$B \to \pi l \nu$ form factors and $|V_{ub}|$ with Möbius domain wall fermions

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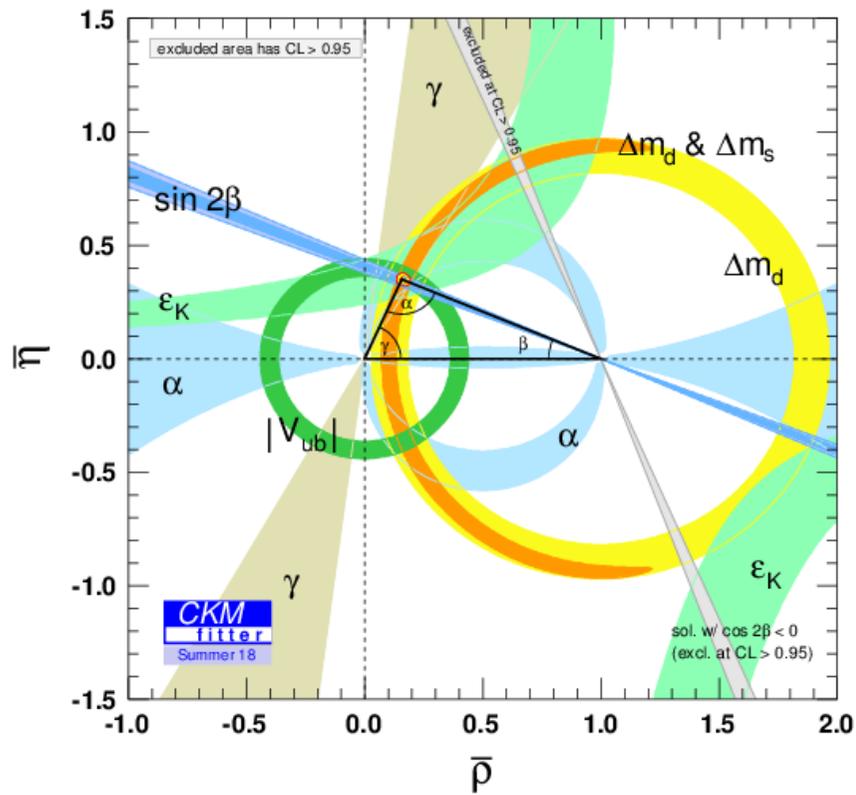with B. Colquhoun, S. Hashimoto, T. Kaneko
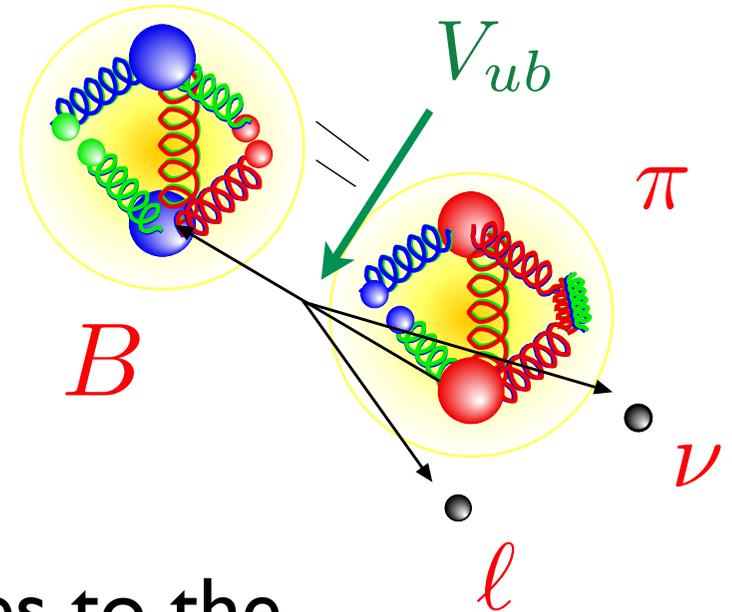
Lattice 2019, Wuhan, China

Outline

- B meson semileptonic decays and CKM matrix elements

- JLQCD lattice ensembles

- Preliminary results

- Summary

Motivation

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \bar{B}_s \rangle & \langle B_s | \bar{B}_s \rangle & \end{pmatrix}$$

CKM matrix element V_{ub}

- CKM matrix element relates to the differential decay rate

$$\frac{\mathrm{d}\Gamma(B \to \pi\ell\nu)}{\mathrm{d}q^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |k_\pi|^3 |f_+(q^2)|^2$$

decay rate (experiment)

CKM matrix element

theory (lattice)

k_π = pion momentum
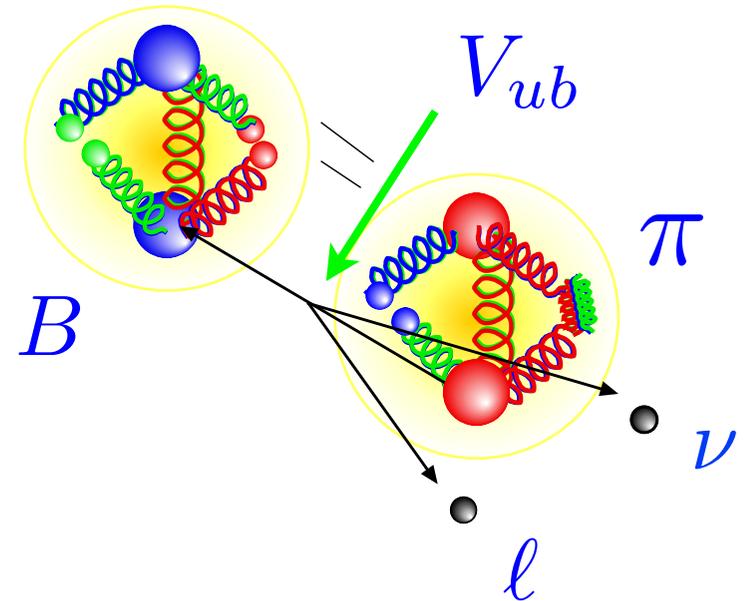q = momentum transfer

Form factors

- For pseudoscalar to pseudoscalar decays

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[(p_B + k_\pi)^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right]$$

$$+ f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2}q^\mu$$

- p_B and k_π are the B and π four momenta
- $q^\mu = p_B^\mu - k_\pi^\mu$ is the momentum transfer
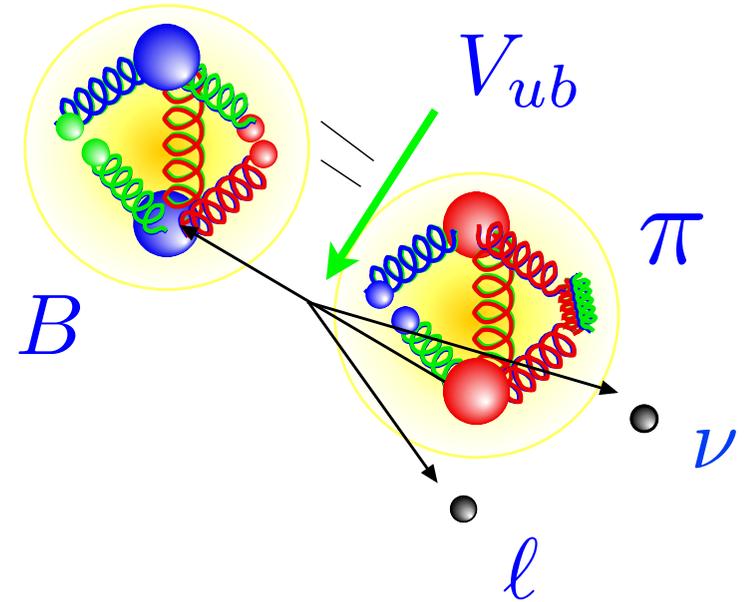- kinematic constraint $f_0(0) = f_+(0)$

Form factors

- In the context of Heavy Quark Effective Theory (HQET), a useful parametrisation is

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{M_B}\left[f_1(v \cdot k_\pi)v^\mu + f_2(v \cdot k_\pi)\frac{k_\pi^\mu}{v \cdot k_\pi} \right]$$

