fit range of MM. Here, $E_0$ and $\Delta E^{(p)}_i$ of MC come from the fit of MM.

## Decay constant

The decay constant, $f_X$ is calculated with the amplitude of meson propagator and the amplitude of meson-current correlator,
$$a^{\frac{3}{2}}f_X = Z_{A^4_{q_h q_\ell}}\sqrt{\frac{2}{M_X}}\frac{A_{\text{MC},0}}{\sqrt{A_{\text{MM},0}}}\,. \tag{7}$$
Here, $A_{\text{MC},0}$ and $A_{\text{MM},0}$ are the amplitude of the ground state for meson-current correlator and meson propagator, respectively. Here, $Z_{A^4_{q_h q_\ell}}$ is the renormalization factor for the heavy-light axial current. This is being calculated nonperturbatively. In the $SU(3)$ flavor breaking ratio $f_{X_s}/f_{X_\ell}$, we assume that the contribution from $Z_{A^4_{q_h q_\ell}}$ factor is negligibly small.

## Preliminary Results and future works

- Now we plot $f_{B_s}/f_B$ and $f_{D_s}/f_D$ which indicate SU(3) flavor symmetry breaking as a function of lattice spacing.
- The decay constant ratios are independent of current improvement.

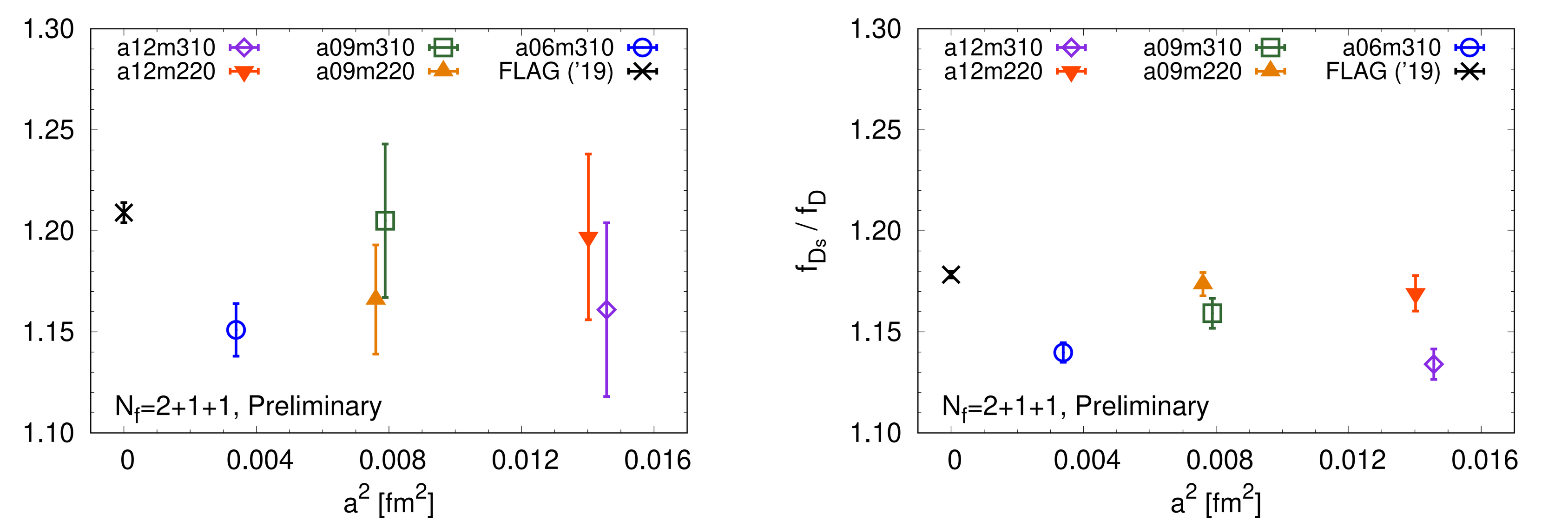


Figure 2: Results on the decay constant ratio. Here, our errors are purely statistical.

- The continuum-chiral extrapolation will be done soon.

## References

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