$B \to D^{(*)} \ell \nu$ form factors from lattice QCD with relativistic heavy quarks

JLQCD Collaboration

T. Kaneko, Y. Aoki, G. Bailas, B. Colquhoun,

H. Fukaya, S. Hashimoto, J. Koponen

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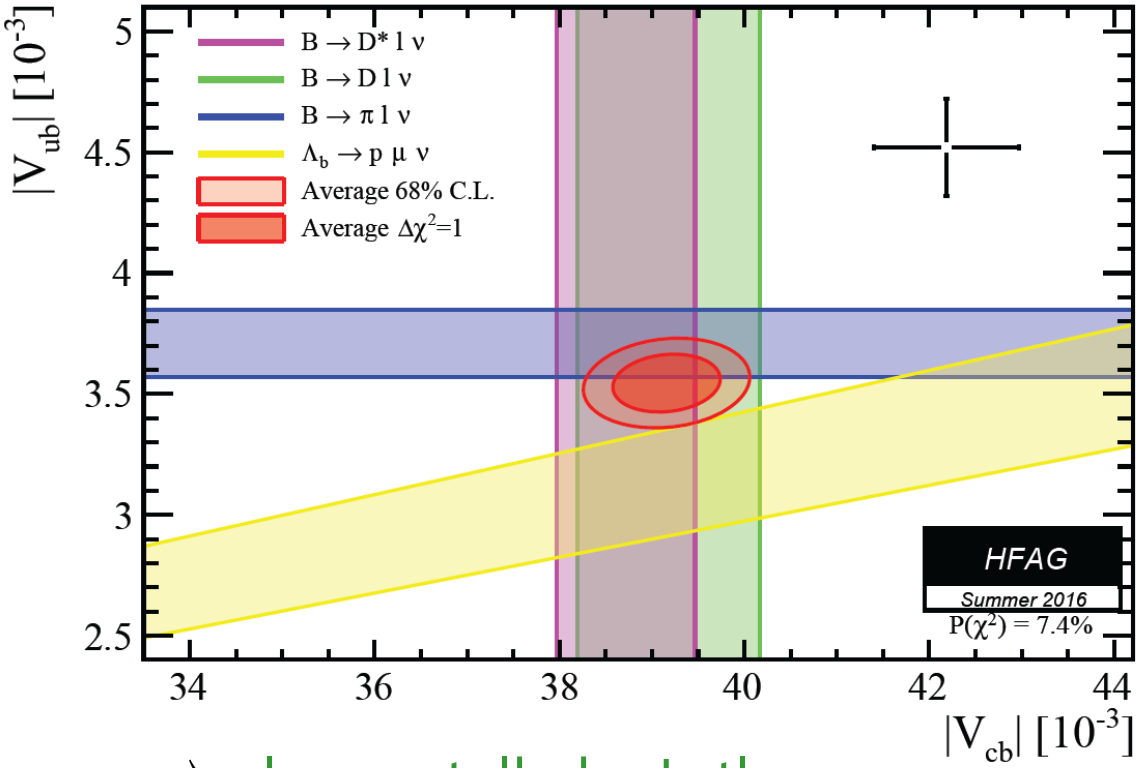
introduction

$|V_{cb}|$ tension

HFLAV'16



$\Delta|V_{cb}| \sim 3\sigma$, 6%

to be understood towards precision NP search in the Belle II era

⇒ plenary talk by Lytle

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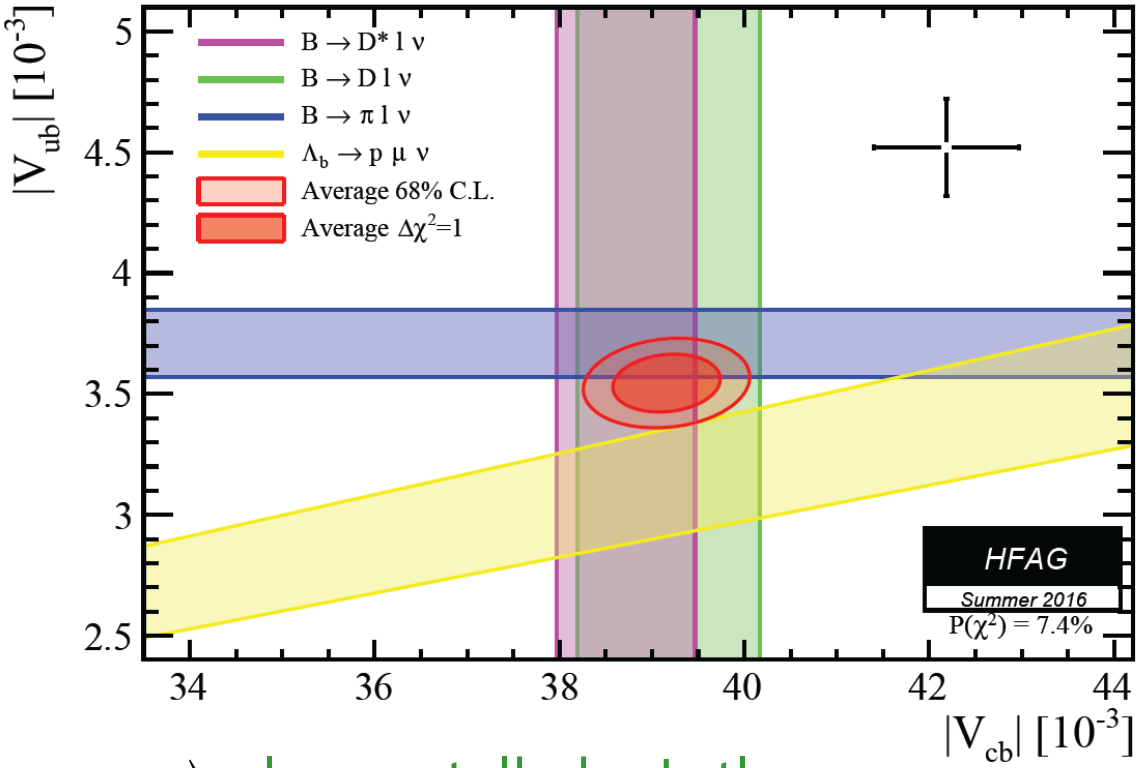

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tension among $B \to D^{(*)}\ell\nu$ analyses?

- Belle tagged / untagged
- form factors (FFs)

  model indep. vs HQS

⇔ FFs @ non-zero recoils

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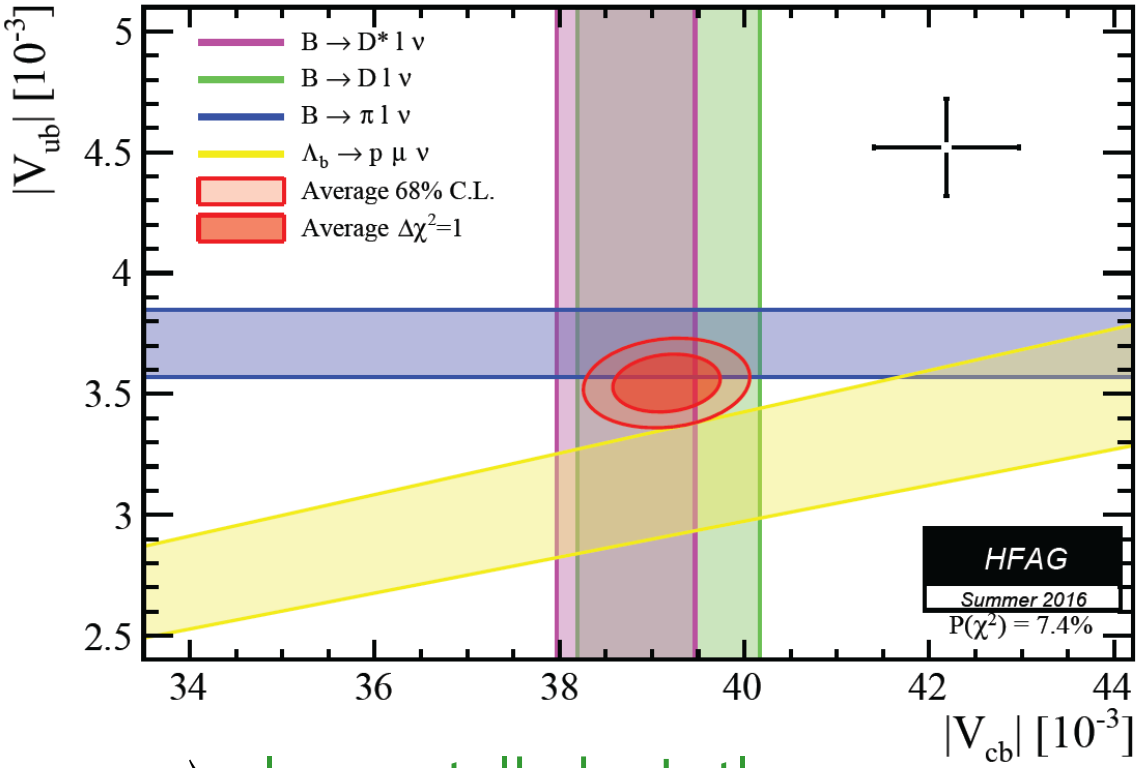

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this talk : JLQCD's calculation of $B \to D^{(*)}\ell\nu$ form factors

