# Charmonium contribution to $B \to K l^+ l^-$: testing the factorization approximation on the lattice

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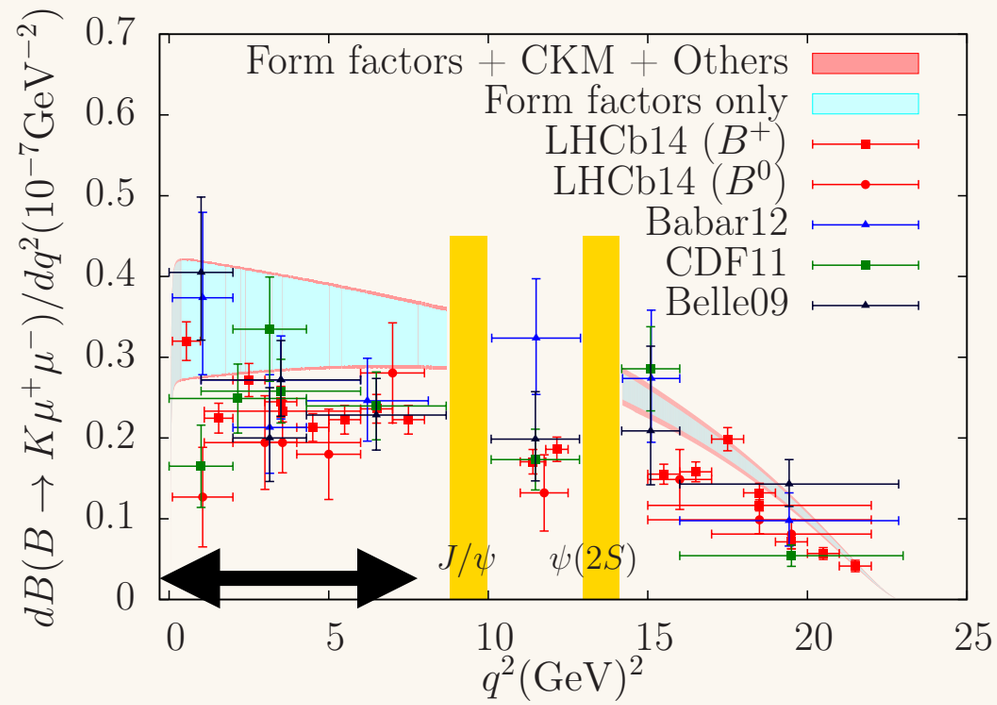
Photo: Tsukuba, Japan, 2019

# ◉ Motivation

(1): FCNC process $B \to K l^+ l^-$ as a clean probe of BSM.
(GIM and loop-suppressed)

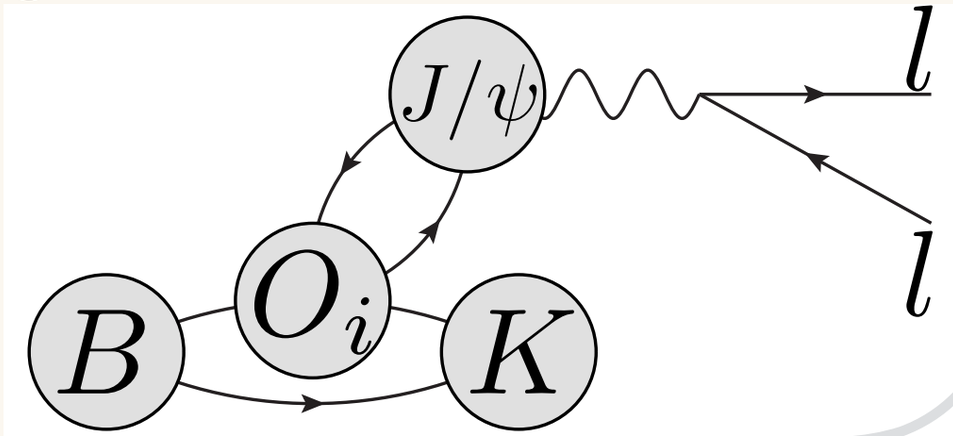
(2): Some anomaly observed by experiment .

$$q^2 < m^2_{J/\psi}$$




[D. Du et al. (Fermilab, MILC) 1510.02349]

Question: Are the estimate of long distance contributions valid?
(From charmonium resonances)
→We calculate the amplitude on the lattice.