- $v^\mu = p_B^\mu/M_B$ is the heavy quark velocity

- pion energy $E_\pi = v \cdot k_\pi = \dfrac{M_B^2 + M_\pi^2 - q^2}{2M_B}$

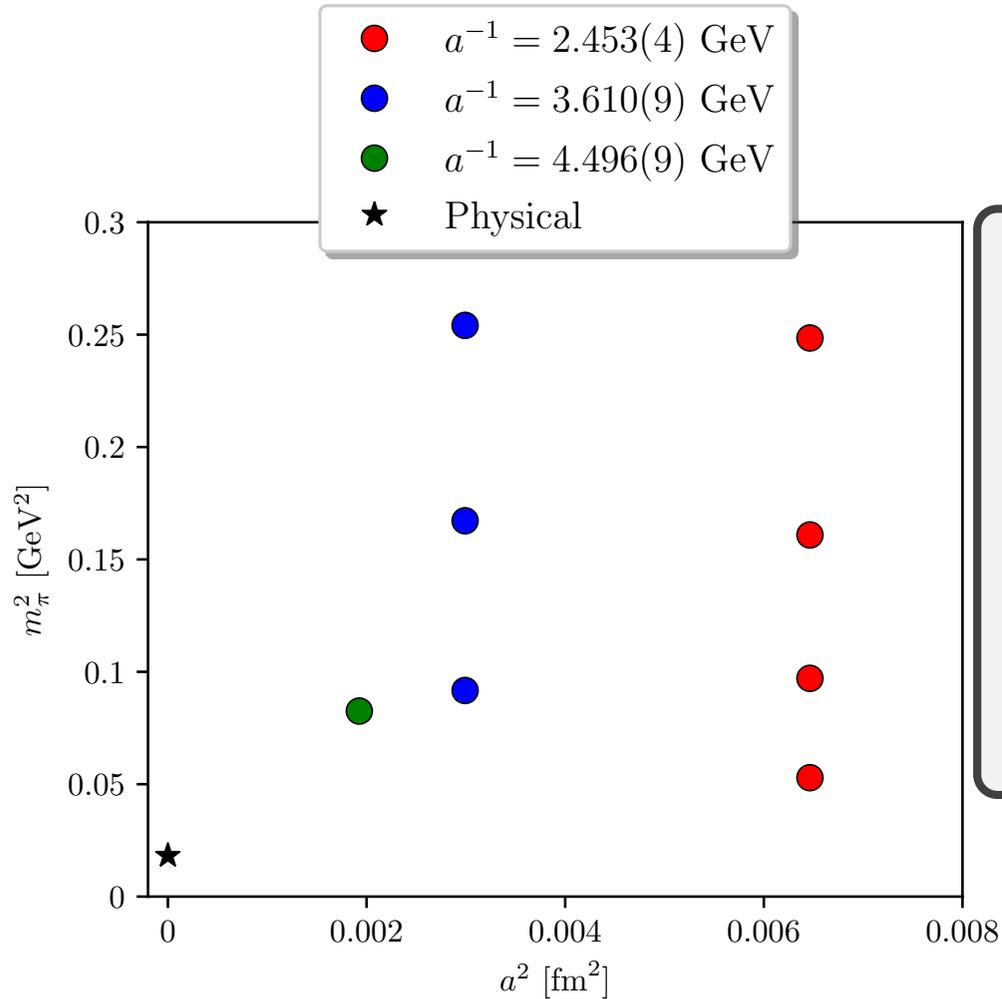# Form factors

V_{ub}

B π ν ℓ

- These two form factor definitions are related by

$$f_+(q^2) = \sqrt{M_B}\left[\frac{f_2(v \cdot k_\pi)}{v \cdot k_\pi} + \frac{f_1(v \cdot k_\pi)}{M_B}\right]$$

$$f_0(q^2) = \frac{2}{\sqrt{M_B}}\frac{M_B^2}{M_B^2 - M_\pi^2}\left[f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)\right.$$
$$\left. - \frac{v \cdot k_\pi}{M_B}\left(f_1(v \cdot k_\pi) + \frac{M_\pi^2}{(v \cdot k_\pi)^2}f_2(v \cdot k_\pi)\right)\right]$$

Lattice configurations

Valence quarks

Legend:
- 🔴 $m_h = m_c$
- 🔵 $m_h = 1.25^2 \times m_c$
- 🟢 $m_h = 1.25^4 \times m_c$
- ★ $m_h = m_b$

Axes: m_c/m_h (vertical), a^2 [fm^2] (horizontal)
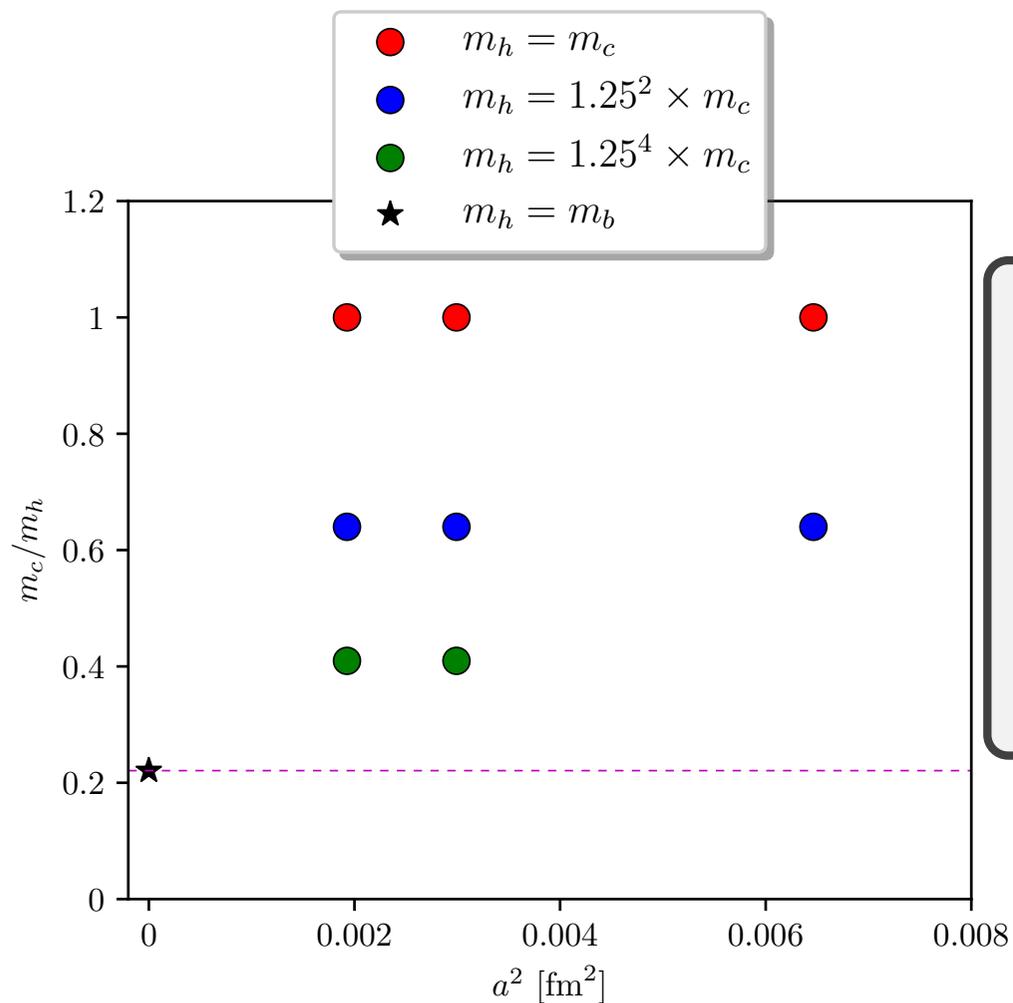
Möbius Domain Wall Fermions
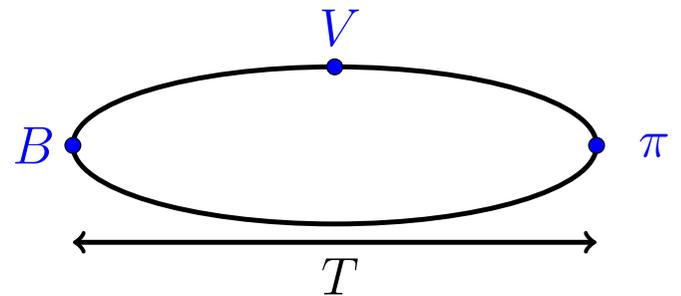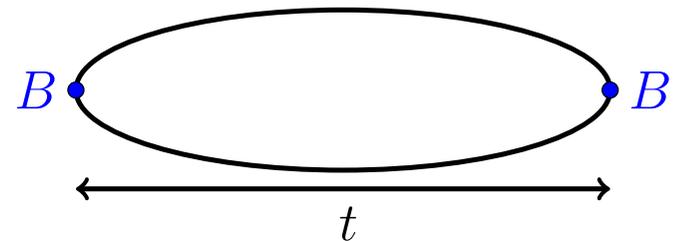
m_l same values as sea quarks

