Extraction of CKM matrix elements from lattice QCD results using dispersion relations

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3. Study of Lattice QCD data
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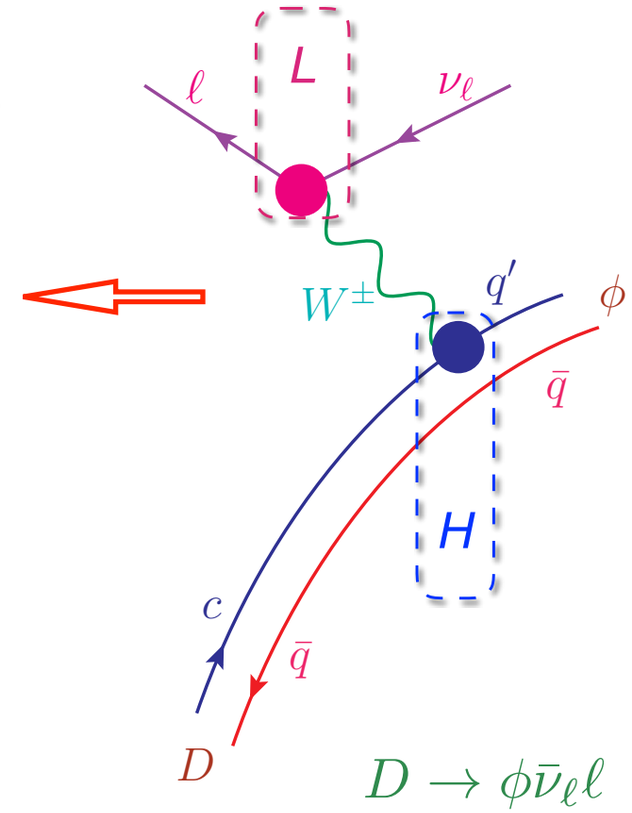
The 37th International Symposium on Lattice Field Theory (Lattice 2019)

Based on works [Eur.Phys.J. C78 (2018) 310] and [arXiv:1906.00727]

INTRODUCTION: DI3 DECAY & FORM FACTORS

- Invariant amplitude: (G_F—Fermi constant, V_{cq}—*CKM* matrix element)

$$\mathcal{M} = \frac{G_F V_{cq}}{\sqrt{2}} \underbrace{\left\{\bar{u}(p_\ell)\gamma^\mu(1-\gamma_5)v(p_\nu)\right\}}_{L^\mu} \underbrace{\left\{\langle\phi(p')|\bar{q}\gamma_\mu(1-\gamma_5)c|D(p)\rangle\right\}}_{H_\mu}$$

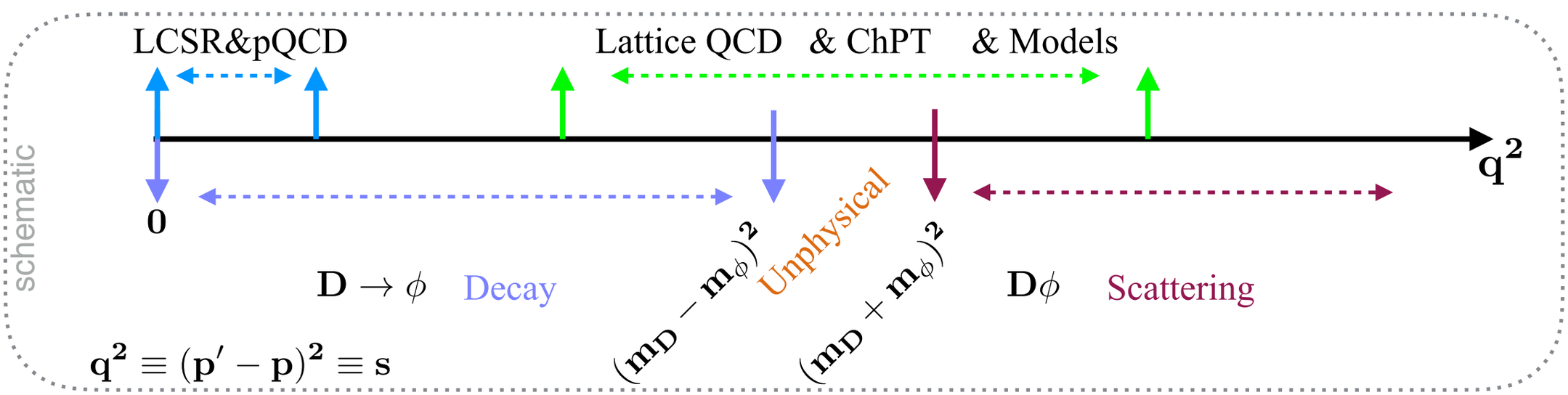


- Hadronic part:

$$H_\mu = f_+(q^2)\left[(p+p')^\mu - \frac{m_D^2 - m_\phi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_D^2 - m_\phi^2}{q^2}q^\mu$$

Vector FF $\mathbf{J^P = 1^-}$ ⟷ $\mathbf{f_+(0) = f_0(0)}$ ⟷ Scalar FF $\mathbf{J^P = 0^+}$

- Motivation: **inspired by [Flynn & Nieves PRD 75 (2007) 074024]**



✓ Study $f_0(q^2)$ in the whole decay region using **Dispersive techniques**

✓ Incorporate information in scattering region using **Watson Theorem**

} **(Muskhelishvili-) Omnes formalism**

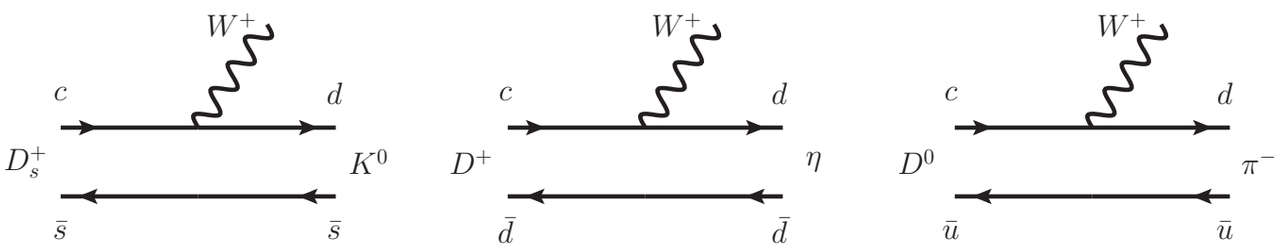
MUSKHELISHIVILI-OMNES (MO) FORMALISM

[N.I. Muskhelishvili, Singular integrals equations, (P. Nordhof 1953)] [R. Omnes, Nuovo Cim. 8, 613 (1958)]

- Scalar form factors in coupled channels:

✓ $\mathbf{c \to d}$ **induced**

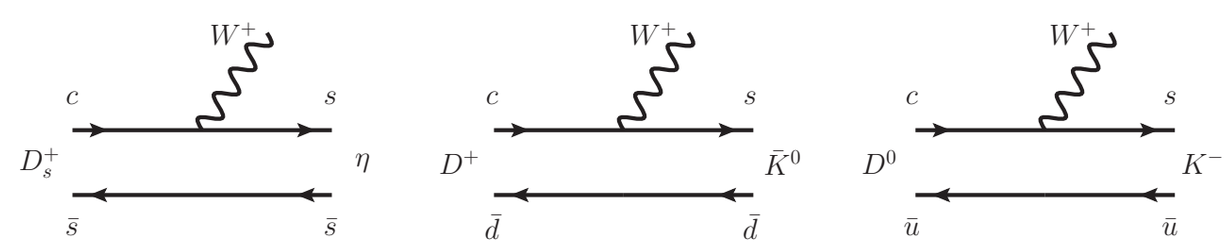
$$\begin{pmatrix} -\sqrt{\frac{3}{2}} f_0^{D^0\to\pi^-}(s) \\ -f_0^{D^+\to\eta}(s) \\ -f_0^{D_s^+\to K^0}(s) \end{pmatrix} = \Omega^{(0,\frac{1}{2})}(s)\cdot\vec{\mathcal{P}}^{(0,\frac{1}{2})}(s)$$



