Radiative leptonic decays on the lattice

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in collaboration with

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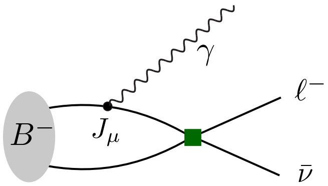
$B^- \to \ell^- \bar{\nu}\gamma$



- Adding a (hard) photon removes the $(m_\ell/m_B)^2$ helicity suppression.

- This is the simplest decay that (for large E_γ) probes the first inverse moment of the B-meson light-cone distribution amplitude,

$$1/\lambda_B = \int_0^\infty \frac{\Phi_{B^+}(\omega)}{\omega}\, \mathrm{d}\omega.$$

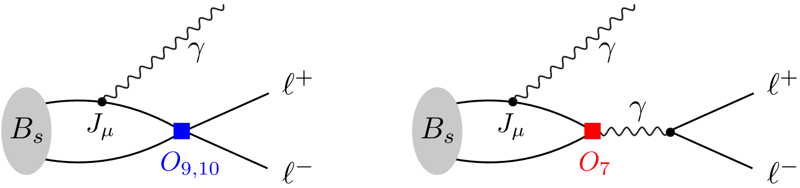
  λ_B is an important input in QCD-factorization predictions for nonleptonic B decays and is poorly known.

  [See, for example, M. Beneke, V. Braun, Y. Ji, Y.-B. Wei, arXiv:1804.04962/JHEP 2018;
  M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, arXiv:hep-ph/9905312/PRL 1999]

- Belle: $\mathcal{B}(B^- \to \ell^- \bar{\nu}\gamma,\ E_\gamma > 1\,\text{GeV}) < 3.0 \times 10^{-6}$ SM: $\mathcal{O}(10^{-6})$

  [arXiv:1810.12976/PRD 2018]

$B_s^0 \to \ell^+\ell^-\gamma$ and $B^0 \to \ell^+\ell^-\gamma$



- Adding a (hard) photon removes the $(m_\ell/m_B)^2$ helicity suppression.

- This decay is sensitive to all operators in the $b \to s\ell^+\ell^-$ weak effective Hamiltonian, including O_9
  ($B_s \to \ell^+\ell^-$ is sensitive to $O_{10,S,P} - O'_{10,S,P}$ only).

- It may be possible to observe $B_s^0 \to \ell^+\ell^-\gamma$ at the LHC
  [F. Dettori, D. Guadagnoli, M. Reboud, arXiv:1610.00629/PLB 2017]

- BaBar: $\mathcal{B}(B^0 \to e^+e^-\gamma) < 1.2 \times 10^{-7}$, $\mathcal{B}(B^0 \to \mu^+\mu^-\gamma) < 1.5 \times 10^{-7}$
  [arXiv:0706.2870/PRD 2008]

Radiative leptonic decays of $D^{\pm}_{(s)}$, $K^{\pm}$, and $\pi^{\pm}$ mesons

- $D_s^+ \to e^+\nu\gamma$: $\mathcal{B}(E_\gamma > 10\,\mathrm{MeV}) < 1.3 \times 10^{-4}$ SM: $\mathcal{O}(10^{-4})$

  [BESIII Collaboration, arXiv:1902.03351]

- $D^+ \to e^+\nu\gamma$: $\mathcal{B}(E_\gamma > 10\,\mathrm{MeV}) < 3.0 \times 10^{-5}$ SM: $\mathcal{O}(10^{-5})$

  [BESIII Collaboration, arXiv:1702.05837/PRD 2017]

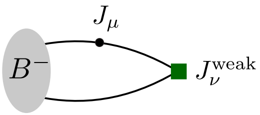
- $K^- \to e^-\bar{\nu}\gamma$, $K^- \to \mu^-\bar{\nu}\gamma$, $\pi^- \to e^-\bar{\nu}\gamma$, $\pi^- \to \mu^-\bar{\nu}\gamma$:

  The partial branching fractions, photon-energy spectra, and angular distributions are known from multiple experiments.

  Contributions from "inner bremsstrahlung," "structure-dependent," and interference terms are distinguished.