$m_h \in \{m_c,\ 1.25^2 m_c,\ 1.25^4 m_c\}$

○ $m_{H_l} \approx 1.95,\ 2.55,\ 3.40$ GeV

○ Fully relativistic formalism
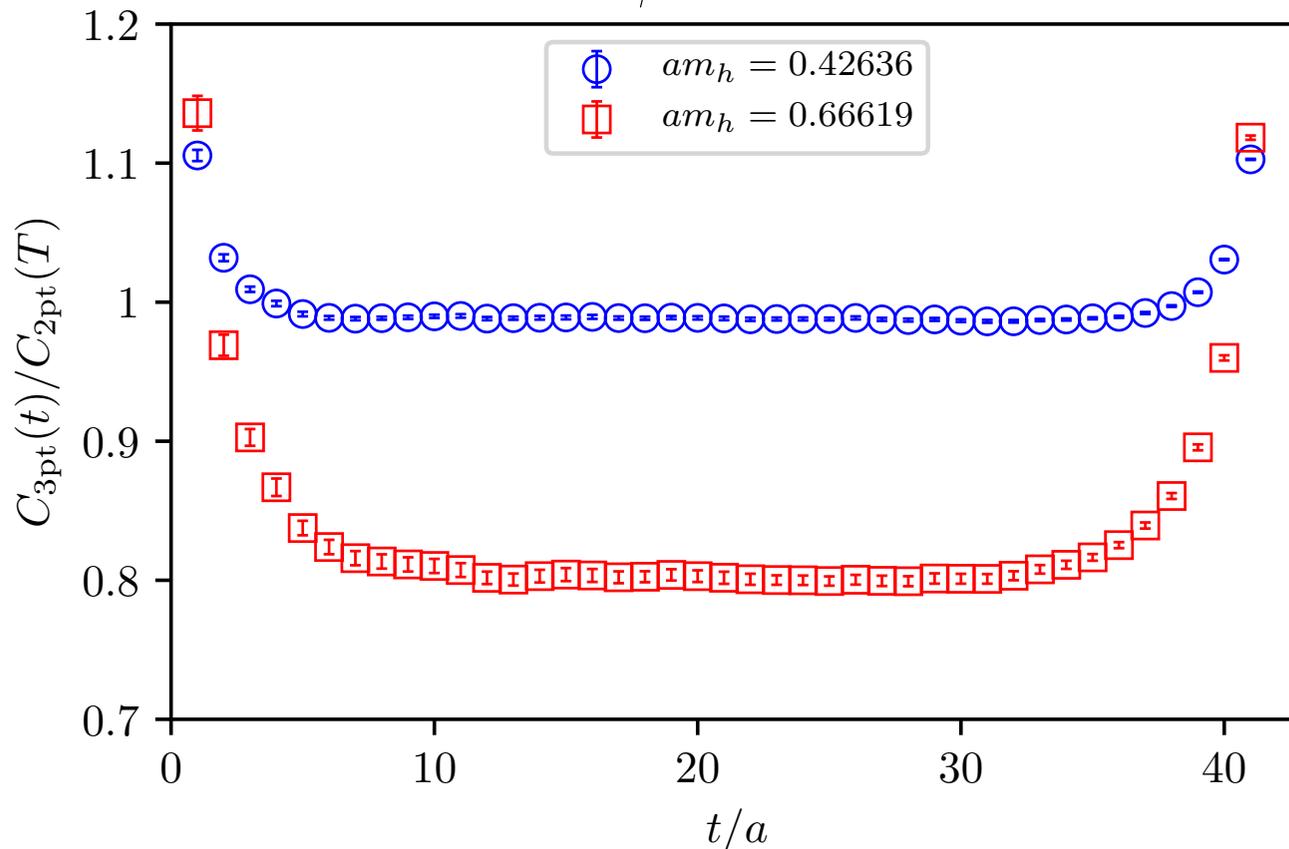
2pt and 3pt correlators

- Smeared sources with $\mathbb{Z}_2$ noise

- Local and smeared sinks

- $1, 2$ or 4 time sources

- B and π mesons separated by time T

- Vector operator V^μ inserted at time $0 < t < T$

Renormalisation

- Renormalise with $Z_{V,bl} = \sqrt{Z_{V,bb} Z_{V,ll}}$

- Require $C_{3\mathrm{pt}}(t)/C_{2\mathrm{pt}}(T) = Z_{V,bb}^{-1}$

- In the continuum limit $Z_{V,bb} = V_{V,ll}$

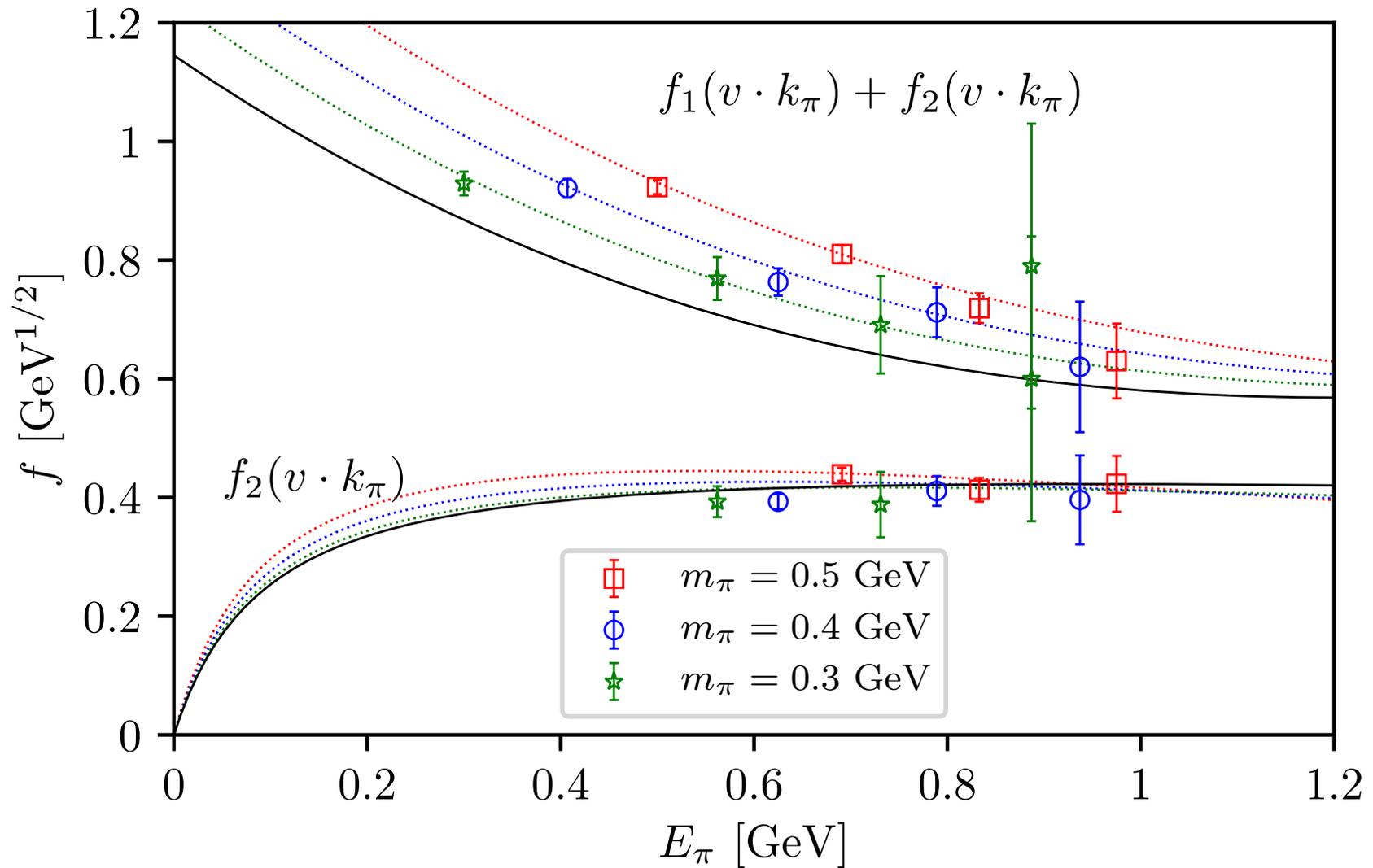# Extrapolating to $a \to 0$, $m_h \to m_b, \, m_l \to m_{u,d}$

$$f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi) = D_0 \left(1 + \sum_{n=1}^{3} D_{E_\pi^n} E_\pi^n + D_{M_\pi} M_\pi^2 \right.$$

$$\left. + \chi_{\log} + D_{E_\pi M_\pi^2} E_\pi M_\pi^2 + \frac{D_{m_h}}{m_h} \right) \left(1 + D_{a^2} a^2 \right)$$

$$f_2(v \cdot k_\pi) = C_0 \frac{E_\pi}{E_\pi + (M_{B^*} - M_B)} \left(1 + C_{E_\pi} E_\pi + C_{M_\pi} M_\pi^2 \right.$$

$$\left. + \chi_{\log} + C_{E_\pi M_\pi^2} E_\pi M_\pi^2 + \frac{C_{m_h}}{m_h} \right) \left(1 + C_{a^2} a^2 \right)$$
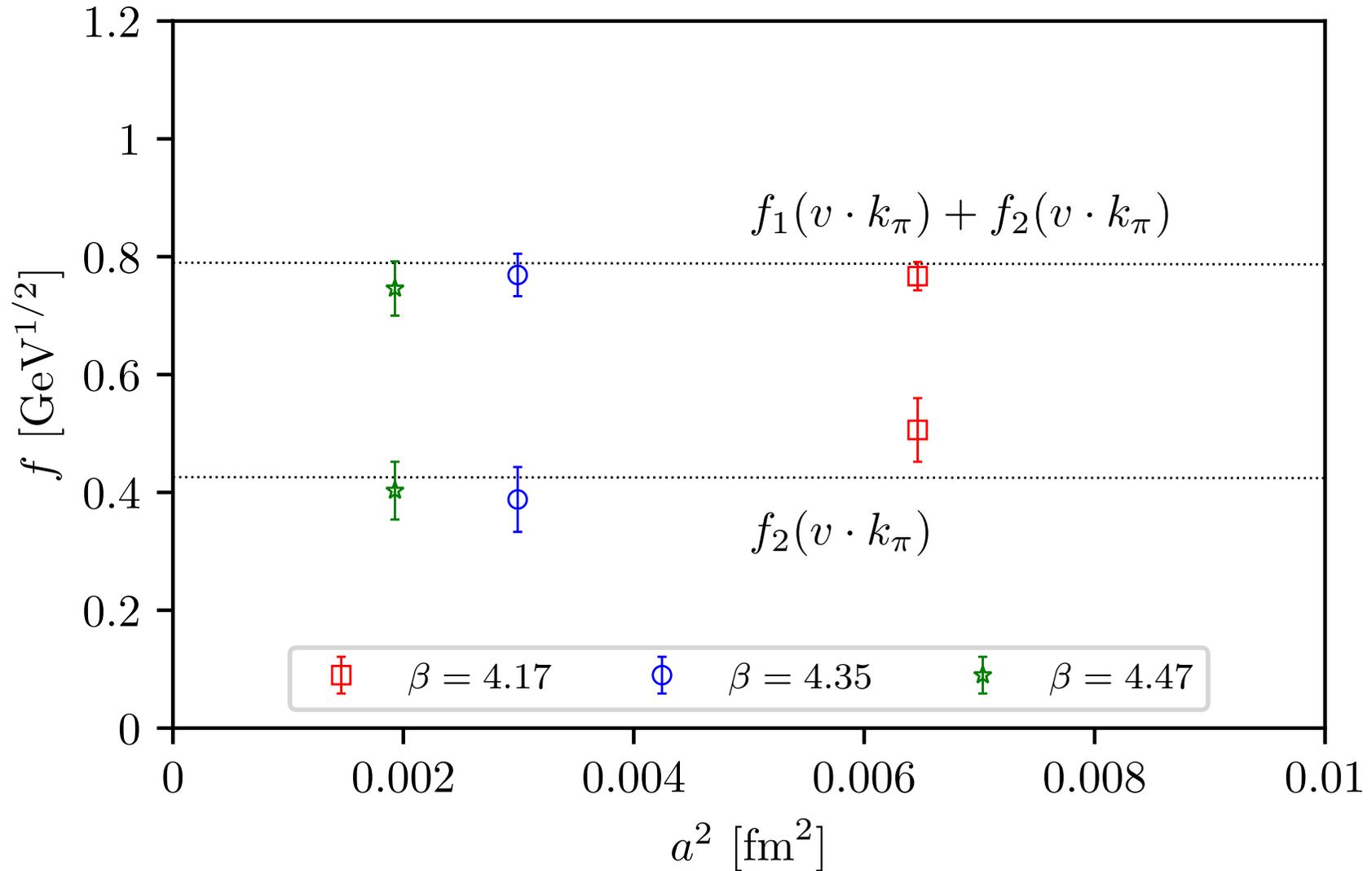
Pole
dominance