$\mathbf{\Delta S = 0,\ \Delta I = \frac{1}{2}}$

✓ $\mathbf{c \to s}$ **induced**

$$\begin{pmatrix} -\sqrt{2} f_0^{D^0\to K^-}(s) \\ f_0^{D_s^+\to\eta}(s) \end{pmatrix} = \Omega^{(1,0)}(s)\cdot\vec{\mathcal{P}}^{(1,0)}(s)$$



$\mathbf{\Delta S = 1,\ \Delta I = 0}$

- MO representation

✓ $\vec{\mathcal{F}}(s) = \Omega(s)\cdot\vec{\mathcal{P}}(s)$

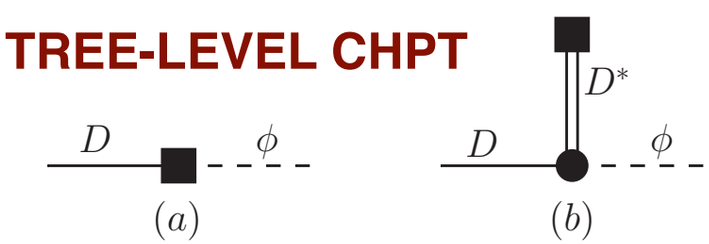
Unitarity

$$\mathrm{Im}\,\vec{\mathcal{F}}(s) = T^*(s)\Sigma(s)\vec{\mathcal{F}}(s)$$

✓ $\Omega(s) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty}\frac{T^*(s)\,\Sigma(s)\,\Omega(s)}{s'-s-i\epsilon}\mathrm{d}s'$ ➡ ***T*: DΦ interactions**

✓ $\vec{\mathcal{P}}(s) = \vec{\alpha_0} + \vec{\alpha_1}\,s\ .$ ➡ **Subtractions** constrained by **chiral matching**

✓ **TREE-LEVEL CHPT**



$$\mathcal{L}_0 = f_D\left(mD^*_\mu - \partial_\mu D\right)u^\dagger J^\mu\ ,$$
$$\mathcal{L}_1 = \beta_1 D\,u\,(\partial_\mu U^\dagger)J^\mu + \beta_2(\partial_\mu\partial_\nu D)\,u\,(\partial^\nu U^\dagger)\,J^\mu\ ,$$
$$\mathcal{L}_{DD^*\phi} = i\,\tilde{g}\left(D^*_\mu u^\mu D^\dagger - D\,u^\mu D^{*\dagger}_\mu\right)\ .$$

➡ Only 2 relevant unknown **LECs**

Dϕ INTERACTIONS IN UCHPT

- Chiral potentials from covariant ChPT & Unitarization

WE USE

✓ **NLO:** [Kolomeitsev & Lutz PLB582 (2004) 39] [Guo, et al, PLB641 (2006) 278] [Liu, et al, PRD87, 014508 (2013)] [Altenbuchinger, et al, PRD89, 014026 (2014)] ……

✓ **NNLO:** [Geng, et al, PRD82, 054022 (2010)] [Du, Guo, Meissner & Yao, JHEP11(2015)058 & EPJC77(2017)728]

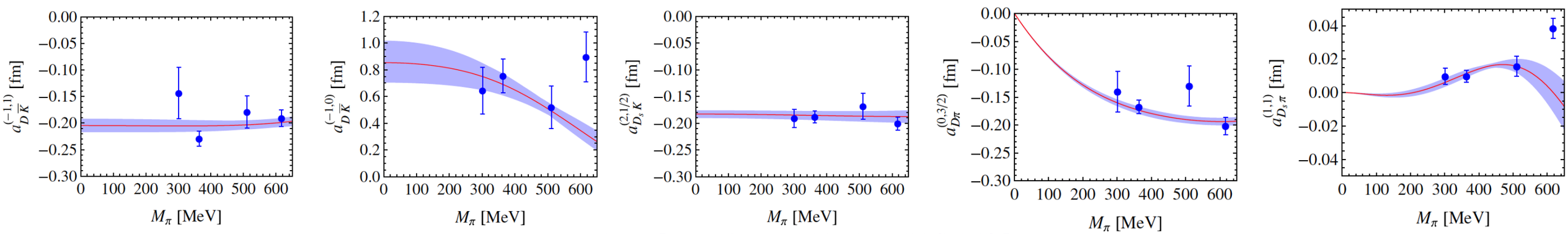
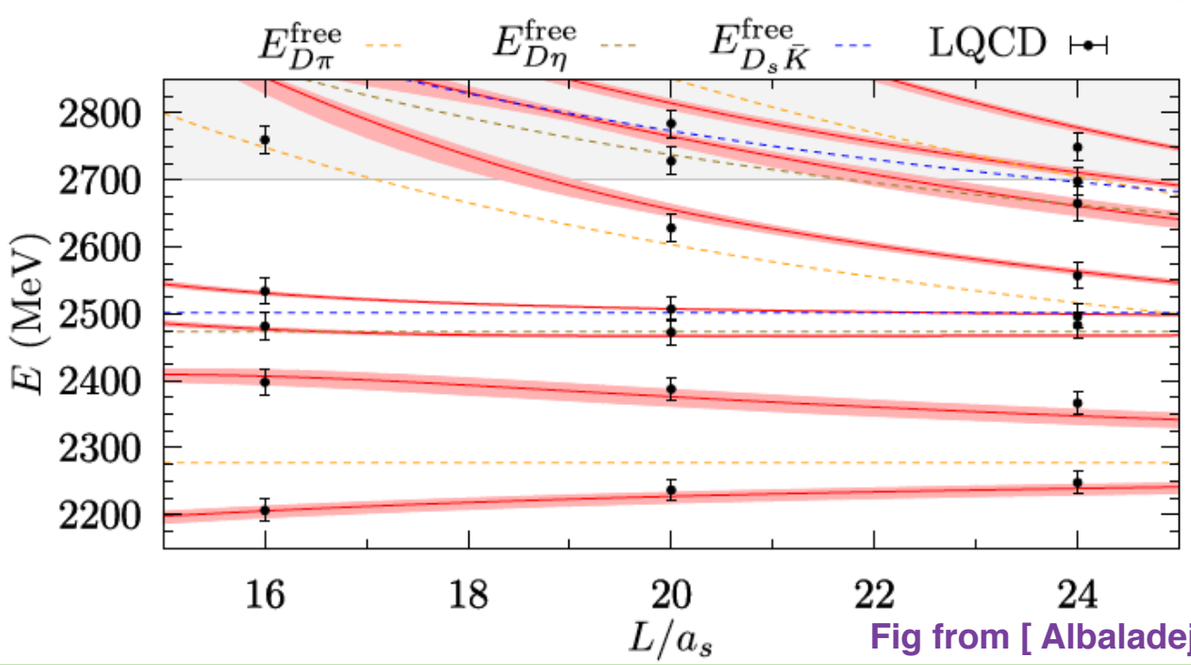


Fig from [Liu, et al, PRD87, 014508 (2013)]