  [M. Bychkov, G. D'Ambrosio (Particle Data Group), "Form Factors for Radiative Pion and Kaon Decays," Section 68 of the Review of Particle Physics, 2018]

Hadronic tensor and form factors in Minkowski space



$$J_\mu = \sum_q e_q\, \bar{q}\gamma_\mu q, \qquad J_\nu^{\rm weak} = \bar{u}\gamma_\nu(1-\gamma_5)b$$

$$\begin{aligned} T_{\mu\nu} &= -i\int d^4x\, e^{ip_\gamma\cdot x}\langle 0|\mathrm{T}\left(J_\mu(x)\, J_\nu^{\rm weak}(0)\right)|B^-(\mathbf{p}_B)\rangle \\ &= \epsilon_{\mu\nu\tau\rho}p_\gamma^\tau v^\rho F_V + i[-g_{\mu\nu}(p_\gamma\cdot v) + v_\mu(p_\gamma)_\nu]F_A - i\frac{v_\mu v_\nu}{p_\gamma\cdot v}m_B f_B \\ &\quad +(p_\gamma)_\mu\text{-terms} \end{aligned}$$

$$(p_B = m_B v)$$

[See, for example, M. Beneke, V. Braun, Y. Ji, Y.-B. Wei, arXiv:1804.04962/JHEP 2018]

Hadronic tensor in Minkowski space: time integrals

$$
\begin{aligned}
T^{<}_{\mu\nu} &= -i\int_{-\infty(1-i\epsilon)}^{0} \mathrm{d}t \quad e^{iE_\gamma t}\int d^3x\, e^{-i\mathbf{p}_\gamma\cdot\mathbf{x}}\langle 0|J_\nu^{\text{weak}}(0)\, J_\mu(t,\mathbf{x})|B^-(\mathbf{p}_B)\rangle \\
&= -\sum_n \frac{1}{2E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)}}\frac{1}{E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B - i\epsilon} \\
&\quad \times\langle 0|J_\nu^{\text{weak}}(0)|n(\mathbf{p}_B-\mathbf{p}_\gamma)\rangle\langle n(\mathbf{p}_B-\mathbf{p}_\gamma)|J^\mu(0)|B(\mathbf{p}_B)\rangle
\end{aligned}
$$

(In infinite volume, the sum over n includes an integral over the continuous spectrum of multi-particle states)

Hadronic tensor in Minkowski space: time integrals

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\begin{aligned}
T^{>}_{\mu\nu} &= -i\int_0^{\infty(1-i\epsilon)} \mathrm{d}t \quad e^{iE_\gamma t}\int d^3x\, e^{-i\mathbf{p}_\gamma\cdot\mathbf{x}}\langle 0|J_\mu(t,\mathbf{x})\, J_\nu^{\text{weak}}(0)|B^-(\mathbf{p}_B)\rangle \\
&= \sum_m \frac{1}{2E_{m,\mathbf{p}_\gamma}}\frac{1}{E_\gamma - E_{m,\mathbf{p}_\gamma} - i\epsilon} \\
&\quad \times\langle 0|J^\mu(0)|m(\mathbf{p}_\gamma)\rangle\langle m(\mathbf{p}_\gamma)|J_\nu^{\text{weak}}(0)|B(\mathbf{p}_B)\rangle
\end{aligned}
$$

(In infinite volume, the sum over n includes an integral over the continuous spectrum of multi-particle states)

Three-point function in Euclidean space

Without time integral:

$$C_{\mu\nu}(t, t_B) \quad = \quad \int d^3x \int d^3y \;\; e^{-i\mathbf{p}_\gamma\cdot\mathbf{x}} e^{i\mathbf{p}_B\cdot\mathbf{y}} \left\langle J_\mu(t, \mathbf{x}) \;\; J_\nu^{\text{weak}}(0, \mathbf{0}) \;\; \phi_B^\dagger(t_B, \mathbf{y}) \right\rangle$$

$$\phi_B \sim \bar{u}\gamma_5 b$$

Three-point function in Euclidean space: time integrals

For large negative t_B,

$$
\begin{aligned}
I^{<}_{\mu\nu}(t_B, T) &= \int_{-T}^{0} dt\;\; e^{E_\gamma t}\; C_{\mu\nu}(t, t_B) \\
&= \langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
&\quad \times \sum_n \frac{1}{2E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)}} \frac{1}{E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B} \\
&\quad \times \langle 0|J_\nu^{\mathrm{weak}}(0)|n(\mathbf{p}_B-\mathbf{p}_\gamma)\rangle\langle n(\mathbf{p}_B-\mathbf{p}_\gamma)|J_\mu(0)|B(\mathbf{p}_B)\rangle \\
&\quad \times \left(1 - e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}\right)
\end{aligned}
$$

Three-point function in Euclidean space: time integrals

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&\quad \times \sum_n \frac{1}{2E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)}} \frac{1}{E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B} \\
&\quad \times \langle 0|J_\nu^{\text{weak}}(0)|n(\mathbf{p}_B - \mathbf{p}_\gamma)\rangle \langle n(\mathbf{p}_B - \mathbf{p}_\gamma)|J_\mu(0)|B(\mathbf{p}_B)\rangle \\
&\quad \times \left(1 - e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}\right)
\end{aligned}$$

The unwanted exponential $e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}$ goes to zero for large T if $E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} > E_B$.

