Study of Intermediate States in the Inclusive Semi-Leptonic $B \to X_c l \nu$ Decay Structure Functions

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Outline

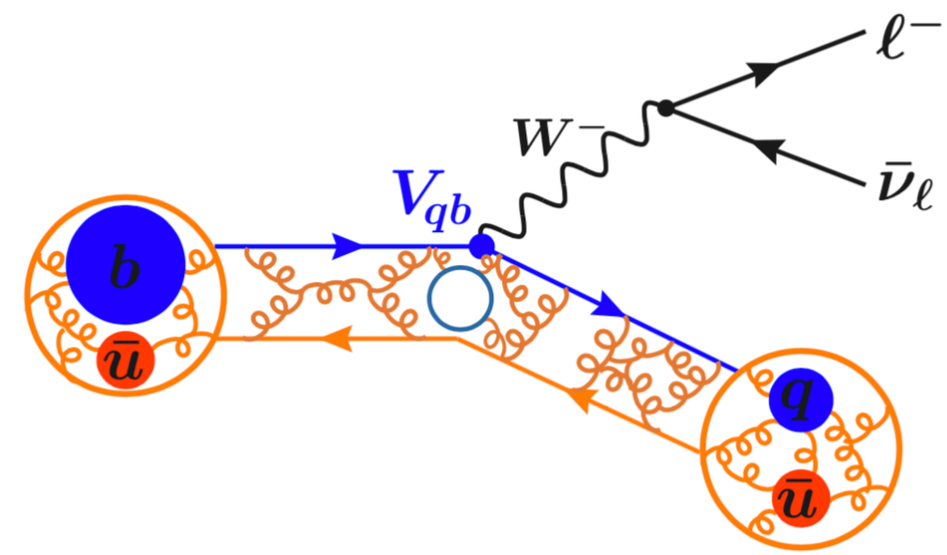
- Theoretical Framework
- Lattice Calculation
- Preliminary Results
  - Zero-Recoil
  - Non Zero-Recoil
- Conclusions and Perspectives

Theoretical Framework

Semileptonic B decays to D^{**} (P-wave)

- We are concerned with semileptonic decays of B meson into **orbitally excited P-wave** D mesons.

We focus on: $B \to D^{**} l \nu_l$



- Particularly interesting because there is a persistent conflict between theory and experiment the so-called **"1/2 versus 3/2 puzzle"**.

- Heavy-Light mesons:

$$B = \{\bar{b}u, \bar{b}d\}$$
$$D = \{\bar{c}u, \bar{c}d\}$$

  - Static Limit: $(m_b, m_c \to \infty)$
  - Finite masses: (m_b, m_c)

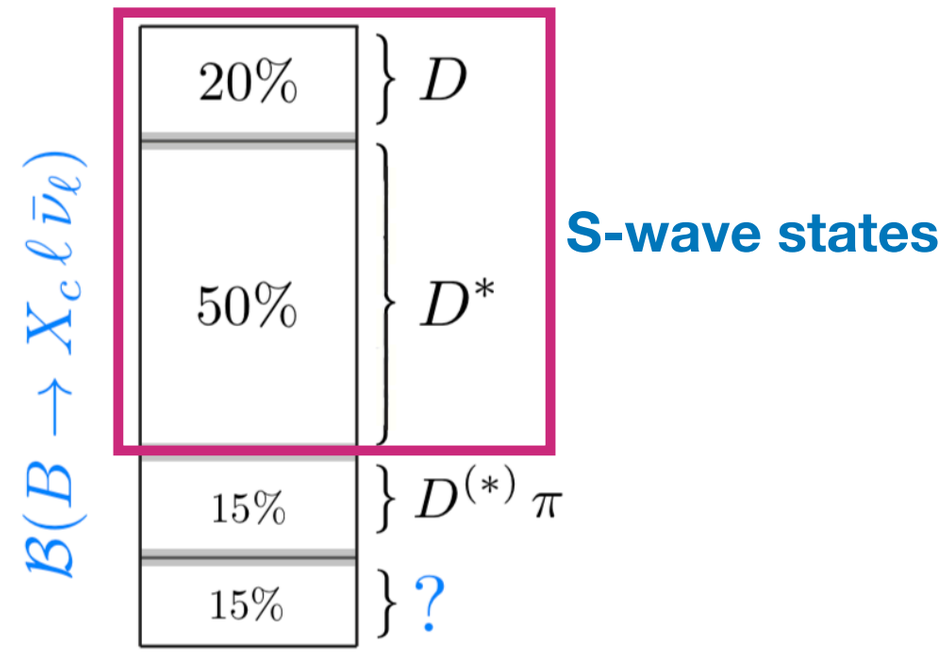
$j^{\mathcal{P}}$	$J^{\mathcal{P}}$	
$(1/2)^- \equiv S$	$0^- \equiv B, D$	
	$1^- \equiv B^*, D^*$	
		Γ (MeV)
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$	236
	$1^+ \equiv D_1^* \equiv D_1^{1/2}$	384
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$	31
	$2^+ \equiv D_2^* \equiv D_2^{3/2}$	47

D^{**}

Classification of heavy-light mesons

The 1/2 versus 3/2 puzzle

- Experiments such as ALEPH, BaBar, BELLE, CDF, DELPHI and others which have studied $B \to X_c l \nu_l$ have found



- The remaining 15% still not well understood.

Ulratsev, PLB 501, 86 (2001)
Le Yaouanc, Oliver, Raynal, PRD67, 114009 (2003)

Theoretical Estimates:

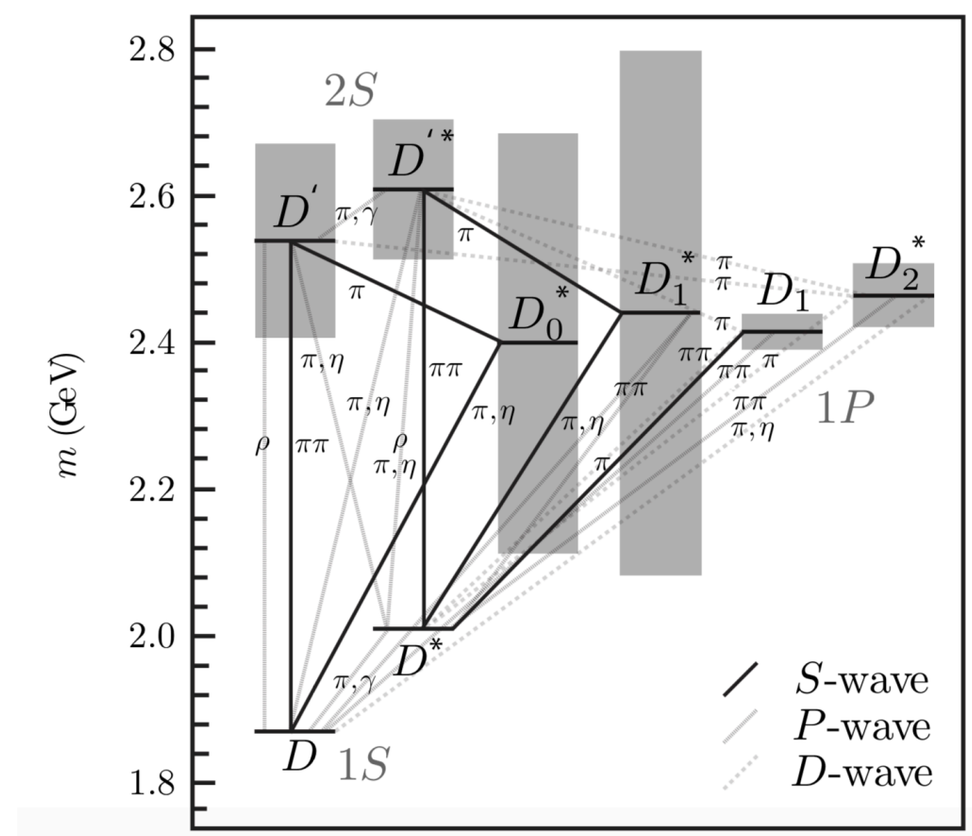
- Heavy quark limit, sum rule, quark model

$$\Gamma(B \to D^{**}_{1/2} l\nu) \ll \Gamma(B \to D^{**}_{3/2} l\nu)$$

Experimental Estimates:

- Bernolochner, Ligeti, Turczyk, PRD85, 090433 (2012)

$$\Gamma(B \to D^{**}_{1/2} l\nu) \approx \Gamma(B \to D^{**}_{3/2} l\nu)$$



Bernlochner, Ligeti, PRD95, 014022 (2017)

Experimental References
arXiv: 0708.1738
arXiv: 0808.0528
arXiv: 0711.3252

Heavy Quark Limit

- Heavy Quark Limit: $(m_b, m_c \to \infty)$
- The relevant matrix elements for decays $B \to D^{**} l\nu$ can be parametrized by two form factors: $\tau_{1/2}$ and $\tau_{3/2}$ → **Isgur-Wise Form Factors**

$$\langle D_0^{1/2}(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \propto \tau_{1/2}(w)(v-v')_\mu$$

$$\langle D_2^{3/2}(v',\varepsilon)|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \propto \tau_{3/2}(w)\left((w+1)\varepsilon^*_{\mu\alpha}v^\alpha - \varepsilon^*_{\alpha\beta}v^\alpha v^\beta v'\right)$$