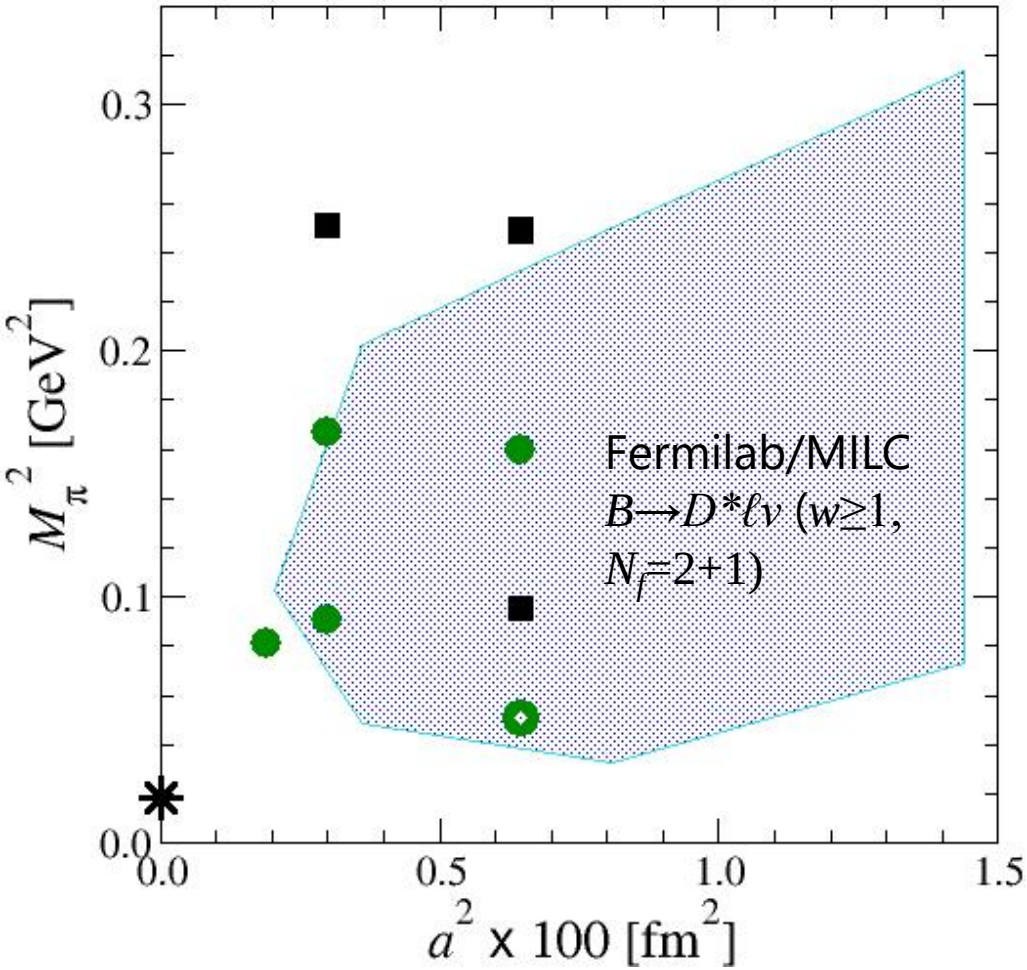
simulation setup

w/ good chiral symmetry

Möbius domain-wall quarks

good chiral symmetry

- simple renormalization
- no O(a) errors



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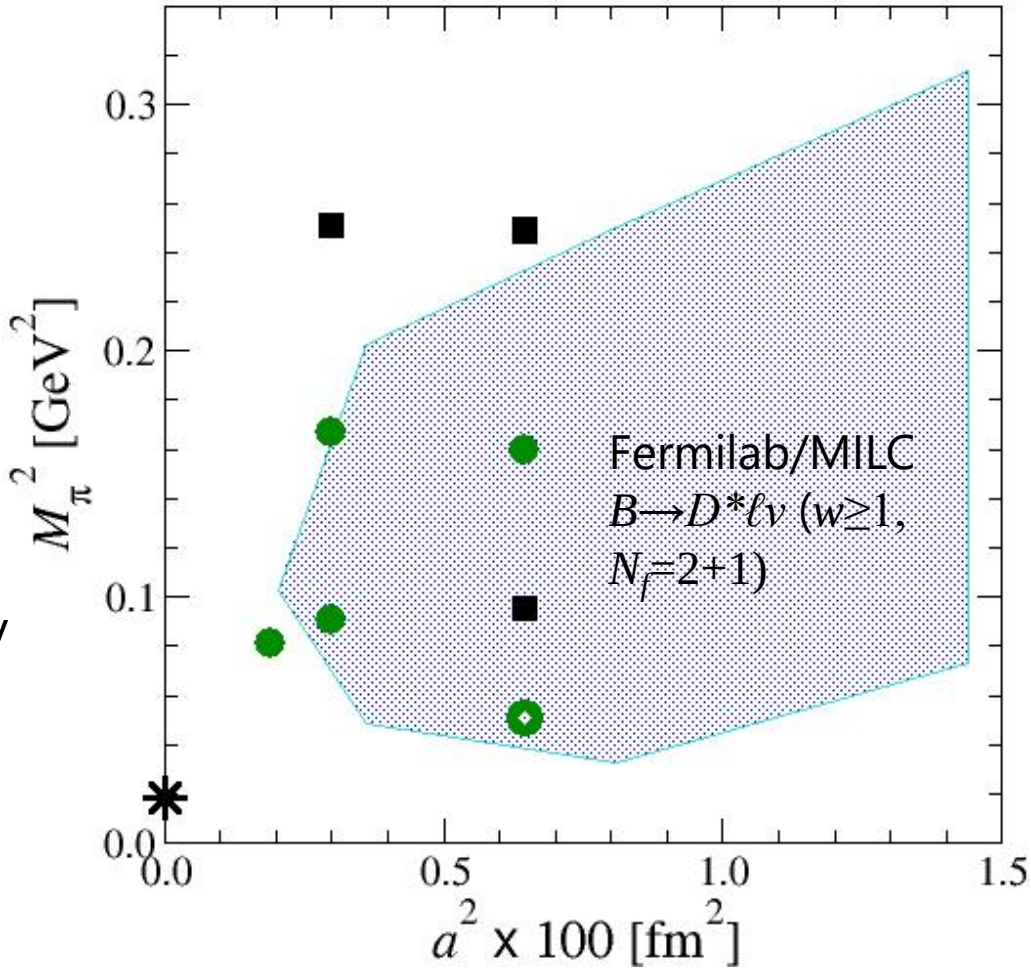
simulation parameters

- a^{-1} ~ 2.5, 3.6, 4.5 GeV
- M_π ~ 230, 300, 400, 500 MeV
- 5,000 HMC traj. for each
- $M_\pi L \geq 4$
- M_π ~ 230 MeV: on-going



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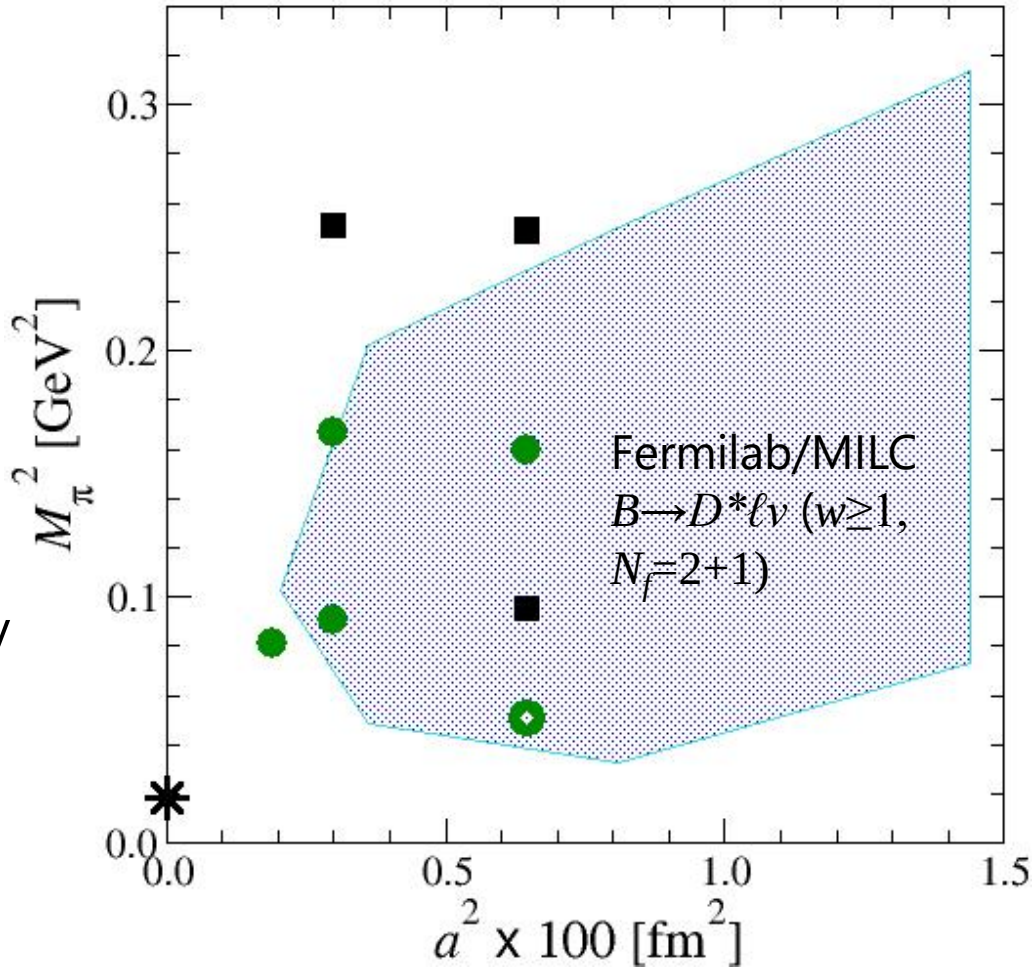
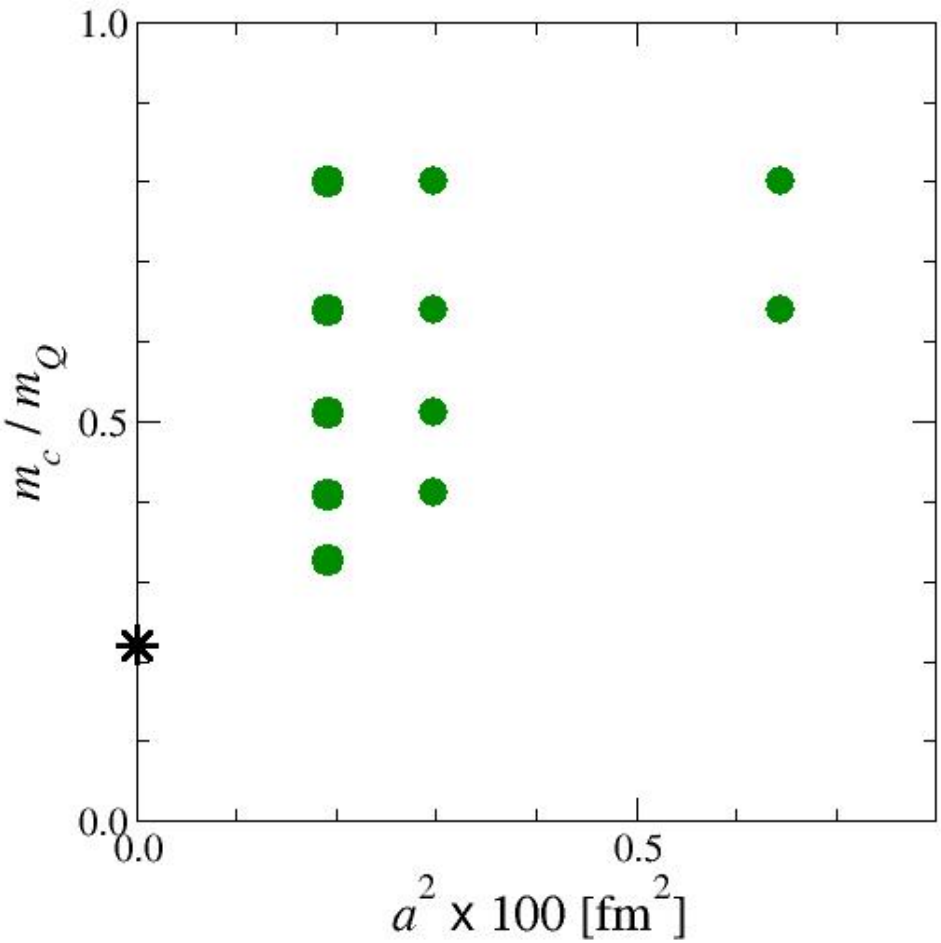