Form factors vs m_π

$$\beta = 4.35;\ \ 1.25^2 \times m_c$$

$f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$

$f_2(v \cdot k_\pi)$

Legend:
- $m_\pi = 0.5$ GeV (red square)
- $m_\pi = 0.4$ GeV (blue circle)
- $m_\pi = 0.3$ GeV (green star)
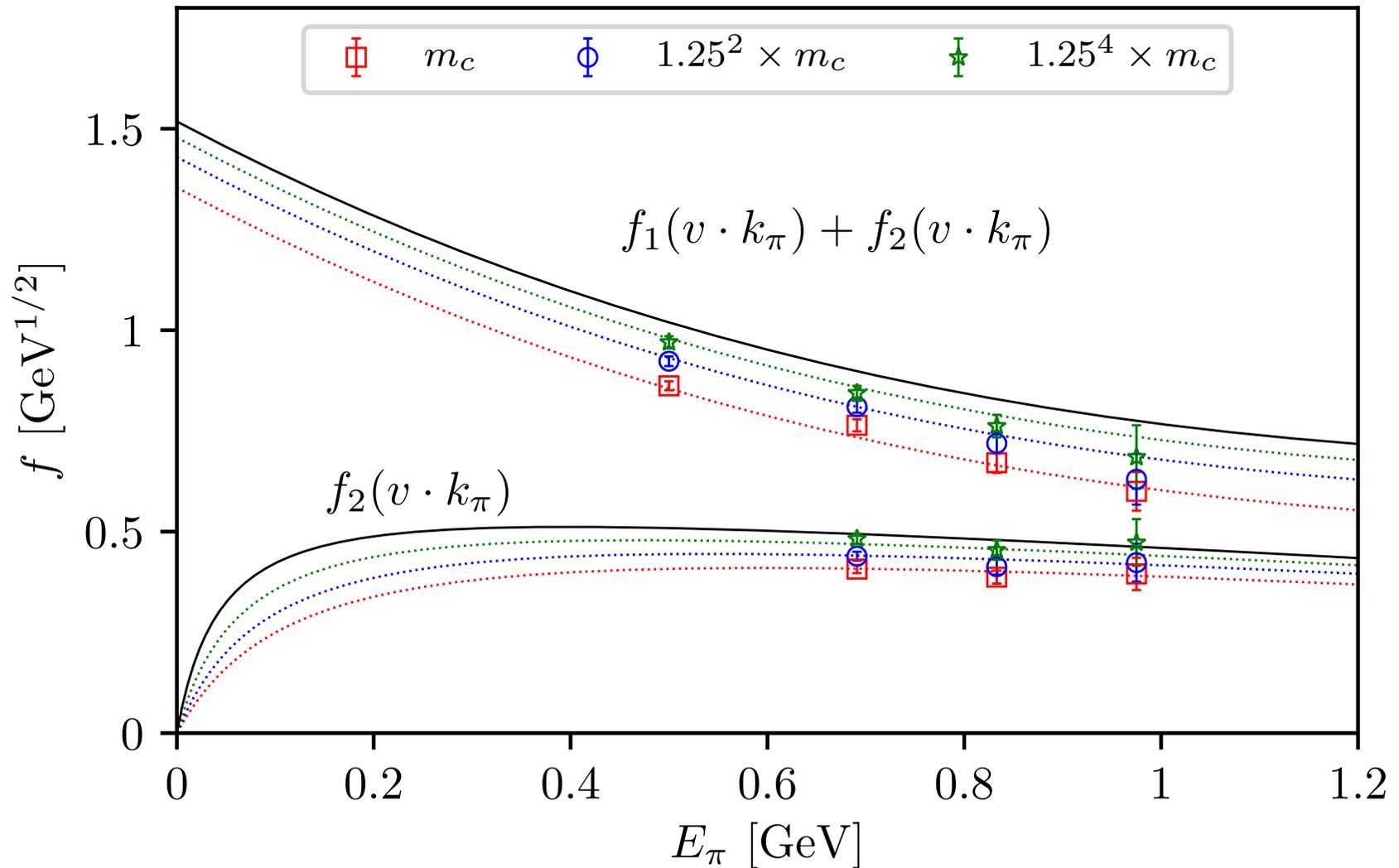
Axes: f [GeV$^{1/2}$] vs E_π [GeV]

Form factors vs a^2

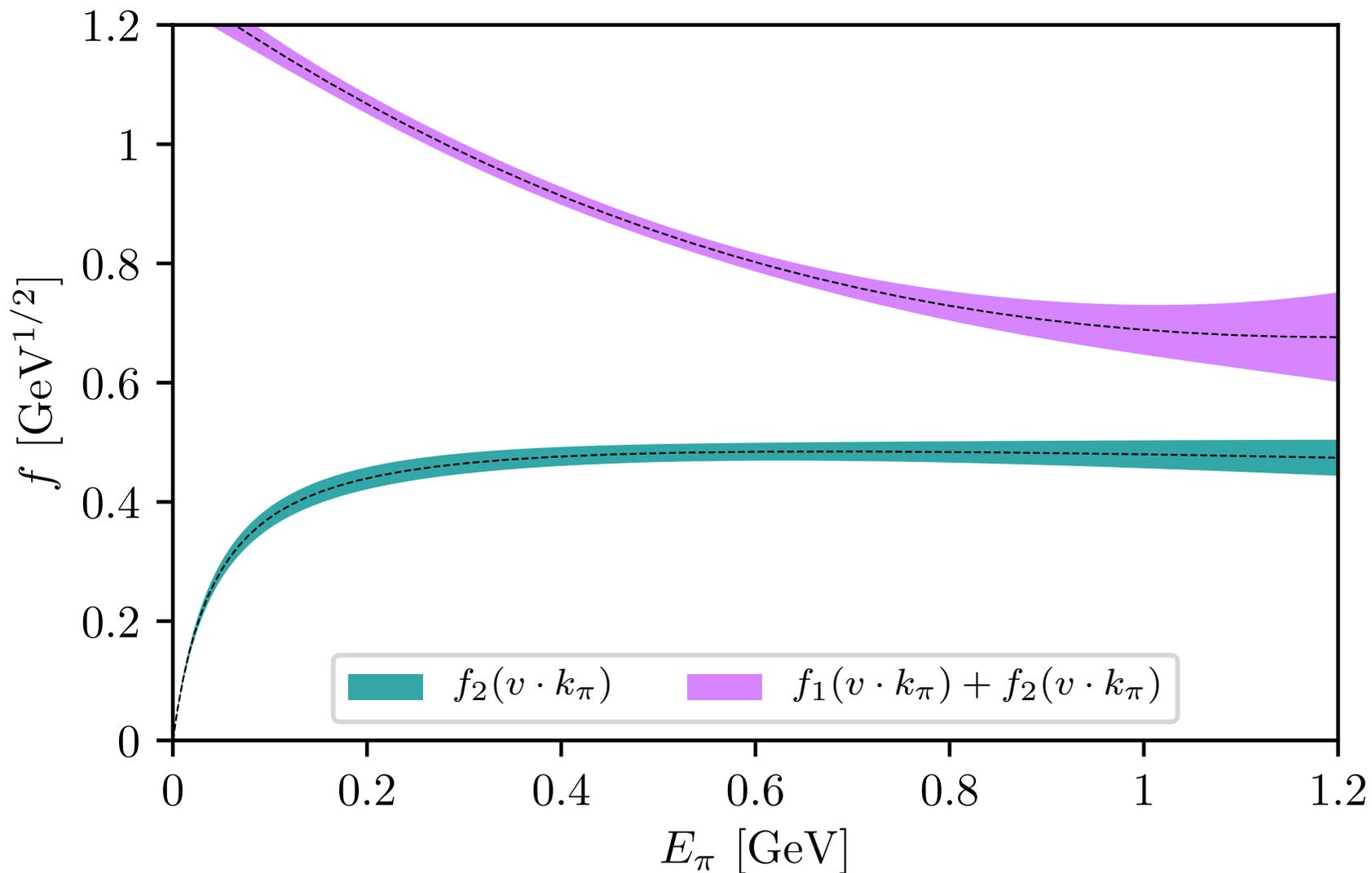$$|\mathbf{p}|^2 = (2\pi/L)^2; \quad m_\pi = 0.3 \text{ GeV}; \quad 1.25^2 \times m_c$$