$w = (v' \cdot v)$

Bjorken and Uraltsev sum rules

$$\rho^2 - 1/4 = 2\sum_m |\tau^{(m)}_{3/2}(1)|^2 + \sum_n |\tau^{(n)}_{1/2}(1)|^2 \quad \textbf{Bjorken}$$

$$1/4 = \sum_m |\tau^{(m)}_{3/2}(1)|^2 - \sum_n |\tau^{(n)}_{1/2}(1)|^2 \quad \textbf{Uraltsev}$$

→ **One may expect saturation from the ground states**

$$1/4 \approx |\tau^{(0)}_{3/2}(1)|^2 - |\tau^{(0)}_{1/2}(1)|^2$$

$$\tau^{(0)}_{1/2}(1) < \tau^{(0)}_{3/2}(1)$$

$$\Gamma(B \to D^{**}_{1/2} l\nu) \ll \Gamma(B \to D^{**}_{3/2} l\nu)$$

Ulratsev, PLB 501, 86 (2001)
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Possible explanations of the 1/2 versus 3/2 puzzle

- The experimental signal for the remaining 15% is rather vague. Then, only a small part might actually be $D_0^{1/2}$ and $D_1^{1/2}$;
- Sum rules might not be saturated by the ground states;
- Sum rules by means of operator product expansion (OPE) works in the static limit and might change for finite heavy quarks masses;
- Sum rules make statements about the zero-recoil $(w = v \cdot v' = 1)$, where the B and D meson have the same velocity; **to obtain decay rates, however one has to integrate over w;**
- Quark models agreed with the sum rule, even when considering finite heavy quark masses.

V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, J.-C. Raynal - Phys.Rev.D56:5668-5680(1997)
D. Ebert, R. N. Faustov, V. O. Galkin - Phys.Lett. B434 (1998) 365-372
D. Ebert, R.N. Faustov, V.O. Galkin - Phys.Rev. D61 (2000) 014016

Lattice Calculation

Lattice calculation

- P-wave states are much harder to calculate S-wave states. We have large noise for excited states, then is hard to identify the plateau.
- We use the forward-scattering matrix elements corresponding to **inclusive** semi-leptonic B meson decay.
- For the inclusive case, we have: $$d\Gamma^{\text{incl}} \sim |\langle X_c l\nu | J | B\rangle|^2 = Im\,\langle B|JJ|B\rangle$$
- Our work is based on a calculation of the four-point function corresponding to the matrix element:

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t) J_\nu(0)|B(\mathbf{0})\rangle$$

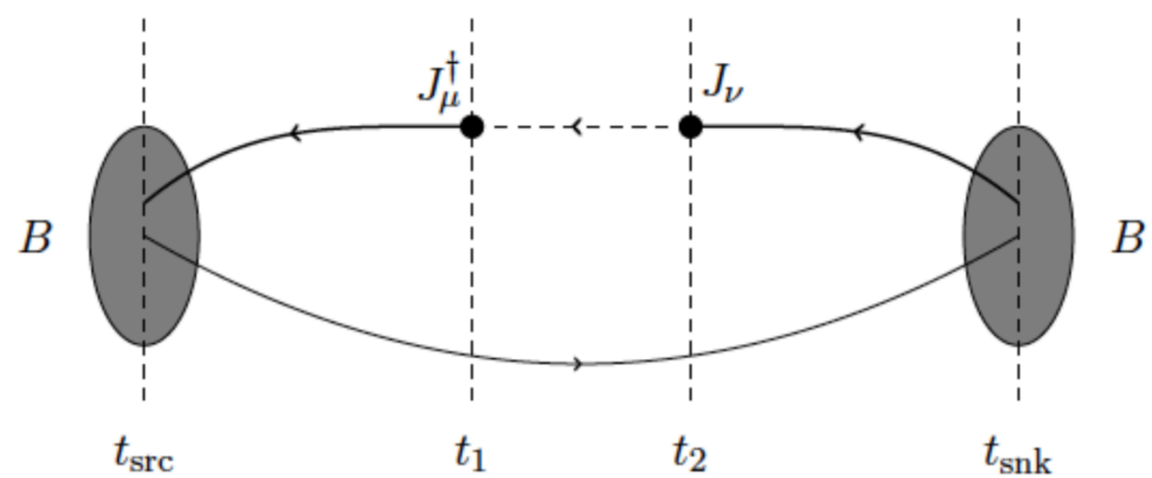
All final states contribute, including D^{**}

Lattice calculation

- **We can extract the matrix element by taking a ratio to two-point correlation function. In practice:**

$$\frac{C_{\mu\nu}^{SJJS}(t_{snk}, t_1, t_2, t_{src})}{C^{SL}(t_{snk}, t_2) C^{LS}(t_1, t_{src})} \rightarrow \frac{\frac{1}{2M_B} \langle B(\vec{0}) | J_\mu(\vec{q}, t_1)^\dagger J_\nu(\vec{q}, t_2) | B(\vec{0}) \rangle}{\frac{1}{2M_B} | \langle 0 | P^L | B(\vec{0}) \rangle |^2}$$

- The position of t_1 is varied between 0 and t_2
- We set t_2 so that it is separated from t_{snk} by 16 to allow the ground state saturation of the final meson.



JLQCD ensemble

- **Mobius Domain-Wall Fermion (2012~)**
  - **2+1 flavor (uds)**
  - Chiral Symmetry
    - Residual Mass < O (1 MeV)
  - Lattice Spacing: 1/a = 2.4, 3.6, 4.5 GeV
  - Volume: L = 2.7 fm ($32^3, 48^3, 64^3$ lattices)
  - ud quark masses: m_π = 230, 300, 400, 500 MeV
  - Statistics: 50 - 400 measurements

- **Valence Quarks**
  - charm/bottom (MDW) + strange (MDW)
  - Bottom is lighter than physical
  - On Oakforest-PAC with IROIRO++

β	a [fm]	$L^3 \times T$	am_{ud}	am_s	am_c	am_b	N_{cfg}
4.35	0.055	$48^3 \times 96$	0.0042	0.0250	0.27287	0.66619	400

- The **charm quark** mass is tuned to its physical value
- The **bottom quark** mass is chosen such that it is **1.56** times heavier than the charm.

- Sea quarks: am_{ud}, am_s
- Valence quarks: am_c, am_b

$$B_s \to D_s^{(*,**)}$$

$B \to \{D, D^*, D^{**}\} l\bar{\nu}$ Form Factors

- **S-wave states:**

$$w = v \cdot v' = v'^0 = \sqrt{1 + \left(\frac{p}{M_{D^{(*)}}}\right)}$$

(D, D^*)

$$\sqrt{M_B M_D}^{-1} \langle D(p')|V_\mu|B(p)\rangle = (v + v')h_+(w) + (v - v')_\mu h_-(w)$$
$$\sqrt{M_B M_D^*}^{-1} \langle D^*(\epsilon, p')|V_\mu|B(p)\rangle = \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} v'^\rho v^\sigma h_V(w)$$
$$\sqrt{M_B M_D^*}^{-1} \langle D^*(\epsilon, p')|A_\mu|B(p)\rangle = -i(w+1)\epsilon^*_\mu h_{A_1}(w) + i(\epsilon^* v)v_\mu h_{A_2}(w) + i(\epsilon^* v)v'_\mu h_{A_3}(w)$$

- **P-wave states:**

For the $\frac{3}{2}^+$ states (D_1, D_2^*)

$$\left.\begin{aligned} \sqrt{M_B M_{D_1}}^{-1} \langle D_1(v', \epsilon)|V_\mu|B(v)\rangle &= f_{V_1}\epsilon^*_\mu + (f_{V_2}v_\mu + f_{V_3}v'_\mu)(\epsilon^* \cdot v) \\ \sqrt{M_B M_{D_1}}^{-1} \langle D_1(v', \epsilon)|A_\mu|B(v)\rangle &= i f_A \varepsilon_{\mu\alpha\beta\gamma}\epsilon^*_\alpha v_\beta v'_\gamma \end{aligned}\right\} 1^+$$

$$\left.\begin{aligned} \sqrt{M_B M_{D_2^*}}^{-1} \langle D_2^*(v', \epsilon)|A_\mu|B(v)\rangle &= k_{A_1}\epsilon^*_{\mu\alpha}v_\alpha + (k_{A_2}v_\mu + k_{A_3}v'_\mu)\epsilon^*_{\alpha\beta}v_\alpha v_\beta \\ \sqrt{M_B M_{D_2^*}}^{-1} \langle D_2^*(v', \epsilon)|V_\mu|B(v)\rangle &= i k_V \varepsilon_{\mu\alpha\beta\gamma}\epsilon^*_{\alpha\sigma}v_\sigma v_\beta v'_\gamma \end{aligned}\right\} 2^+$$

For the $\frac{1}{2}^+$ states (D_0^*, D_1^*)

