Calculation of the $K_L - K_S$ mass difference with physical quark masses

Bigeng Wang
RBC-UKQCD Collaborations

Department of Physics
Columbia University in the City of New York

Lattice 2019

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Motivation

Physics:

- $\Delta m_K = m_{K_L} - m_{K_S}$ is generated by neutrual Kaon mixing through **weak interaction**
- $\Delta m_{K,exp} = m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12} MeV$
  Sensitive to BSM effects
- **Highly non-perturbative**

Status of the calculation:

- **"Long-distance contribution at the $K_L - K_S$ mass difference"**,
  N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. D 88(2013), 014508
- **"Results for Δm_K for physical quark masses"**
  Presented in Lattice 2018
  All diagrams included on a $64^3 \times 128$ lattice with **physical masses** on **129** configurations: $\Delta m_k = 7.0(1.7)_{stat}(1.8)_{sys} \times 10^{-12}$ MeV
- Here I present an update of the analysis methods used and results having smaller errors with **152** configurations.

From Correlators to Δm_K^{lat}

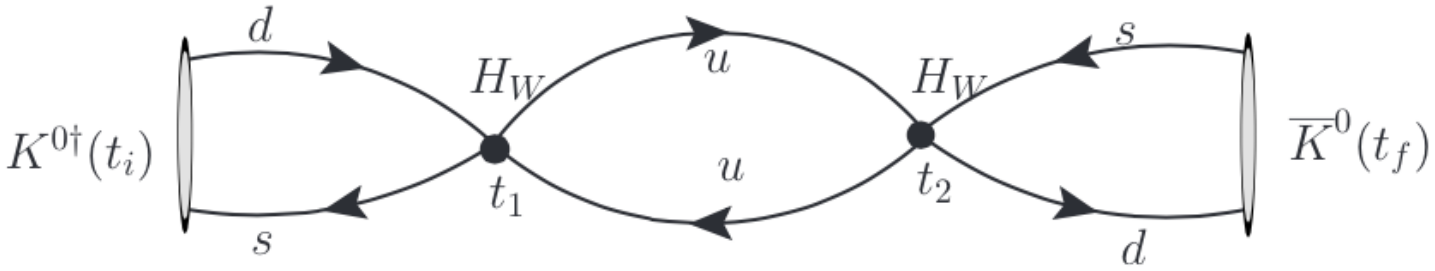
- Δm_K is given by:

$$\begin{aligned}\Delta m_K &\equiv m_{K_L} - m_{K_S} \\ &= 2\mathcal{P}\sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}\end{aligned} \tag{1}$$

- What we measure on lattice are:

$$G(t_1, t_2, t_i, t_f) \equiv \langle 0|T\{\bar{K}^0(t_f)H_W(t_2)H_W(t_1)K^0(t_i)\}|0\rangle \tag{2}$$

$$\rightarrow G(\delta) = N_K^2 e^{-m_K(t_f - t_i)}\sum_n \langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle e^{(m_K - E_n)\delta}$$



Extract Δm_K from Double-integrated Correlators

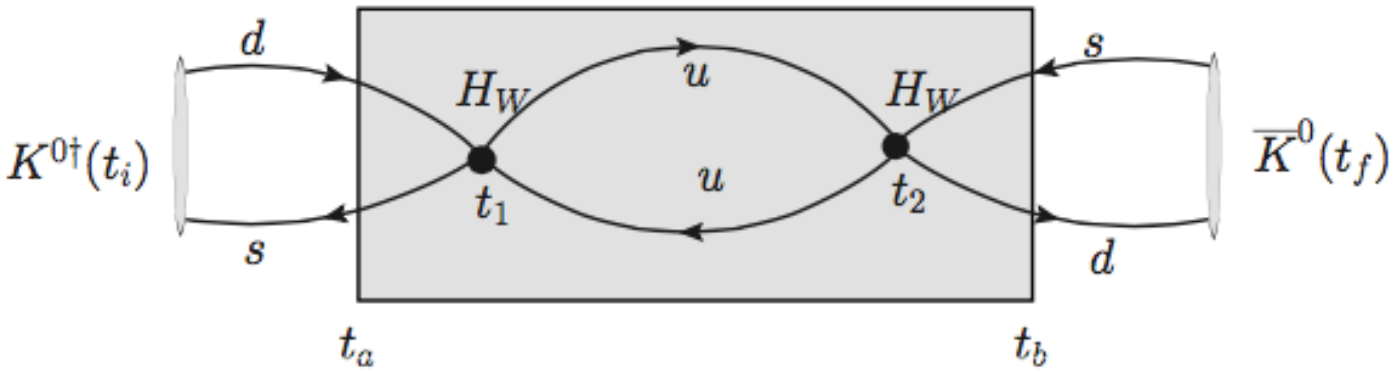
- The double-integrated correlator is defined as:

$$\mathcal{A} \equiv \frac{1}{2!} \sum_{t_2=t_a}^{t_b} \sum_{t_1=t_a}^{t_b} \langle 0| T\{\bar{K}^0(t_f) H_W(t_2) H_W(t_1) K^0(t_i)\}|0\rangle \quad (3)$$

- If we insert a complete set of intermediate states, we find:

$$\mathcal{A} = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n} \{-T + \frac{e^{(m_K-E_n)T} - 1}{m_K - E_n}\} \quad (4)$$

with $T \equiv t_b - t_a + 1$.



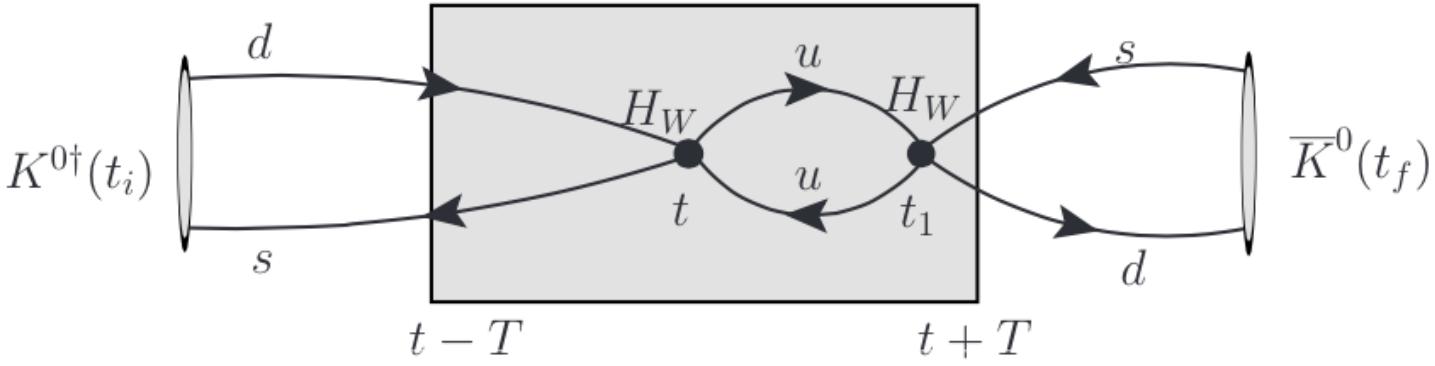
Extract Δm_K from Single-integrated Correlators

- The single-integrated correlator is defined as:

$$\mathcal{A}^s(t,T) \equiv \frac{1}{2!} \sum_{t_1=t-T}^{t+T} \langle 0| T\{\bar{K}^0(t_f) H_W(t_1) H_W(t) K^0(t_i)\}|0\rangle \quad (5)$$

- If we insert a complete set of intermediate states, we find:

$$\mathcal{A}^s = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}\left(-1+e^{(m_K-E_n)(T+1)}\right) \quad (6)$$



Subtraction of the light states

- Either Double- or Single-integrated Method requires subtraction of the terms from light states:

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \} \tag{7}$$

$$\mathcal{A}^s = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{ -1 + e^{(m_K - E_n)(T+1)} \} \tag{8}$$

- For $|n\rangle$(in our case $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$) with $E_n < m_K$ or $E_n \sim m_K$: the exponential terms will be significant. We can:
  - freedom of adding $c_s \bar{s} d$, $c_p \bar{s} \gamma^5 d$ operators to the weak Hamiltonian Here we choose:

$$\langle 0 | H_W - c_p \bar{s} \gamma_5 d | K^0 \rangle = 0, \langle \eta | H_W - c_s \bar{s} d | \bar{K}^0 \rangle = 0$$