⇒ this talk: preliminary results

simulation setup

fully relativistic lattice QCD



w/ Möbius DW heavy quarks

- simple renormalization
- $m_Q < m_b$ ⇒ need extrapolation

  m_Q / m_c = 1.25, 1.25^2, …

  and $m_Q < 0.8\ a^{-1}$

simulation setup

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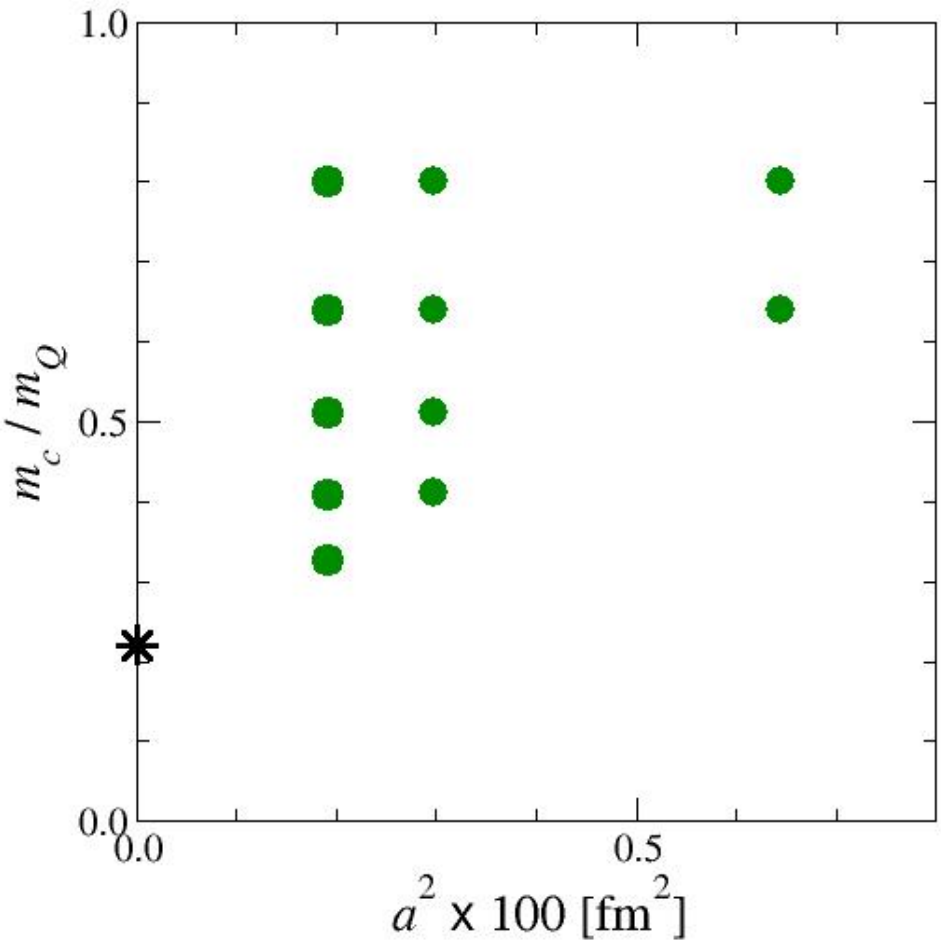

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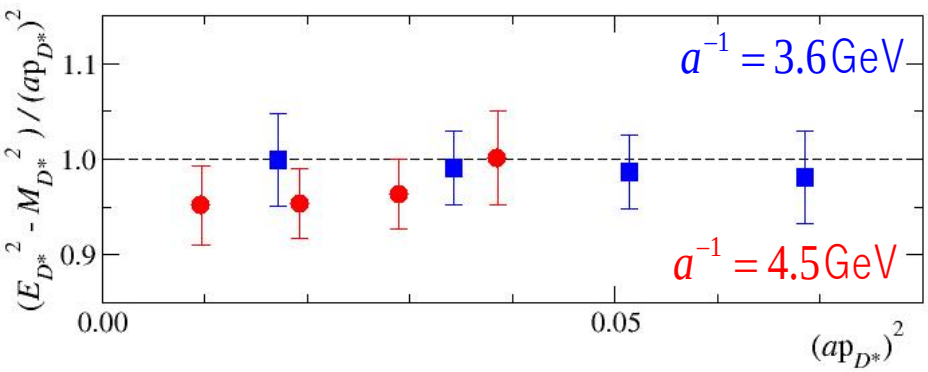

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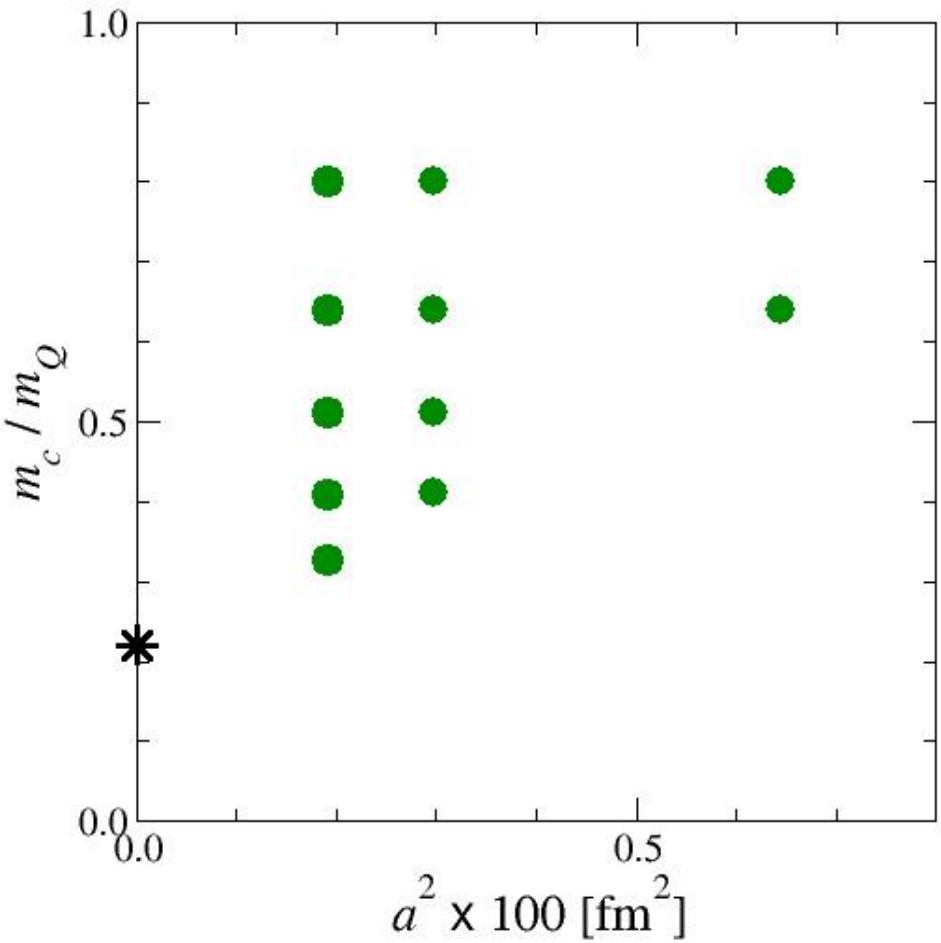

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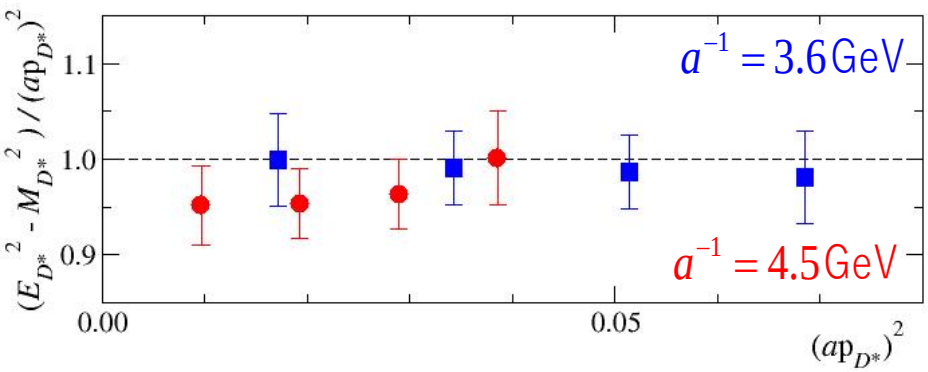


independent calculation w/ (very) different systematics

$B \to D^{(*)} \ell \nu$ form factors

In the SM

$$\left\langle D(p') \middle| V_\mu \middle| B(p) \right\rangle = (v+v')_\mu \, h_+(w) + (v-v')_\mu \, h_-(w)$$

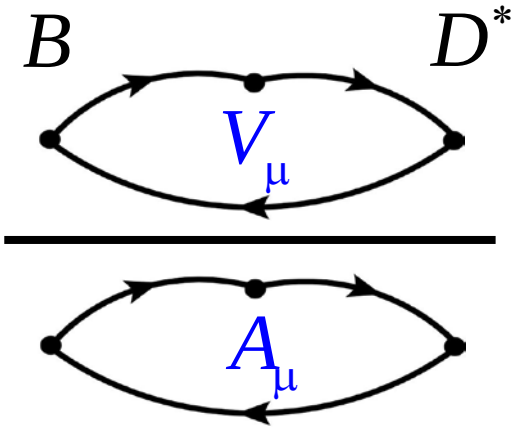
$$\left\langle D^*(p',\varepsilon') \middle| V_\mu \middle| B(p) \right\rangle = i\varepsilon_{\mu\nu\rho\sigma} \varepsilon'^{*\nu} v'^{\rho} v^{\sigma} h_V(w)$$

$$\left\langle D^*(p',\varepsilon') \middle| A_\mu \middle| B(p) \right\rangle = \varepsilon'^*_\mu (1+w) h_{A_1}(w)$$

$$- \varepsilon'^* v \left\{ v_\mu h_{A_2}(w) + v_\mu h_{A_2}(w) \right\}$$

$$v = p/M_B, \;\; v' = p'/M_{D^{(*)}}, \; w = vv' \geq 1$$

ratio method (Hashimoto et al. '99)



$$= \frac{\left\langle D^* \left| V_\mu^{(\mathrm{lat})} \right| B \right\rangle}{\left\langle D^* \left| A_\mu^{(\mathrm{lat})} \right| B \right\rangle} \rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