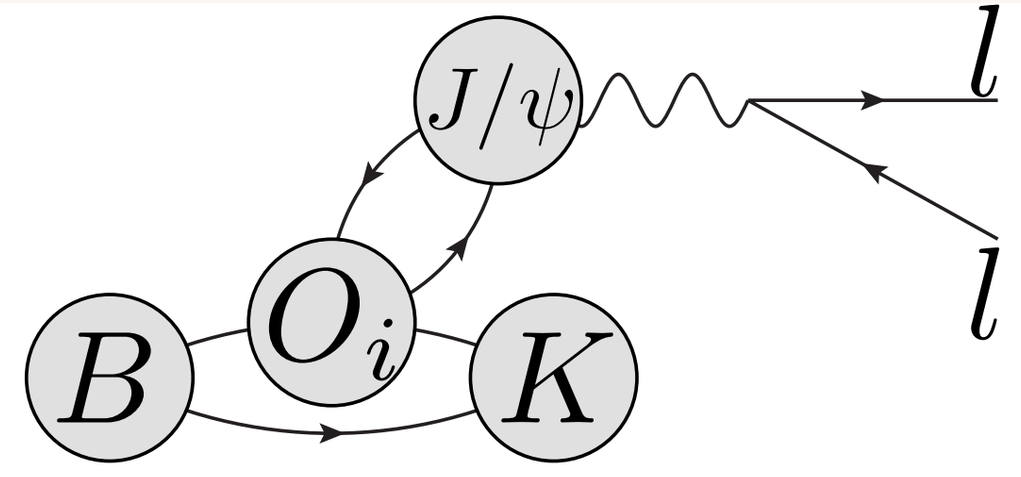


## ◉ Charmonium resonance part

◇ We focus on the diagram containing the $c\overline{c}$ loop.
It may have a large contribution, which has so far been estimated using the factorization approximation.

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \sum_{i=1}^{2} (V_{us}^* V_{ub} C_i O_i^u + V_{cs}^* V_{cb} C_i O_i^c) - V_{ts}^* V_{tb} \sum_{i=3}^{10} C_i O_i \right)$$

$$O_1^c = (\overline{s}_i \gamma_\mu P_- c_j)(\overline{c}_j \gamma_\mu P_- b_i)$$
$$O_2^c = (\overline{s}_i \gamma_\mu P_- c_i)(\overline{c}_j \gamma_\mu P_- b_j)$$




→ These two operators are responsible for the $c\overline{c}$ loop

# ◉ Decay amplitudes

◇ The calculation is similar in topology to the one for $B \to K l^+ l^-$. But only one topology is relevant in our case.

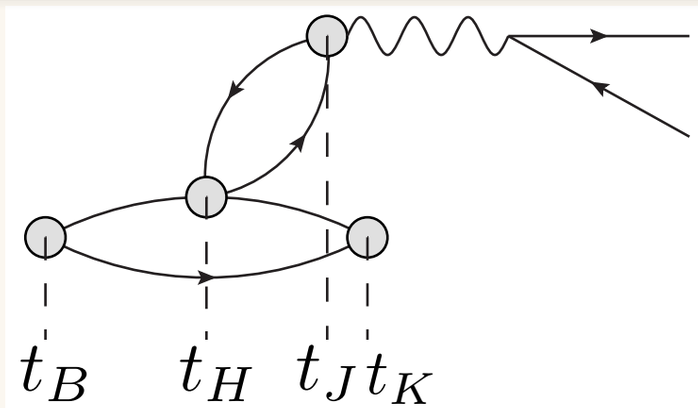
[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \, \langle \pi(\mathbf{p}) | \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] | K(\mathbf{k}) \rangle$$

$$\mathcal{A}_\mu\left(q^2\right) = \int \mathrm{d}^4x \, \langle K(\mathbf{k}) | \mathrm{T}\left[J_\mu(0) H_{\mathrm{eff}}(x)\right] | B(\mathbf{p}) \rangle$$

◇ This amplitude is obtained from a four-point function:

$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) \simeq \int_{t_J - T_a}^{t_J + T_b} \mathrm{d}t_H \left[ \right.$$