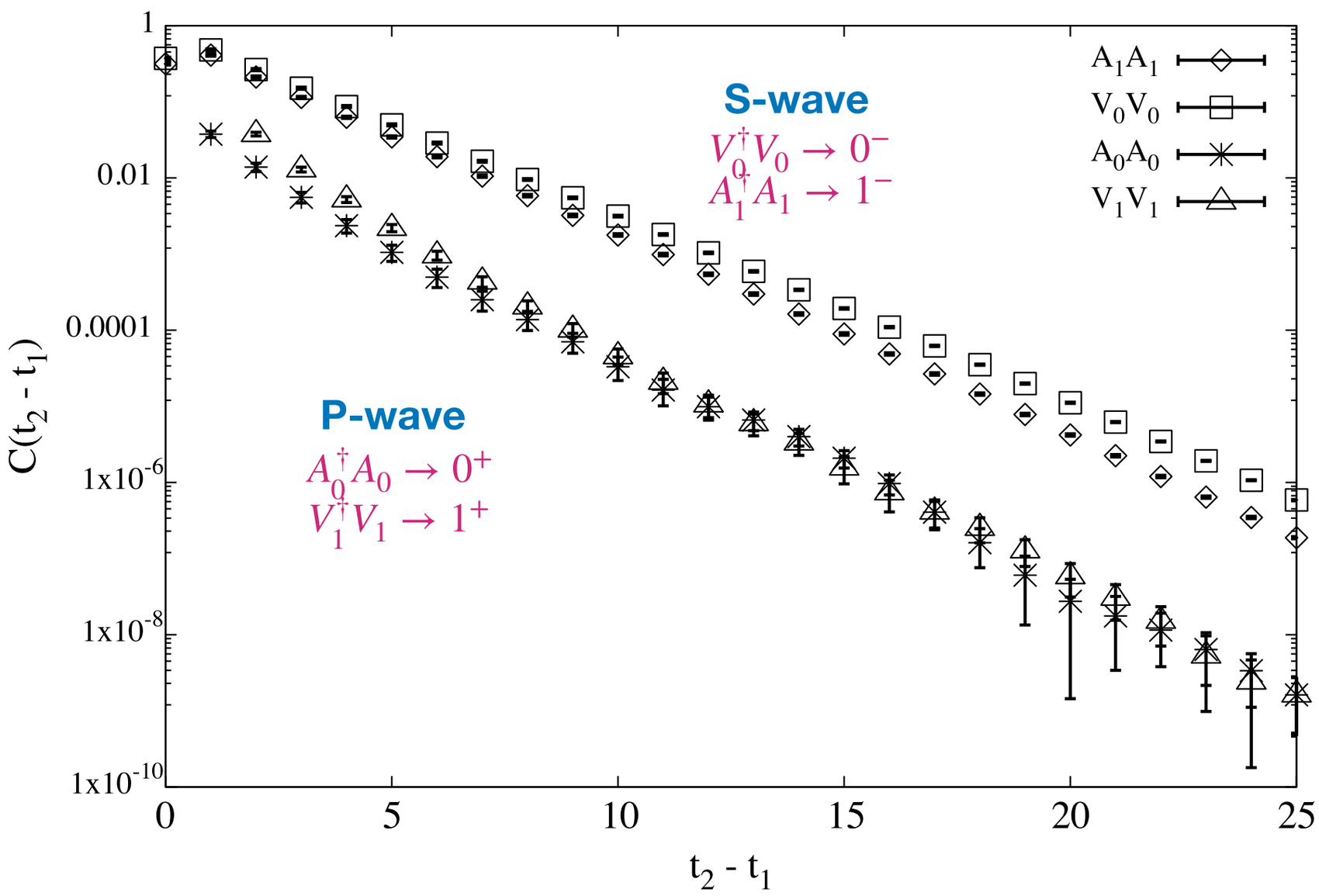
$$\left.\begin{aligned} \langle D_0^*(v')|V_\mu|B(v)\rangle &= 0 \\ \sqrt{M_B M_{D_0^*}}^{-1} \langle D_0^*(v')|A_\mu|B(v)\rangle &= g_+(v_\mu + v'_\mu) + g_-(v_\mu - v'_\mu) \end{aligned}\right\} 0^+$$

$$\left.\begin{aligned} \sqrt{M_B M_{D_1^*}}^{-1} \langle D_1^*(v', \epsilon)|V_\mu|B(v)\rangle &= g_{V_1}\epsilon^*_\mu + (g_{V_2}v_\mu + g_{V_3}v'_\mu)(\epsilon^* \cdot v) \\ \sqrt{M_B M_{D_1^*}}^{-1} \langle D_1^*(v', \epsilon)|A_\mu|B(v)\rangle &= i g_A \varepsilon_{\mu\alpha\beta\gamma}\epsilon^*_\alpha v_\beta v'_\gamma \end{aligned}\right\} 1^+$$

Zero-Recoil

Four-point correlation functions

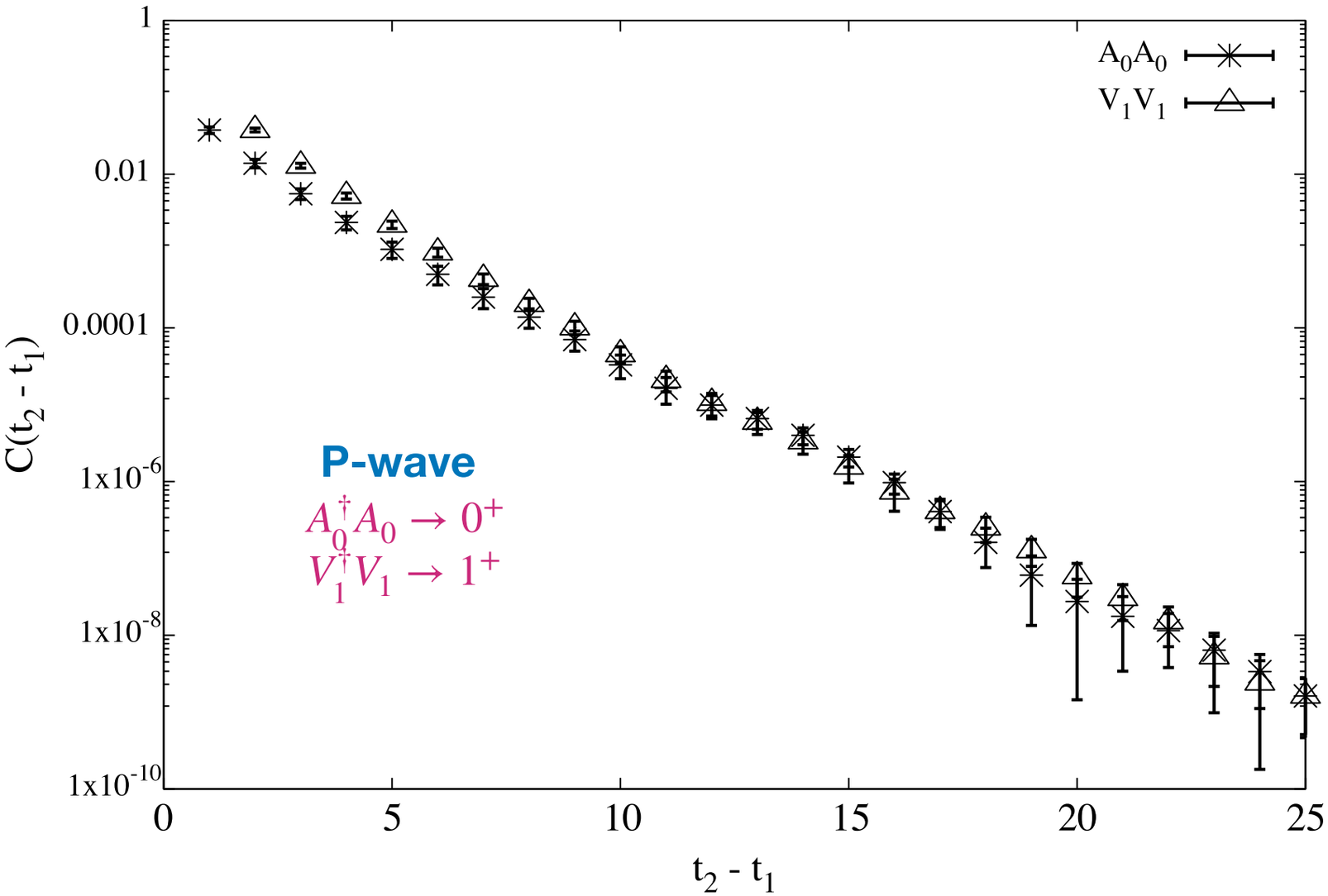
- **Zero-Recoil**: due to the parity symmetry we can distinguish the S-wave states and P-wave states by choosing $JJ = V_0^\dagger V_0, A_1^\dagger A_1, \cdots$



$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv B, D$
	$1^- \equiv B^*, D^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$
	$1^+ \equiv D_1' \equiv D_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$
	$2^+ \equiv D_2^* \equiv D_2^{3/2}$

- The spatial AA channel corresponds to the D_s^* meson in the final state.
- The temporal VV channel probes the D meson.