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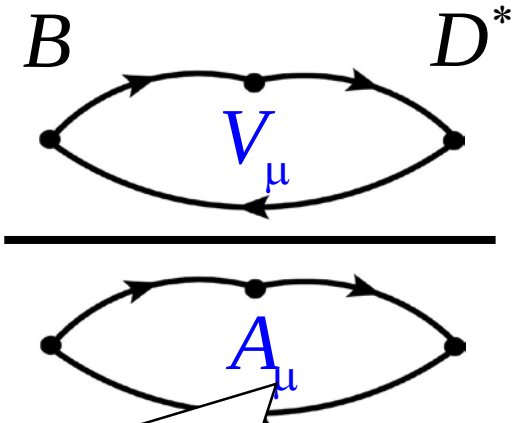

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$\left\langle B \,|\, O_B^\dagger \right\rangle$, $\exp\left[-M_B \Delta t\right]$, $\cdots$ cancel

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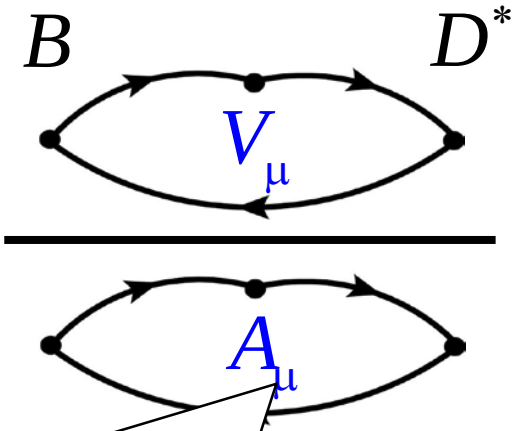

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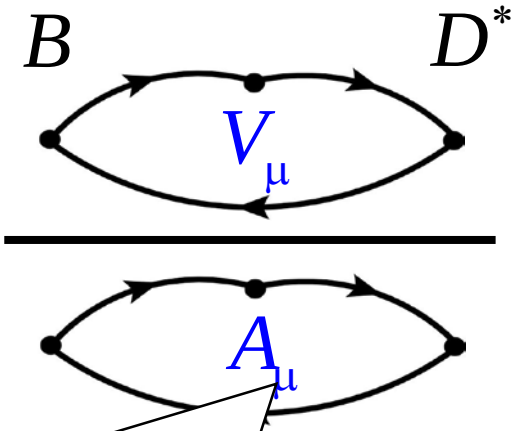

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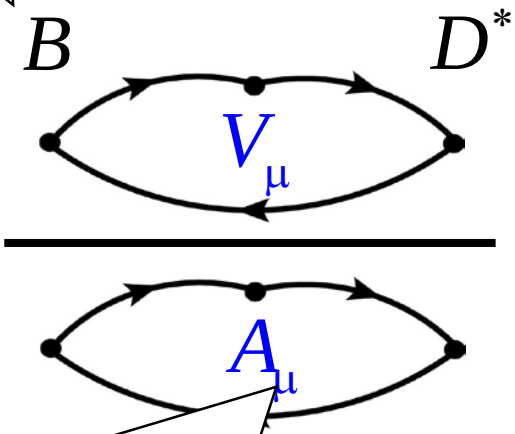
can calculate SM FFs w/o explicit renormalization

$$\left\{h_{A_2}(w), h_{A3}(w), h_V(w)\right\} \Big/ h_{A_1}(w), \quad h_{A_1}(w) \Big/ h_{A_1}(1), \quad h_{A_1}(1) \Big/ \sqrt{F_B^{\mathrm{EM}}(1) F_{D^*}^{\mathrm{EM}}(1)}$$

ratio method (Hashimoto et al. '99)

$\mathbf{p} = \mathbf{0}$



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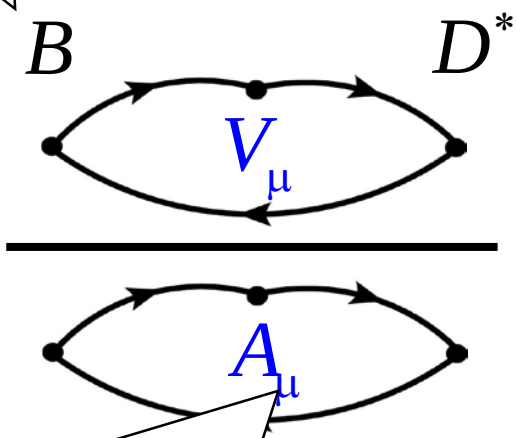
$\mathbf{p} = \mathbf{0}$

physical m_c, $|\mathbf{p}'|^2 = 0, 1, 2, 3, 4$ in units of $(2\pi/L)^2$



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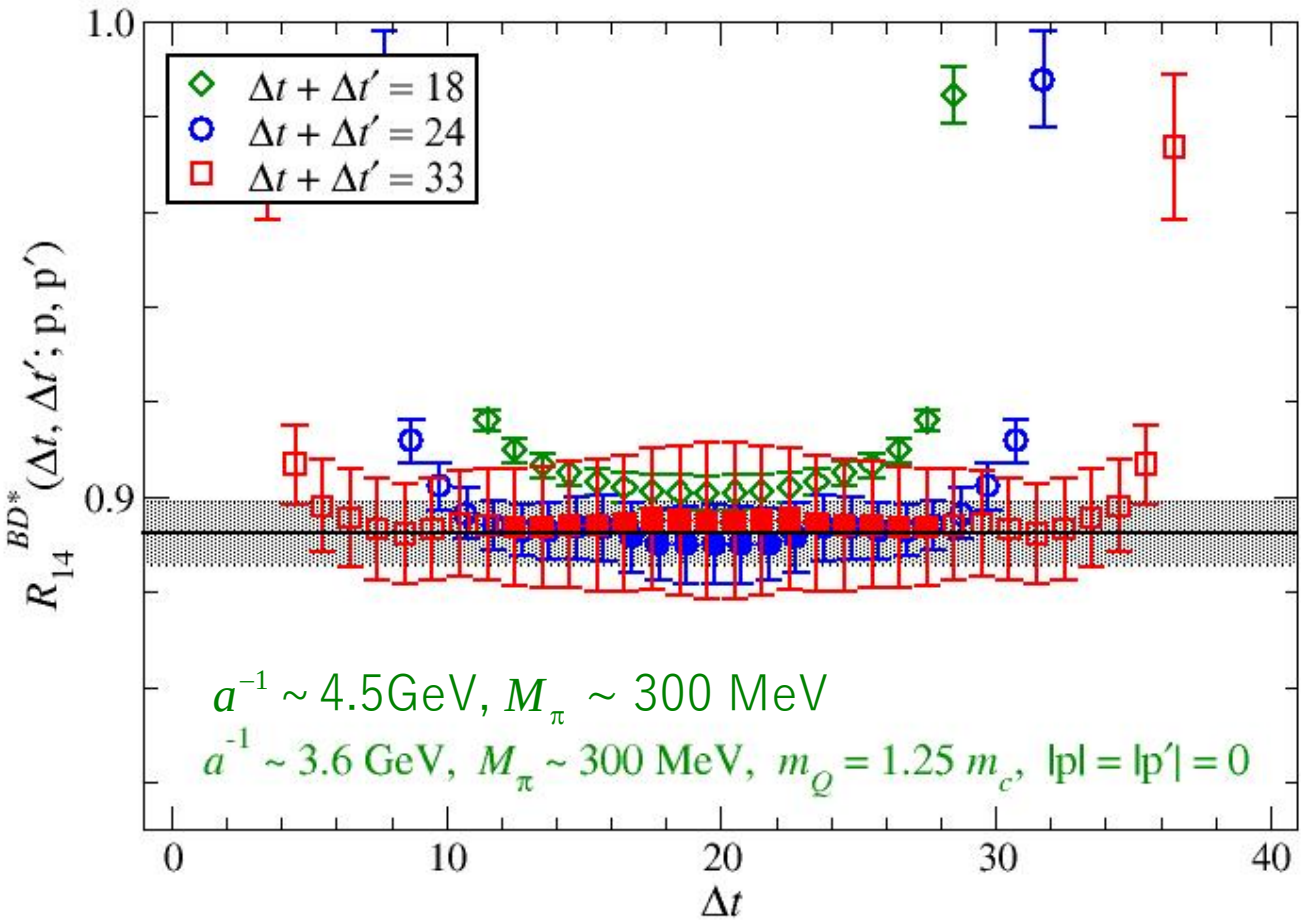
excited state contamination