Form factors vs m_h
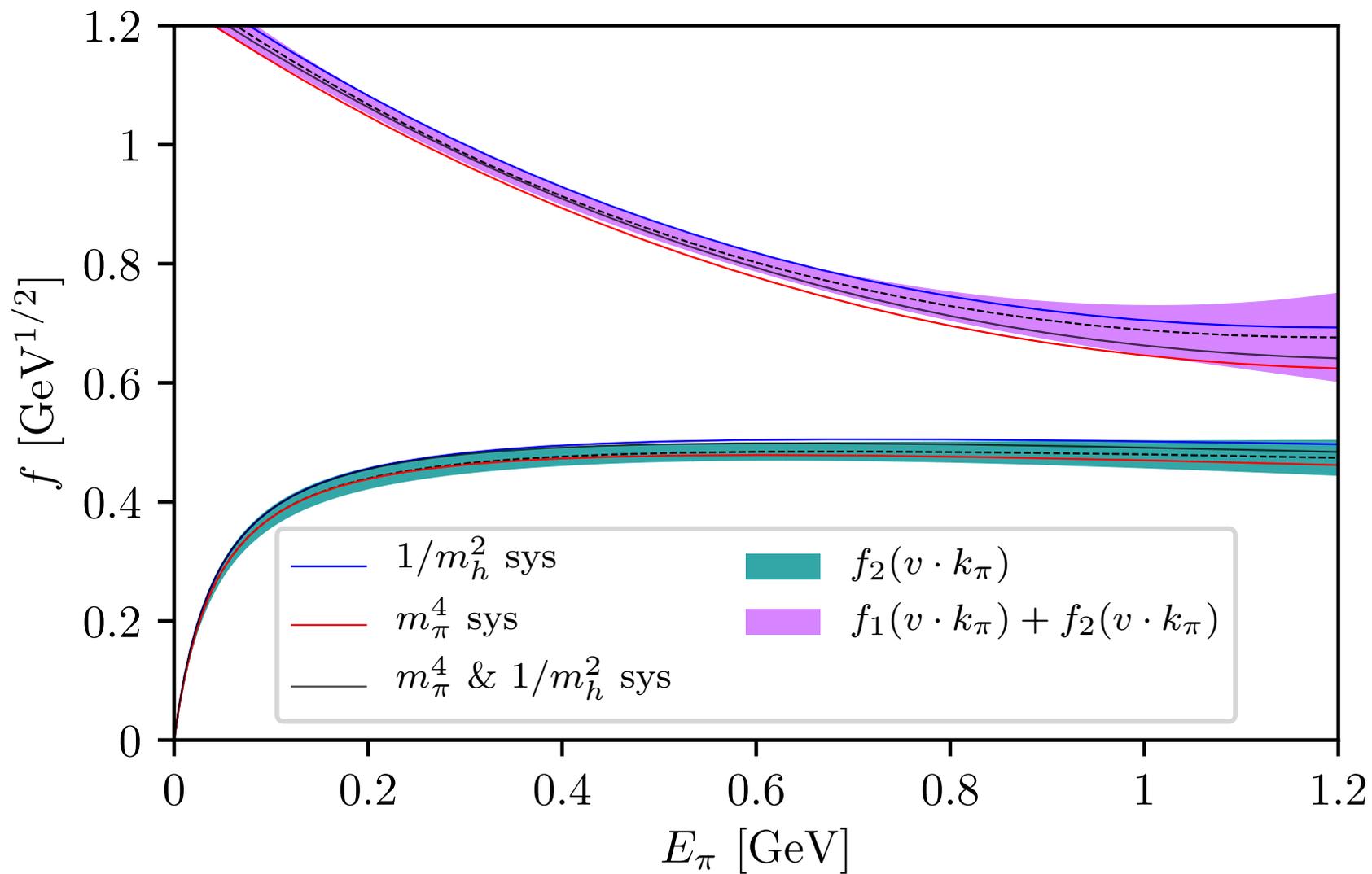

$\beta = 4.35; \ m_\pi = 0.5$ GeV

Form factors $f_1 + f_2$ and f_2

Systematic effects

- Systematic effects vary across q^2

- Adding higher powers of E_π (E_π^4 and E_π^2 for f_1+f_2 and f_2 respectively): a couple of %

- Adding M_π^4 terms: around -5%

- Adding $1/m_h^2$ terms: around +5%

- No chiral log: <1%

Systematic effects

z-expansion

semileptonic region

- q^2 range is large, and z is often used instead:

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$
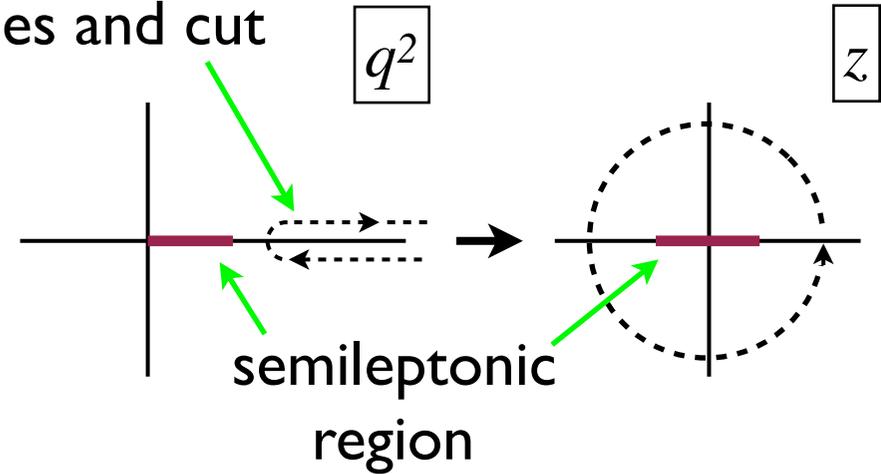
where $t_+ = (M_B + M_\pi)^2$ and we choose

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Full kinematic range is now $-0.3 < z < 0.3$
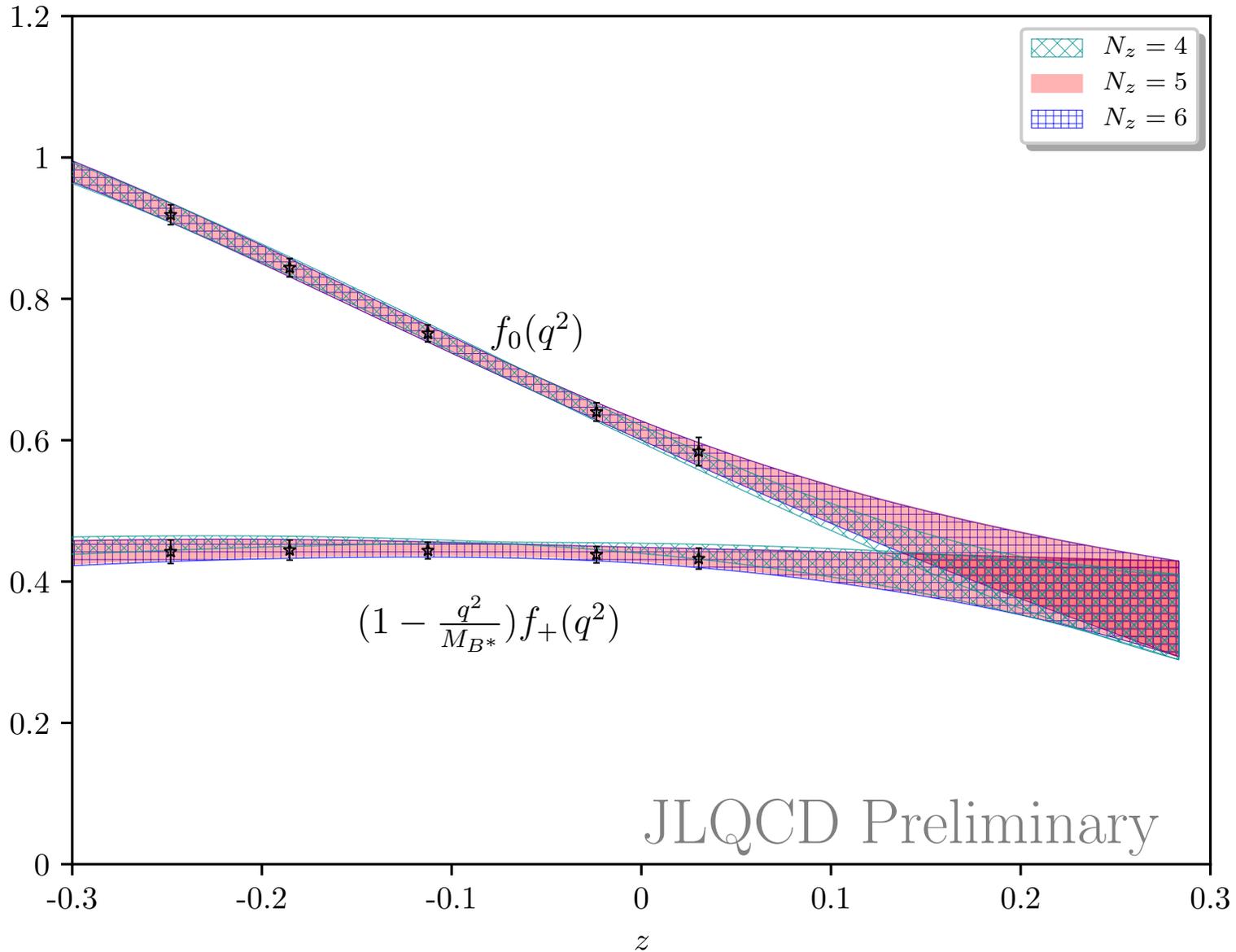
- Fit functions for the form factors are