  - subtract contributions from other states($|\pi\rangle$, $|\pi\pi\rangle$) explicitly

Operators of Δm_K^{lat} calculation

- The $\Delta S = 1$ effective Weak Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \tag{9}$$

where the $Q_i^{qq'}{}_{i=1,2}$ are current-current opeartors, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j)$$

$$Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)$$

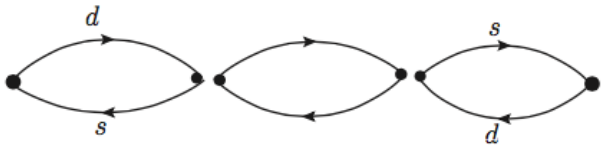
- There are four states need to subtracted: $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$. We add $c_s \bar{s} d$, $c_p \bar{s} \gamma^5 d$ operators to weak operators to make:

$$\langle 0|Q_i - c_{pi} \bar{s} \gamma_5 d|K^0\rangle = 0, \langle \eta|Q_i - c_{si} \bar{s} d|K^0\rangle = 0 \tag{10}$$

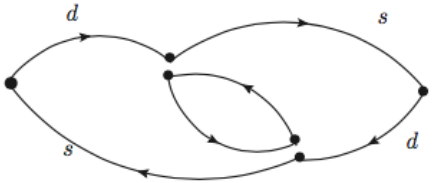
$$Q'_i = Q_i - c_{pi} \bar{s} \gamma_5 d - c_{si} \bar{s} d \tag{11}$$

Diagrams in the Calculation of Δm_K^{lat}

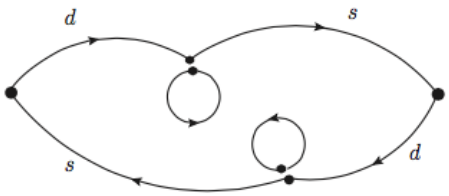
- For contractions among Q_i, there are four types of diagrams to be evaluated.



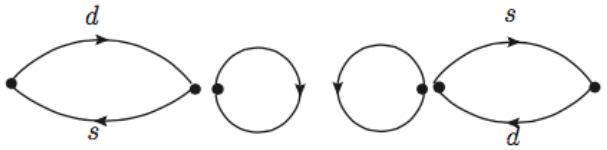
type 1



type 2

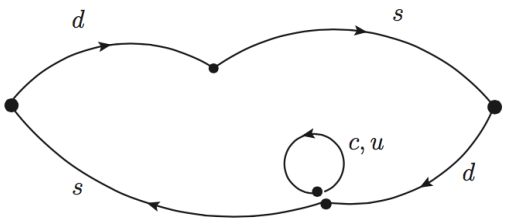


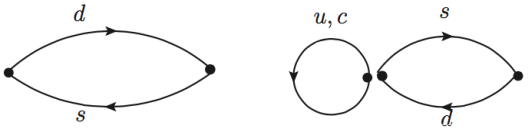
type 3



type 4

- In addition, there are "mixed" diagrams from the contractions between the $c_s \bar{s} d$ $c_p \bar{s} \gamma^5 d$ operators and Q_i operators.





From Δm_K^{lat} to Δm_K

To get Δm_k from Δm_K^{lat}, we also need to consider:

- Ultraviolet divergences as the two H_W approach each other: GIM mechanism removes both quadratic and logarithmic divergences $\rightarrow$ charm quark propagators(for valence charm we used $am_c \simeq 0.31$)
- Renormalization of Lattice operator $Q_{1,2}$ in 3 steps:
  - Non-perturbative Renormalization: from lattice to RI-SMOM
  - Perturbation theory: from RI-SMOM to $\overline{MS}$

    C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 014001
  - Use Wilson coefficients in the $\overline{MS}$ scheme

    G. Buchalla, A.J. Buras and M.E. Lautenbacher, arXiv:hep-ph/9512380

Details of the Calculation

- $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV) and $a^{-1} = 2.36$GeV

N_f	β	am_l	am_h	$\alpha = b + c$	L_s
2+1	2.25	0.0006203	0.02539	2.0	12

- Data:
  - Sample AMA Correction and Super-jackknife Method

data type	CG stop residual
sloppy	$1e-4$
exact	$1e-8$

Data Set	# of Sloppy	# of Correction	# of Type12
Lattice 2018	113	16	17
Total	116	**36**	**36**

  - Disconnected Type4 diagrams:
    save left- and right-pieces separately and use multiple source-sink separation for fitting.