- Applications of NLO potentials by Liu, et al

Energy Level **(S,I)=(0,1/2)** **Two-pole structure of D*(2400)**



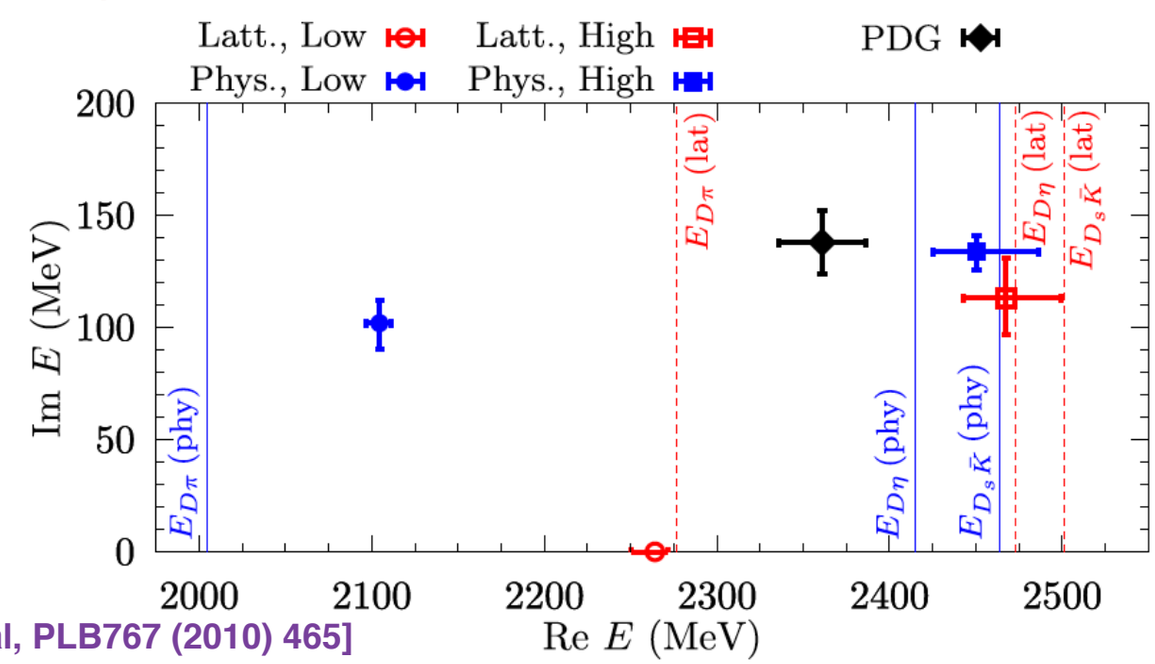


Fig from [Albaladejo, et al, PLB767 (2010) 465]

MO PROBLEM: I. INPUTS

- T-matrix:

$s_{\rm thr}$ — Threshold

$$T^U = \frac{V}{1 - VG}$$

ChPT & Unitarization

$s_{\rm m}$ — Matching point

T^H

Constructed from S-Matrix parameters with **Asymptotic Condition:**

$$X_i(s) = X_i(\infty) + \left[X_i(s_m) - X_i(\infty)\right] \frac{2}{1 + (s/s_m)^{3/2}}$$

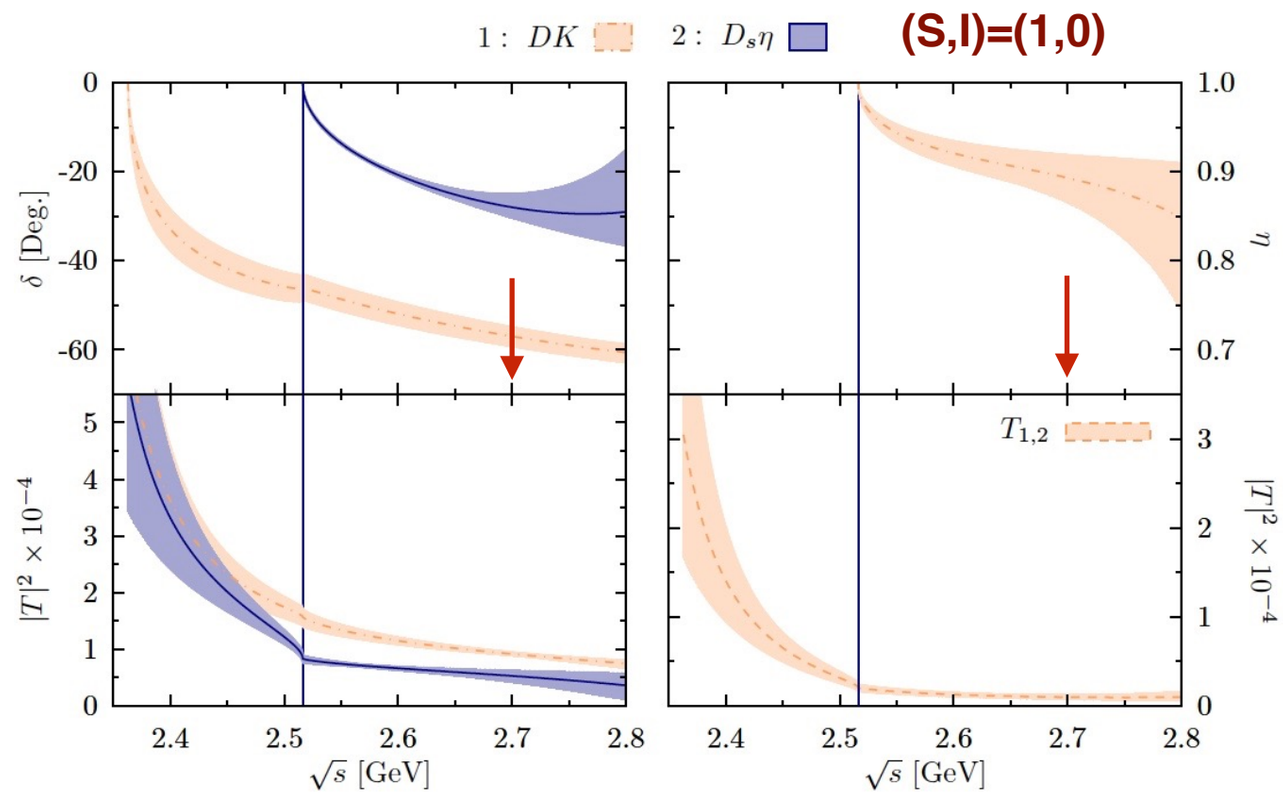
$$X \in \{\delta, \eta\} \qquad \delta_1(\infty) = n\pi \qquad \delta_{i\neq 1}(\infty) = 0$$

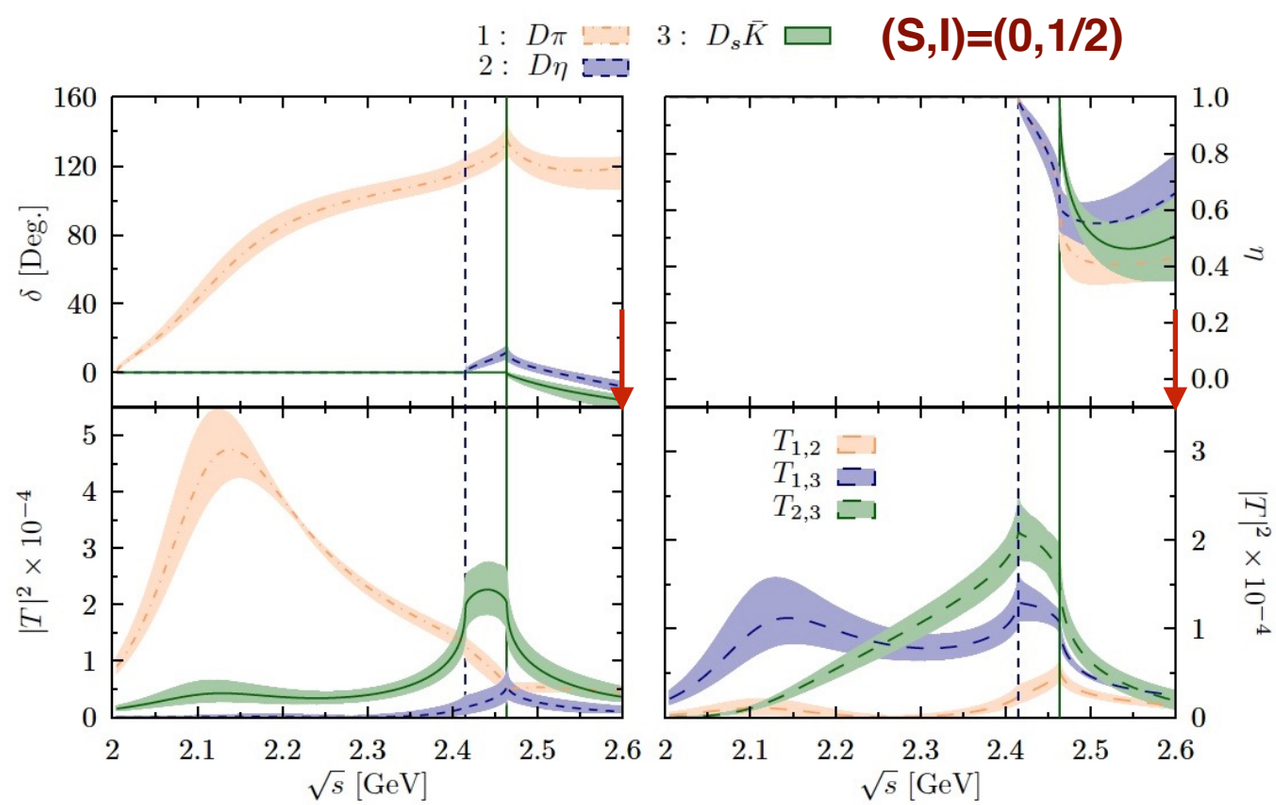
S-Matrix parametrisation

$$S_{ii}(s) = \eta_j e^{2i\delta_i} \qquad S_{ij}(s) = \gamma_{ij} e^{i\phi_{ij}}$$