$$\left. \right]$$

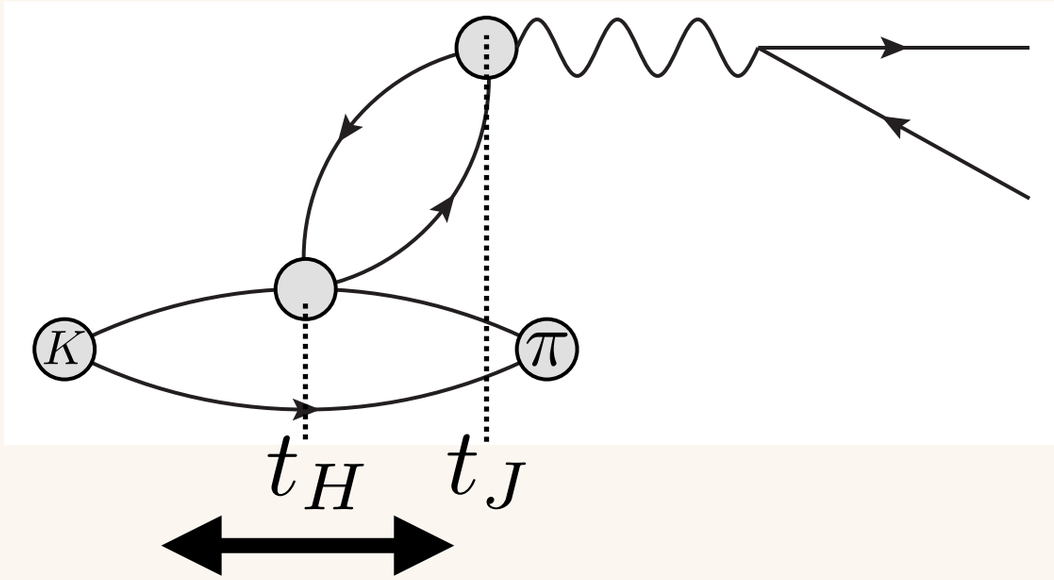
$$(0 \ll t_J - T_a \leq t_J + T_b \ll t_K)$$

# ◉ Divergence to be avoided at previous work

◇ $K \to \pi l^+ l^-$ case

[N.H. Christ et al. (RBC, UKQCD) 1507.03094]

$I_\mu =$




$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{k}) \left| J_\mu(0) \right| E, \mathbf{p} \rangle \langle E, \mathbf{p} | H_{\mathrm{eff}}(0) | K(\mathbf{p}) \rangle}{E_K(\mathbf{p}) - E} \left(1 - e^{[E_K(\mathbf{p}) - E] T_a}\right)$$

◇ There are intermediate states with energy $E$ $\quad E_K > E$

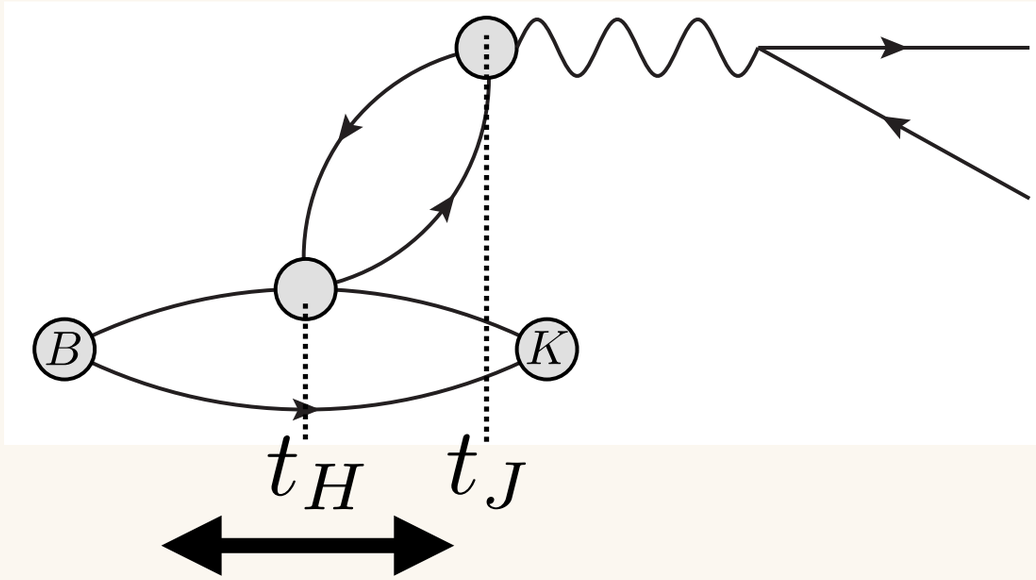
→ Since $T_a \to \infty$ ,they must be subtracted.

(e.g. $K \to \pi, \pi\pi, \pi\pi\pi$)

## ◉ →Artificial divergence does not exist

◇ $B \to K l^+ l^-$ case

$I_\mu =$




$$I_\mu\left(T_a, T_b, \mathbf{p}, \mathbf{k}\right) = -\int_0^\infty \mathrm{d}E \frac{\rho_S(E)}{2E} \frac{\langle K(\mathbf{k})\left|J_\mu(0)\right| E, \mathbf{p}\rangle \langle E, \mathbf{p}|H_{\mathrm{eff}}(0)|B(\mathbf{p})\rangle}{E_B(\mathbf{p}) - E} \left(1 - e^{[E_B(\mathbf{p}) - E]T_a}\right)$$

◇ We take unphysical light bottom and heavy up down quarks.