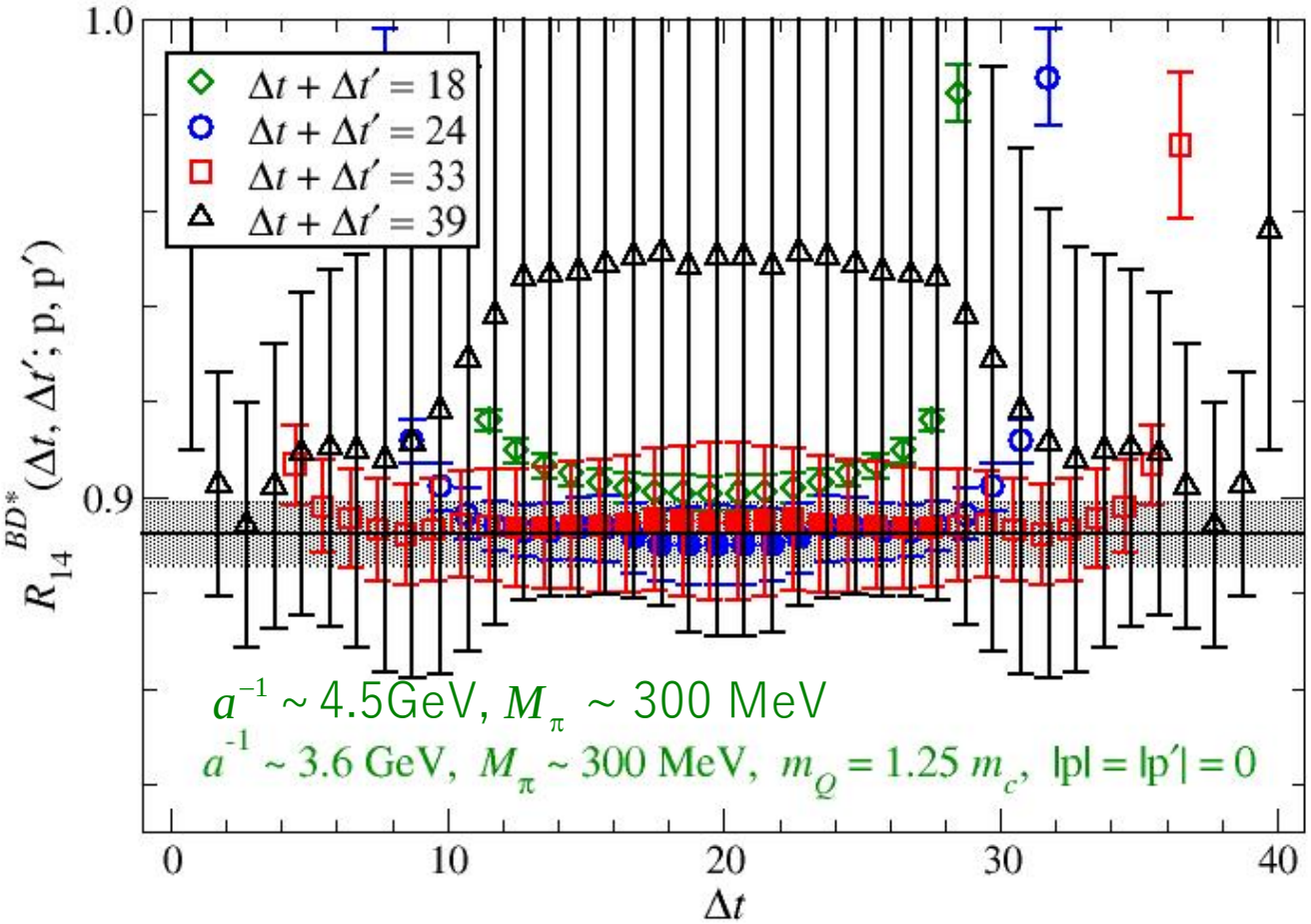
sequential source

⇒ source-sink separation $\Delta t+\Delta t'$ fixed

larger $\Delta t+\Delta t'$

- smaller excited contamination
- poorer statistical accuracy

excited state contamination



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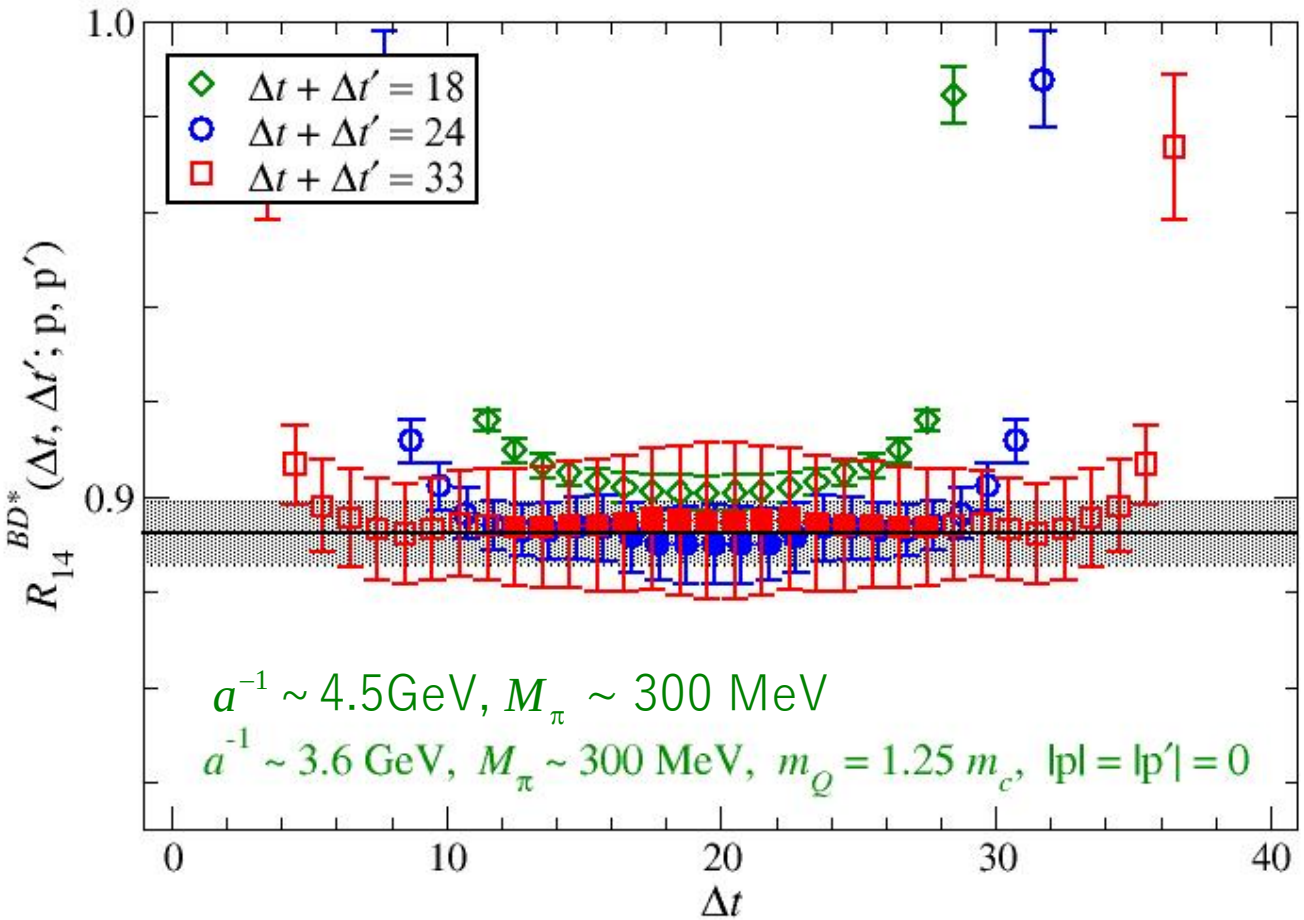

sequential source

⇒ source-sink separation $\Delta t+\Delta t'$ fixed

larger $\Delta t+\Delta t'$

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- poorer statistical accuracy

4 values of $\Delta t+\Delta t'$ ⇒ confirm ground state saturation

(0.7 – 2.0 fm) ⇒ simultaneous fit to extract FFs

continuum + chiral extrapolation

simple form based on NLO HMChPT (Randall-Wise '92, Savage'01)

$$h_{A1}(w) = 1 + a(m_c) + b_{\log}\bar{F}_{\log}(M_\pi, \Delta_c, \Lambda_\chi)$$
$$+ c_\pi M_\pi^2 + c_{\eta s} M_{\eta s}^2 + c_w (w-1) + \frac{c_b}{m_b} + c_a a^2$$
$$+ d_w (w-1)^2 + \frac{d_b}{m_b^2} + \cdots$$

$$b_{\log} = \frac{g_{D^*D\pi}^2}{16\pi^2 f^2}\Delta_c^2 b_\pi$$

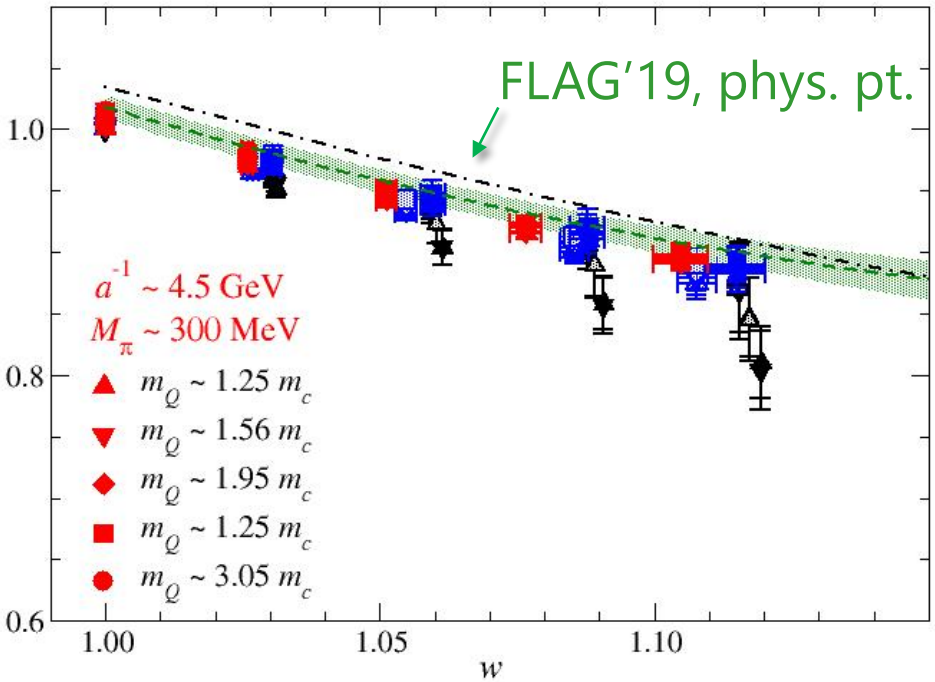
$g_{D^*D\pi}$ ~ 0.53(8) (Fermilab/MILC '14 ← Becirevic-Safilippo'13, Can et al. '12, Detmold et al. '12)

$\Delta_c = M_{D^*} - M_D$ = expt'

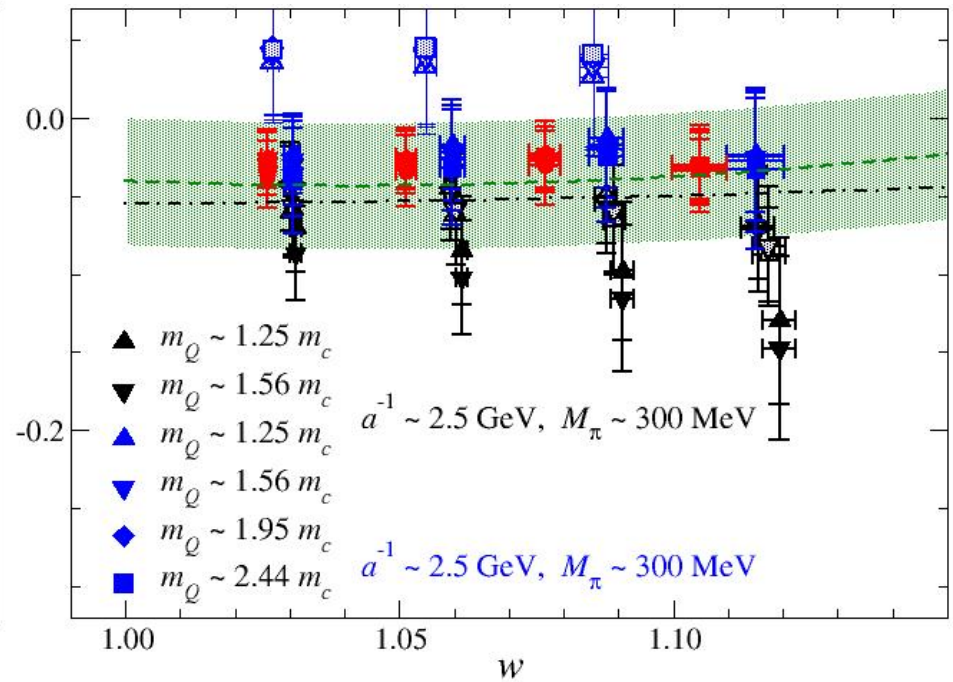
preliminary analysis : statistical error only