$$S_{ij}(s) = \delta_{ij} + 2i\sigma_i^{1/2}\sigma_j^{1/2} T_{ij}(s)$$

∞

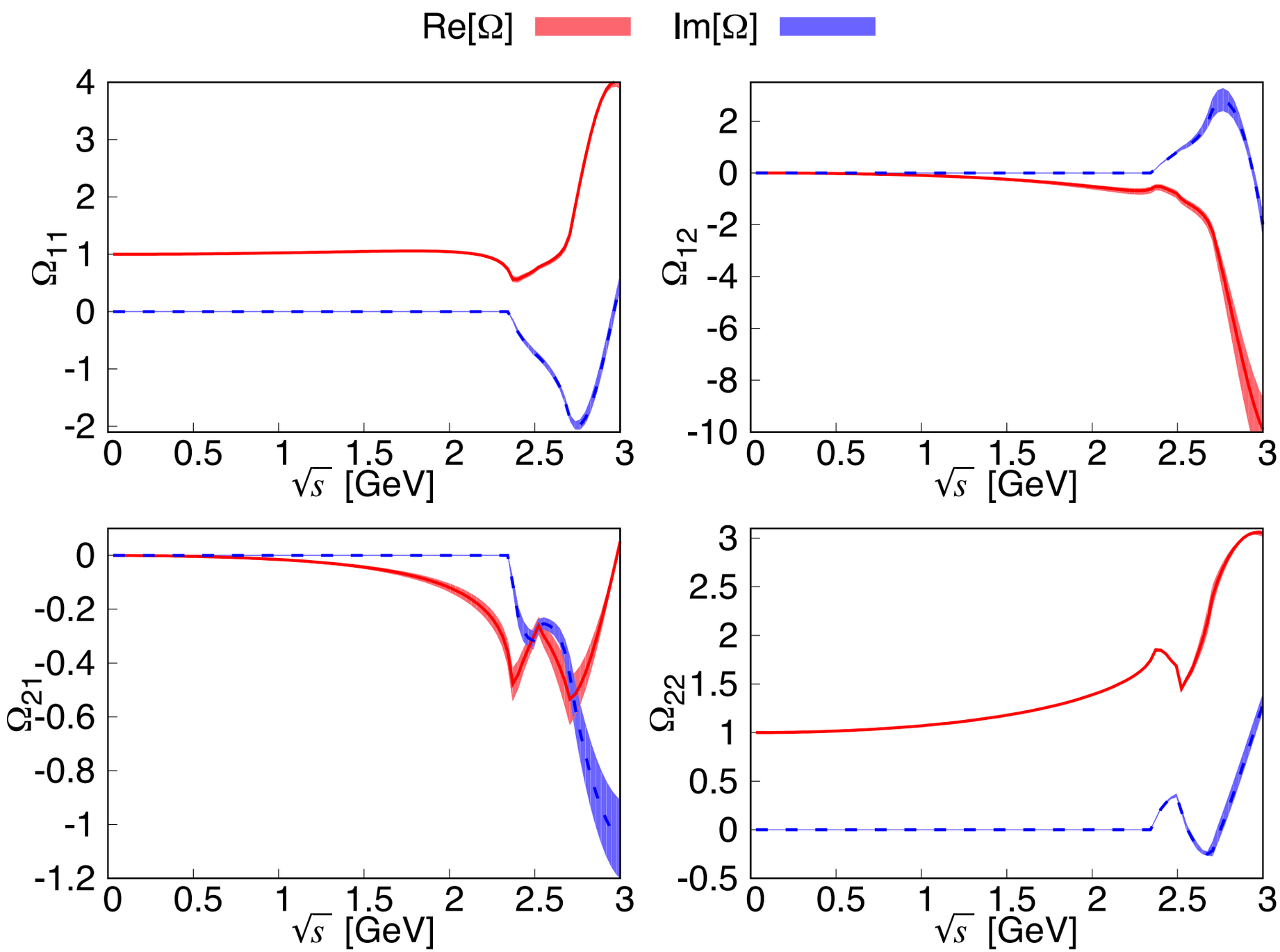




MO PROBLEM: II. SOLUTION

- MO-matrix for channel (S,I)=(1,0):

✓ **Normalised at s=0**

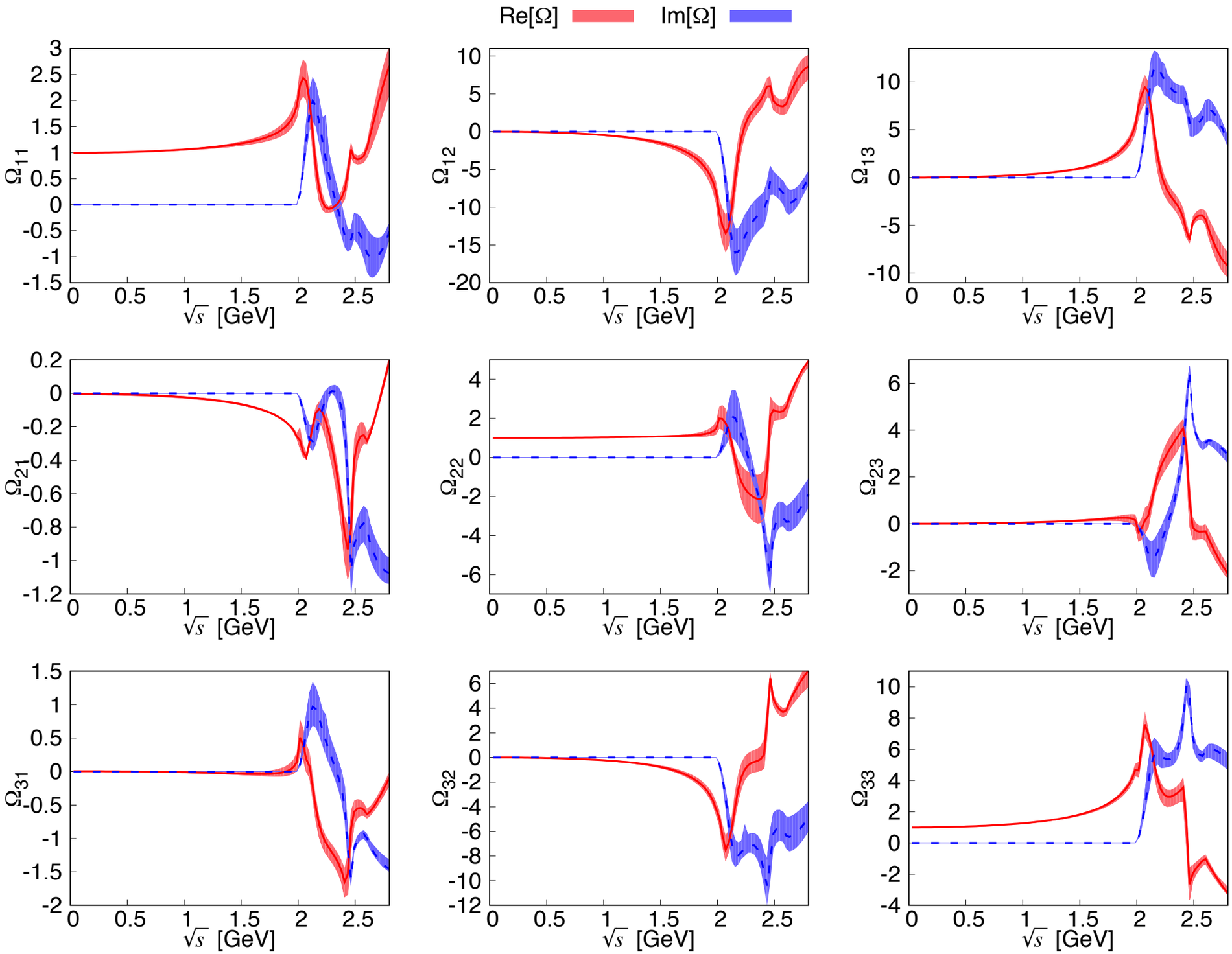


Error propagated from the uncertainties of LECs in chiral potential

MO PROBLEM: II. SOLUTION

- MO-matrix for channel (S,I)=(0,1/2):