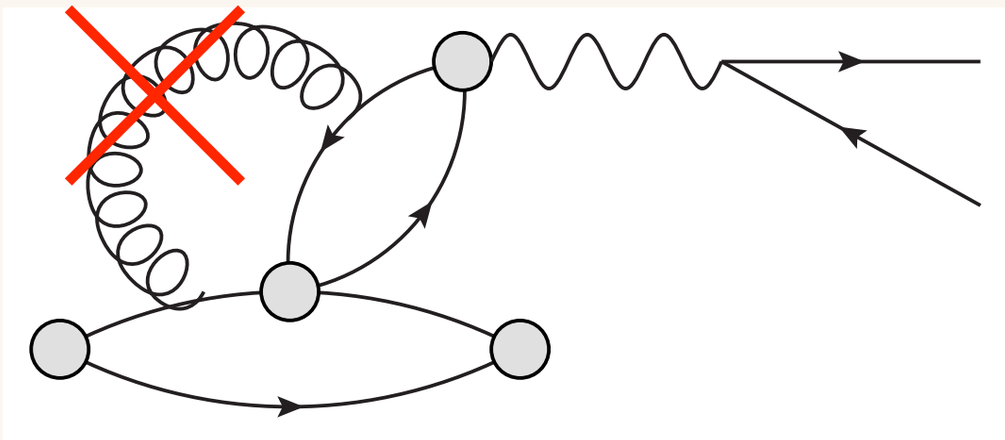
$$E_B < E_{J/\psi} + E_K$$

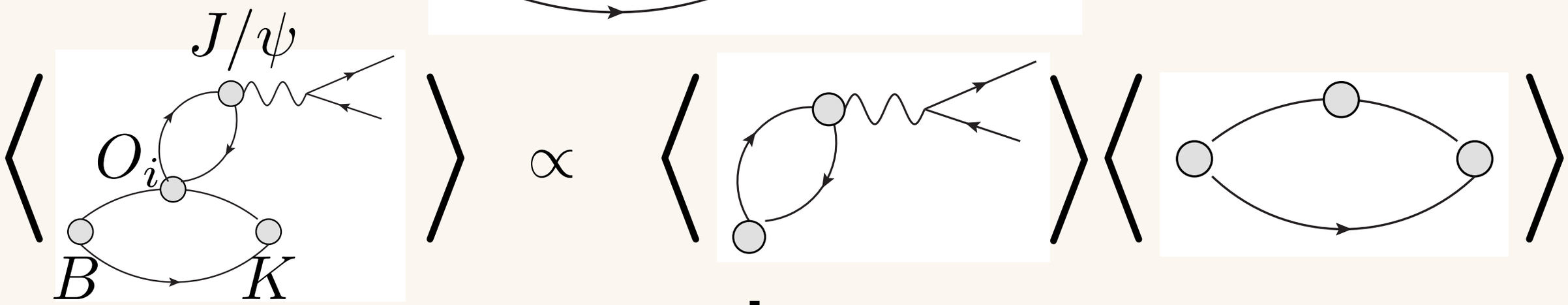
→ The intermediate states always have a larger energy, thus no divergence

# Factorization method for $B \to K l^+ l^-$ decay

# ◉ Factorization

◇ Assumption to neglect the gluon exchange contribution






$$\langle P_K | J_\nu^{\bar{c}c} (\bar{c}_i \gamma_\mu P_- c_i)(\bar{s}_j \gamma_\mu P_- b_j) | P_B \rangle = \frac{1}{(\mathrm{Vol.})} \langle 0 | J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} | 0 \rangle \langle P_K | V_\mu | P_B \rangle$$

→ We test this relation and assumption.

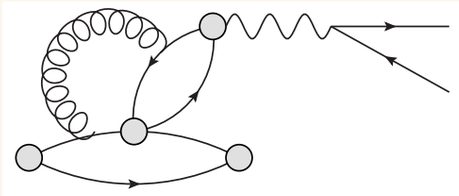
## ◉ Test of factorization in $K \to \pi\pi$ .

[P.A. Boyle et al. (RBC, UKQCD) 1212.1474]

◇ Factorizable operator $O_F$ and non-factorizable $O_{NF}$

$$O_F^{(1)} = \left(\bar{l}_i \gamma_\mu P_- l_i\right)\left(\bar{l}_j \gamma_\mu P_- s_j\right)$$

$$O_{NF}^{(8)} = \left(\bar{l}_i \left[T^a\right]_{ij} \gamma_\mu P_- l_j\right)\left(\bar{l}_k \left[T^a\right]_{kl} \gamma_\mu P_- s_l\right)$$



Fierz transformation

$$O_1^l = O_F^{(1)} \qquad O_2^l = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)}$$

◇ Neglecting the non−factorizable operator $O_{NF}^{(8)}$

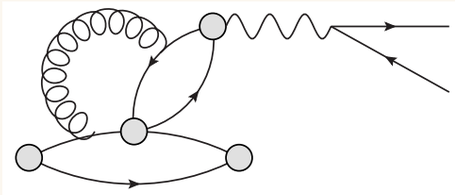
$K \to \pi\pi$, Lattice. $\quad O_2^l \simeq -0.7 O_1^l$