$B \to D\ell\nu$ form factors

h_+ vs w



h_- vs w



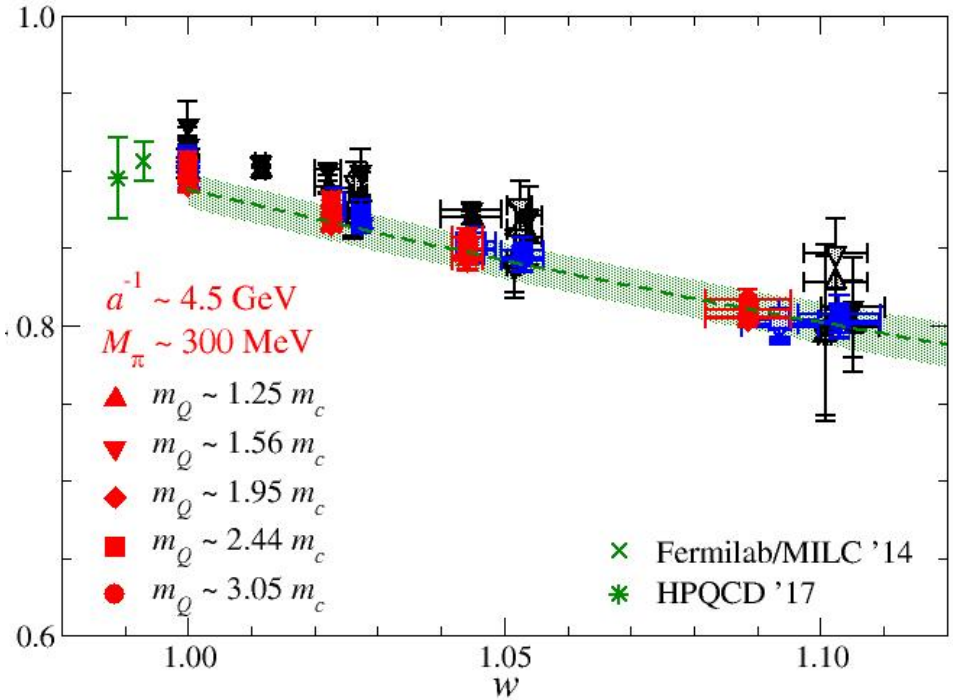
- mildly depend on a, M_π, m_s, m_Q $\Rightarrow$ $\geq$ 50% error except c, c_w, c_b (h_+)

  $\Rightarrow$ in reasonable agreement w/ FLAG4 (Aoki et al. '19)

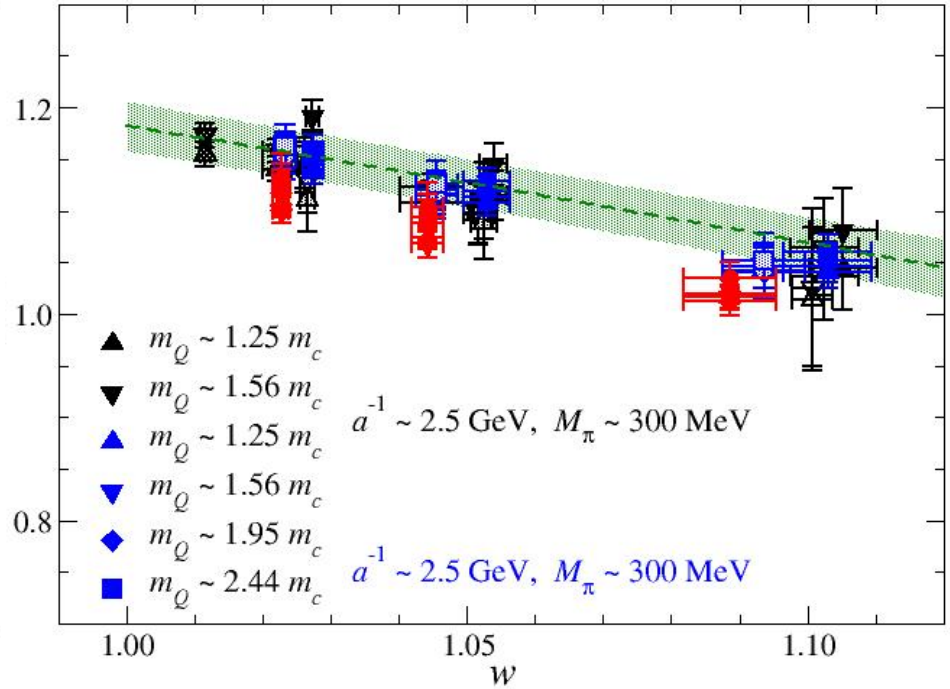
- typical statistical accuracy: $\Delta h_+ \leq$ 1 - 3%, $\Delta h_- \geq$ 50 %

$B \to D^* \ell \nu$ form factors

h_{A1} vs w



h_V vs w

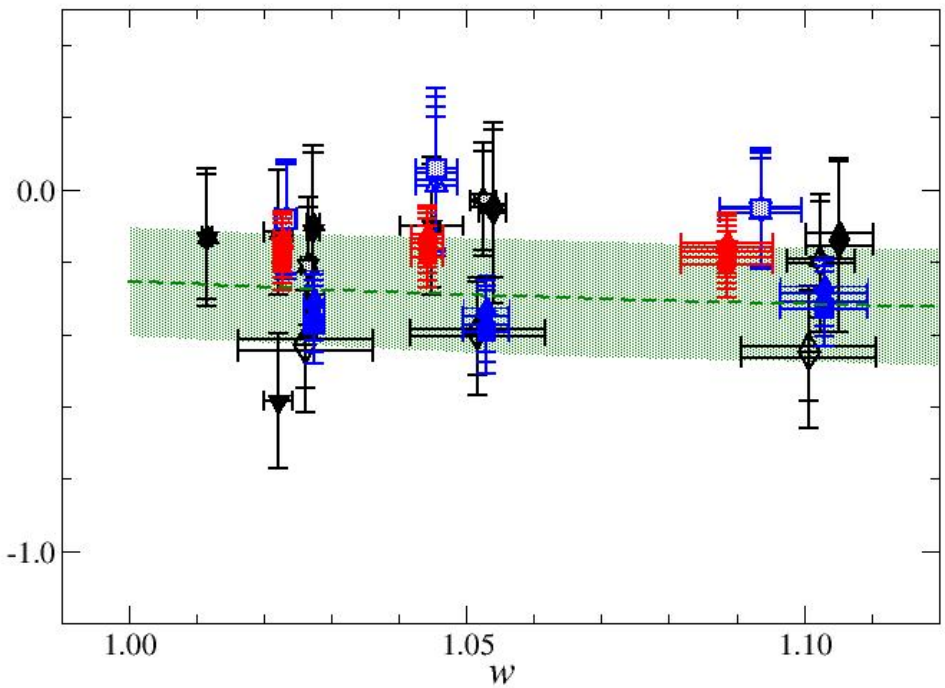


- mild a, M_π, m_s, m_Q dependence $\Rightarrow$ $\geq$ 50 % error except c, c_w, c_b
- typical statistical accuracy: Δh_{A1} ~ 1 - 2 %, Δh_V ~ 2 - 3 %

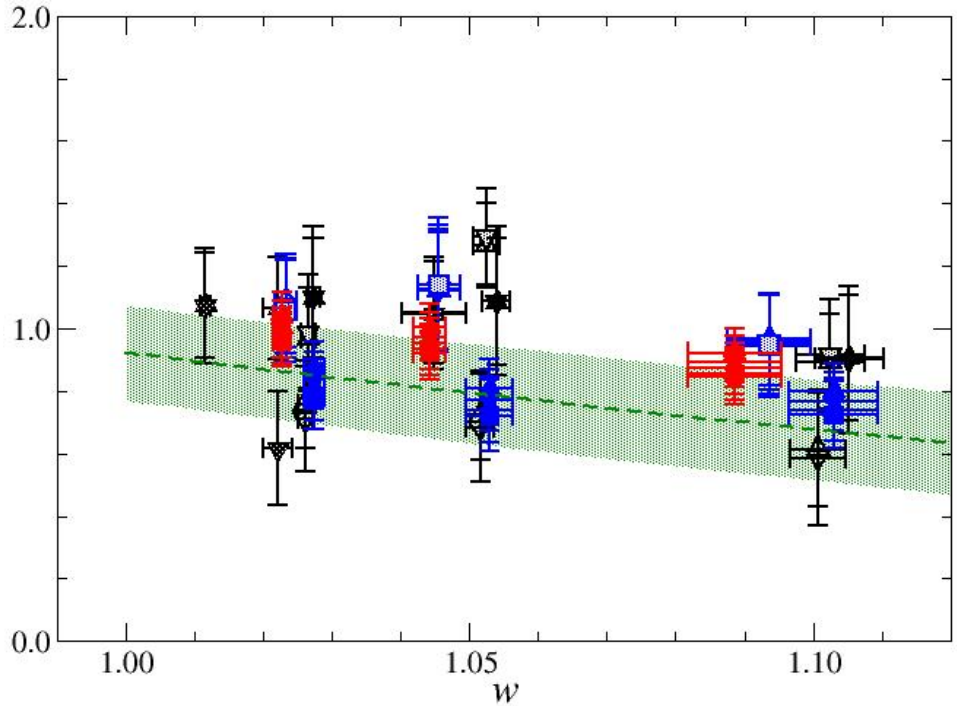
$$\left\langle D^*(p',\varepsilon')\middle|V_\mu\middle|B(p)\right\rangle \Rightarrow h_V(w) \quad \left\langle D^*(p',\varepsilon')\middle|A_\mu\middle|B(p)\right\rangle \Rightarrow h_{A_1}(w) \quad (\mathbf{p}' \perp \boldsymbol{\varepsilon}')$$