✓ **Normalised at s=0**



Error propagated from the uncertainties of LECs in chiral potential

FIT TO LATTICE QCD DATA

- Latest data (8+8) with hypercubic effects by ETM [PRD96(2017)054514]

$$\begin{pmatrix} -\sqrt{\frac{3}{2}} f_0^{D^0\to\pi^-}(s) \\ -f_0^{D^+\to\eta}(s) \\ -f_0^{D_s^+\to K^0}(s) \end{pmatrix} = \Omega^{(0,\frac{1}{2})}(s)\cdot\vec{\mathcal{P}}^{(0,\frac{1}{2})}(s) \qquad \begin{pmatrix} -\sqrt{2} f_0^{D^0\to K^-}(s) \\ f_0^{D_s^+\to\eta}(s) \end{pmatrix} = \Omega^{(1,0)}(s)\cdot\vec{\mathcal{P}}^{(1,0)}(s)$$

$$\downarrow$$

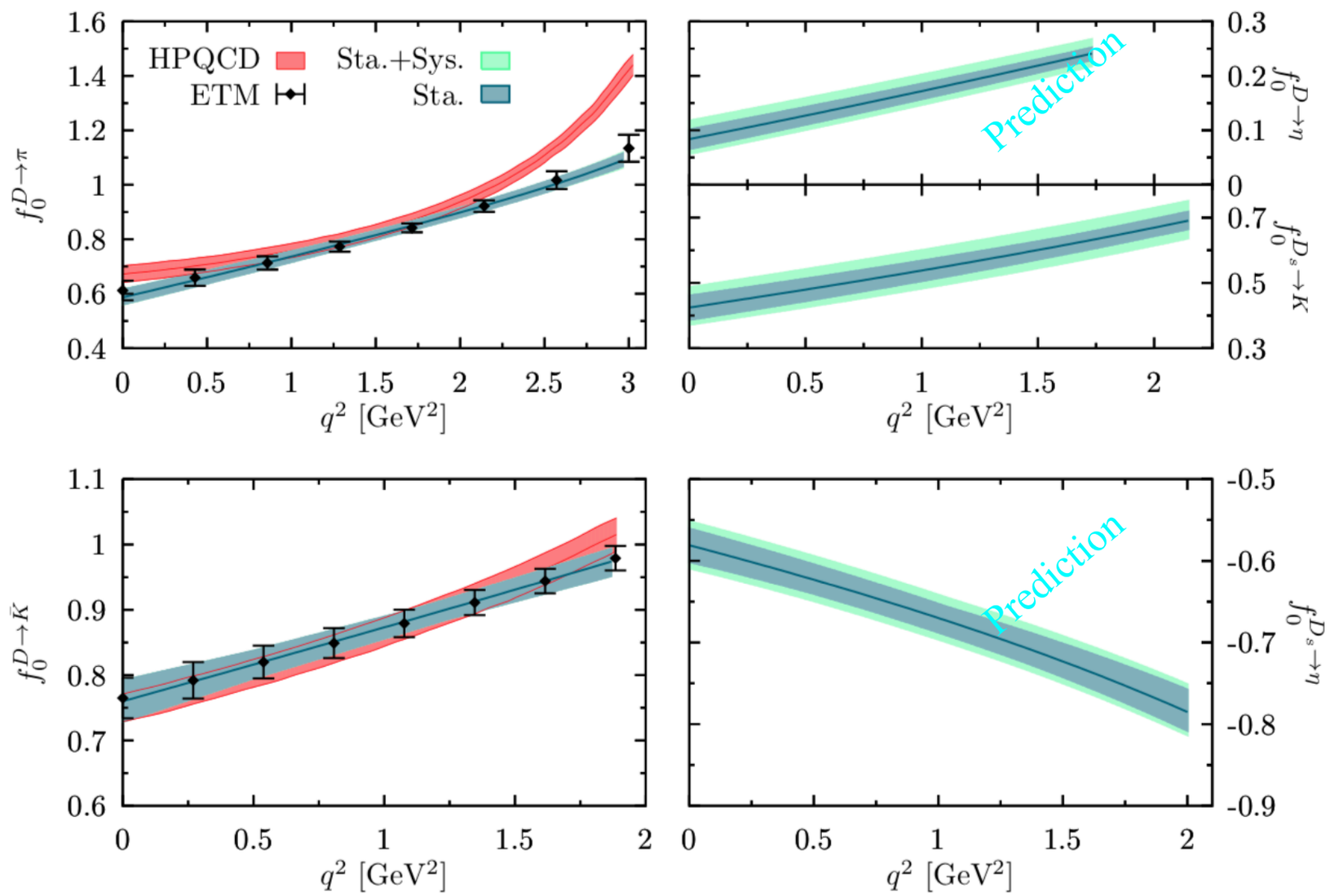
$$\frac{\beta_0\,\vec{\Gamma}}{s-s_p} + \vec{\mathcal{P}}^{(1,0)}(s)$$

- Contribution of $D_s^*(2317)$ below threshold

RESULTS

HPQCD ETM Sys. Fit

	Value	Correlation matrix			
χ^2	1.82	β_1	β_2	β_0	δ_χ
β_1	0.08(2)	1	0.96	−0.03	0.68
β_2	0.07(1)		1	0.09	0.84
β_0	0.12(1)			1	0.11
δ_χ	−0.21(2)				1



Hypercubic effects should be taken into account in lattice simulation!

HPQCD: [Na, et al, PRD82(2010)114506]
[Na, et al, PRD84(2011)114505]

EXTENDED TO BOTTOM SECTOR: HQFS & SCALING

- Heavy Quark Flavor Symmetry (HQFS)
  - ➡ Formalism applies to bottom sector straightforwardly
  - ➡ However, LECs need to be adjusted

$b \to u$ **induced**

$$\begin{pmatrix} \sqrt{\frac{3}{2}} f_0^{\bar{B}^0 \to \pi^+}(s) \\ f_0^{B^- \to \eta}(s) \\ f_0^{\bar{B}_s^0 \to K^+}(s) \end{pmatrix} = \Omega_{\bar{B}}^{(0,\frac{1}{2})}(s) \cdot \vec{\mathcal{P}}_{\bar{B}}^{(0,\frac{1}{2})}(s)$$

- Scaling

$D(\bar{B})-\phi$ Interactions

$$h_{0,1,2,3} \sim m_Q \ , \qquad h_{4,5} \sim \frac{1}{m_Q}$$

$$h_{0,1,2,3}^B = \frac{\bar{m}_B}{\bar{m}_D} h_{0,1,2,3}^D \ , \qquad h_{4,5}^B = \frac{\bar{m}_D}{\bar{m}_B} h_{4,5}^D$$

$D(\bar{B}) \to \phi$ Decay

$$\beta_1 \sim m_Q^{\frac{1}{2}} \qquad \beta_2 \sim m_Q^{-\frac{3}{2}}$$

$$\beta_1^B = \sqrt{\frac{\bar{m}_B}{\bar{m}_D}} \beta_1^D (1+\delta) \ , \qquad \beta_2^B = \sqrt{\frac{\bar{m}_D^3}{\bar{m}_B^3}} \beta_2^D (1-3\delta)$$