Three-point function in Euclidean space: time integrals

For large negative t_B,

$$\begin{aligned}
I^{<}_{\mu\nu}(t_B, T) &= \int_{-T}^{0} dt \;\, e^{E_\gamma t}\, C_{\mu\nu}(t, t_B) \\
&= \langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
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The unwanted exponential $e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}$ goes to zero for large T if $E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} > E_B$.

Because the states $|n(\mathbf{p}_B-\mathbf{p}_\gamma)\rangle$ have the same quark-flavor quantum numbers as the B meson, we have $E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} \geq E_{B,(\mathbf{p}_B-\mathbf{p}_\gamma)} = \sqrt{m_B^2 + (\mathbf{p}_B-\mathbf{p}_\gamma)^2}$.

Three-point function in Euclidean space: time integrals

For large negative t_B,

$$
\begin{aligned}
I_{\mu\nu}^{<}(t_B, T) &= \int_{-T}^{0} dt\ e^{E_\gamma t}\, C_{\mu\nu}(t, t_B) \\
&= \langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
&\quad \times \sum_n \frac{1}{2E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)}} \frac{1}{E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B} \\
&\quad \times \langle 0|J_\nu^{\text{weak}}(0)|n(\mathbf{p}_B-\mathbf{p}_\gamma)\rangle\langle n(\mathbf{p}_B-\mathbf{p}_\gamma)|J_\mu(0)|B(\mathbf{p}_B)\rangle \\
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$$

The unwanted exponential $e^{-(E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} - E_B)T}$ goes to zero for large T if $E_\gamma + E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} > E_B$.

Because the states $|n(\mathbf{p}_B - \mathbf{p}_\gamma)\rangle$ have the same quark-flavor quantum numbers as the B meson, we have $E_{n,(\mathbf{p}_B-\mathbf{p}_\gamma)} \geq E_{B,(\mathbf{p}_B-\mathbf{p}_\gamma)} = \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2}$.

The inequality becomes $\sqrt{\mathbf{p}_\gamma^2} + \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2} > \sqrt{m_B^2 + \mathbf{p}_B^2}$.

This is in fact always satisfied (as long as $\mathbf{p}_\gamma \neq 0$).

Three-point function in Euclidean space: time integrals

For large negative t_B,

$$
\begin{aligned}
I^{>}_{\mu\nu}(t_B, T) &= \int_0^T dt\; e^{E_\gamma t}\, C_{\mu\nu}(t, t_B) \\
&= -\langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
&\quad \times \sum_n \frac{1}{2E_{m,\mathbf{p}_\gamma}} \frac{1}{E_\gamma - E_{m,\mathbf{p}_\gamma}} \\
&\quad \times \langle 0|J_\mu(0)|m(\mathbf{p}_\gamma)\rangle \langle m(\mathbf{p}_\gamma)|J_\nu^{\mathrm{weak}}(0)|B(\mathbf{p}_B)\rangle \\
&\quad \times \left(1 - e^{(E_\gamma - E_{m,\mathbf{p}_\gamma})T}\right)
\end{aligned}
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$$
\begin{aligned}
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&= -\langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle \frac{1}{2E_B} e^{E_B t_B} \\
&\quad \times \sum_n \frac{1}{2E_{m,\mathbf{p}_\gamma}} \frac{1}{E_\gamma - E_{m,\mathbf{p}_\gamma}} \\
&\quad \times \langle 0|J_\mu(0)|m(\mathbf{p}_\gamma)\rangle \langle m(\mathbf{p}_\gamma)|J_\nu^{\text{weak}}(0)|B(\mathbf{p}_B)\rangle \\
&\quad \times \left(1 - e^{(E_\gamma - E_{m,\mathbf{p}_\gamma})T}\right)
\end{aligned}
$$

The unwanted exponential $e^{(E_\gamma - E_{m,\mathbf{p}_\gamma})T}$ goes to zero for large T if $E_{m,\mathbf{p}_\gamma} > E_\gamma$.