# ◉ Factorization

◇ Factorizable operator $O_F$ and non-factorizable $O_{NF}$

$$O_F^{(1)} = (\overline{c}_i \gamma_\mu P_- c_i)\,(\overline{s}_j \gamma_\mu P_- b_j)$$

$$O_{NF}^{(8)} = \left(\overline{c}_i \left[T^a\right]_{ij} \gamma_\mu P_- c_j\right)(\overline{s}_k \left[T^a\right]_{kl} \gamma_\mu P_- b_l)$$



Fierz transformation

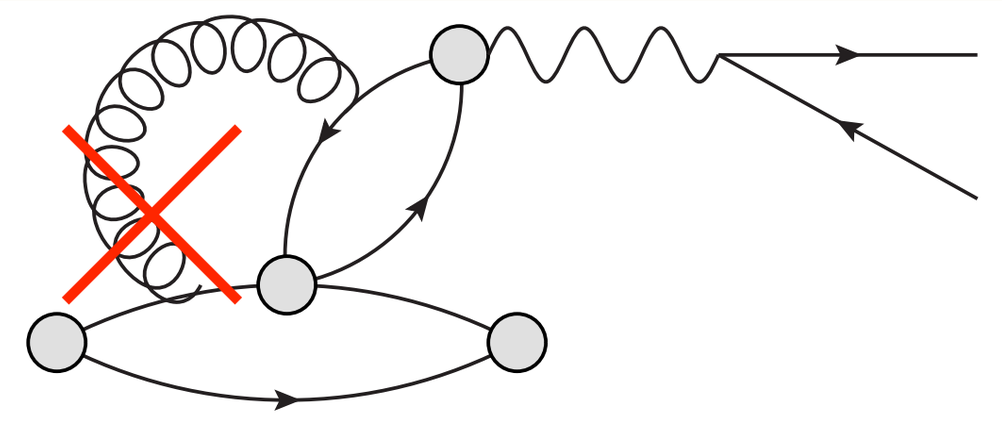
$$O_1^c = O_F^{(1)} \qquad\qquad O_2^c = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)}$$

◇ Assume non−factorizable operator $O_{NF}^{(8)}$ could be ignored

→ We test this assumption $O_2^c = \frac{1}{3} O_1^c$ .

# ◉ More on factorization (Perturbation)

→ A test of the relation $O_2^c = \frac{1}{3} O_1^c$ .



◇ Perturbatively, one can estimate the size of the non-factorizable contribution.

$$O_2^c = \left( \frac{1}{3} + \frac{O_{NF}^{(8)}}{O_F^{(1)}} \right) O_F^{(1)}$$

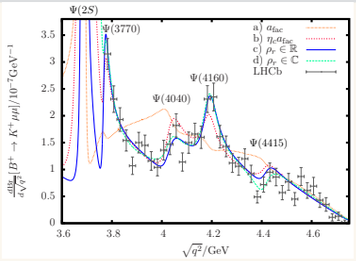
$$3 \frac{O_{NF}^{(8)}}{O_F^{(1)}} \simeq 3 \frac{\alpha_s(\mu)}{4\pi} \simeq 0.06 \qquad (6\%)$$

# ◉ Factorization in perturbation

**Perturbation**

◇ Non-factorizable contribution is sizable.

◇ We could estimate the contribution as $\eta_c \simeq -0.5$.




[J. Lyon and R. Zwicky 1406.0566]

**Experiment**

◇ $\eta_c \simeq -2.5$ well represents results from experiments.

**Lattice**

→ We test naive factorization $O_2^c = \frac{1}{3} O_1^c$ as a first step.