- Combined fit: lattice data from both D and B sectors & LCSR results

$$\chi^2 = (\chi^2_{\rm cov})^{\bar{B}\to\pi} + (\chi^2_{\rm cov})^{\bar{B}_s\to K} + (\chi^2)^{\bar{B}\to\pi}_{\rm LCSR} + (\chi^2)^{B_s\to K}_{\rm LCSR} + (\chi^2_{\rm cov})^{D\to\pi} + (\chi^2_{\rm cov})^{D\to\bar{K}}$$

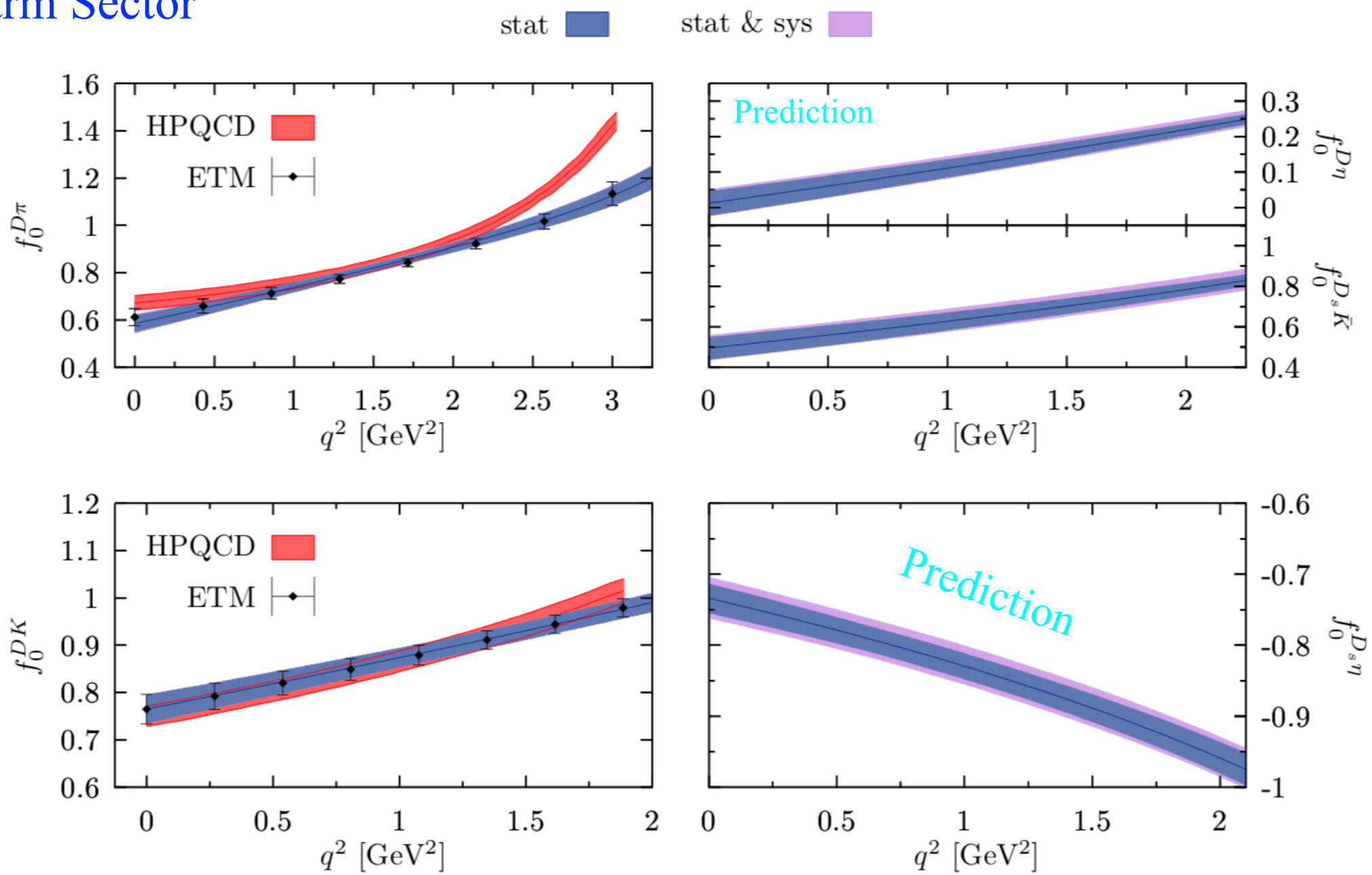
RBC&UKQCD — **[Flynn, et al, PRD91(2015)074510]**

LCSR — **[Bharucha, JHEP1205(2012)092]** **[Duplancic, et al, JHEP0804(2008)014]**

ETM — **[Arxiv:1706.03017]**

EXTENDED TO BOTTOM SECTOR: COMBINED FIT

- Charm Sector



✓ Here almost the same resutls are obtained as the fit only to data of charm sector

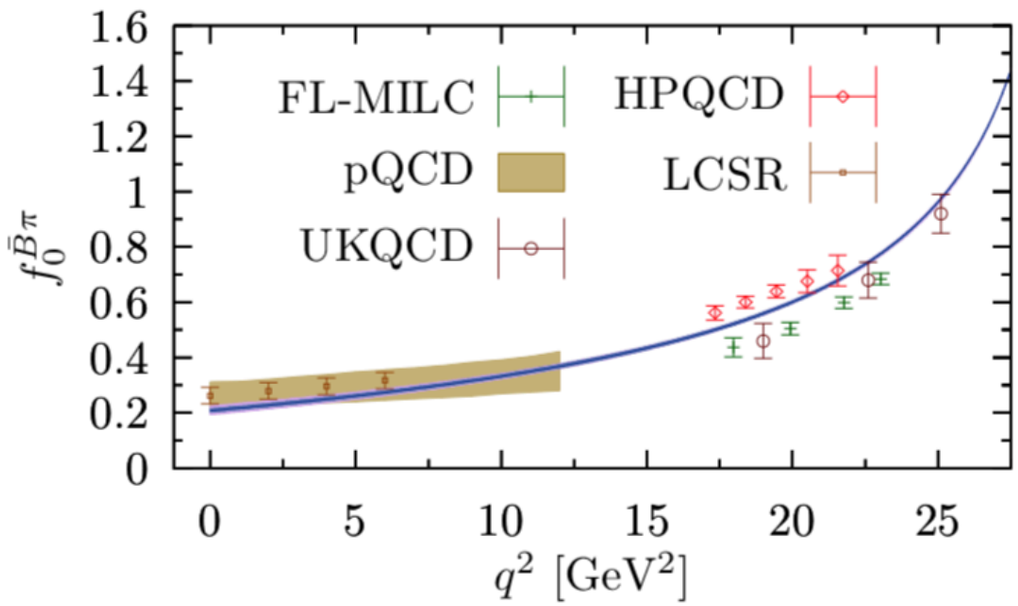
$$f_+^{D\to\pi}(0)|V_{cd}| = 0.1426(19) \qquad |V_{cd}| = 0.244(22)$$

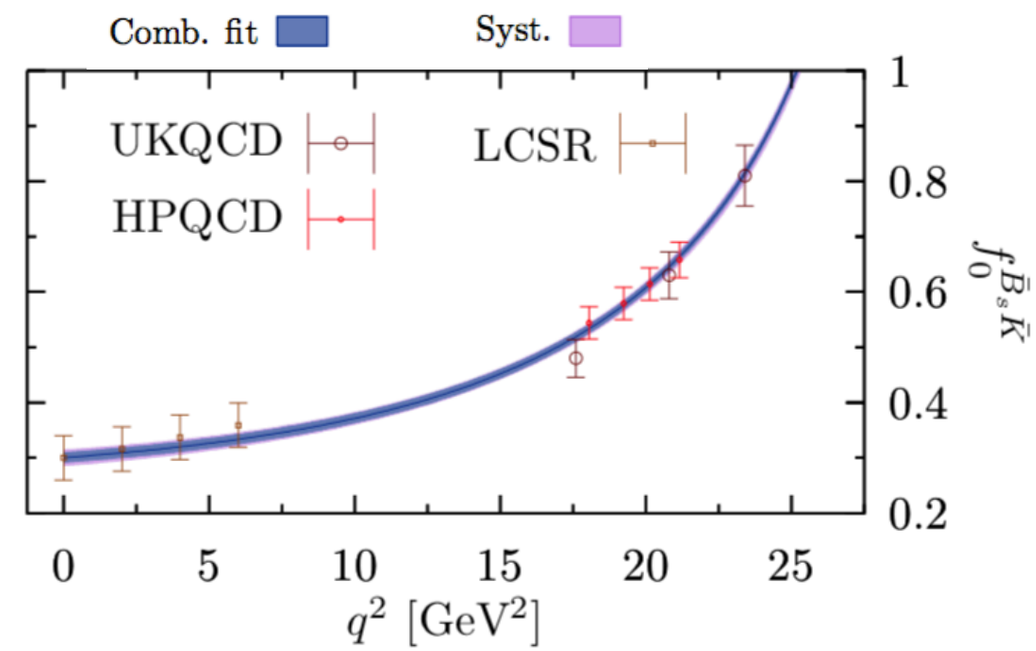
$$f_+^{D\to\bar{K}}(0)|V_{cs}| = 0.7226(34) \qquad |V_{cs}| = 0.945(41)$$

