S-wave $\pi\pi$ I=0 and I=2 scattering at physical pion mass

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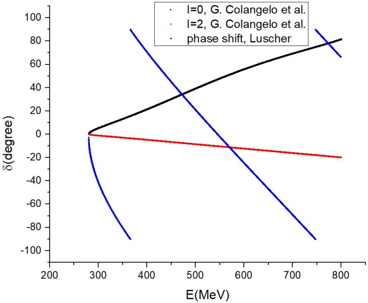
Outline

Why $\pi\pi$ scattering?

- Need $\pi\pi$ energy and amplitude in $K \to \pi\pi$ calculation
- 2015 results on $K \to \pi\pi$ relies on a $\pi\pi$ energy which is 3σ(7σ with more statistics) higher than the phenomenological prediction
- Phase shift is important when computing LL-factor, which is necessary to get $K \to \pi\pi$ matrix element
- We start first lattice calculation on $\pi\pi$ scattering with physical pion mass around kaon mass

- Luscher's formula (Single channel, BC, CM momentum dependent)
  $tan\delta = \frac{\pi^{3/2} q}{Z_{00}^{0,G}(1,q^2)}$
- Schenk's ansatz with Colangelo's parametrization[1]
  $tan\delta_I = \sqrt{1 - \frac{4M_\pi^2}{s}}(A_I + B_I q^2 + C_I q^4 + D_I q^6)(\frac{4M_\pi^2 - s_I}{s - s_I})$



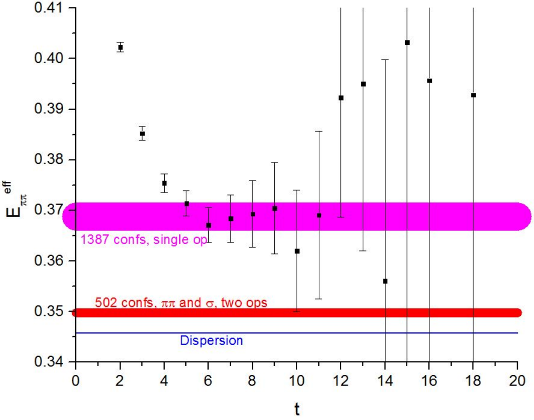
S-wave phase shift and Luscher's formula in stationary calculation.

[1] G. Colangelo, Nuclear Physics B 603 (2001) 125 - 179

- G-parity boundary condition
  Helps with $K \to \pi\pi$ calculation, ground state π has momentum $(\pm\frac{\pi}{L}, \pm\frac{\pi}{L}, \pm\frac{\pi}{L})$
- All to all propagator
  Better overlap between interpolating operator and meson ground state, 900 low modes plus 1536 random modes from time/flavor/color/spin dilution, 1s hydrogen wave smearing function, pion mesonfield with different choices of momentum
- Time separated pipi operator
  Two pions are time separated by 4
- Moving frame calculation
  We can recombine π operators with different momenta to do calculation with different CM momentum
- Adding more operators
  In stationary I=0 calculation, we add Sigma operators which looks like $(\bar{u}u + \bar{d}d)$
  In both stationary/moving, I=0/2 calculation, we add "311" π operator with momentum $(\pm\frac{3\pi}{L}, \pm\frac{\pi}{L}, \pm\frac{\pi}{L})$ to construct $\pi\pi$ operator with different momenta

Why multiple operators

- 2015 result (216 confs) and later results (1386 confs) shows a huge discrepancy with experimental and dispersion prediction.
- We introduce sigma operator which partly solves the problem.
- Adding more operators to further suppress excited state contamination?



Moving frame calculation

Intro

- Three CM momenta: $(\pm 2, 0, 0)\pi/L$, $(\pm 2, \pm 2, 0)\pi/L$, $(\pm 2, \pm 2, \pm 2)\pi/L$ and their permutation.
- Three operators: $\pi\pi(111, 111)$, $\pi\pi(311, 311)$ and $\pi\pi(111, 311)$.
- Together with stationary case, it allows us to calculate phase shift at four different energy.
- Moving frame calculation is more vulnerable to excited state contamination error due to the denser spectrum of state.