$B \to D^* \ell \nu$ form factors

h_{A2} vs w



h_{A3} vs w



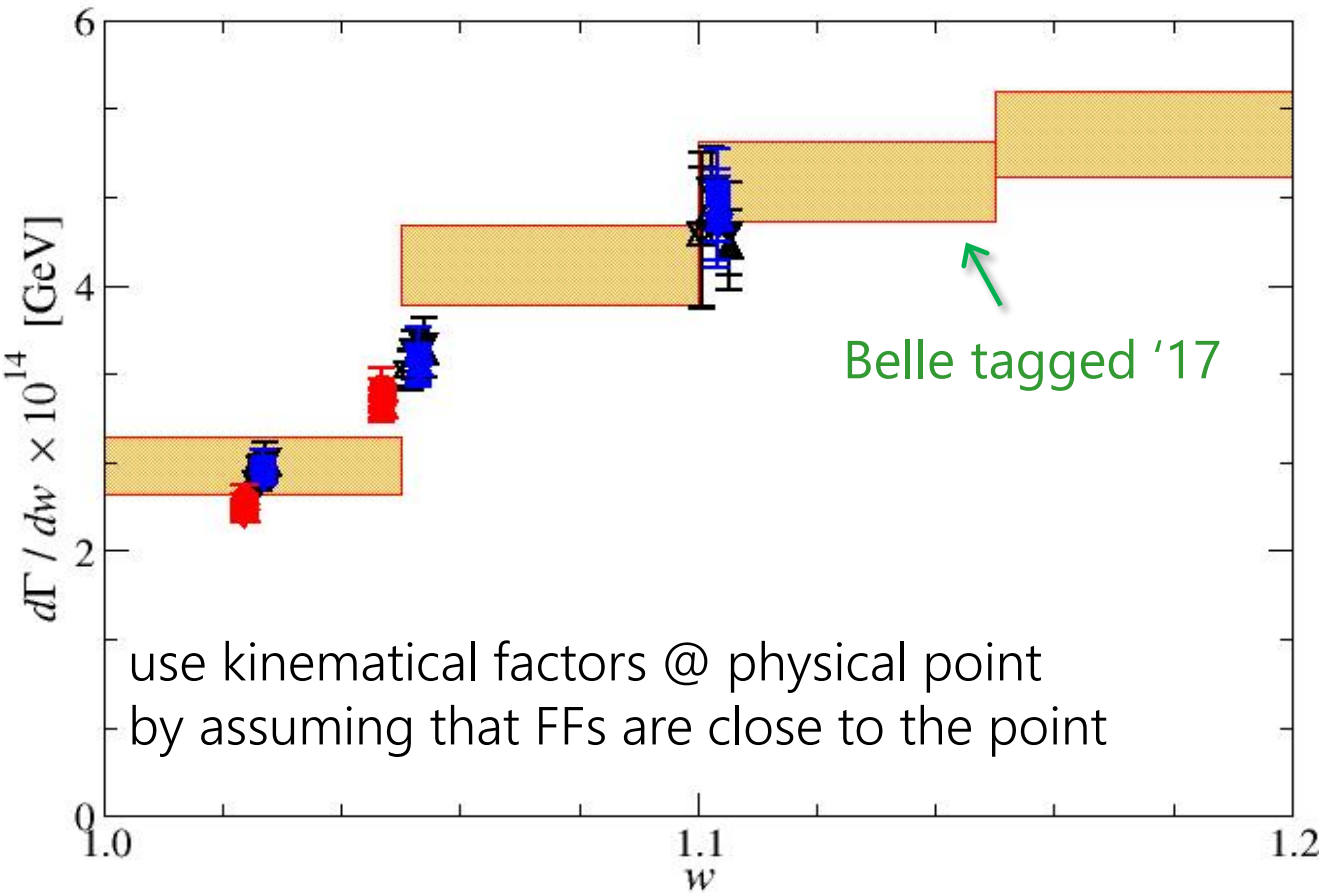
- h_{A1}, h_{A3}, h_V $(\to \xi) \sim O(1)$, h_{A2} $(\to 0)$ ~ 0
- typical accuracy: $\Delta h_{A2} \geq 50$ %, $\Delta h_{A3} \sim 20$ %

$$\left\langle D^*(p',\varepsilon') \middle| A_\mu \middle| B(p) \right\rangle \Rightarrow \left\{ h_{A_1}(w), h_{A_2}(w), h_{A_3}(w) \right\}$$

$B \to D^* \ell \nu$ differential decay rate

in the limit $m_l^2 = 0$

$$\frac{d\Gamma}{dw} \propto \left[2\frac{1-2wr+r^2}{(1-r)^2}\left\{1+\frac{w-1}{w+1}R_1(w)\right\} + \left\{1+\frac{w-1}{1-r}\left(1-R_2(w)\right)\right\}^2 \right] h_{A1}(w)^2$$



$$R_1 = \frac{h_V}{h_{A1}}$$

$$R_2 = \frac{r h_{A2} + h_{A3}}{h_{A1}}$$

can predict $d\Gamma/dw$ with accuracy comparable to exp.

phenomenological parametrizations

- Boyd-Grinstein-Lebed (BGL) parametrization of FFs '97
  - generic, based on analyticity + unitarity $\Rightarrow$ many parameters
- Caprini-Lellouch-Neubert (CLN) '98
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  - $h_{A1} / f_+^{B\to D}$, R_1, R_2 from HQET, dispersive bounds on $f_+^{B\to D}$

$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2_{D^*} z + \left(53\rho^2_{D^*} - 15\right) z^2 - \left(231\rho^2_{D^*} - 91\right) z^3\right)$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

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- Belle untagged '18 $\Rightarrow$ CLN: 38.4(0.9) BGL: 38.3(1.0)

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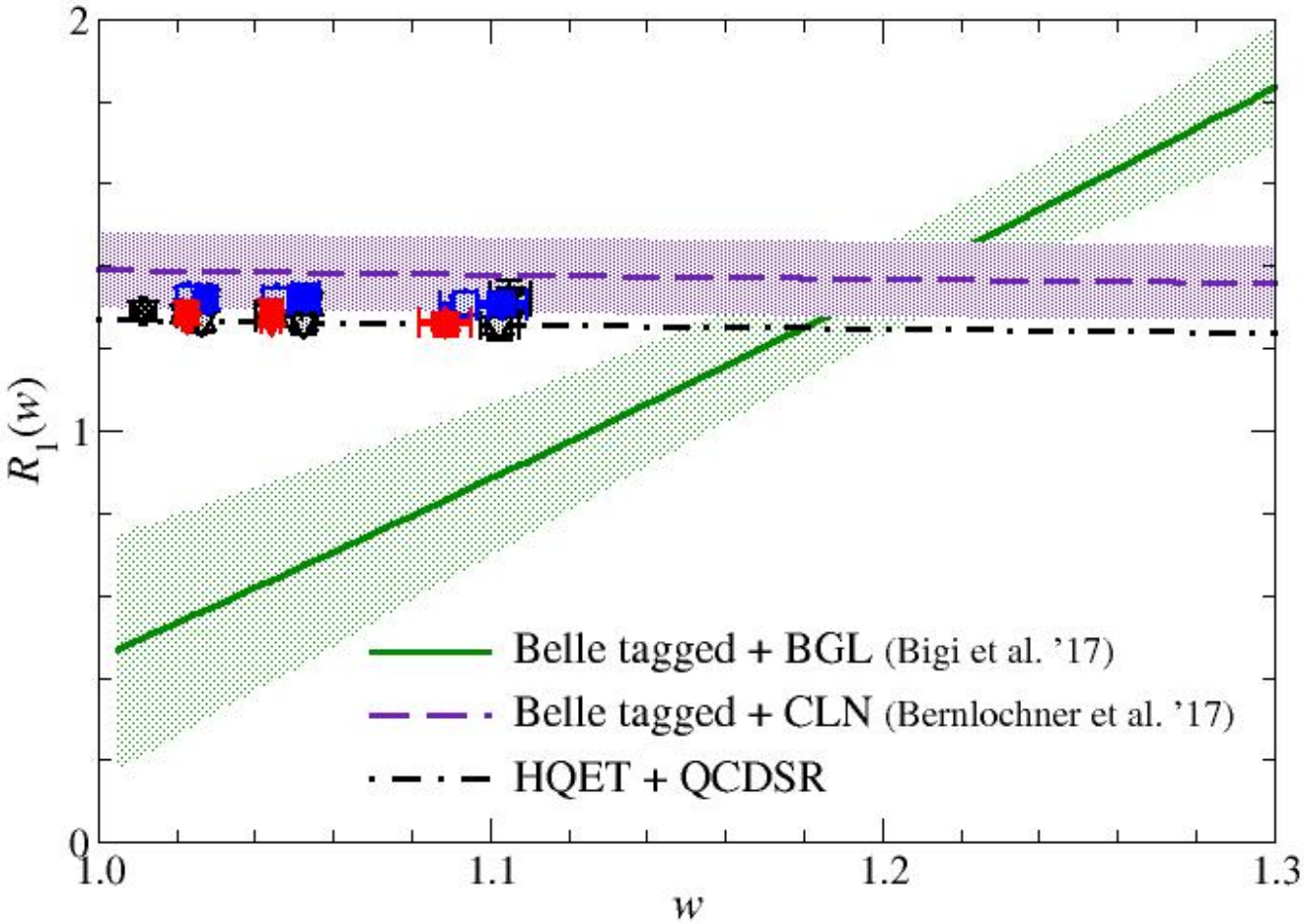
compare lattice data w/ HQET, BGL, CLN analyses

LQCD vs BGL vs CLN vs HQET

$$R_1 = h_V / h_{A1}$$



w/ Belle tagged '17

BGL vs CLN, HQET

Bernlochner et al.

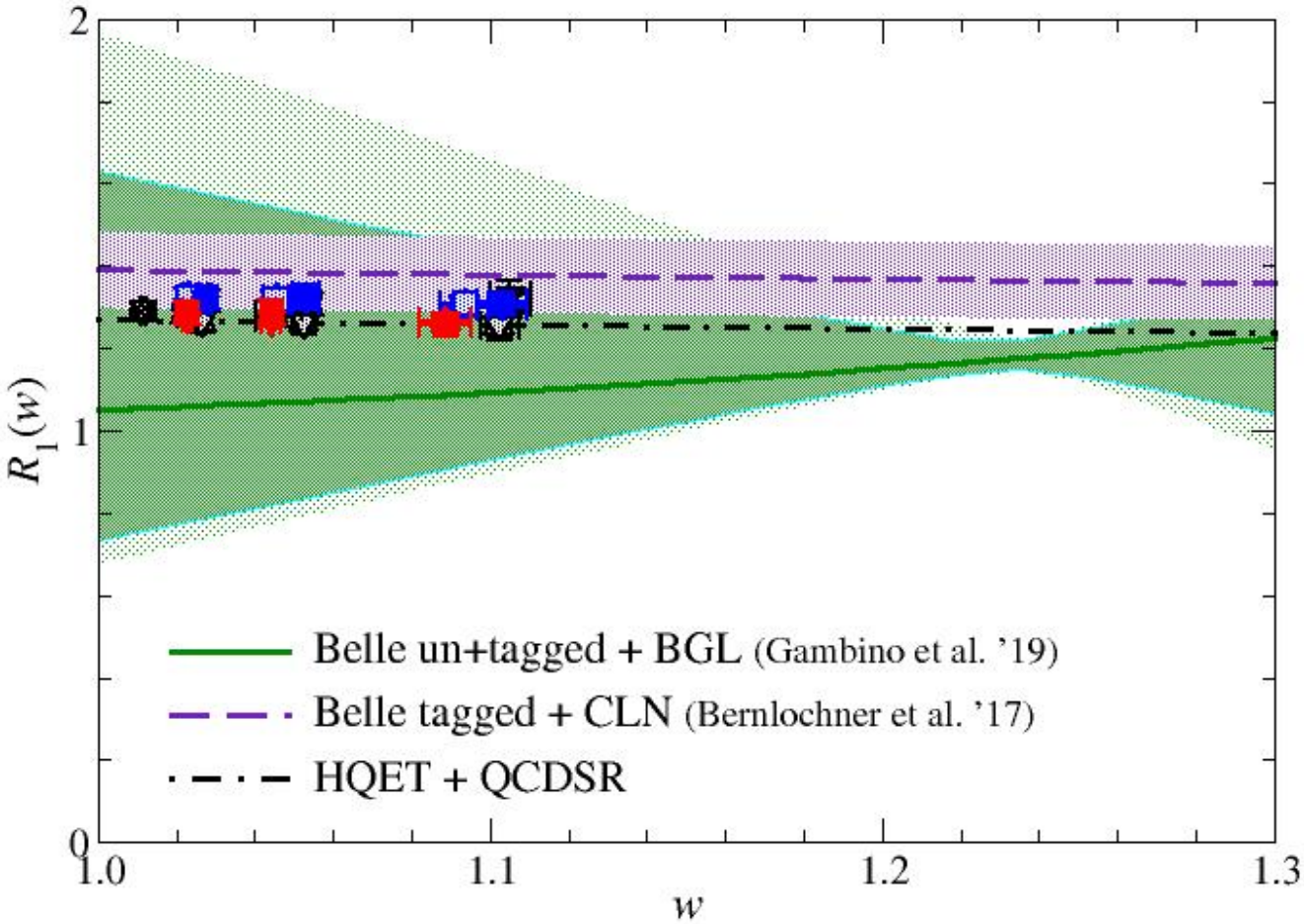
Bigi et al.

