$B_c \longrightarrow B_{s(d)}$ form factors with NRQCD and Heavy-HISQ

Laurence Cooper

DAMTP, University of Cambridge

L.J.Cooper@damtp.cam.ac.uk

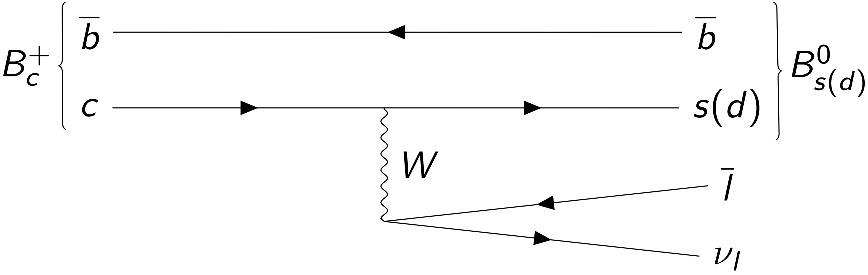
Christine Davies, Judd Harrison, Javad Komijani, Matthew Wingate

17th June 2019

Overview

1. Introduction: decay rates, matrix elements, form factors
2. Lattice Methodology: heavy valence quarks, lattices, PCVC, correlator fitting
3. Results: form factor fitting, Z values, form factor data, physical-continuum fits

Tree-level weak decay of c inside pseudoscalar B_c^+



$0 < q^2 < t_- = (M_{B_c} - M_{B_{s(d)}})^2 \ll 4\ \ [\mathrm{GeV}]^2 \sim M^2_{\mathrm{res}}$.

Similar to $c \longrightarrow s(d)$ in $D \longrightarrow K(\pi)$ (*see B. Chakraborty, 4:30pm Tues*).

Aim to compare our current best approaches for heavy quarks on the lattice.

$B_c \longrightarrow B_s(d)$ form factors

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V|^2}{24\pi^3}\Delta(q^2)^3|f_+(q^2)|^2,$$

where V is the associated CKM matrix element,

$$\Delta(q^2) = |\mathrm{p}_2| = \sqrt{\left(\frac{M_{B_c}^2 + M_{B_{s(d)}}^2 - q^2}{2M_{B_c}}\right)^2 - M_{B_{s(d)}}^2},$$

and f_+ is one of two form factors that parametrise the weak matrix element

$$\langle B_{s(d)}(\mathrm{p}_2)|V^\mu|B_c(\mathrm{p}_1)\rangle = f_0(q^2)\left[\frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2}q^\mu\right] + f_+(q^2)\left[p_2^\mu + p_1^\mu - \frac{M_{B_c}^2 - M_{B_{s(d)}}^2}{q^2}q^\mu\right].$$

Flavour formalisms

HISQ for sea (MILC 2+1+1 configs) and valence l, s and c

- Staggering transformation trivialises spin structure of Fermion matrix
- Systematic elimination of all a^2 errors: discretisation and unphysical taste-changing interactions at tree-level and one-loop

NRQCD for valence b

- Quarks are fairly non-relativistic in B mesons
- Non-relativistic effective theory
- Propagator solves an IVP, cheap to compute