Four-point correlation functions



P-wave states

$$\frac{1}{\sqrt{m_{D_0^*} m_B}} \langle D_0^* | A^0 | B \rangle = 2 g_+(1),$$

$$\frac{1}{\sqrt{m_{D_1^*} m_B}} \langle D_1^* | V^k | B \rangle = g_{V_1}(1)\, \epsilon^{*k},$$

$$\frac{1}{\sqrt{m_{D_1} m_B}} \langle D_1 | V^k | B \rangle = f_{V_1}(1)\, \epsilon^{*k}$$

Extract directly by fitting

Form Factors

$$g_+(1), g_{V_1}(1), f_{V_1}(1)$$

$$C^{A_0 A_0}(t) = |g_+(1)|^2 e^{-m_{D_0^*} t}$$

$$C^{V^k V^k}(t) = \frac{|g_{V_1}(1)|^2}{4} e^{-m_{D_1^*} t} + \frac{|g_{f_1}(1)|^2}{4} e^{-m_{D_1} t}$$

Heavy quark expansion

- Relation to the **Isgur-Wise form factors** (heavy quark limit) is obtained by the heavy quark expansion

$$\sqrt{6}\, f_{V_1}(w) = -\left[w^2 - 1 + 8\,\varepsilon_c(\bar{\Lambda}' - \bar{\Lambda})\right]\tau(w) + \ldots \qquad \frac{3}{2}^+$$

$$g_+(w) = -\frac{3}{2}\,(\varepsilon_c + \varepsilon_b)\,(\bar{\Lambda}^* - \bar{\Lambda})\,\zeta(w) + \ldots \qquad \frac{1}{2}^+$$

$$g_{V_1}(w) = \left[w - 1 + (\varepsilon_c - 3\,\varepsilon_b)\,(\bar{\Lambda}^* - \bar{\Lambda})\right]\zeta(w) + \ldots$$

Adam K. Leibovich, Zoltan Ligeti, Iain W. Stewart, Mark B. Wise - Phys.Rev.D57:308-330 (1998)

- The 1/m expansion:

$$\varepsilon_c = \frac{1}{2m_c} \qquad \varepsilon_b = \frac{1}{2m_b}$$

$$m_c = 1.4 \text{ GeV}$$

$$m_b = (1.25)^4 m_c \text{ GeV}$$

- The energy of the light degrees of $m_Q \to \infty$ →

Parameter	$\bar{\Lambda}$	$\bar{\Lambda}'$	$\bar{\Lambda}^*$
Value [GeV]	0.40	0.80	0.76

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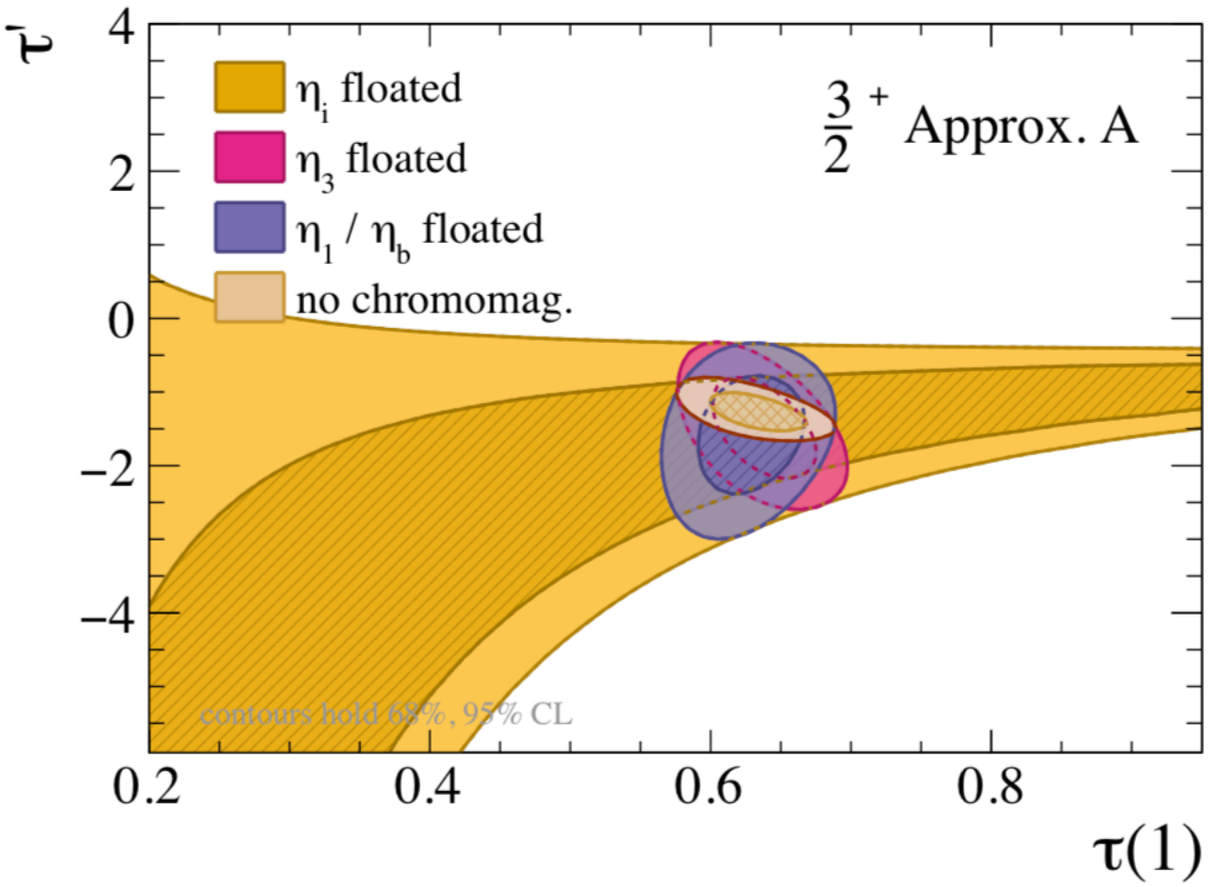
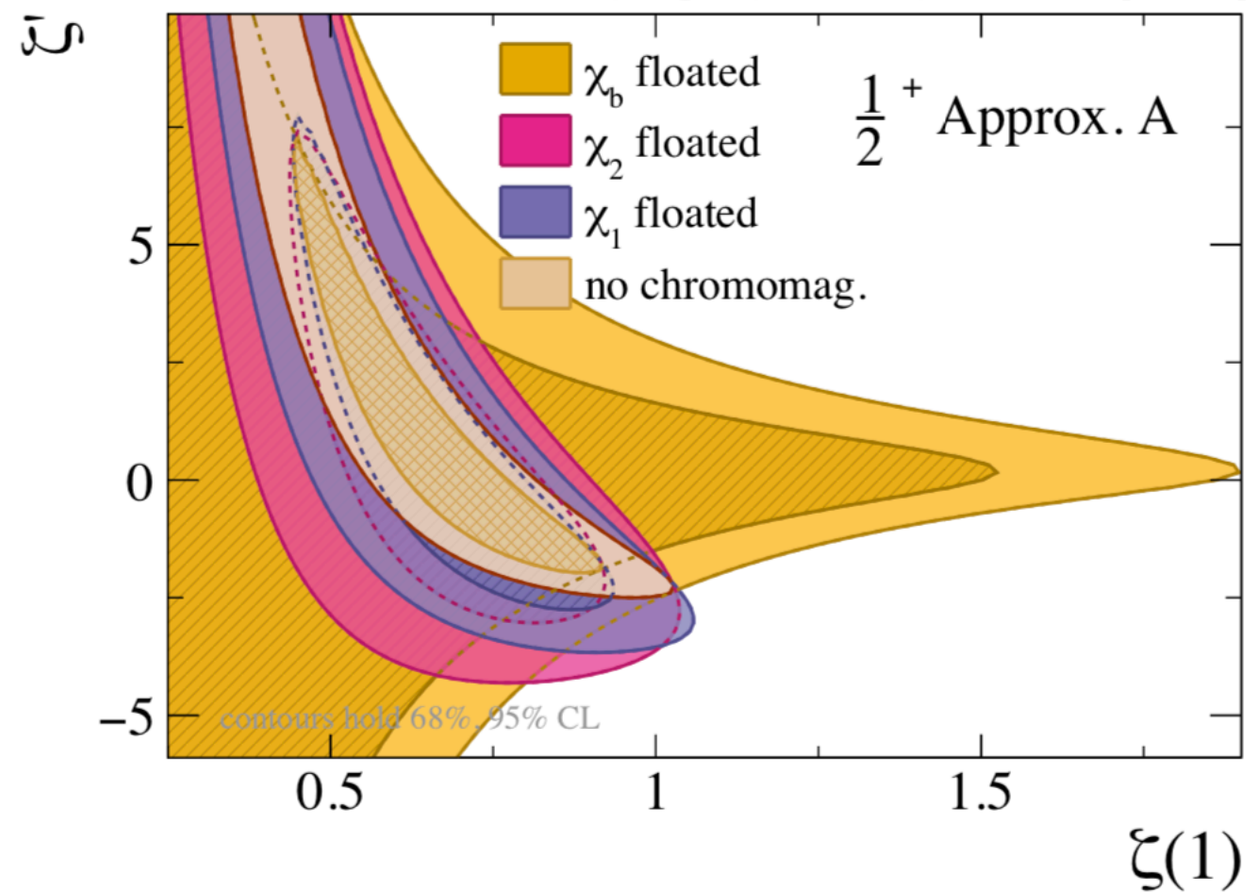
Results