Because the states $|m(\mathbf{p}_\gamma)\rangle$ have a nonzero mass, this is always satisfied.

In summary, for $\mathbf{p}_\gamma \neq 0$,

$$T_{\mu\nu} = -\lim_{T\to\infty}\lim_{t_B\to-\infty}\frac{2E_B\, e^{-E_B t_B}}{\langle B(\mathbf{p}_B)|\phi_B^\dagger(0)|0\rangle} I_{\mu\nu}(t_B, T).$$

Parameters of our initial runs

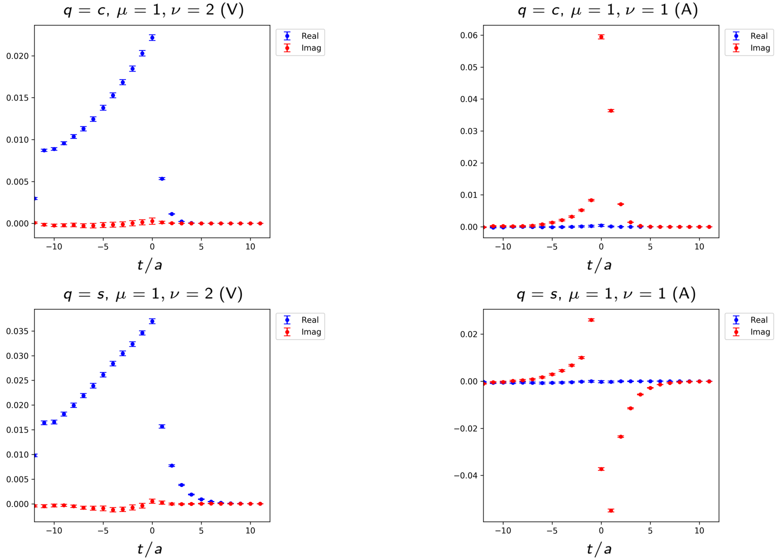
We initially consider $D_s^+ \to \ell^+ \nu \gamma$ and $K^- \to \ell^- \bar{\nu} \gamma$ decays, with the following setup:

- $\mathbb{Z}_2$ random-wall source at location of weak current
- Disconnected diagrams neglected
- RBC/UKQCD ensembles, $24^3 \times 64$, $a \approx 0.11$ fm, $m_\pi \approx 340$ MeV and $48^3 \times 96$, $a \approx 0.11$ fm, $m_\pi \approx 140$ MeV
- Up/down/strange valence quarks: same domain-wall action as sea quarks
- Charm valence quarks: Möbius domain-wall with "stout" smearing
- "Mostly nonperturbative" renormalization
- $\mathbf{p}_{K/D_s} = 0$, $\mathbf{p}_\gamma^2 \in \{1, 2, 3, 4, 5\} \left(\frac{2\pi}{L}\right)^2$
- All-mode averaging with 16 sloppy and 1 exact samples per configuration

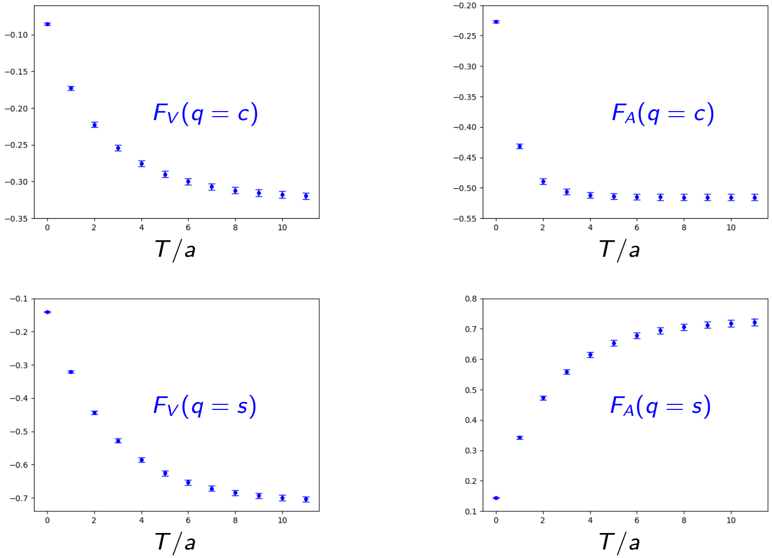
I will show preliminary results from only 25 configurations of the $24^3 \times 64$ ensemble.