Grinstein-Kobach

our data favor the CLN results

LQCD vs BGL vs CLN vs HQET

$$R_1 = h_V / h_{A1}$$



w/ Belle tagged '17

BGL vs CLN, HQET

Bernlochner et al.

Bigi et al.

Grinstein-Kobach

our data favor the CLN results

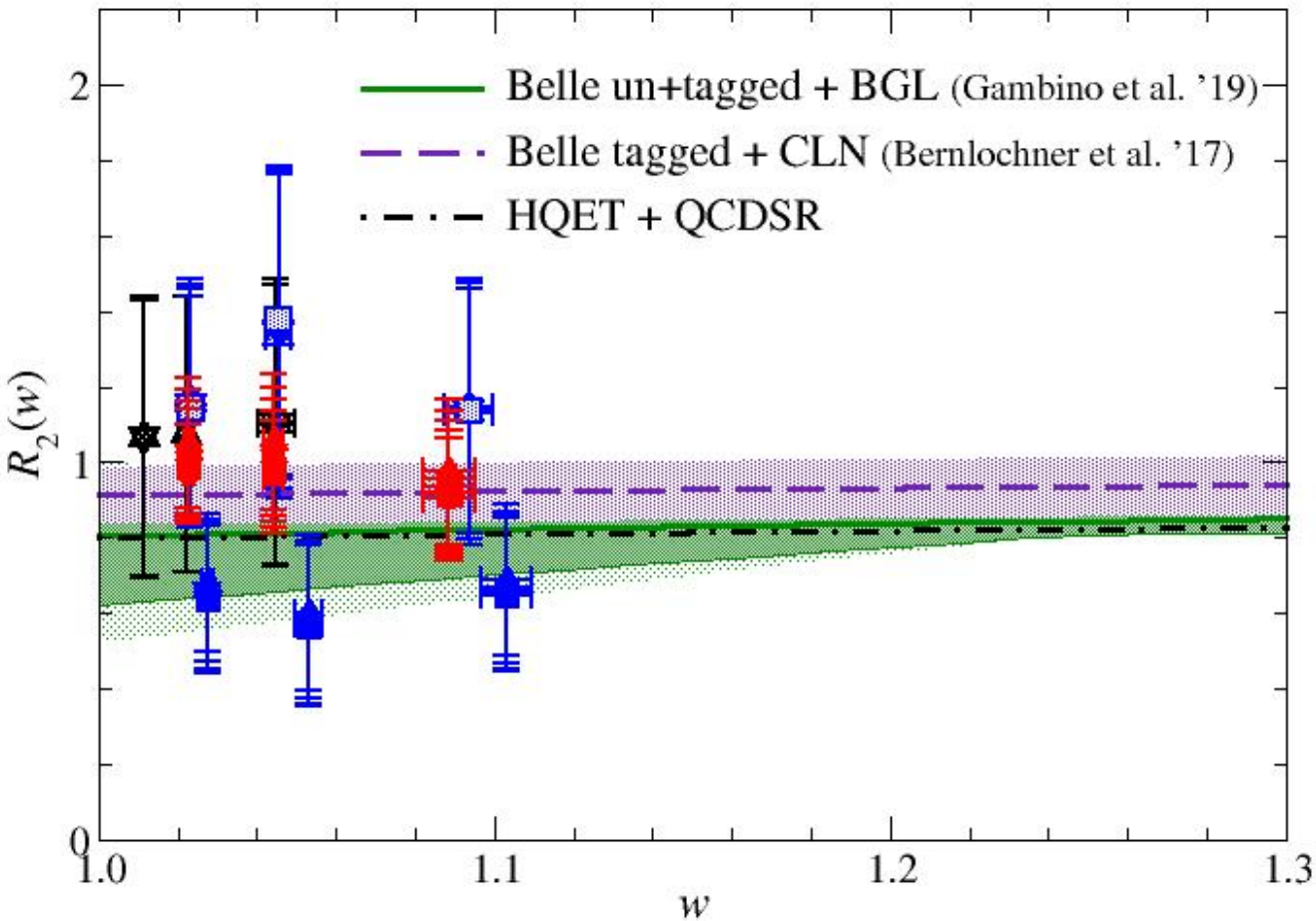
Gambino et al '19

tagged+untagged

consistency among LQCD, BGL, CLN, HQET

LQCD vs BGL vs CLN vs HQET

$$R_2 = (rh_{A2}+h_{A3}) / h_{A1}$$



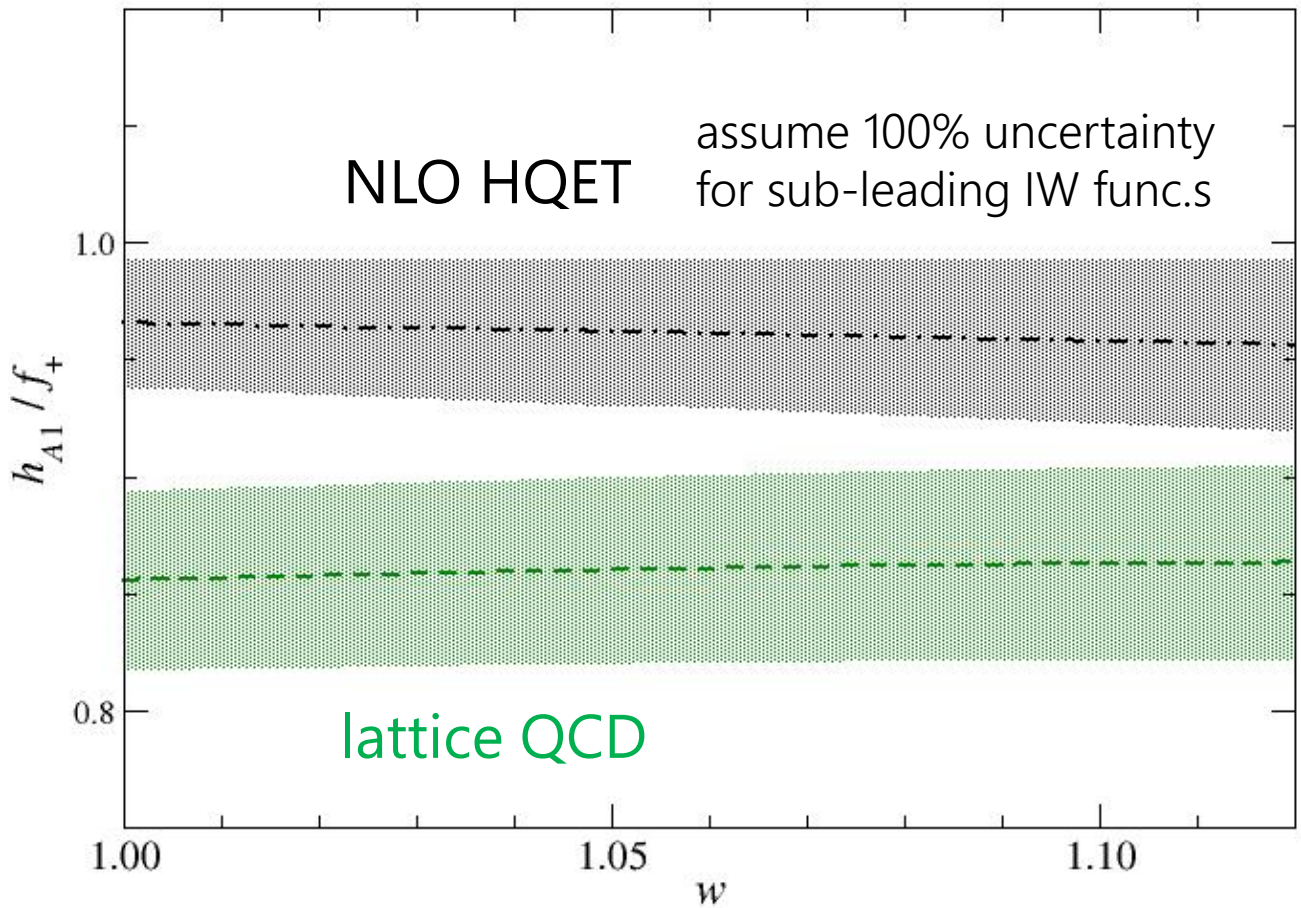
no big difference among CLN, BGL, HQET...

larger uncertainty from h_{A2}, h_{A3}

less accurate, but consistent w/ BGL, CLN, HQET

LQCD vs HQET

$$h_{A1}\,[B\to D^*]\ /\ f_+\,[\mathrm{B}\to\mathrm{D}]$$



used to derive

CLN param. for h_{A1}

$V_1,\ h_{A1}/V_1 \Rightarrow h_{A1}$

not necessarily imply $|V_{cb}|$ tension → normalization is fit parameter

10% difference in normalization, but not FF shape

Summary

$B \to D^{(*)} \ell \nu$ form factors from relativistic QCD

- several source-sink separations : good accuracy
- small a, m_b, M_π, m_s dependence → controlled extrapolation
- consistency w/ NLO HQET, CLN and BGL (except h_{A1} / f_+)
- extension to BSM form factors : on-going
- wide applications to B physics w/ relativistic b quarks
  - J. Koponen (Mon 16:30-) : $B \to \pi \ell \nu$
  - G. Bailas (Tue 14:40-) : $B \to X_c \ell \nu$, $D^{**} \ell \nu$
  - K. Nakayama (Tue 15:00-) : $B \to K ll$
  - T. Ishikawa (Mon 16:50-) : NPR of bilinear / four fermion