Bernlochner, Ligeti, PRD95, 014022 (2017)





Zero-Recoil:

$\zeta(1)$	$0.77(13)$
$\tau(1)$	$0.69(15)$

$$\tau^{(0)}_{3/2}(1) = \left(\frac{1}{\sqrt{3}}\tau(1)\right) = 0.45(7)$$

$$\tau^{(0)}_{1/2}(1) = \left(\frac{1}{2}\zeta(1)\right) = 0.39(6)$$

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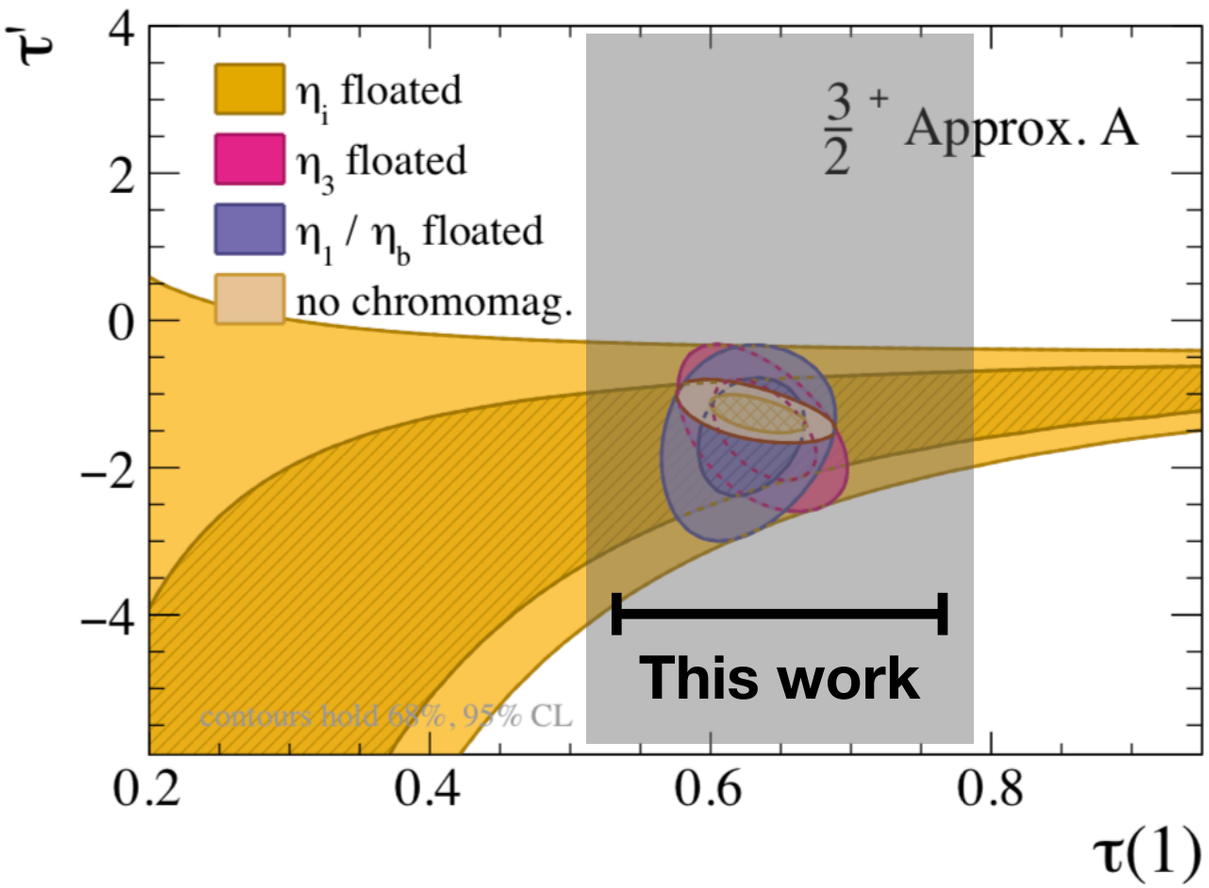

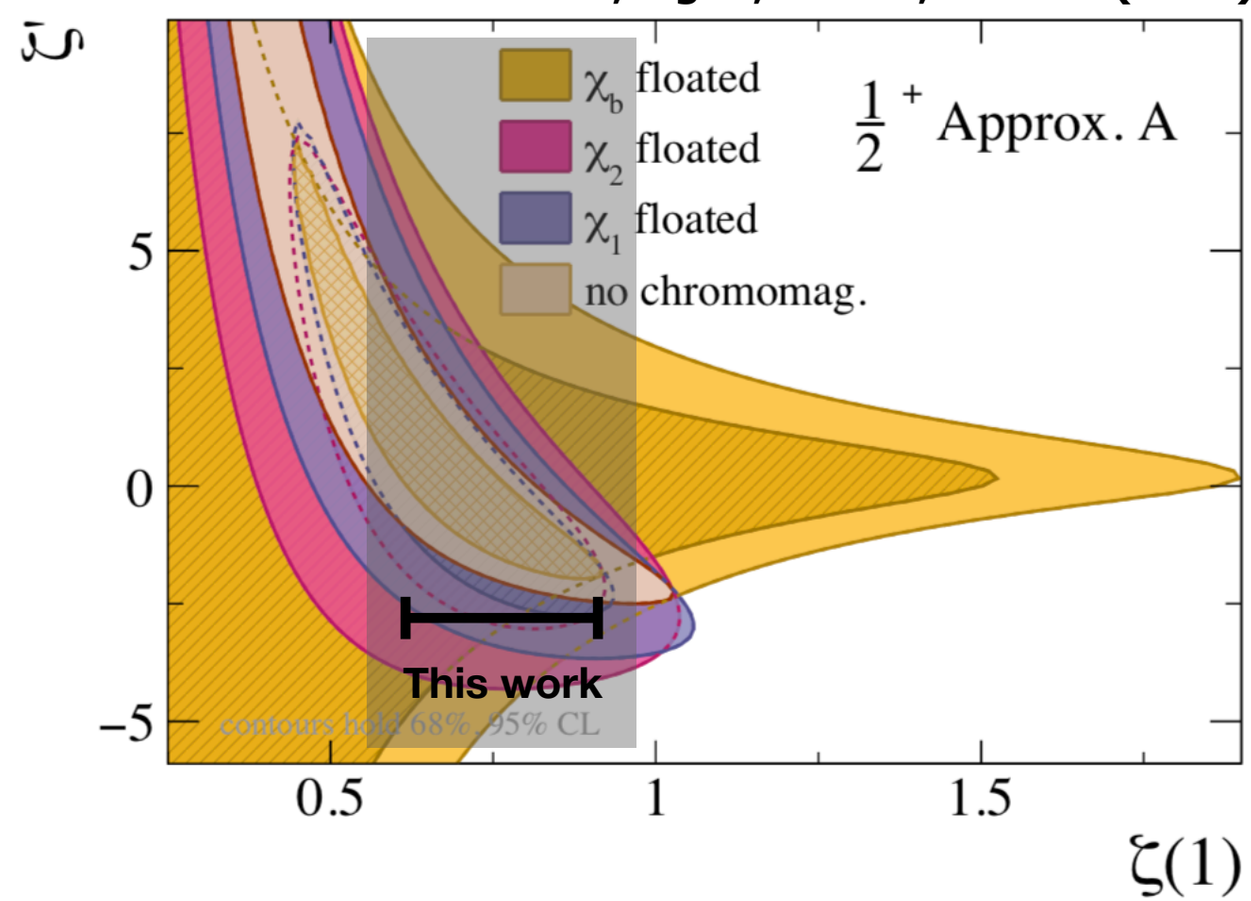

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Comparison to other theoretical estimates

	$\tau^{(0)}_{3/2}(1)$	$\tau^{(0)}_{1/2}(1)$
Relativistic Model[1]	0.6	0.32
Quark Model[2]	0.54	0.22
Lattice[3]	0.53(2)	0.30(3)
This work	0.45(7)	0.39(6)

Experimental Estimates

$$\Gamma(B \to D^{**}_{1/2} l\nu) \approx \Gamma(B \to D^{**}_{3/2} l\nu)$$

Uraltsev sum rules

One may expect saturation from the ground states:

$$\tau^{(0)}_{1/2}(1) < \tau^{(0)}_{3/2}(1)$$

$$\Gamma(B \to D^{**}_{1/2} l\nu) < \Gamma(B \to D^{**}_{3/2} l\nu)$$

$$0.25 \approx |\tau^{(0)}_{3/2}(1)|^2 - |\tau^{0}_{1/2}(1)|^2$$

This work:

$$0.050(78) \approx |\tau^{(0)}_{3/2}(1)|^2 - |\tau^{0}_{1/2}(1)|^2$$

(1) ETM Collaboration: Benoit Blossier, Marc Wagner, Olivier Pene - HEP 0906:022 (2009)
(2) Hai-Yang Cheng, Chun-Khiang Chua, Chien-Wen Hwang - Phys.Rev. D69 (2004) 074025
(3) V. Morénas, A. Le Yaouanc, L. Oliver, O. Pène, J.-C. Raynal - Phys.Rev.D56:5668-5680 (1997)