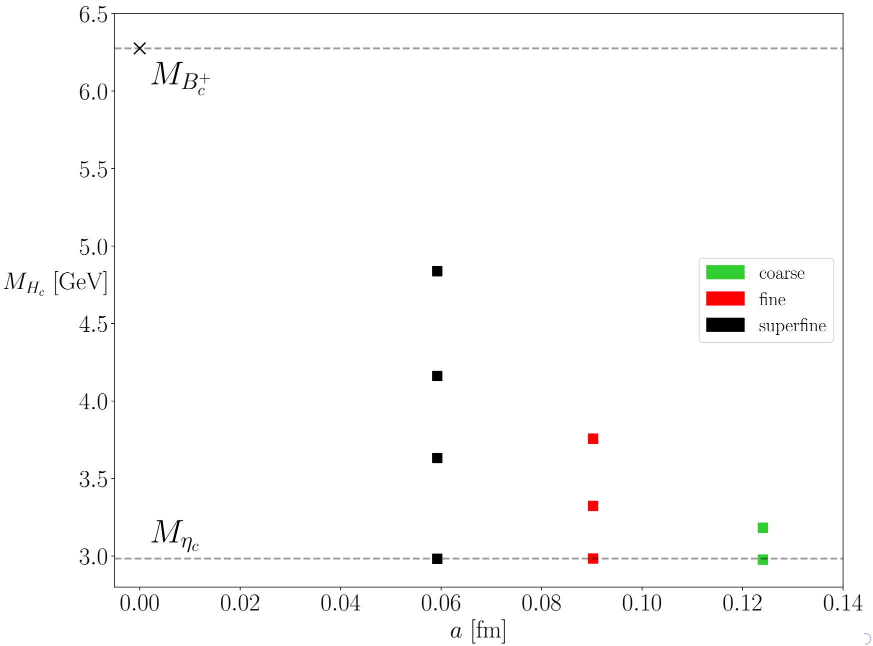
Heavy-HISQ for valence b

- Use masses beyond m_c with finer lattices
- Use HQET to extrapolate mass to m_b
- Use $m_l/m_s = 0.2$ only to keep computations cheap

Lattice parameters

set	handle	w_0/a	$N_x^3 \times N_t$	n_{cfg}	am_l^{sea}	am_s^{sea}	am_c^{sea}
1	very-coarse	1.1119(10)	$16^3 \times 48$	1000	0.013	0.065	0.838
2	very-coarse-physical	1.1367(5)	$32^3 \times 48$	500	0.00235	0.00647	0.831
3	coarse	1.3826(11)	$24^3 \times 64$	1053	0.0102	0.0541	0.635
4	coarse-physical	1.4149(6)	$48^3 \times 64$	1000	0.00184	0.0507	0.628
5	fine	1.9006(20)	$32^3 \times 96$	504	0.0074	0.037	0.440
6	superfine	2.896(6)	$48^3 \times 144$	250	0.0048	0.024	0.286

Table: Sets 1 to 5 are used by the NRQCD calculation, whilst Heavy-HISQ data is obtained on sets 3, 5 and 6.


6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
M_{H_c} [GeV]
$M_{B_c^+}$
M_{η_c}
coarse
fine
superfine
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
a [fm]

PCVC

Continuum Partially Conserved Vector Current (PCVC) Ward identity

$$\partial_\mu V^\mu = (m_c - m_{s(d)})S$$

relates the c to $s(d)$ vector current V^μ and scalar density S.

Using only local lattice operators, need a single q^2 independent renormalisation factor Z satisfying

$$q_\mu \langle B_{s(d)}|V^\mu|B_c\rangle Z = (m_c - m_{s(d)})\langle B_{s(d)}|S|B_c\rangle.$$

Hence, f_0 is solely determined by the scalar density matrix element through

$$f_0(q^2) = \langle B_{s(d)}|S|B_c\rangle \frac{m_c - m_{s(d)}}{M^2_{B_c} - M^2_{B_{s(d)}}}$$

Correlators

$$\langle \mathcal{O}_{B_c}(t)\mathcal{O}^\dagger_{B_c}(0)\rangle = \sum_i b[i]^2 e^{-E_b[i]t} - \sum_i b_o[i]^2(-1)^t e^{-E_{b_o}[i]t}$$

$$\langle \mathcal{O}_{B_{s(d)}}(t)\mathcal{O}^\dagger_{B_{s(d)}}(0)\rangle = \sum_i a[i]^2 e^{-E_a[i]t} - \sum_i a_o[i]^2(-1)^t e^{-E_{a_o}[i]t}$$

$$\begin{aligned}\langle \mathcal{O}_{B_{s(d)}}(T)J(t)\mathcal{O}^\dagger_{B_c}(0)\rangle = &\sum_{i,j} a[i]e^{-E_a[i]t}V_{nn}[i,j]b[j]e^{-E_b[j](T-t)} \\ &- \sum_{i,j}(-1)^{T-t}a[i]e^{-E_a[i]t}V_{no}[i,j]b_o[j]e^{-E_{b_o}[j](T-t)} \\ &- \sum_{i,j}(-1)^t a_o[i]e^{-E_{a_o}[i]t}V_{on}[i,j]b[j]e^{-E_b[j](T-t)} \\ &+ \sum_{i,j}(-1)^T a_o[i]e^{-E_{a_o}[i]t}V_{oo}[i,j]b_o[j]e^{-E_{b_o}[j](T-t)}\end{aligned}$$