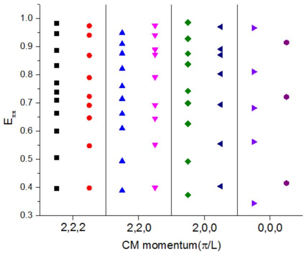


Figure: Density of spectrum of $\pi\pi$ state. Left: I=0; Right: I=2

- We perform 1,2 and 3 state, correlated fit, each state is represented by a cosh function, including around the world constant for I=2, drop it for I=0.

Stationary: two operators, S-wave $\pi\pi$(111,111) and $\pi\pi$(311,311)
Moving: three operators, S-wave $\pi\pi$(111,111), $\pi\pi$(111,311) and $\pi\pi$(311,311)

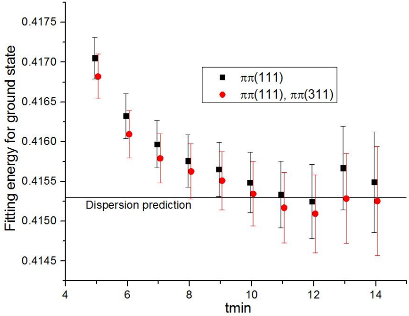


Figure: Stationary

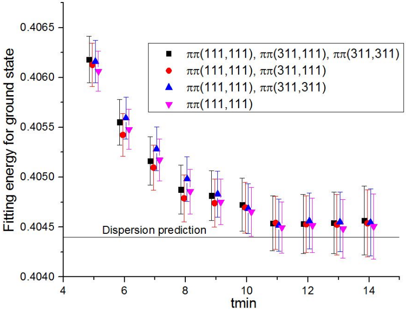


Figure: Moving, 200

- Introducing the second and third operator slightly lowers ground state energy in stationary frame and has no effect in moving frame.

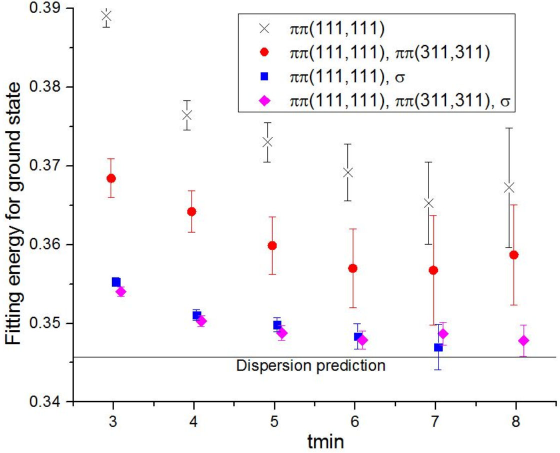
0,0,0	$state_0$	$state_1$
op_0	1.0(0.0)	0.072(56)
op_1	-0.068(3)	1.0(0.0)

2,0,0	$state_0$	$state_1$	$state_2$
op_0	1.0(0.0)	0.049(3)	0.037(8)
op_1	0.032(0.000)	1.0(0.0)	0.043(11)
op_2	$-13(0)\times10^{-4}$	0.069(2)	1.0(0.0)

- Overlap matrix is nearly diagonal, which suggests extra operators are not very useful with current statistical accuracy
- There is a constant term describing the around the world effect. This term is significantly resolvable from 0 (about 60σ) therefore necessary in fitting.
- All fitting has extremely good pvalue (about 0.5).