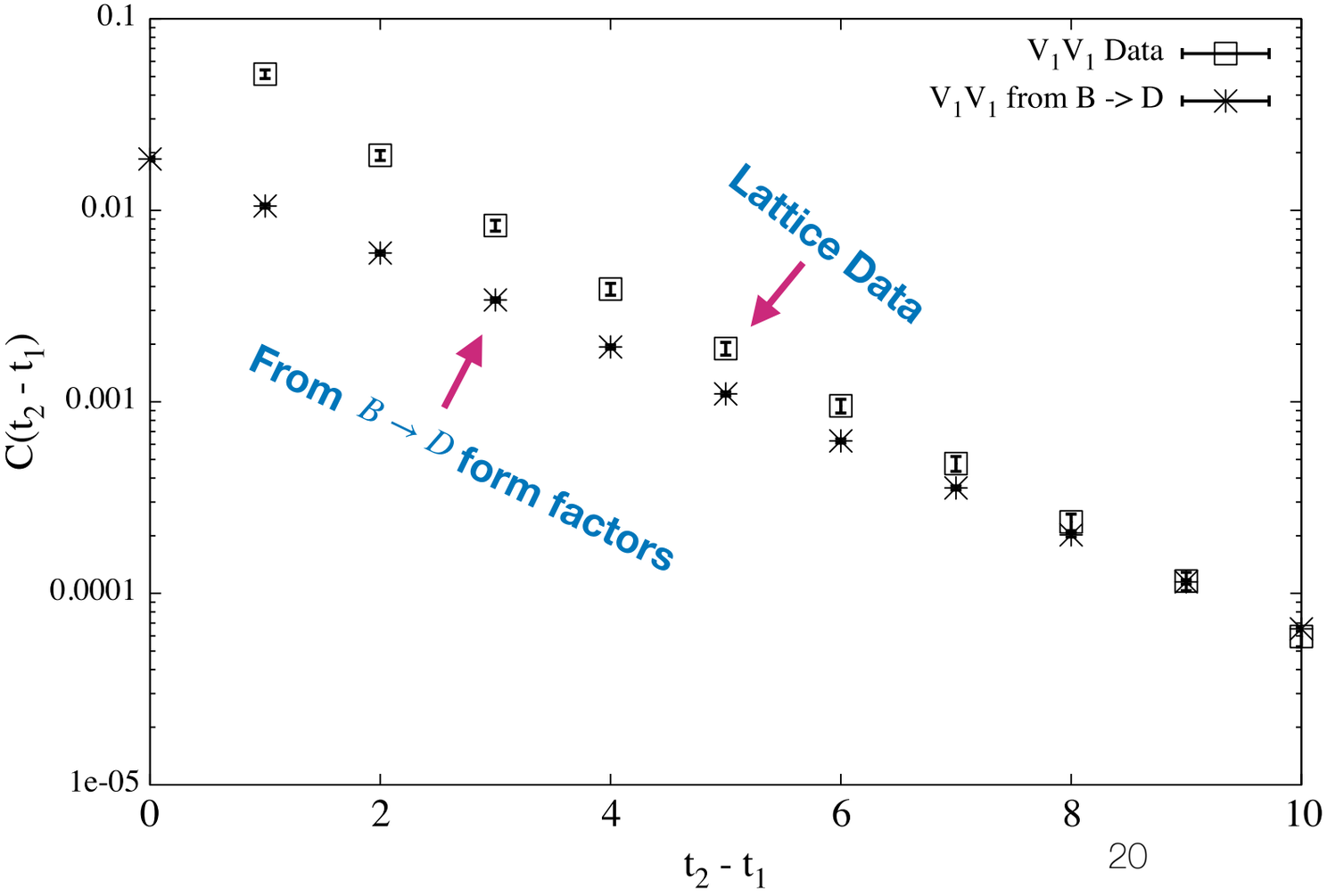
Non Zero-Recoil

Non Zero-Recoil

- Different from the zero-recoil case: Now, we have S-wave contributions as well as P-wave contributions
- We subtract the S-wave contributions
- S-wave form factors computed by JLQCD Collaboration (T. KANEKO et al.)
- In this work: $p' = \frac{2\pi}{L}(0,0,1)$

but for B_d, not B_s

Four-point correlation function



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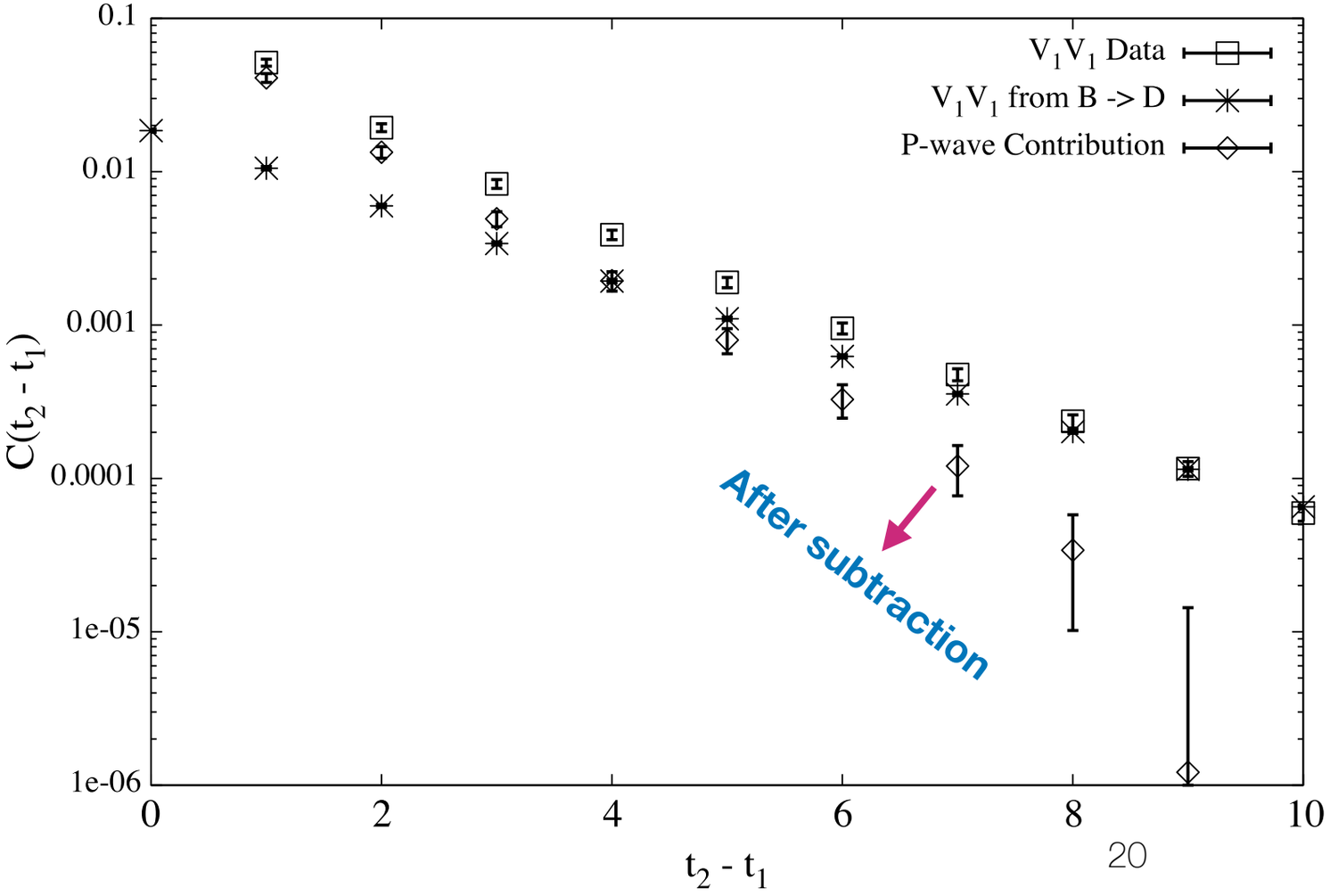