# Preliminary result for the test of factorization

# ◉ Current status

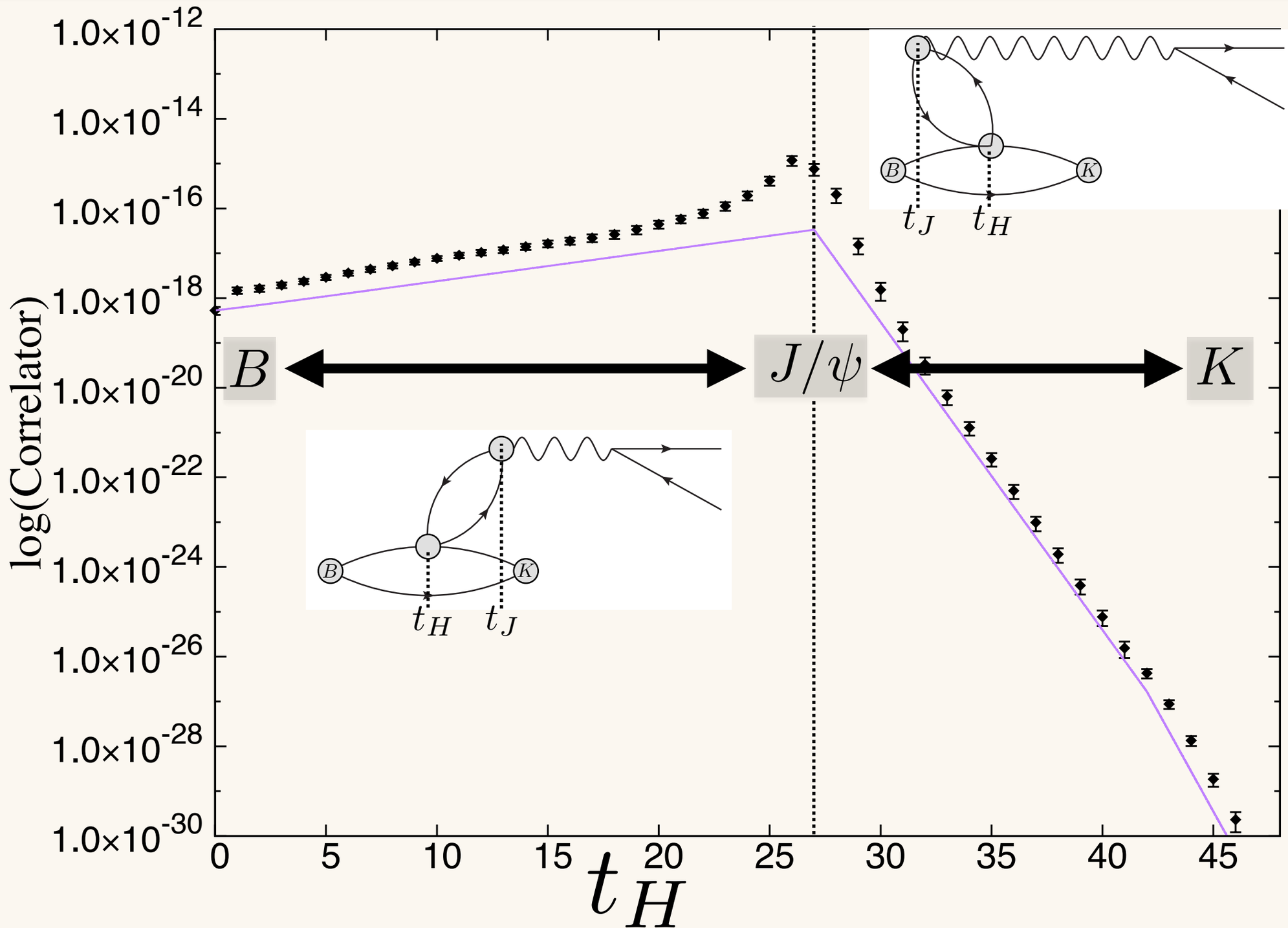
| $\beta$ | $a^{-1}$ [GeV] | $L^3 \times T(\times L_s)$ | $am_{val}$ | $am_c$ | $am_b$ |
|---|---|---|---|---|---|
| 4.35 | 3.610(9) | $48^3 \times 96(\times 8)$ | 0.025 | 0.27287 | 0.66619 |

| ap | # Conf. | $m_\pi$[MeV] | $E_K$ [MeV] | $E_{J/\psi}$ [GeV] | $m_B$ [GeV] |
|---|---|---|---|---|---|
| $-\frac{2\pi}{L}$ (1,0,0) | 390 | 714(1) | 854(3) | 3.128(1) | 3.44(1) |
| $-\frac{2\pi}{L}$ (1,1,0) | 400 | 714(1) | 969(9) | 3.158(1) | 3.44(1) |

- Heavy up and down mass same with strange.
- Light bottom mass: $m_b = (1.25)^4 m_c$
- Finite momentum at final state $\mathbf{k} = \left(-\frac{2\pi}{L}, 0, 0\right), \left(-\frac{2\pi}{L}, -\frac{2\pi}{L}, 0\right)$

# ◉ 4 point functions

$$\Gamma_\mu^{(4)}\left(t_H, t_J, \mathbf{p}, \mathbf{k}\right) = \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{y}} \left\langle \phi_K\left(t_K, \mathbf{k}\right) \mathrm{T}\left[J_\mu\left(t_J, \mathbf{y}\right) H_{\mathrm{eff}}\left(t_H, \mathbf{x}\right)\right] \phi_B^\dagger(0, \mathbf{p}) \right\rangle$$