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z - 1} b_n \left[z^n - (-1)^{n - N_z} \frac{n}{N_z} z^{N_z} \right]$$
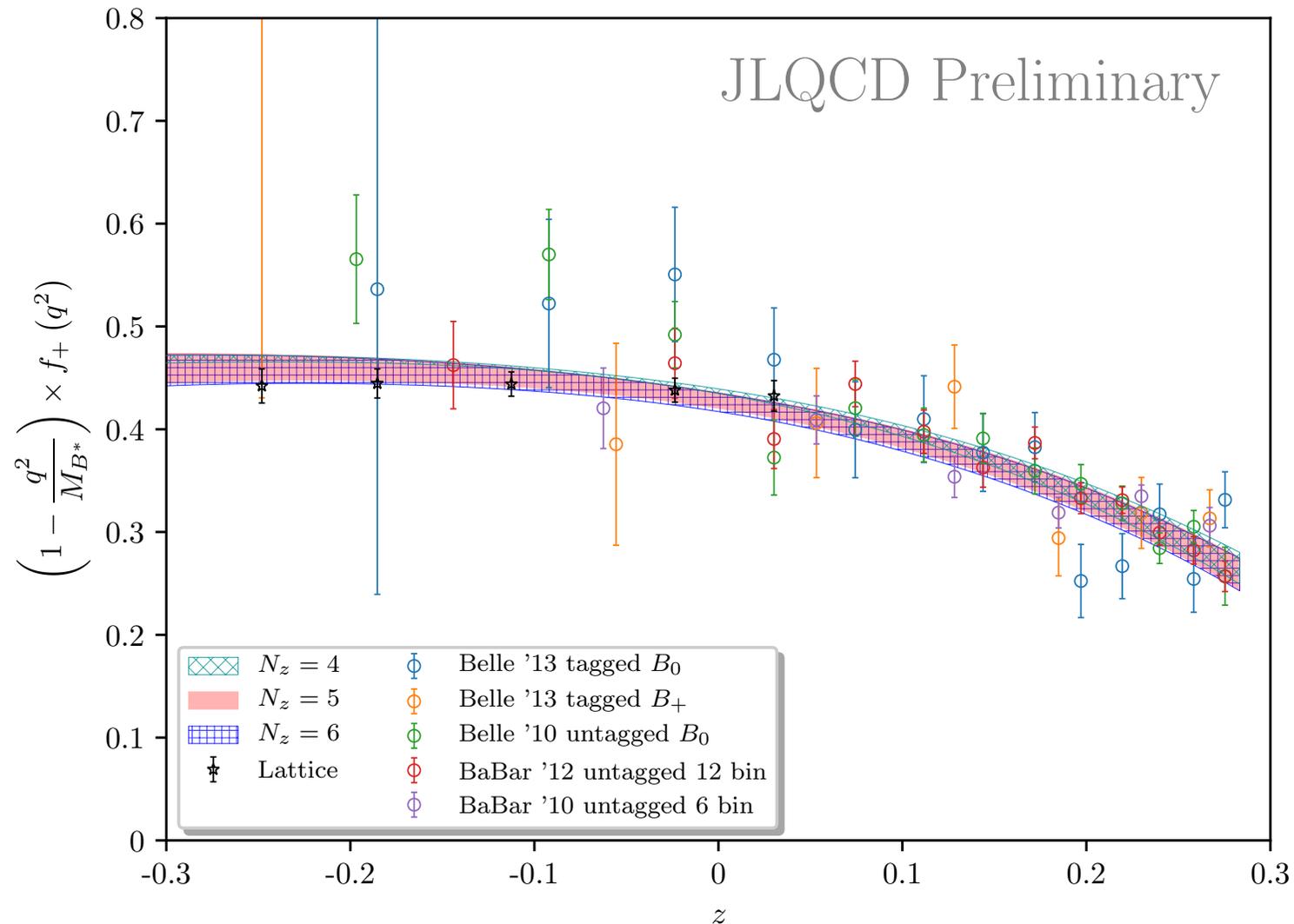
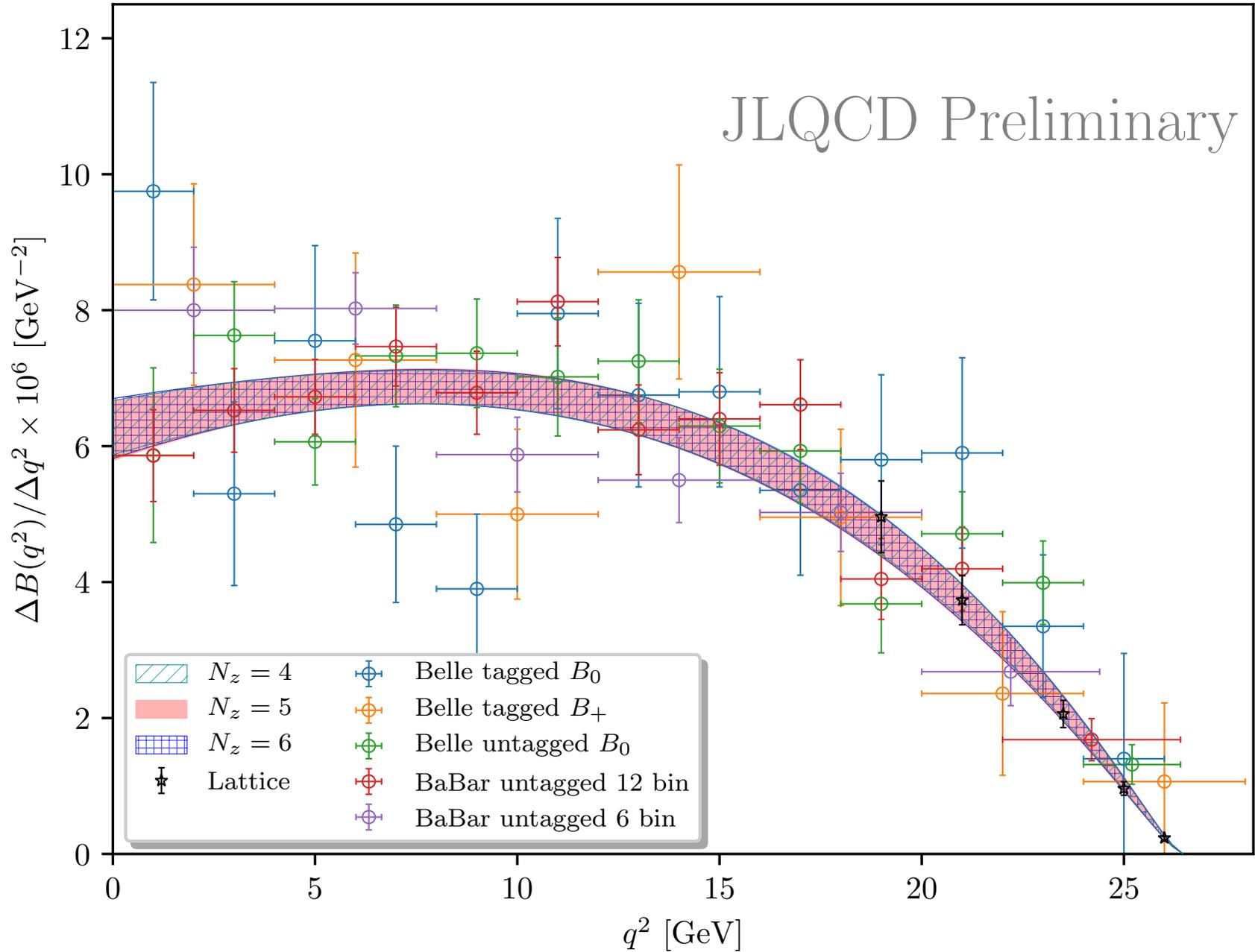$$f_0(z) = \sum_{n=0}^{N_z - 1} a_n z^n$$
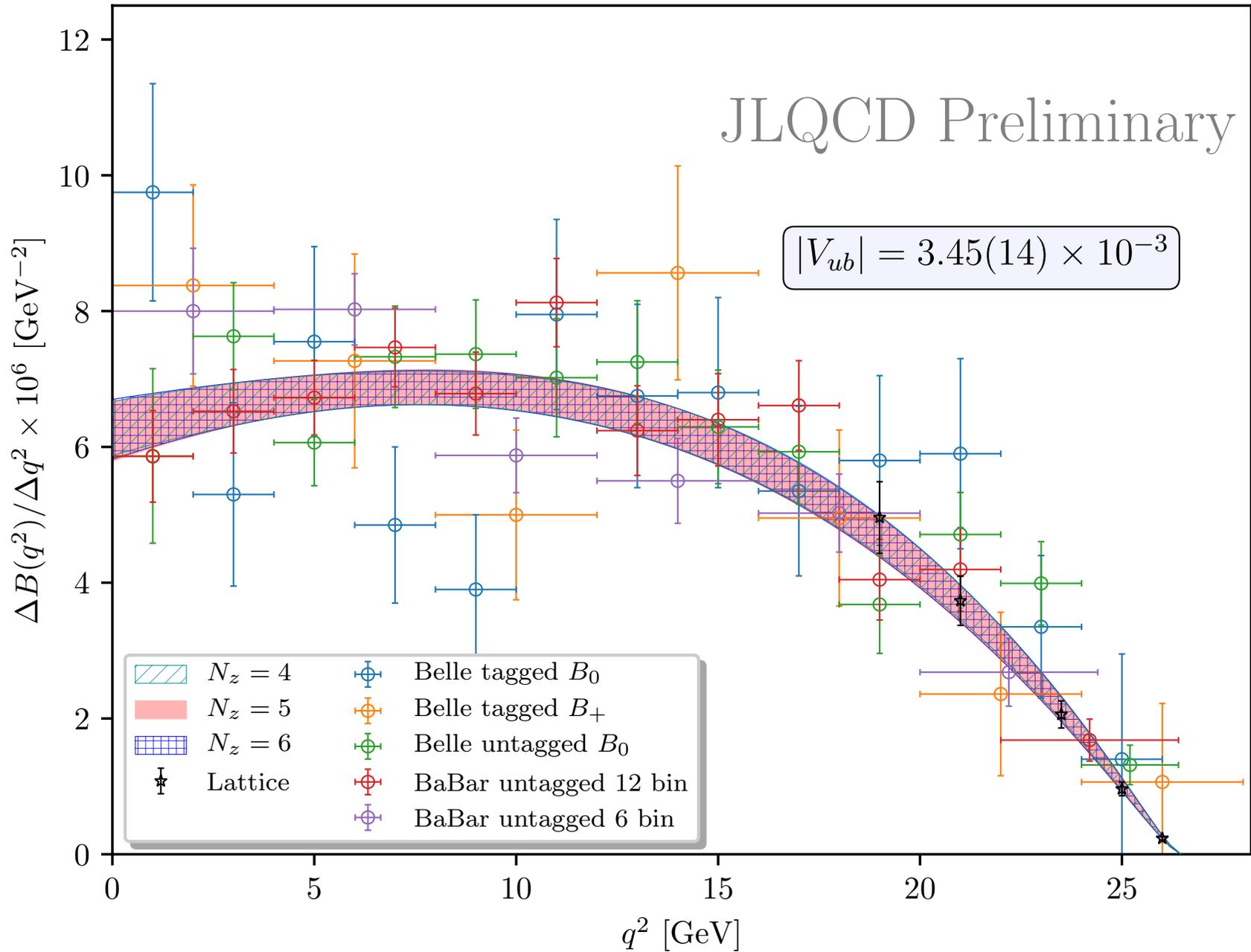
z-parameter expansion & fit