CKM →

EXTENDED TO BOTTOM SECTOR: COMBINED FIT

- Bottom Sector

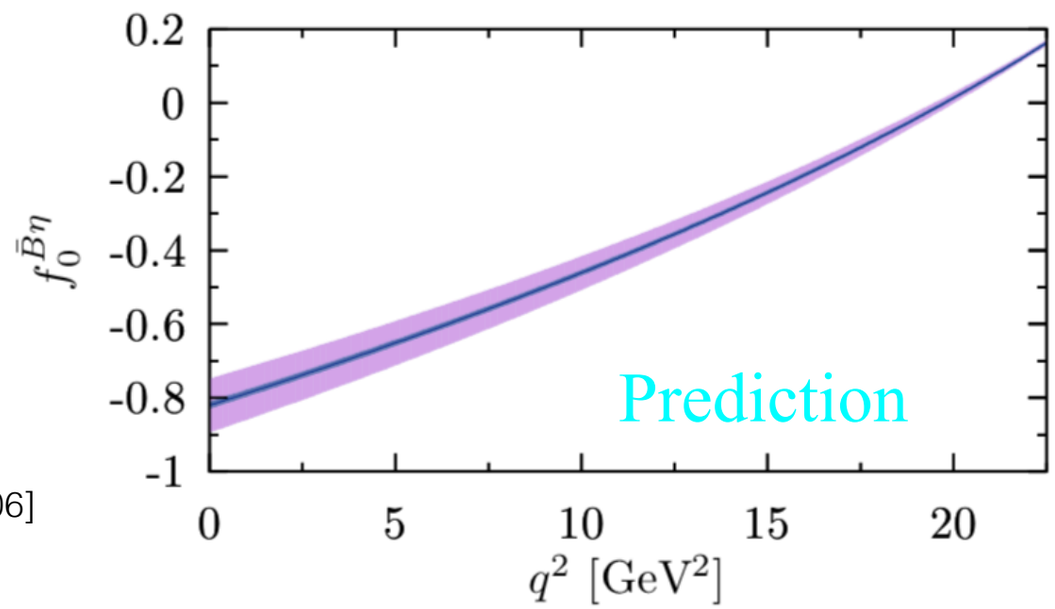




FL+MILC:
[Bailey, et al, PRD92(2015)014024]

pQCD:
[Li, et al, PRD85(2012)074004]
[Wang, et al, PRD86(2012)114025]

HPQCD:
[Dalgic, et al, PRD73(2006)074502]
[Bouchard, et al, PRD90(2014)054506]



$\frac{\chi^2}{dof} = 2.77$	
β_0	0.152(14)(13)
β_1^B	0.22(4)(4)
β_2^B	0.0346(16)(15)
δ	0.138(21)(18)
δ'	−0.18(4)(2)

$$f_+^{\bar{B}\to\pi}(0)|V_{ub}| = (8.9 \pm 0.3) \times 10^{-4}$$

CKM ↓

$$10^3|V_{ub}| = 4.3(7)$$

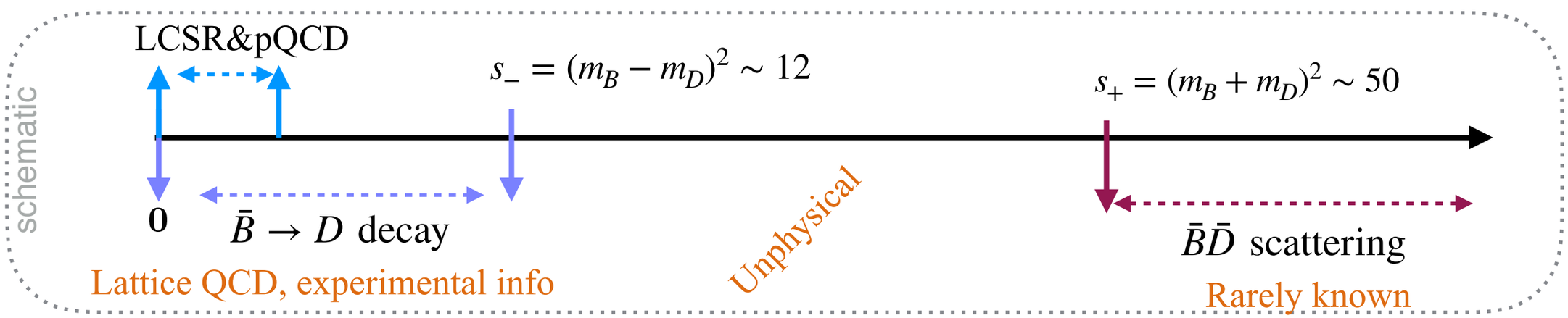
✓ Lattice QCD data in both sectors are well described simultaneously.

✓ Scalar FF are predicted in the whole kinematical region.

A NEW PARAMETRIZATION: B TO D TRANSITION

- Tensions
  - inclusive v.s. exclusive
    1. HFLAV average $|V_{cb}|_{\rm in} = (42.19 \pm 0.78) \times 10^{-3}$ [Amhis et al, EPJC77(2017)]
    2. Belle determination $|V_{cb}|_{\rm ex,CLN} = (39.86 \pm 1.33) \times 10^{-3}$ [Glattauer et al, PRD93(2016).]
  - theoretical v.s. experimental
    1. FL-MILC: $\mathcal{R}_D = 0.299(11)$; HPQCD: $\mathcal{R}_D = 0.300(8)$. [Bailey et al, PRD92(2015); Na et al, PRD92(2015)]
    2. BABAR : $\mathcal{R}_D = 0.440(58)(42)$; Belle: $\mathcal{R}_D = 0.375(64)(26)$ [Lees et al, PRL109(2012); Hushcle et al, PRD92(2015).]

- Kinematics for B to D decay

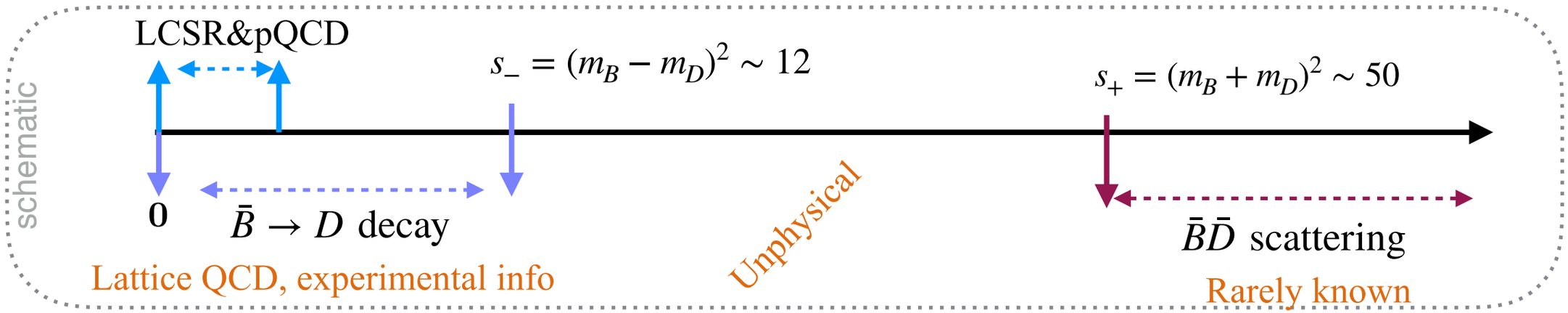


✓ Study *FF* in the whole decay region using **Dispersive techniques**
✓ Extract information from decay region to faraway scattering region