## ◉ Renormalization Constant

$$\langle O_1 \rangle_R = Z_{11}\langle O_1 \rangle + Z_{12}\langle O_2 \rangle$$
$$\langle O_2 \rangle_R = Z_{21}\langle O_1 \rangle + Z_{22}\langle O_2 \rangle$$

◇ Determined in the shceme to match the tree level (see T.Ishikawa's talk)

[Monday, 16:50, [150],Standard Model parameter and renormalization]

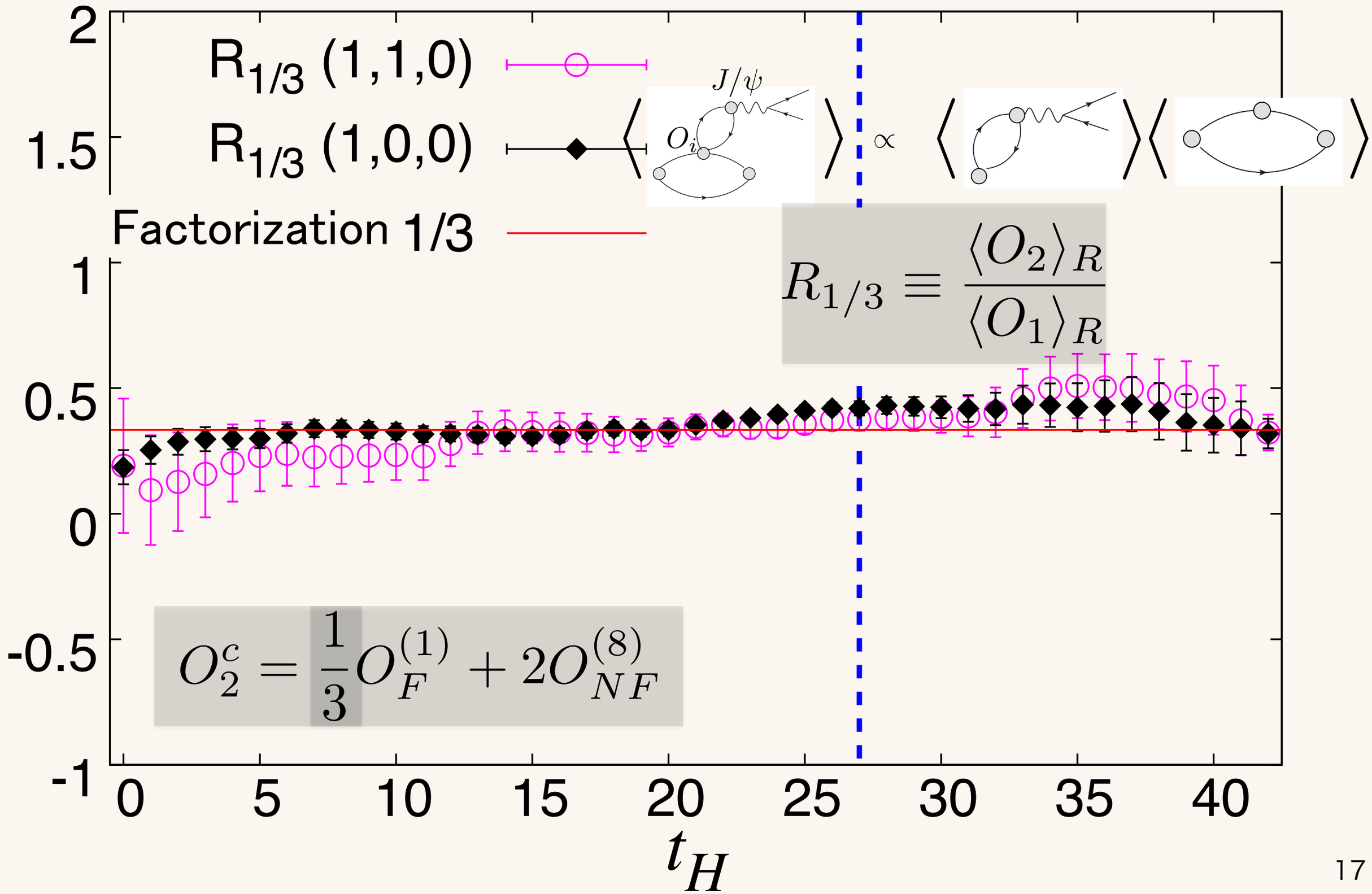
$$Z_{11} = Z_{22} = 0.669(11) \qquad Z_{12} = Z_{21} = 0.093(4)$$