$D_s^+ \to \ell^+ \nu \gamma$ three-point functions: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -12$

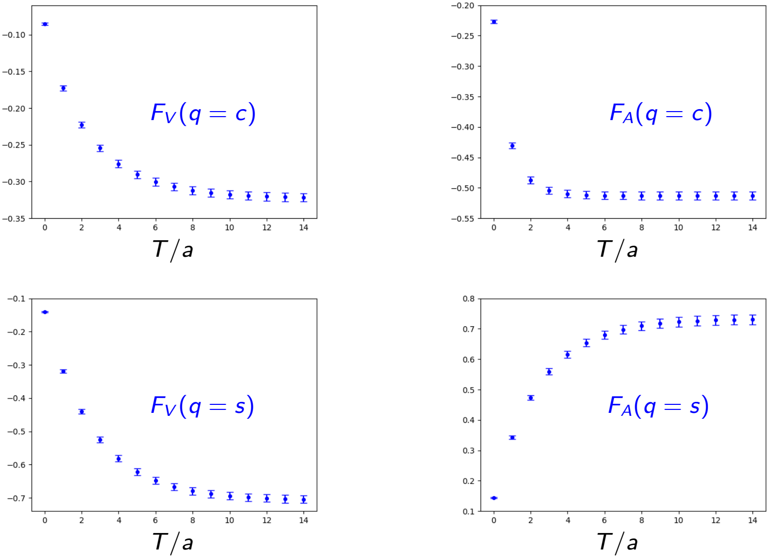
The following plots show $-\dfrac{2\,E_{D_s}\,e^{-E_{D_s} t_{D_s}}}{\langle D_s(\mathbf{p}_{D_s})|\phi^\dagger_{D_s}(0)|0\rangle} C_{\mu\nu}(t, t_{D_s})$ as a function of t.



$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -12$



$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -15$



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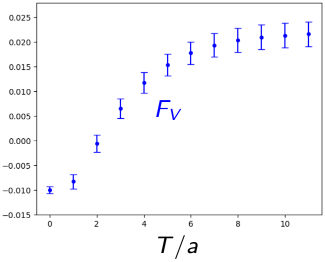
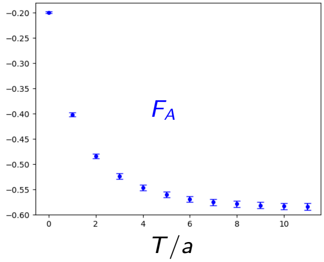
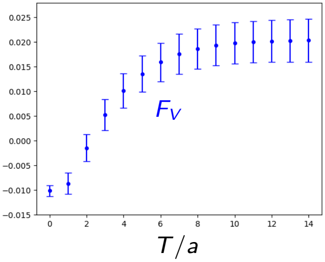
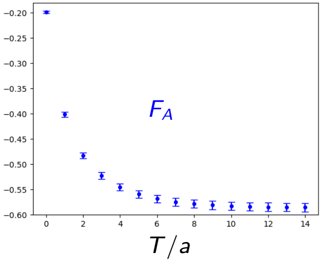




$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -15$





$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -12$

Recall

$$\begin{aligned} T_{\mu\nu} &= \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i[-g_{\mu\nu}(p_\gamma \cdot v) + v_\mu (p_\gamma)_\nu] F_A - i \frac{v_\mu v_\nu}{p_\gamma \cdot v} m_{D_s} f_{D_s} \\ &\quad + (p_\gamma)_\mu\text{-terms} \end{aligned}$$

$\longrightarrow$ also extract f_{D_s} as a cross-check

$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -12$

Recall

$$T_{\mu\nu} = \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i[-g_{\mu\nu}(p_\gamma \cdot v) + v_\mu (p_\gamma)_\nu] F_A - i \frac{v_\mu v_\nu}{p_\gamma \cdot v} m_{D_s} f_{D_s} + (p_\gamma)_\mu\text{-terms}$$

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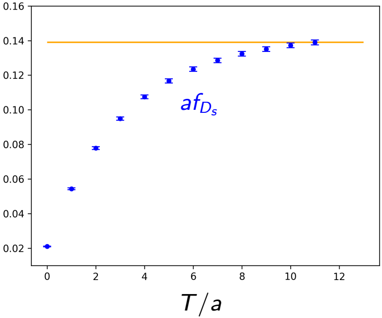