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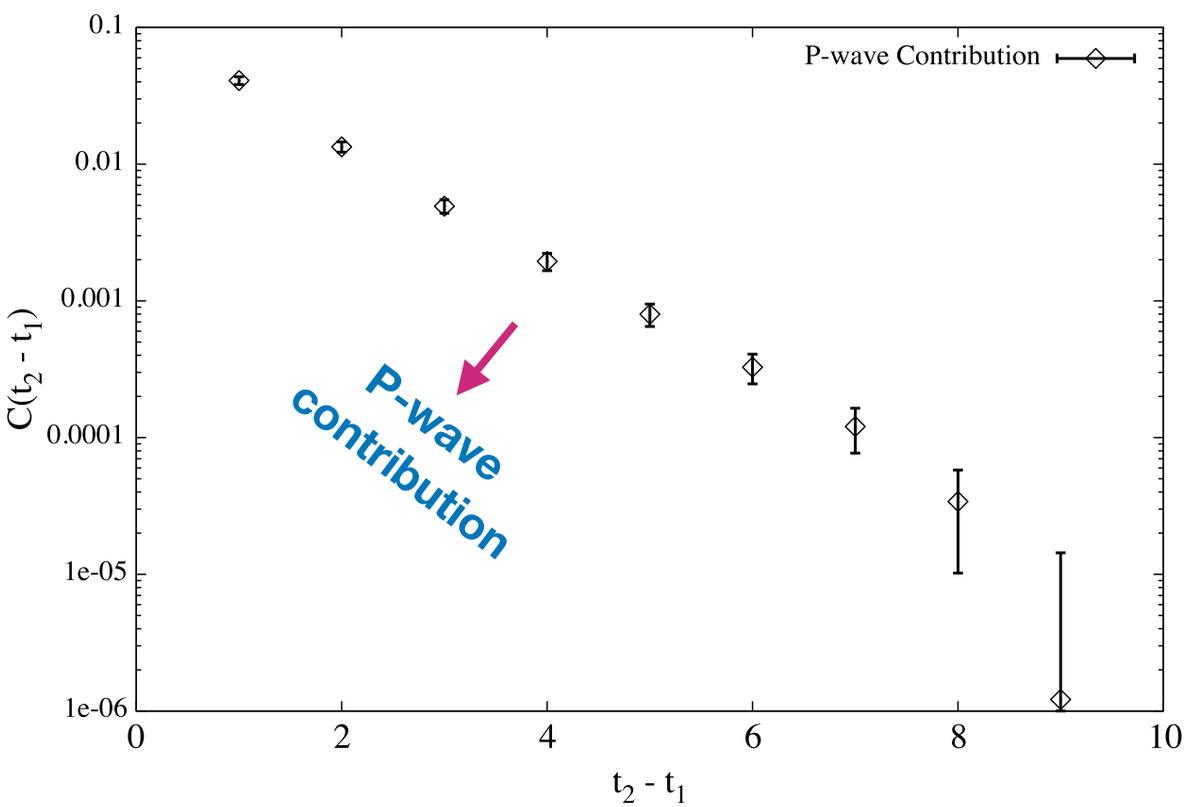
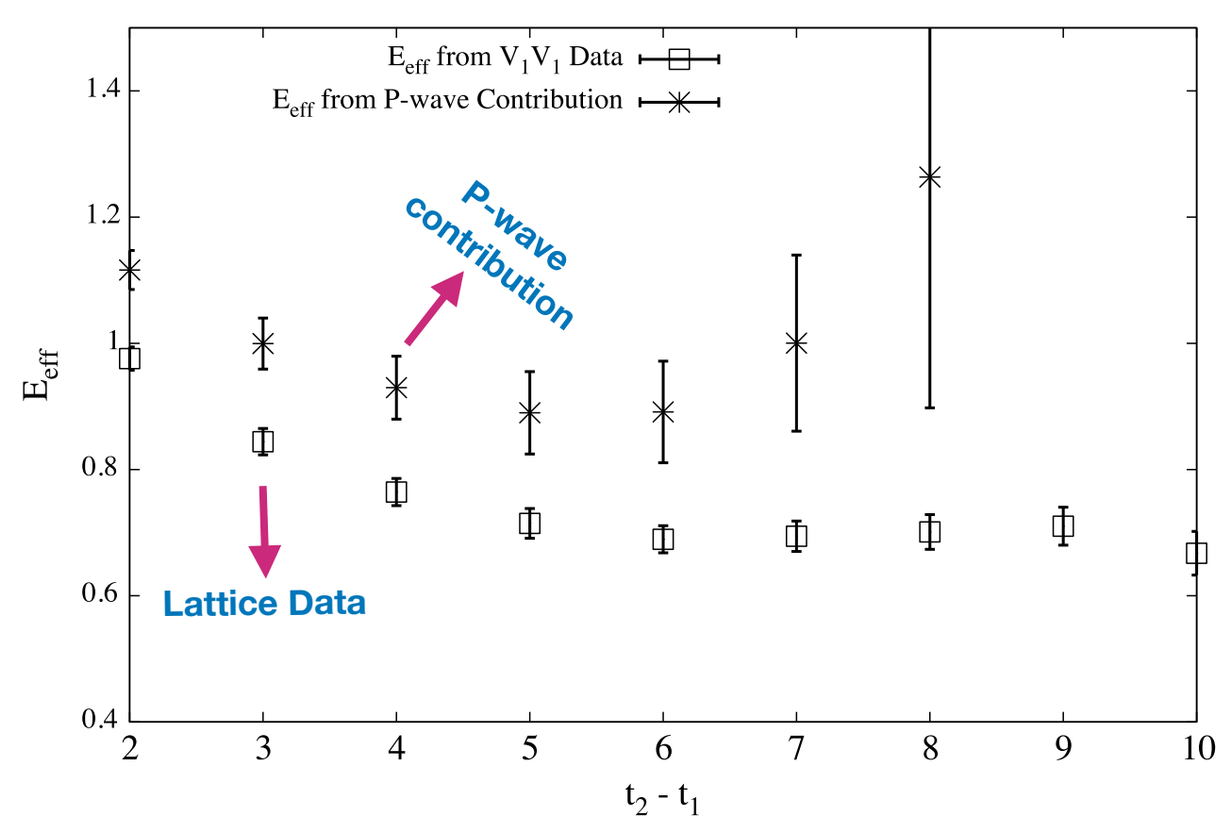


Effective Energy



Non-Zero Recoil

- For the **P-wave states** we have:

$$\left.\sqrt{M_B M_{D_1}}^{-1}\langle D_1(v',\epsilon)|V_\mu|B(v)\rangle = f_{V_1}\epsilon^*_\mu + (f_{V_2}v_\mu + f_{V_3}v'_\mu)(\epsilon^*\cdot v)\right\}\ \frac{3}{2}^+$$

$$\left.\begin{aligned}\sqrt{M_B M_{D_0^*}}^{-1}\langle D_0^*(v')|A_\mu|B(v)\rangle &= g_+(v_\mu + v'_\mu) + g_-(v_\mu - v'_\mu)\\ \sqrt{M_B M_{D_1^*}}^{-1}\langle D_1^*(v',\epsilon)|V_\mu|B(v)\rangle &= g_{V_1}\epsilon^*_\mu + (g_{V_2}v_\mu + g_{V_3}v'_\mu)(\epsilon^*\cdot v)\end{aligned}\right\}\ \frac{1}{2}^+$$

- Doing a fit on the different channels we get:

Form Factors

$$f_{V_1}(w), f_{V_3}(w) g_+(w), g_{V_1}(w), g_{V_3}(w)$$

- Roughly speaking: $< B | V_1^\dagger V_1 | B > \sim |f_{V1}|^2 + |g_{V1}|^2$

- **Heavy quark expansion** to related then to $\tau(w)$ and $\zeta(w)$

Approximation A: neglect $(w-1)^2, (w-1)\varepsilon_{b,c}$ **and** $\varepsilon^2_{b,c}$

$$\left.\begin{aligned}f_{V_1} &= \frac{1}{\sqrt{6}}\left[(1-w^2)\tau - \epsilon_c 4(w+1)(w\bar{\Lambda}' - \bar{\Lambda})\tau\right]\\ wf_{V_1} + (w^2-1)f_{V_3} &= \frac{w}{\sqrt{6}}\left[(1-w^2)\tau - \varepsilon_c 4(w+1)(w\bar{\Lambda}' - \bar{\Lambda})\tau\right] + \frac{w^2-1}{\sqrt{6}}(w-2)\tau\end{aligned}\right\}\ \frac{3}{2}^+$$

$$\left.\begin{aligned}g_{V_1} &= (w-1)\zeta + \varepsilon_c(w\bar{\Lambda}^* - \bar{\Lambda})\zeta - \varepsilon_b[\bar{\Lambda}^*(2w+1) - \bar{\Lambda}(w+2)]\zeta\\ wg_{V_1} + (w^2-1)g_{V_3} &= -(w^2-1)\zeta\\ g_+ &= \zeta\left[-3\varepsilon_c\left(\frac{w\bar{\Lambda}^* - \bar{\Lambda}}{w+1}\right) - \varepsilon_b\frac{\bar{\Lambda}^*(2w+1) - \bar{\Lambda}(w+2)}{w+1}\right]\end{aligned}\right\}\ \frac{1}{2}^+$$

Non-Zero Recoil

- For the **P-wave states** we have:

$$\left.\begin{aligned}\sqrt{M_B M_{D_1}}^{-1}\langle D_1(v',\epsilon)|V_\mu|B(v)\rangle &= f_{V_1}\epsilon^*_\mu + (f_{V_2}v_\mu + f_{V_3}v'_\mu)(\epsilon^*\cdot v)\\ \sqrt{M_B M_{D_0^*}}^{-1}\langle D_0^*(v')|A_\mu|B(v)\rangle &= g_+(v_\mu + v'_\mu) + g_-(v_\mu - v'_\mu)\\ \sqrt{M_B M_{D_1^*}}^{-1}\langle D_1^*(v',\epsilon)|V_\mu|B(v)\rangle &= g_{V_1}\epsilon^*_\mu + (g_{V_2}v_\mu + g_{V_3}v'_\mu)(\epsilon^*\cdot v)\end{aligned}\right\}\ \frac{3}{2}^+ \quad \left.\begin{aligned}&\\ &\\ &\end{aligned}\right\}\ \frac{1}{2}^+$$