Update of the results

2-point and 3-point results **preliminary**

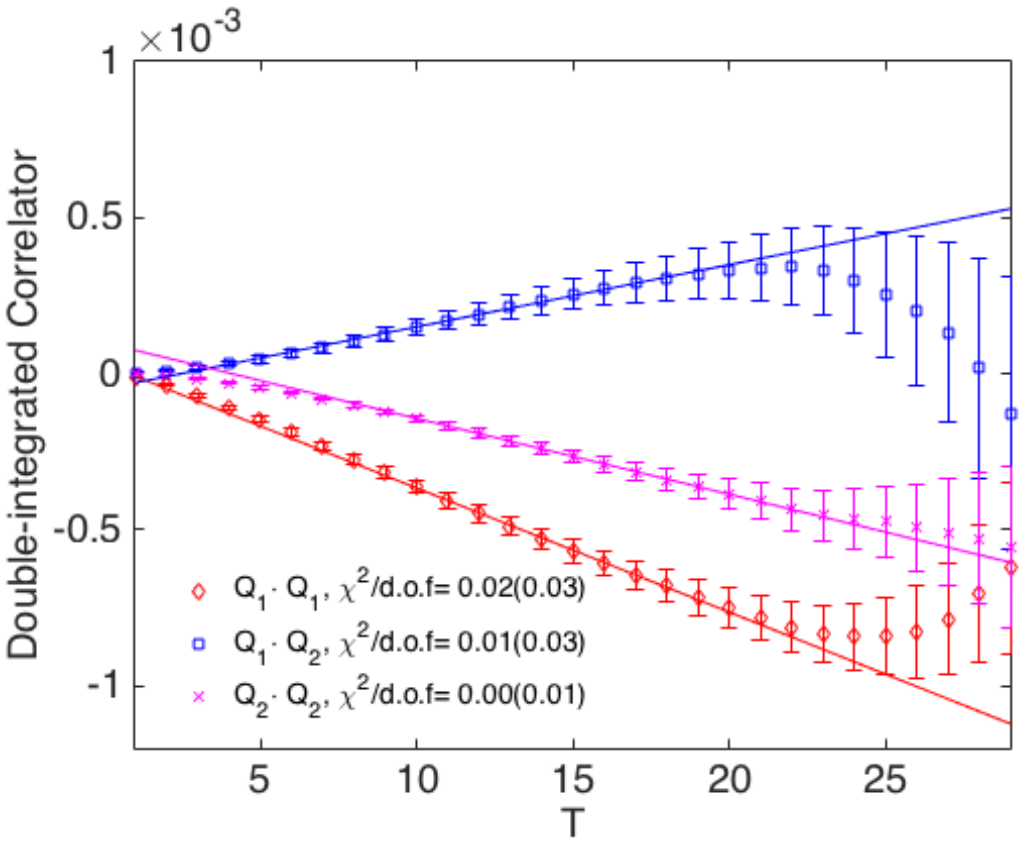
- Meson masses are consistent with physical values

m_π	m_K	m_η	$m_{\pi\pi,I=0}$
0.0574(1)	0.2104(1)	0.258(16)	0.1138(5)
135.5(2)	496.5(2)	609.9(37.8)	268.5(1.3)

- $c_s's$ and $c_p's$ will be multilpied by the "mixing" diagrams and the errors from $c_s's$ and $c_p's$ will be carried all along.