Stationary $\pi\pi$ I=0

Three operators: S-wave $\pi\pi(111)$, $\pi\pi(311)$, σ



- In stationary I=0, introducing the σ operator helps a lot in suppressing excited state error, but the effect of $\pi\pi(311)$ operator is not very obvious.
- Overlap matrix is far from diagonal, which suggests σ operator is very useful, the small overlap between $\pi\pi(311)$ operator and ground state suggests that operator is not as useful as σ operator

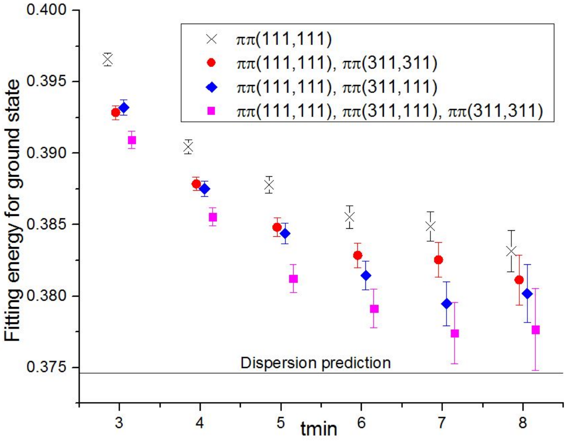
0,0,0	state_0	state_1	state_2
$\pi\pi(111, 111)$	1.0(0.0)	0.47(2)	0.31(7)
$\pi\pi(311, 311)$	0.053(9)	-0.84(12)	1.0(0.0)
σ	1.0(0.0)	-0.83(3)	-0.87(22)

- There is a constant term describing the around the world effect. This term is statistically consistent with 0 from fitting, so we drop it and perform fitting without this constant in exchange of better statistical error.

Moving $\pi\pi$ I=0

CM momentum $(2, 0, 0)\frac{\pi}{L}$

Three operators, S-wave $\pi\pi$(111,111), $\pi\pi$(111,311) and $\pi\pi$(311,311)



- In moving I=0, introducing the second and third operator lowers the energy by roughly 1.5σ, which is not as helpful as stationary I=0.
- The overlap between $\pi\pi(311, 311)$ operator and state 0 and 1 are very small, which suggests this operator is not very useful

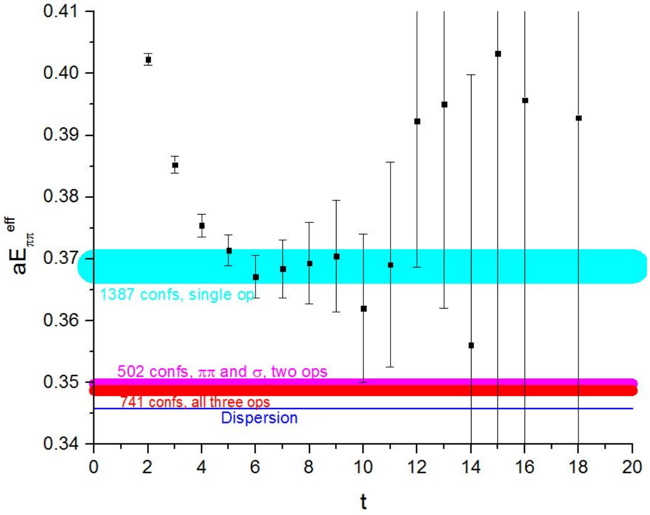
2,0,0	$state_0$	$state_1$	$state_2$
$\pi\pi(111, 111)$	1.0(0.0)	-0.31(5)	0.14(2)
$\pi\pi(111, 311)$	0.09(2)	1.0(0.0)	-0.30(20)
$\pi\pi(311, 311)$	0.01(1)	0.09(5)	1.0(0.0)

- There is a constant term describing the around the world effect. This term is statistically consistent with 0 from fitting, so we drop it and perform fitting without this constant in exchange of smaller statistical error.