Combined fit

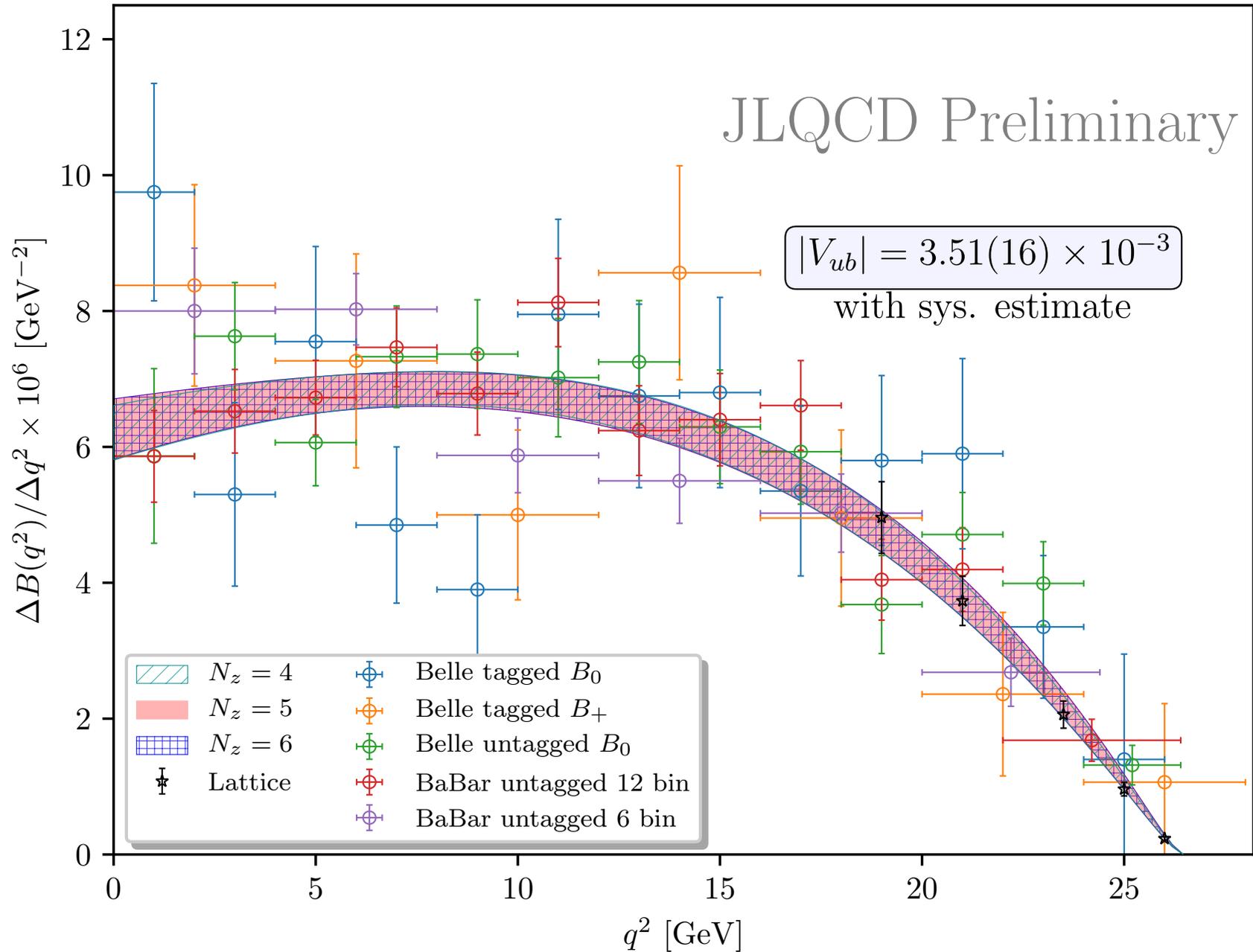- Use z-expansion to fit both lattice and exptl. data, including V_{ub} as a fit parameter
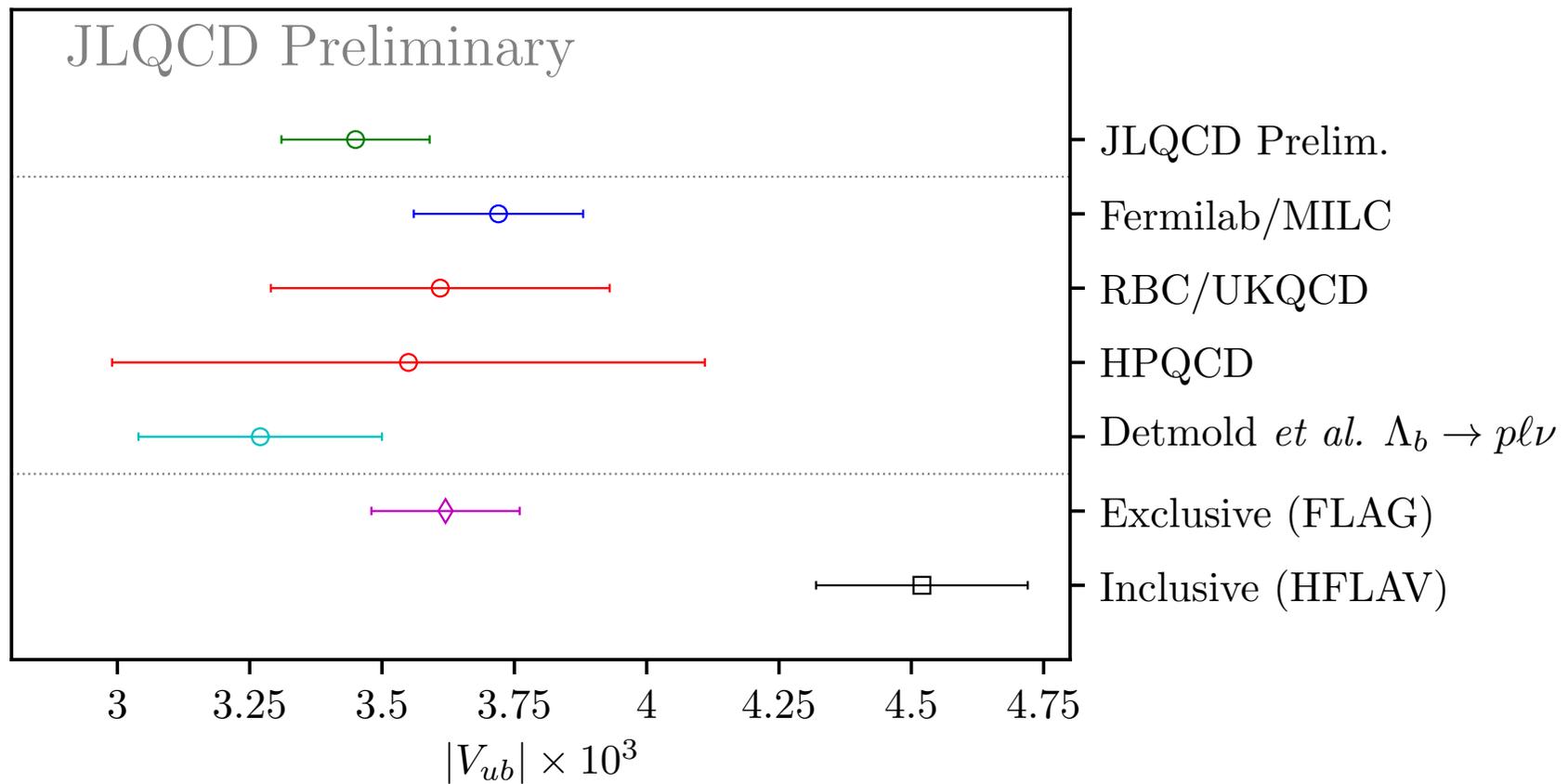
Combined fit