$c_{s1,\eta}$	$c_{s2,\eta}$	$c_{p1,vac}$	$c_{p2,vac}$
2.13(33) $\times 10^{-4}$	-3.16(25)$\times 10^{-4}$	1.472(2) $\times 10^{-4}$	2.807(2) $\times 10^{-4}$
$\langle \pi\pi_{I=0} \vert Q_1' \vert K^0 \rangle$	$\langle \pi\pi_{I=0} \vert Q_2' \vert K^0 \rangle$	$\langle \pi \vert Q_1' \vert K^0 \rangle$	$\langle \pi \vert Q_2' \vert K^0 \rangle$
-8.7(1.5)$\times 10^{-5}$	9.5(1.5) $\times 10^{-5}$	7.7(2.5)$\times 10^{-4}$	-4.1(1.6)$\times 10^{-4}$

Double-integrated correlators **preliminary**



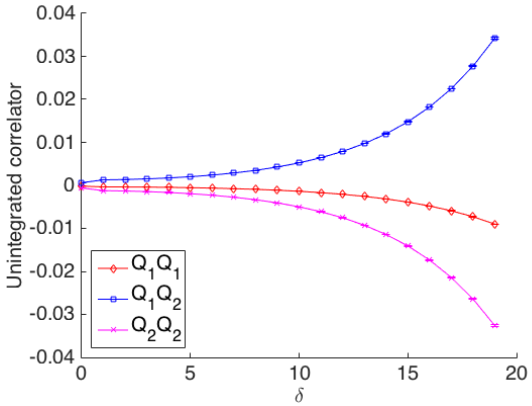
- Fitting range: 10:20
- All diagrams, uncorrelated fit
- $\Delta m_K = 8.1(1.2) \times 10^{-12}\text{MeV}$

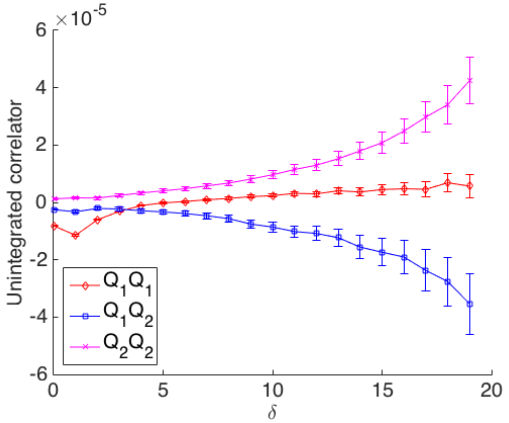
$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle K^0 | H_W | n \rangle \langle n | H_W | \bar{K}^0 \rangle}{m_K - E_n} \{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \} \tag{12}$$

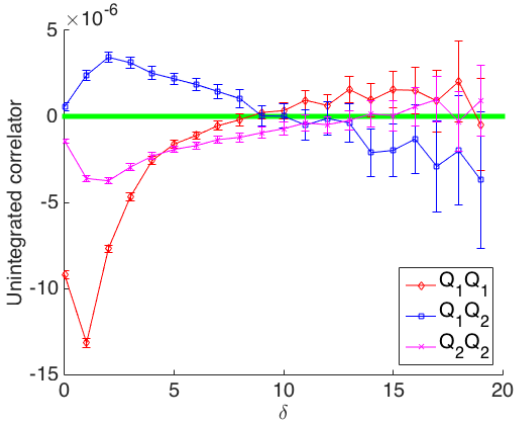
Single-integrated correlators **preliminary**

$$G(\delta) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle e^{(m_K - E_n)\delta} \qquad (13)$$

Unintegrated $\rightarrow \langle 0 | Q_i' | K^0 \rangle = 0$, $\langle \eta | Q_i' | K^0 \rangle = 0 \rightarrow$ Subtract $\langle \pi | Q_i' | K^0 \rangle$