$$O_1^c = O_F^{(1)} \qquad O_2^c = \frac{1}{3}O_F^{(1)} + 2O_{NF}^{(8)}$$

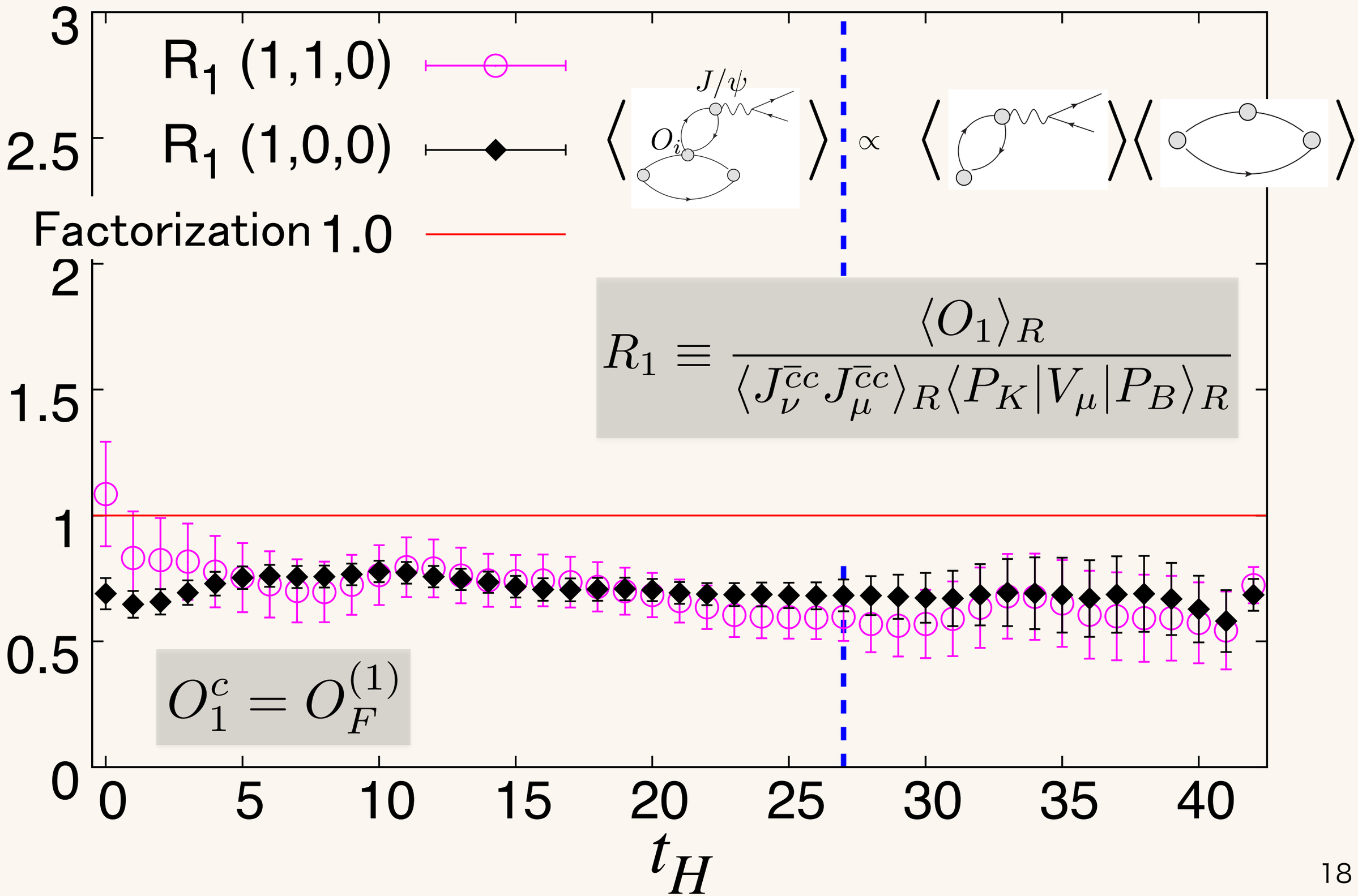
$$R_{1/3} \equiv \frac{\langle O_2 \rangle}{\langle O_1 \rangle} \to \frac{\langle O_2 \rangle_R}{\langle O_1 \rangle_R}$$

$$R_1 \equiv \frac{\langle O_1 \rangle}{\langle J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} \rangle \langle P_K | V_\mu | P_B \rangle} \to \frac{\langle O_1 \rangle_R}{\langle J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} \rangle_R \langle P_K | V_\mu | P_B \rangle_R}$$

# ◉ Factorization of 4 point functions



# ◉ Factorization of 4 point functions



## ◉ Summary

◇ We studied the $c\overline{c}$ loop contribution to $B \to K l^+ l^-$.

◇ The lattice calculation is similar to $K \to \pi l^+ l^-$ ; we take an unphysically smaller bottom quark mass to avoid the divergence.

◇ We test the factorization assumption; found a sizeable violation.

◇ Estimate of the physical amplitude is yet to be performed.