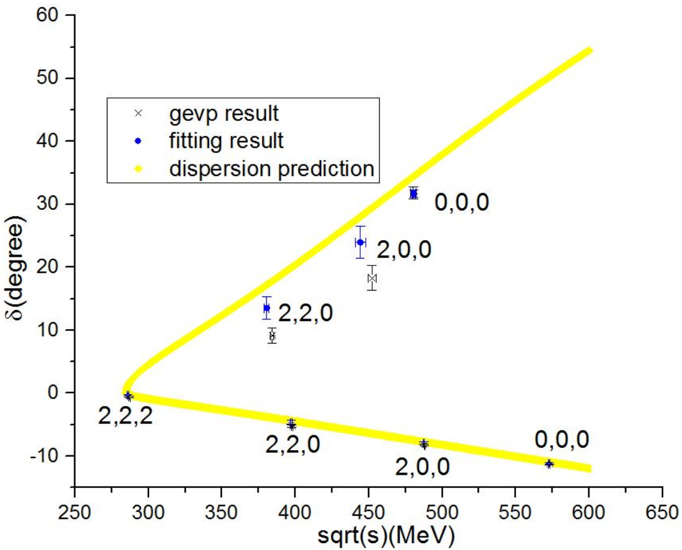
Summary of multiple operators

- Extra state significantly increase the number of parameter: from 3(2) to 9(6) to 18(12)
- Multiple operators are not always helpful, for example multiple operators in I=2 and moving I=0 are not as useful as they are in stationary I=0.
- Choose operator carefully. The reason multiple operators are helpful in stationary I=0 is because we introduce the σ operator.
- Sometimes multiple operators might introduce bad effect into calculation. For example the dimension of covariance matrix in moving frame calculation can be 6 times bigger then single operator, which might destabilize the covariance matrix.
- We also use GEVP method to analyze the same sets of data, and all of them are consistent with fitting result except moving I=0, where our results are inconsistent with dispersion prediction.

Summary of multiple operators



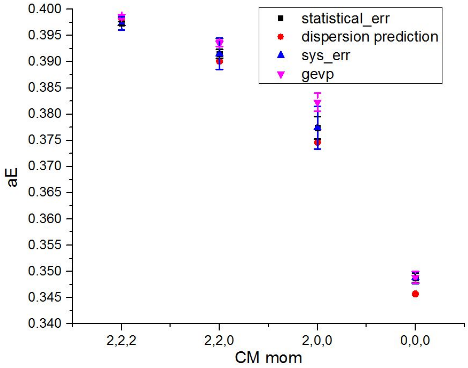
Summary of multiple operators



Some of the calculations give apparently inconsistent results with dispersion prediction and GEVP.
Possible source of systematic error:

- Finite lattice spacing error
- Finite volume effect
- Excited state contamination

The first and second error contribute little to final phase shift
Due to the small off-diagonal term in overlap matrix, there could be large excited state contamination from states which mainly couples to one operator.
This error can be estimated by fitting our data to a fit function where we introduce another higher energy state, whose energy is frozen by dispersion prediction.



After including systematic error from excited state contamination, our results in moving frame are now consistent with dispersion predictions.
We can do the same thing for I=2, but the results suggests that in that case, this systematic error are very small so that can be neglected.

Conclusion

What do we get:

- We should choose the operators carefully.
- Results of $\pi\pi_{I=2}$, both moving and stationary frame, are consistent with dispersion prediction.
- Extra operators help reducing the excited state contamination error in stationary $\pi\pi_{I=0}$ calculation so that result are almost consistent with dispersion prediction.
- To solve the inconsistency between our results and dispersion prediction in $\pi\pi_{I=0}$ moving frame, currently we need to introduce a tricky and huge systematic error which comes from excited state error.

Outlook:

- Consider the effect of auto-correlation by binning and try to find a new method to reduce the effect of binning on resolution of correlation matrix (See Chris's talk next).
- Adding new operators in moving frame (a moving σ operator is now implemented, considering its success in stationary calculation).
- Finish k$\rightarrow \pi\pi$ calculation with k$\rightarrow \sigma$ diagrams included.