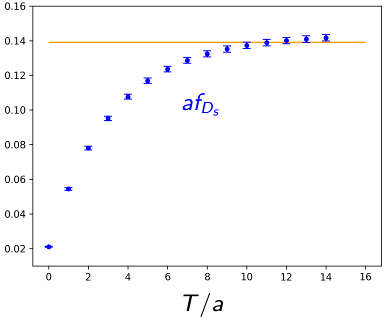
Yellow line = FLAG 2019 average [arXiv:1902.08191]

$D_s^+ \to \ell^+\nu\gamma$ form factors vs T: $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$, $t_{D_s}/a = -15$

Recall

$$T_{\mu\nu} = \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i[-g_{\mu\nu}(p_\gamma \cdot v) + v_\mu (p_\gamma)_\nu] F_A - i \frac{v_\mu v_\nu}{p_\gamma \cdot v} m_{D_s} f_{D_s}$$
$$+(p_\gamma)_\mu\text{-terms}$$

$\longrightarrow$ also extract f_{D_s} as a cross-check



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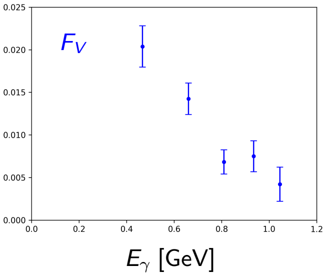
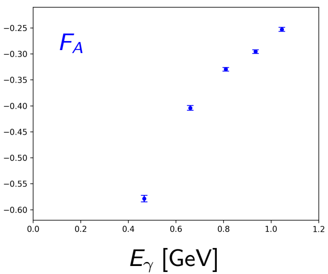
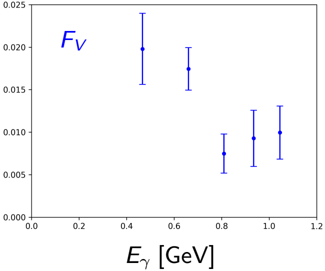
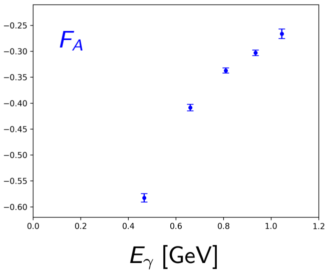
$D_s^+ \to \ell^+\nu\gamma$ form factors vs E_γ: $t_{D_s}/a = -12$, $T/a = 8$



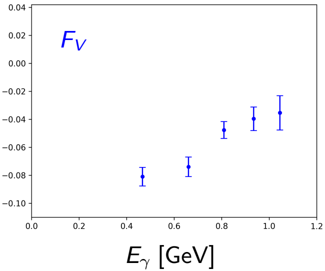


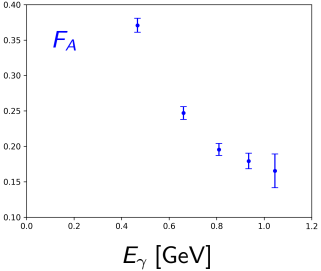
$D_s^+ \to \ell^+\nu\gamma$ form factors vs E_γ: $t_{D_s}/a = -15$, $T/a = 10$



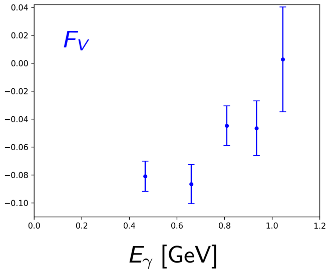


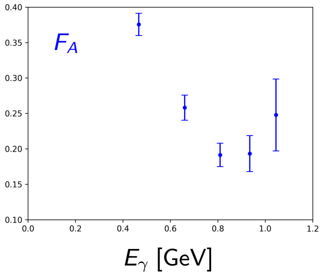
$K^- \to \ell^- \bar{\nu}\gamma$ form factors vs E_γ: $t_K/a = -12,\ T/a = 8$





$K^- \to \ell^- \bar{\nu}\gamma$ form factors vs E_γ: $t_K/a = -15,\ T/a = 10$





Conclusions and Outlook

- Radiative leptonic decays can be calculated on the lattice. We have preliminary results for $D_s^+ \to \ell^+ \nu \gamma$ and $K^- \to \ell^- \bar{\nu} \gamma$.

- For $K^- \to \ell^- \bar{\nu} \gamma$, we need to reach lower photon energies. We are investigating moving frames (i.e., nonzero $\mathbf{p}_K$) and runs with larger volume.

- To study the $B_{(s)}$ radiative leptonic decays with the domain-wall action for the heavy quark, we will need to extrapolate in the mass.

  We are also considering the "RHQ" action (anisotropic clover), but the problem is that this action is only on-shell improved.