Form factor fit functions: NRQCD

$$f(q^2) = P(q^2)\sum_{n=0}^{3} b^{(n)} z'^n$$

$$P(q^2)^{-1} = 1 - \frac{q^2}{M_{\text{res}}^2}$$

$$t_\pm = (M_{B_c} \pm M_{B_{s(d)}})^2$$

$$z' = kz = k\frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

$$\begin{aligned} b^{(0)} = A^{(0)}\Big\{ & 1 + B^{(0)}(am_c/\pi)^2 + C^{(0)}(am_c/\pi)^4 \\ & + \kappa_{(1)}\frac{\delta m_l^{\text{sea}}}{10 m_s^{\text{tuned}}} + \kappa_{(2)}\frac{\delta m_s^{\text{sea}}}{10 m_s^{\text{tuned}}} + \kappa_{(3)}\frac{\delta m_c^{\text{sea}}}{m_c^{\text{tuned}}} \\ & + \kappa_{(4)}\frac{\delta m_s^{\text{val}}}{10 m_s^{\text{tuned}}} + \kappa_{(5)}\frac{\delta m_c^{\text{val}}}{m_c^{\text{tuned}}} + \kappa_{(6)}\frac{\delta m_b^{\text{val}}}{m_b^{\text{tuned}}}\Big\} \\ b^{(n\geq 1)} = A^{(n)}\Big\{ & 1 + B^{(n)}(am_c/\pi)^2 + C^{(n)}(am_c/\pi)^4\Big\}. \end{aligned}$$

Form factor fit functions: Heavy-HISQ

$$f(q^2) = P(q^2) \sum_{n,i,j,k=0}^{3} A^n_{ijk} z'^n \left(\frac{am_c}{\pi}\right)^{2i} \left(\frac{am_h}{\pi}\right)^{2j} \Delta^k_M \mathcal{N}^n_{\text{mis}}$$

where for $k = 0$, $\Delta^k_M = 1$ and for $k \neq 0$

$$\Delta^k_M = \left(\frac{\Lambda_{\text{QCD}}}{M_{H_c}}\right)^k - \left(\frac{\Lambda_{\text{QCD}}}{M_{B_c}}\right)^k$$