} **towards a new parametrization**

A NEW PARAMETRIZATION: B TO D TRANSITION

- B to D form factors



$$s' \geq s_+ \gg s_- \geq s_0 \Longrightarrow$$

Dispersive representation

$$f(s) = f(s_0)\exp\left[\frac{s - s_0}{\pi}\int_{s_+}^{\infty}\frac{\mathrm{d}s'}{s' - s_0}\frac{\alpha(s')}{s' - s - i\epsilon}\right]$$

$\alpha(s')$: phase of form factor near threshold
= phase shift (elastic region)

The new parametrization

$$f(s) = f(s_0)\prod_{n=0}^{\infty}\exp\left[\frac{s - s_0}{s_+}\mathscr{A}_n(t_0)\frac{s^n}{s_+^n}\right]$$

$$\mathscr{A}_n(s_0) \equiv \frac{1}{\pi}\int_{s_+}^{\infty}\frac{\mathrm{d}s'}{s' - s_0}\frac{\alpha(s')}{(s'/s_0)^{n+1}}$$

The coefficients $\mathscr{A}_n$ are called Phase Moments.

physically meaningful → important information on $\bar{B}\bar{D}$ interations

A NEW PARAMETRIZATION: B TO D TRANSITION

- Description of lattice QCD and experimental data

The new parametrization

$$f(s) = f(s_0)\prod_{n=0}^{\infty}\exp\left[\frac{s-s_0}{s_+}\mathscr{A}_n(t_0)\frac{s^n}{s_+^n}\right]$$

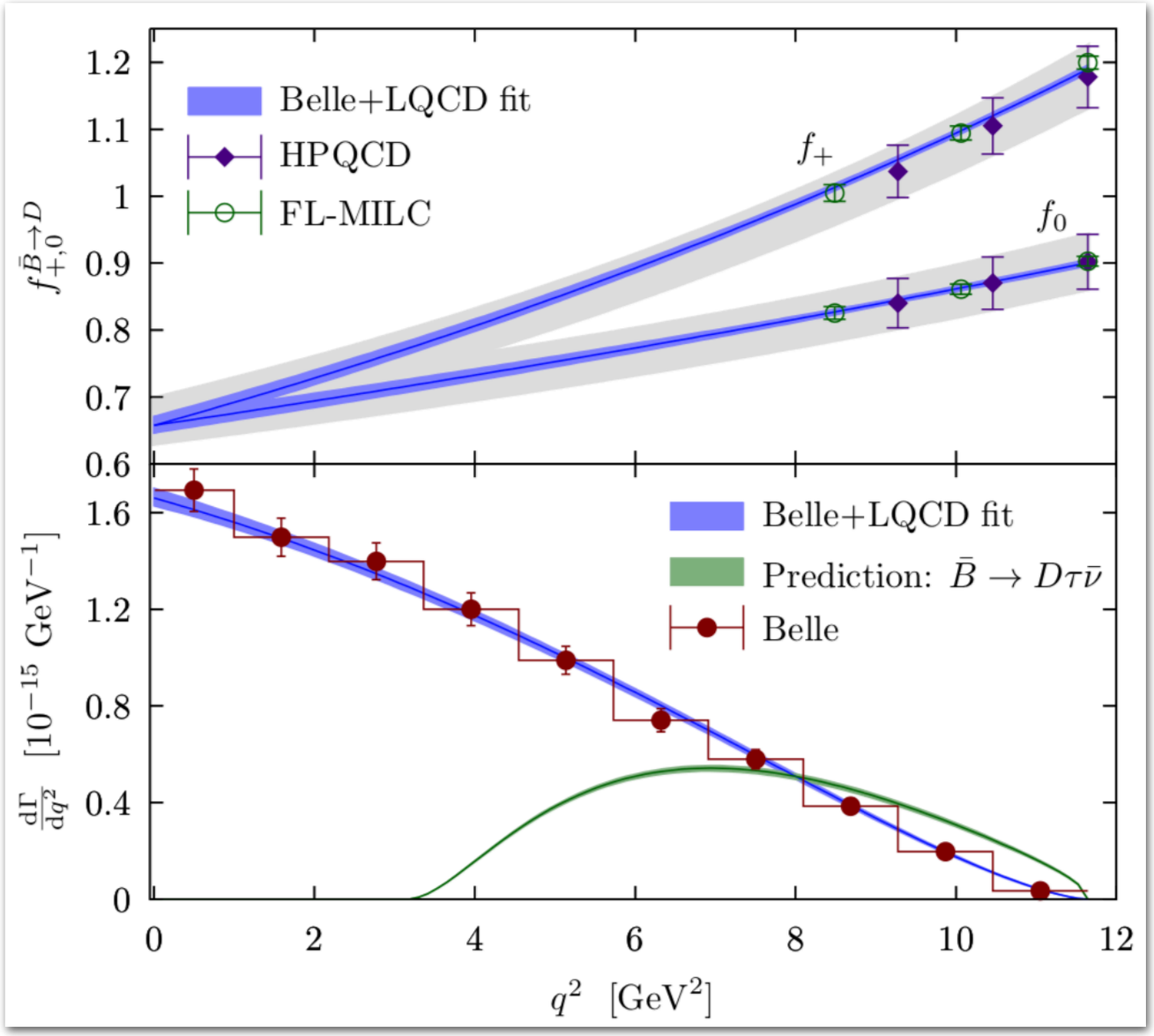
$$\mathscr{A}_n(s_0) \equiv \frac{1}{\pi}\int_{s_+}^{\infty}\frac{\mathrm{d}s'}{s'-s_0}\frac{\alpha(s')}{(s'/s_0)^{n+1}}$$

Fit results

$$\frac{\chi^2}{dof} = \frac{6.47}{22-4} \simeq 0.36$$

$f_0(0)$	0.658(17)
$\mathcal{A}_0^0$	1.38(12)
$\mathcal{A}_0^+$	2.60(12)
$\lvert V_{cb}\rvert \times 10^3$	41.01(75)

$$\mathscr{R}_D = \frac{BR(\bar{B}\to D\tau\bar{\nu}_\tau)}{BR(\bar{B}\to D\ell\bar{\nu}_\ell)} = 0.301(5)$$



[arXiv:1906.00727]

✓ n=0 is sufficient

✓ Independent of subtraction point s_0: s_0=0

✓ At least one bound and one virtual BD 0^+ states

SUMMARY AND OUTLOOK

- Muskhelishvili-Omnes representation of scalar form factor
  - ➡ Based on axiomatical principles: unitarity & analyticity
  - ➡ Elegant bridge connecting heavy-to-light decay (outcome) with heavy-light scattering (input)
  - ➡ Include coupled-channel effects
- Successfully describe results of LQCD and LCSR in *D*&*B* sectors
  - ➡ Communicate information between *D* and *B* sectors by imposing HQFS
  - ➡ Constrain parameters by using chiral symmetry of light quarks
  - ➡ Obtain scalar FFs in the fitted channels in the whole kinematical region
  - ➡ Predict scalar FFs in the other channels related by chiral symmetry
  - ➡ Extract all the heavy-to-light CKM elements
- The new parametrization for B to D semileptonic decay

More efficient than traditional parametrizations like BGL(BCL), CLN approaches:

- less free parameters & physically meaningful
- more precise results:
  $|V_{cb}| = (41.01 \pm 0.75) \times 10^{-3}$ & $\mathcal{R}_D = \frac{\mathcal{BR}(\bar{B}\to D\tau\bar{\nu}_\tau)}{\mathcal{BR}(\bar{B}\to D\ell\bar{\nu}_\ell)} = 0.301(5)$

Universial for any other semileptonic processes induced by $b \to c$ transition:

- $\bar{B} \to D^*\ell\bar{\nu}$ decay
- $\bar{\Lambda}_b \to \Lambda_c^{(*)}\ell\bar{\nu}$ decay

Thank you for your attention!