- Doing a fit on the different channels we get:

Form Factors

$$f_{V_1}(w), f_{V_3}(w) g_+(w), g_{V_1}(w), g_{V_3}(w)$$

- Roughly speaking: $< B | V_1^\dagger V_1 | B > \sim |f_{V1}|^2 + |g_{V1}|^2$

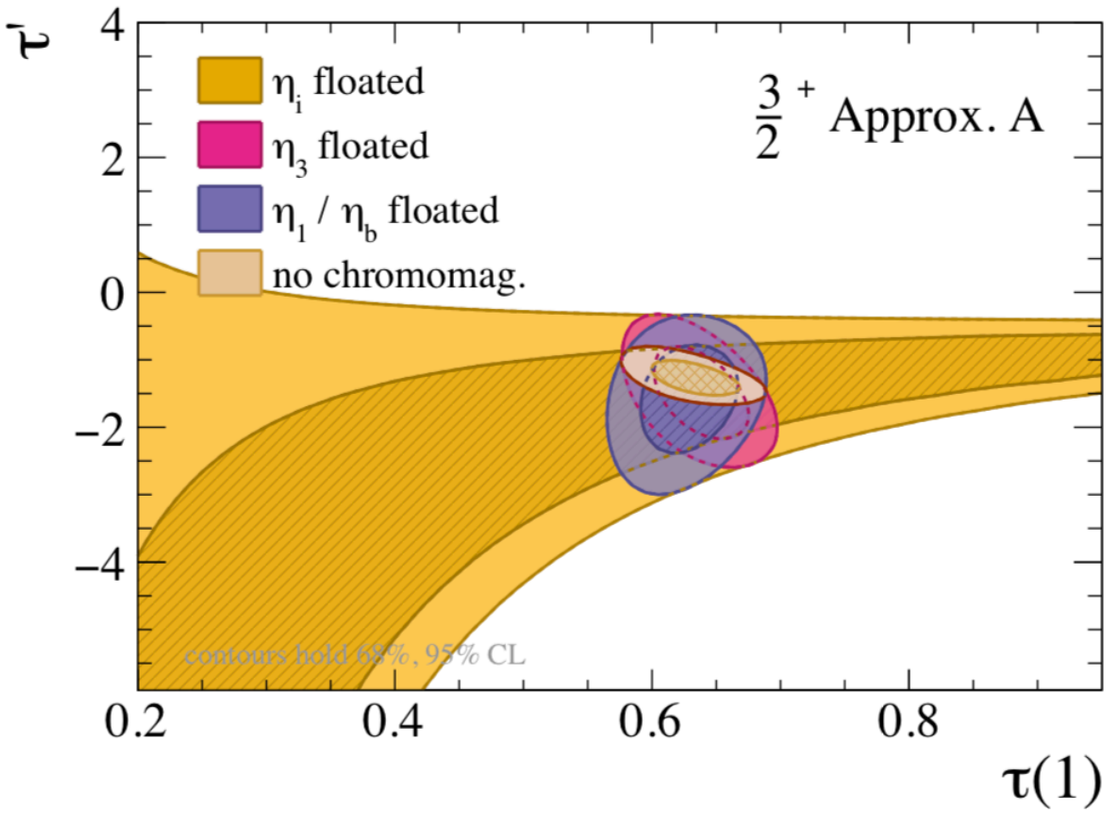
- **Heavy quark expansion** to related then to $\tau(w)$ and $\zeta(w)$

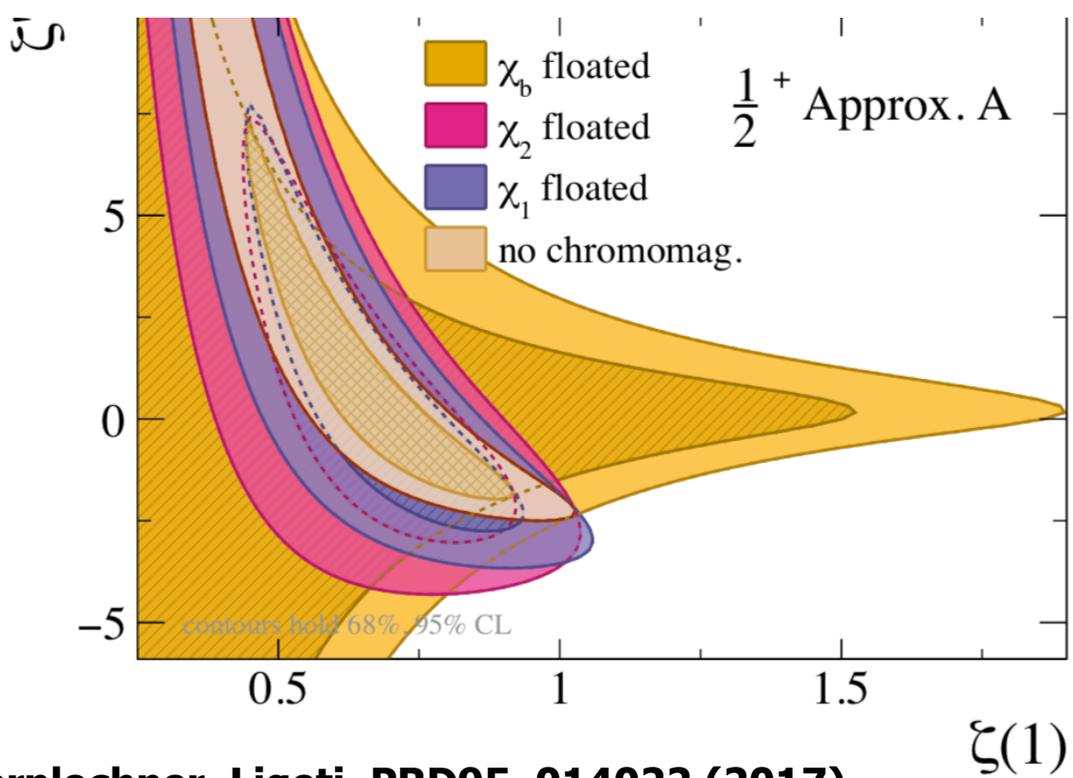
Approximation A: neglect $(w-1)^2, (w-1)\varepsilon_{b,c}$ **and** $\varepsilon^2_{b,c}$

$$\left.\begin{aligned}f_{V_1} &= \frac{1}{\sqrt{6}}\left[(1-w^2)\tau - \epsilon_c 4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau\right]\\ wf_{V_1} + (w^2-1)f_{V_3} &= \frac{w}{\sqrt{6}}\left[(1-w^2)\tau - \varepsilon_c 4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau\right] + \frac{w^2-1}{\sqrt{6}}(w-2)\tau\end{aligned}\right\}\ \frac{3}{2}^+$$

$$\left.\begin{aligned}g_{V_1} &= (w-1)\zeta + \varepsilon_c(w\bar\Lambda^* - \bar\Lambda)\zeta - \varepsilon_b[\bar\Lambda^*(2w+1) - \bar\Lambda(w+2)]\zeta\\ wg_{V_1} + (w^2-1)g_{V_3} &= -(w^2-1)\zeta\\ g_+ &= \zeta\left[-3\varepsilon_c\left(\frac{w\bar\Lambda^* - \bar\Lambda}{w+1}\right) - \varepsilon_b\frac{\bar\Lambda^*(2w+1) - \bar\Lambda(w+2)}{w+1}\right]\end{aligned}\right\}\ \frac{1}{2}^+$$

Results





$V_1 V_1 \to \tau(w) = 0.539(33)$
$V_3 V_3 \to \tau(w) = 0.455(27)$
$A_0 A_0 \to \zeta(w) = 1.21(14)$

The leading order Isgur-Wise functions can be parametrized as

$$\tau(w) = \tau(1)[1 + \tau'(w-1)]$$
$$\zeta(w) = \zeta(1)[1 + \zeta'(w-1)]$$

$V_1 V_1 \to \tau'(w) = -7.8(6.3)$
$V_3 V_3 \to \tau'(w) = -12.2(5.3)$
$A_0 A_0 \to \zeta'(w) = 21(12)$

Bernlochner, Ligeti, PRD95, 014022 (2017)

Conclusions and Perspectives

Conclusions and Perspectives

- We presented our Lattice computation on the inclusive structure function that contains contributions from from S-wave states and P-wave states;
- Our data is consistent with previous JLQCD Collaboration analysis: both S-wave states and P-wave states;
- **P-wave contribution can be extracted;**
- **Zero-Recoil**: Our estimations for the Isgur-Wise form factors are in agreement with phenomenological results;
- More like $\tau_{3/2} \sim \tau_{1/2}$ rather than $\tau_{3/2} \gg \tau_{1/2}$;
- **Non-Zero Recoil:** Our results present a large error, but nonetheless they are consistent with phenomenological results;
- **Next Steps:**
  - Corresponding $B_s \to D_s^* l\nu$ calculation;
  - Higher momenta;
  - Other approximations for heavy quark expansions;
  - Other values for m_b

Thank you for your attention!

Study of Intermediate States in the Inclusive Semi-Leptonic $B \to X_c l \nu$ Decay Structure Functions

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