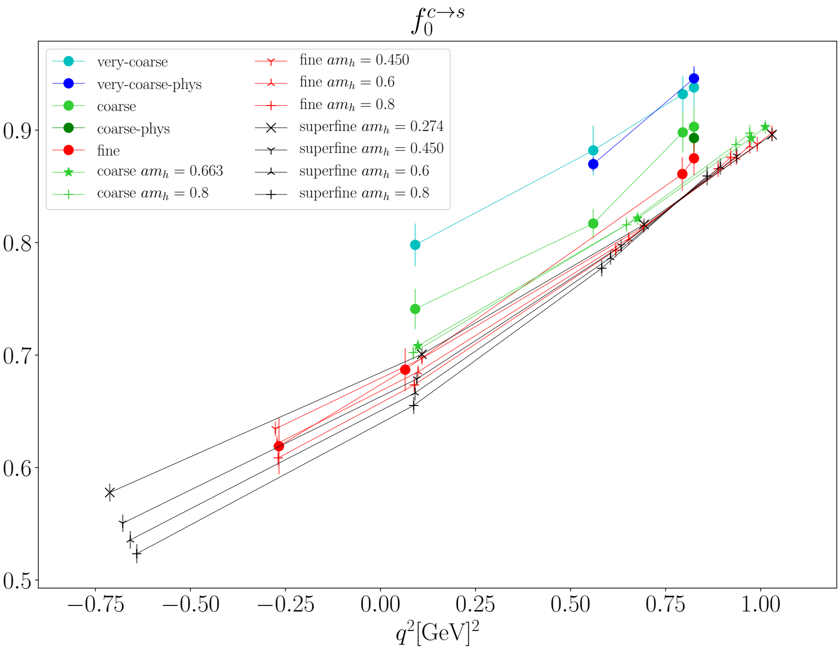
Vector current renormalisation Z

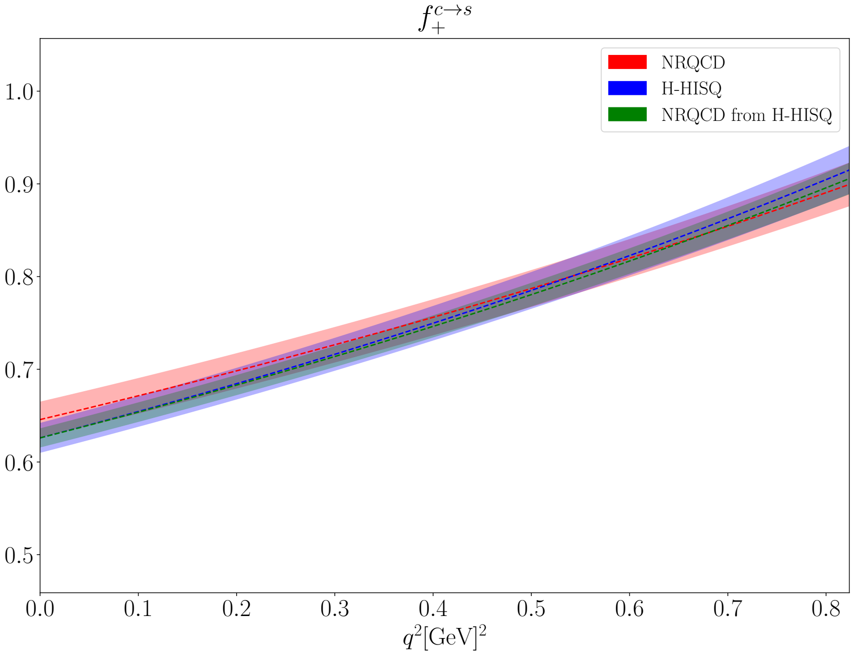
set	handle	$Z_{c\to s}$
1	very-coarse	1.0338(65)
2	very-coarse-physical	1.0325(41)
3	coarse	1.0187(59)
4	coarse-physical	1.0035(67)
5	fine	0.997(11)

Table: $Z_{c\to s}$ from NRQCD

set $\backslash am_h$	0.274	0.450	0.6	0.663	0.8
3	-	-	-	1.0165(99)	1.017(11)
5	-	1.0057(96)	1.004(10)	-	1.003(11)
6	0.9963(91)	0.994(10)	0.993(11)	-	0.992(12)

Table: $Z_{c\to s}$ from Heavy-HISQ


$f_0^{c \to s}$
very-coarse
very-coarse-phys
coarse
coarse-phys
fine
coarse $am_h = 0.663$
coarse $am_h = 0.8$
fine $am_h = 0.450$
fine $am_h = 0.6$
fine $am_h = 0.8$
superfine $am_h = 0.274$
superfine $am_h = 0.450$
superfine $am_h = 0.6$
superfine $am_h = 0.8$
0.5
0.6
0.7
0.8
0.9
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
$q^2[\mathrm{GeV}]^2$



$f_+^{c \to s}$
NRQCD
H-HISQ
NRQCD from H-HISQ
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$q^2[\mathrm{GeV}]^2$

Preliminary results for $|V|^{-2}\Gamma$

- $|V_{cs}|^{-2}\Gamma(B_c \longrightarrow B_s\bar{l}\nu_l) = 3.51(16) \times 10^{-11}$ MeV
- $|V_{cd}|^{-2}\Gamma(B_c \longrightarrow B_d\bar{l}\nu_l) = 2.28(11) \times 10^{-12}$ MeV