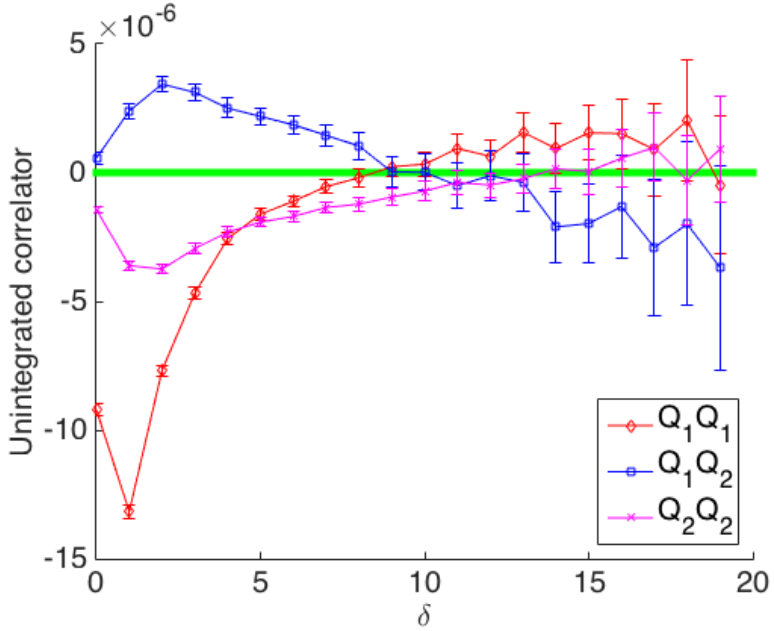


$$Q_i' = Q_i - c_{pi} \bar{s} \gamma_5 d - c_{si} \bar{s} d$$

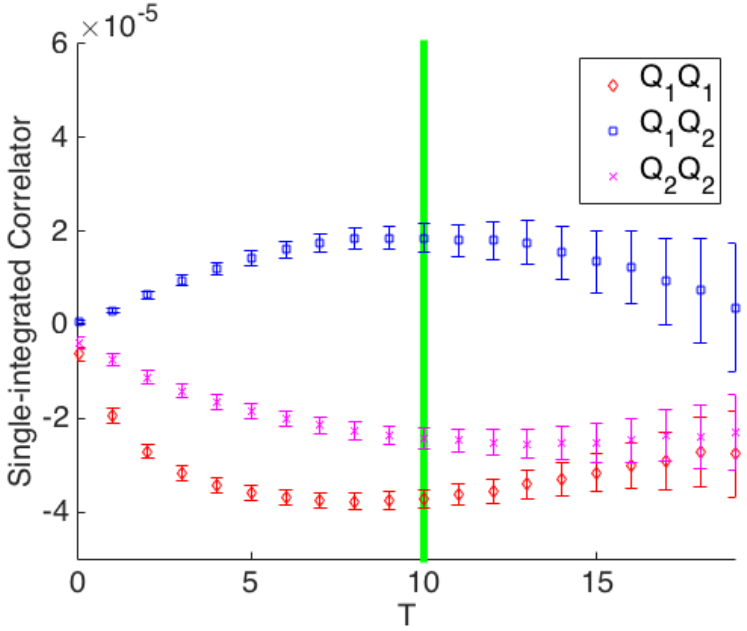
Next step: integrate and obtain Δm_K

Note: Need to add back contributions to Δm_K from subtracted states.

Single-integrated correlators: All diagrams, uncorrelated, **preliminary**



(a) unintegrated results with π subtraction



(b) After integrating to large T, converged

Choosing T=10, as the integration upper limit:

$$\Delta m_K = 7.9(0.7) \times 10^{-12}\text{MeV}$$

Results **preliminary**

- Δm_K values obtained from 2 analysis methods

Method	Double-int	Single-int
$\Delta m_K/10^{-12}$ MeV	8.1(1.2)	7.9(0.7)

- Systematice errors:
  - **Finite-volume corrections**: small compared to statistical errors
    "Effects of finite volume on the $K_L - K_S$ mass difference"
    N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170
    $\Delta m_K(FV) = -0.22(7) \times 10^{-12}$MeV
  - **Discretization effects** are the **largest** source of systematic error:
    - Heavy charm quark, $\sim (m_c a)^2$ gives 25%
    - Another estimate based on HVP calculation is $\sim$ 15%

Conclusion and Outlook

- Our **preliminary** result based on 152 configurations is

$$\Delta m_K = 7.7(0.7)_{stat}(2.0)_{sys} \times 10^{-12} MeV$$

to be compared to the experimental value

$$(\Delta m_K)^{exp} = 3.483(6) \times 10^{-12} MeV$$

- Outlook
  - Better estimate of the discretization error:
    Continue the calculation of Δm_K on Summit:
    - On finer lattice($96^3 \times 192$, $a^{-1} = 2.8$ GeV) $\rightarrow$ smaller $m_c a$.
  - Continue the check of the measurement on lattice and data analysis, though the code was checked by Jianglei, Ziyuan and myself before.