Combined fit

Combined fit

JLQCD Preliminary

$|V_{ub}| = 3.51(16) \times 10^{-3}$

with sys. estimate

Legend:
- $N_z = 4$
- $N_z = 5$
- $N_z = 6$
- Lattice
- Belle tagged B_0
- Belle tagged B_+
- Belle untagged B_0
- BaBar untagged 12 bin
- BaBar untagged 6 bin

Y-axis: $\Delta B(q^2)/\Delta q^2 \times 10^6 \ [\text{GeV}^{-2}]$

X-axis: $q^2 \ [\text{GeV}]$
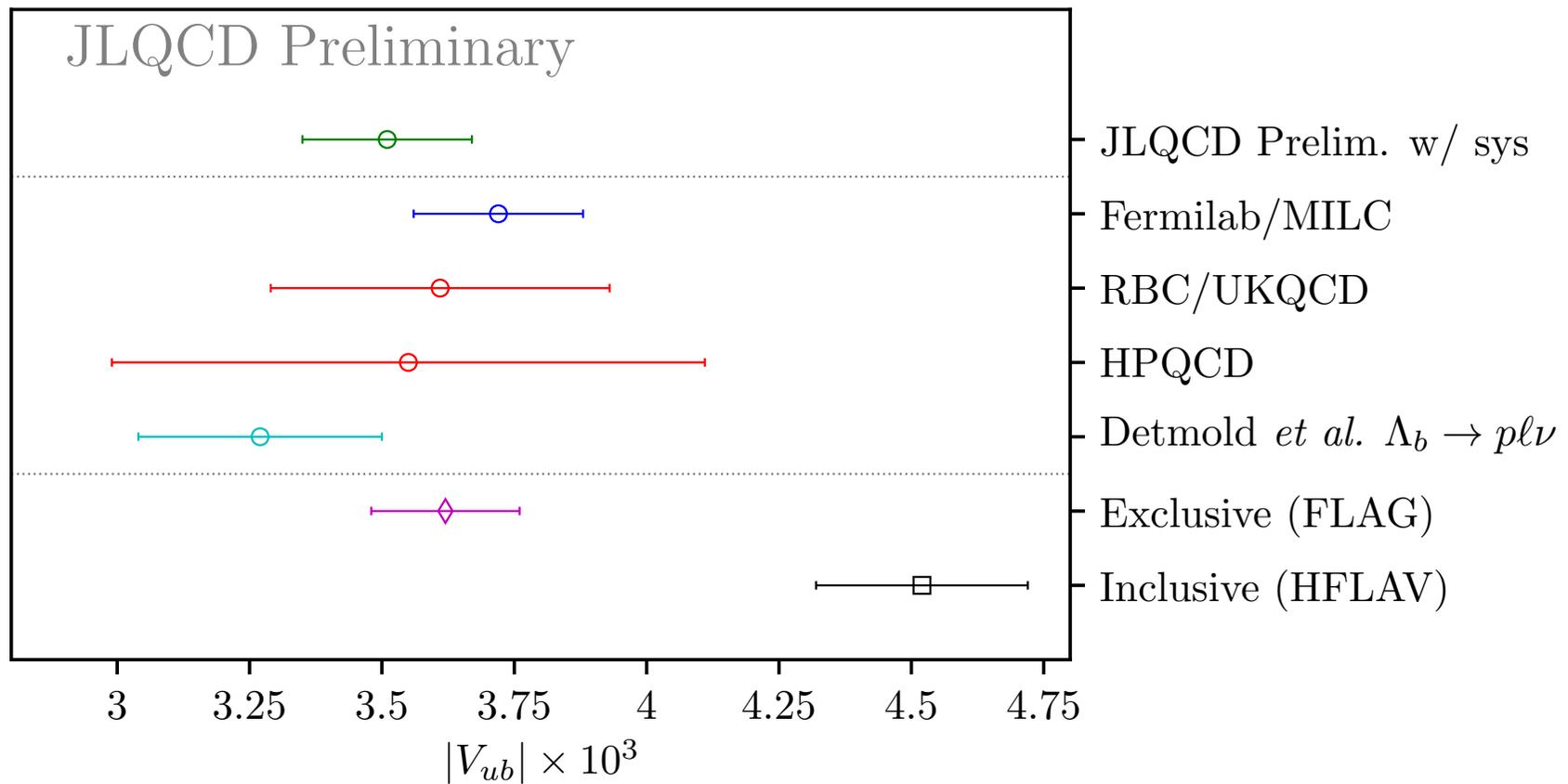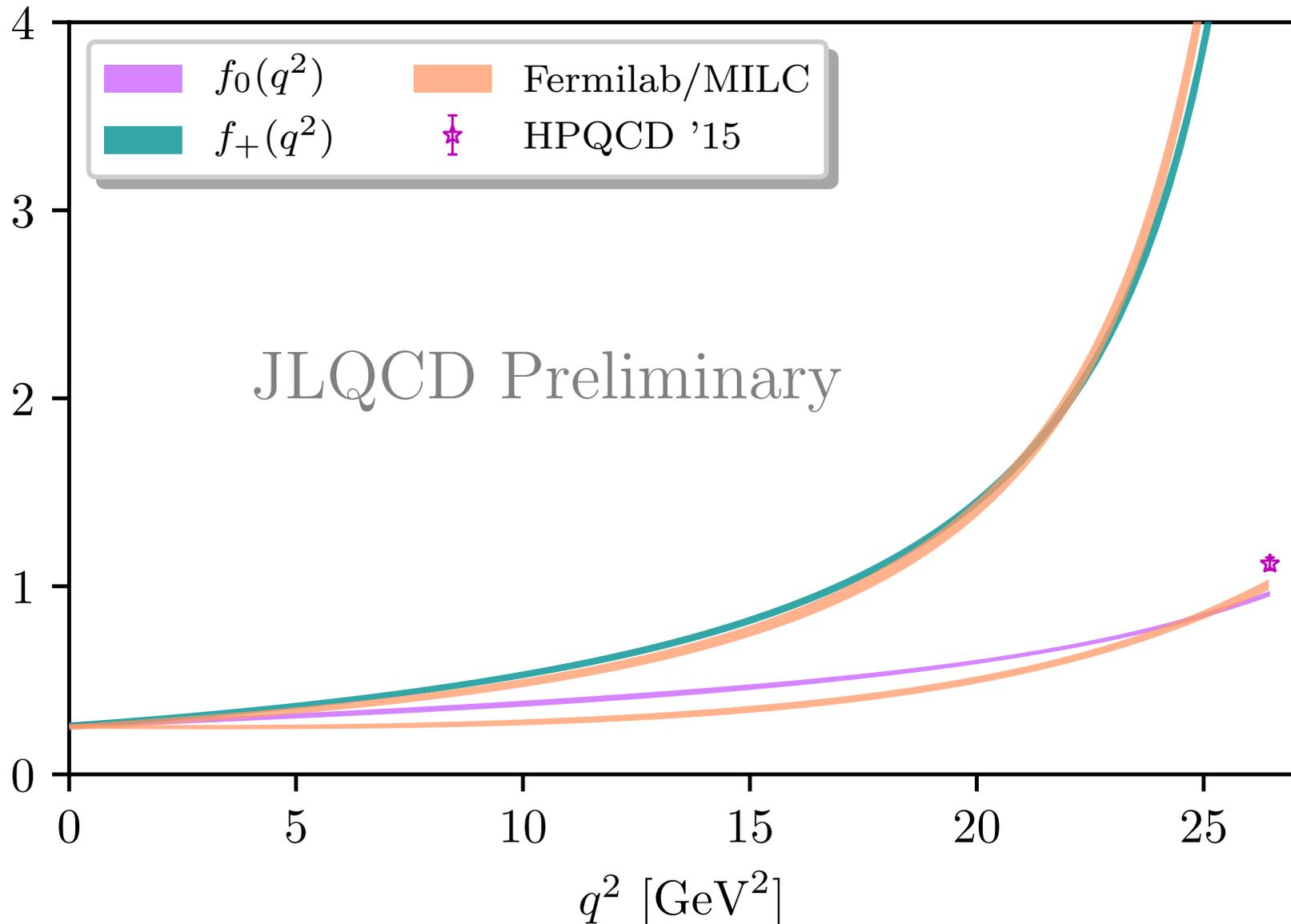
CKM element V_{ub}

CKM element V_{ub}

Form factor comparison

Summary

- $B \to \pi l \nu$ using Möbius Domain Wall fermions for all quarks
  - heavy quark treated relativistically
  - use multiple $m_h > m_c$ and extrapolate to m_b
  - use multiple light quark masses and extrapolate to physical M_π

- Fit form factors calculated on the lattice together with experimental partial branching fractions to get $|V_{ub}|$

Thank you!

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